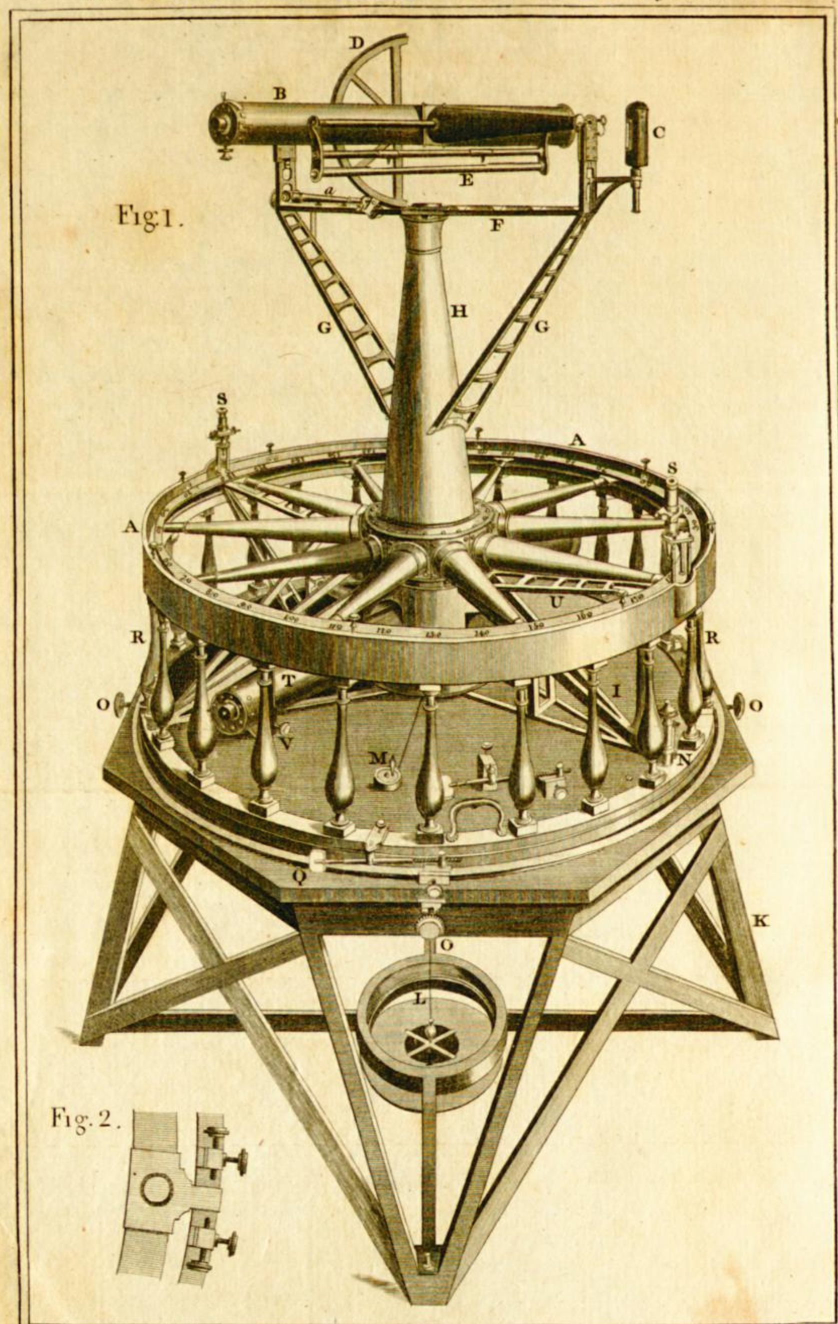
Geometrical and graphical essays containing a general description of the mathematical instruments used in geometry, civil and military surveying, levelling, and perspective

Adams, George

London, 1797

The GREAT THEODOLITE, *by Ramsden*.



Used by the late Gen.l Roy &c. in the great English Trigonometrical Operations.

London. Printed for & Published by W. and S. Jones, 135 Holborn, as the Act directs 1st April 1797.

GEOMETRICAL

AND

GRAPHICAL ESSAYS,

CONTAINING,

A GENERAL DESCRIPTION

OF THE

MATHEMATICAL INSTRUMENTS

USED IN

GEOMETRY, CIVIL AND MILITARY SURVEYING,
LEVELLING, AND PERSPECTIVE;

WITH MANY NEW

PRACTICAL PROBLEMS.

ILLUSTRATED BY THIRTY-FOUR COPPER PLATES.

BY THE LATE

GEORGE ADAMS,

MATHEMATICAL INSTRUMENT MAKER TO HIS MAJESTY, &c.

THE SECOND EDITION,

CORRECTED AND ENLARGED BY

WILLIAM JONES,

MATHEMATICAL INSTRUMENT MAKER.

LONDON:

PRINTED BY J. DILLON, AND CO.

AND SOLD BY W. AND S. JONES, OPTICIANS,
HOLBORN, LONDON.

1797.

TO

THE MOST NOBLE

CHARLES, DUKE OF RICHMOND,

AND LENNOX,

Master General of the Ordnance, &c.

THESE ESSAYS

ARE

WITH GREAT RESPECT,

JUSTLY INSCRIBED,

BY HIS GRACE'S

Most obedient,

Humble Servant,

GEORGE ADAMS.

PREFACE.

THOSE who have had much occasion to use the mathematical instruments constructed to facilitate the arts of drawing, surveying, &c. have long complained that a treatise was wanting to explain their use, describe their adjustments, and give such an idea of their construction, as might enable them to select those that are best adapted to their respective purposes.

This complaint has been the more general, as there are few active stations in life whose professors are not often obliged to have recourse to mathematical instruments. To the civil, the military, and the naval architect, their use must be familiar; and they are of equal, if not of more importance to the engineer, and the surveyor; they are the means by which the abstract parts of the mathematics are rendered useful in life, they connect theory with practice, and reduce speculation to use.

Monsieur *Bion*'s treatise on the construction of mathematical instruments, which was translated into English by Mr. *Stone*, and published in 1723, is the only regular treatise* we have upon this subject; the numerous improvements that have been made in instruments since that time, have

* I do not speak of Mr. Robertson's work, as it is confined wholly to the instruments contained in a case of drawing instruments.

rendered this work but of little use. It has been my endeavour by the following Essays to do away this complaint; and I have spared no pains to render them intelligible, and make them useful. Though the materials, of which they are composed, lie in common, yet it is presumed, that essential improvements will be found in almost every part.

These Essays begin by defining the necessary terms, and stating a few of those first principles on which the whole of the work is founded: they then proceed to describe the mathematical drawing instruments; among these, the reader will find an account of an improved pair of triangular compasses, a small pair of beam compasses with a micrometer screw, four new parallel rules, and other articles not hitherto described: these are followed by a large collection of useful geometrical problems; I flatter myself, that the practitioner will find many that are new, and which are well adapted to lessen labour and promote accuracy. In describing the manner of dividing large quadrants, I have first given the methods used by instrument makers, previous to the publication of that of Mr. *Bird*, subjoining his mode thereto, and endeavouring to render it more plain to the artist by a different arrangement. This is succeeded by geometrical and mechanical methods of describing circles of every possible magnitude; for the greater part of which I am indebted to *Joseph Priestley*, Esq. of Bradford, Yorkshire, whose merit has been already noticed by an abler pen than mine.* From this, I proceed to give a short view of elliptic and other compasses, and a description of *Suardi's geometric pen*, an instrument not known in this country, and

* Priestley's Perspective.

whose curious properties will exercise the ingenuity of mechanics and mathematicians.

Trigonometry is the next subject; but as this work was not designed to teach the elements of this art, I have contented myself with stating the general principles, and giving the canons for calculation, subjoining some useful and curious problems, which, though absolutely necessary in many cases that occur in county and marine surveying, have been neglected by every practical writer on this subject, except Mr. *Mackenzie*,* and *B. Donn*.† Some will also be found, that are even unnoticed by the above-mentioned authors.

Our next article treats of surveying, and it is presumed the reader will find it a complete, though concise system thereof. The several instruments now in use, and the methods of adjusting them, are described in order; and I think it will appear evident, from a view of those of the best construction, that large estates may be surveyed and plotted with greater accuracy than heretofore.

The great improvements that have been made within these few years in the art of dividing, have rendered observers more accurate and more attentive to the necessary adjustments of their instruments, which are not now considered as perfect, unless they are so constructed, that the person who uses them can either correct or allow for the errors to which they are liable. Among the various improvements which the instruments of science have received from Mr. *Ramsden*, we are to reckon those of the theodolite here described; the surveyor will find also the description of a small quadrant that should be constantly used with the

* Treatise on Maritime Surveying.
† Donn's Geometrician.

chain, improvements in the circumferentor, plain-table, protractor, &c. In treating of surveying, I thought to have met with no difficulty; having had however no opportunity of practice myself, I had recourse to books; a multiplicity have been written upon this subject, but they are for the most part imperfect, irregular, and obscure. I have endeavoured (with what success must be left to the reader's judgment) to remove their obscurities, to rectify their errors, and supply their deficiencies; but whatever opinion he may form of my endeavours, I can venture to say, he will be highly gratified with the valuable communications of Mr. *Gale*,* and Mr. *Milne*, here inserted, and which I think will contribute more to the improvement of the art of surveying, than any thing it has received since its original invention.

The reader will, I hope, excuse me, if I stop a moment to give him some account of Mr. *Gale*'s improvements; they consist, first, in a new method of plotting, which is performed by scales of equal parts, without a protractor, from the northings and southings, eastings and westings, taken out of the table which forms the appendix to this work;† this method is much more accurate than that in common use, because any small inaccuracy that might happen in laying down one line is *naturally corrected* in the next; whereas, in the common method of plotting by scale and protractor, any inaccuracy in a former line is *naturally communicated* to all the succeeding lines. The next improvement consists in a new method of determining the *area*, with superior accuracy, from

* A gentleman well known for his ingenious publications on finance.

† The table is printed separate, that it may be purchased, or not, as the surveyor sees convenient.

the northings, southings, eastings, and westings, without any regard to the plot or draught, by an easy computation.

As the measuring a strait line with exactness is one of the greatest difficulties in surveying, I was much surprised to find many land surveyors using only a chain; a mode in which errors are multiplied without a possibility of their being discovered, or corrected. I must not forget to mention here, that I have inserted in this part Mr. *Break*'s method of surveying and planning by the plain table, the bearings being taken and protracted at the same instant in the field upon one sheet of paper; thus avoiding the trouble and inconvenience of shifting the paper: this is followed by a small sketch of maritime surveying; the use of the pantographer, or pantagraph; the art of levelling, and a few astronomical problems, with the manner of using Hadley's quadrant and sextant; even here some suggestions will be found that are new and useful.

I have now to name another gentleman, who has contributed to render this work more perfect than it would otherwise have been, and it is with pleasure I return my best thanks to Mr. *Landman*, Professor of fortification and artillery to the Royal Academy at Woolwich, for his communications, more particularly for the papers from which the course of practical geometry on the ground was extracted. If the professors of useful sciences would thus liberally co-operate for their advancement, the progress thereof would be rapid and extensive. This course will be found useful not only to the military officer, but would make a useful and entertaining part of every gentleman's education. I found it necessary to abridge the papers Mr. *Landman* lent me, and leave out the calculations, as the work had already swelled to a larger size than was originally intended, though printed on a page unusually full.

The work finishes with a small tract on perspective, and a description of two instruments desighed to promote and facilitate the practice of that useful art. It is hoped, that the publication of these will prevent the public from being imposed upon by men, who, under the pretence of secresy, enhance the value of their contrivances. I knew an instance where 40l. was paid for an instrument inferior to the most ordinary of the kind that are sold in the shops. Some pains have been taken, and no small expense incurred, to offer something to the public superior in construction, and easier to use, than any instrument of the kind that has been hitherto exhibited.

I have been anxious and solicitous not to neglect any thing that might be useful to the practitioner, or acceptable to the intelligent. In a work which embraces so many subjects, notwithstanding all the care that has been taken, many defects may still remain; I shall therefore be obliged to any one who will favour me with such hints or observations, as may tend towards its improvement.

A list of the authors I have seen is subjoined to this Preface. I beg leave to return my thanks to the following gentlemen for their hints and valuable communications, the Rev. Mr. *Hawkins*, *J. Priestley*, Esq. Mr. *Gale*, Mr. *Milne*, Dr. *Rotherham*, Mr. *Heywood*, Mr. *Landman*, and Mr. *Beck*, a very ingenious artist.

ADVERTISEMENT

BY

THE EDITOR.

THE first edition of these Essays having, like the rest of the late ingenious Author's works, received much share of public approbation and encouragement; and being myself a joint proprietor with my brother, S. JONES, *of the copyright of all his publications, I conceive, that I cannot employ the few leisure moments, after the business of the day, better, than by revising, correcting, and improving those works that require reprinting. The present is the first of my editing: considerable errors in the former edition have been corrected; more complete explanations of instruments given, and many particulars of new and useful articles, not noticed by the Author, with notes, &c. are inserted in their proper places. The additions and amendments are, upon the whole, such, as I presume, without any pretension to superior abilities on my part, will again render the work deserving of the notice of all students and practitioners in the different professional branches of practical geometry. The principal additions I have made are the following:*

Description of a new pair of pocket Compasses, containing the ink and pencil points in its two legs—Improved Perambulator—Way Wiser—Improved Surveying Cross—Improved Circumferentor—Complete portable Theodolite—Great Theodolite, by *Ramsden*—Pocket box Sextant—Artificial Horizon—Pocket Spirit Levels—A Pair of Perspective Compasses—*Keith's* improved Parallel Scale—New Method of Surveying and keeping a Field Book—Gunner's Callipers—Gunner's Quadrant—Gunner's Level, &c.

March 30, 1797. *W. JONES.*

LIST
OF AUTHORS,
CONSULTED FOR THIS WORK.

Bion . . . . Construction, &c. of Mathematical Instruments, ... London, 1723
Break . . . . System of Land Surveying, ... London, 1778
Bonnycastle . . Introduction to Mensuration . . . . . London, 1787
Cunn . . . . Treatise on the Sector, ... London, 1729
Clavius. . . . . . Astrolabium Tribus Libris Explicatum, ... Moguntiæ, 1611
Cagnoli . . . . . Traite de Trigonometrie, ... Paris, 1786
De la Grive . . Manuel de Trigonometrie . . . . Paris, 1754
Donn . . . . Geometrician . . . . London, 1775
Daudet . . . . . . Introduction a la Geometrie, ... Paris, 1780
Dalrymple . . . Essay on Nautical Surveying, ... London, 1786
Eckhardt . . . . Description d'un Graphometre, . . A la Haye, 1778
Gardner . . . . . Practical Surveying, ... London, 1737
Gibson . . . . . . Treatise of Surveying, ... Dublin, 1763
Hutton . . . . . . Treatise on Mensuration, ... London, 1788
Hume . . . . Art of Surveying, ... London, 1763
Hammond . . . Practical Surveyor, ... London, 1765
Le Febvre . . . . Oeuvres Complettes, ... Maestricht, 1778
Love . . . . Art of Surveying, ... London, 1786
Mackenzie . . . Treatise of Maritime Surveying, . . London, 1774
Mandey . . . . . Marrow of Measuring, ... London, 1717
Nicholson . . . Navigator's Assistant, ... London, 1784
Noble . . . . . . Essay on Practical Surveying, ... London, 1774
Payne . . . . . . Elements of Geometry, ... London, 1767
—— . . . . . . ———— Trigonometry, ... London, 1772
Picard . . . . . . Traite du Nivelment, ... Paris, 1784
Robertson . . . Treatise of Mathematical Instruments, ... London, 1775
Spiedell . . . . . Geometrical Extraction, ... London, 1657
Talbot . . . . . . Complete Art of Land Measuring, London, 1784

Hutton's Mathematical and Philosophical Dictionary, 4to. 1795.

WORKS

By the late Author *of these* Essays, *now published and sold by* W. *and* S. Jones, *Holborn, London.*

I. AN ESSAY ON ELECTRICITY, explaining clearly and fully the Principles of that useful Science, describing the various Instruments that have been contrived either to illustrate the Theory, or render the Practice of it entertaining. The different Modes in which the Electrical Fluid may be applied to the human Frame for Medical Purposes, are distinctly and clearly pointed out, and the necessary Apparatus explained. To which is added AN ESSAY ON MAGNETISM. *Third Edition,* 8vo. Price 6s. illustrated with seven Plates.

II. AN ESSAY ON VISION, briefly explaining the Fabric of the Eye, and the nature of vision; intended for the Service of those whose Eyes are weak and impaired, enabling them to form an accurate Idea of the State of their Sight, the Means of preserving it, together with proper Rules for ascertaining when Spectacles are necessary, and how to choose them without injuring the Sight.

III. ASTRONOMICAL AND GEOGRAPHICAL ESSAYS, containing, 1. A full and comprehensive View, on a new Plan, of the general Principles of Astronomy, with a large Account of the Discoveries of Mr. Herschel. 2. The Use of the Celestial and Terrestrial Globes, exemplified in a greater Variety of Problems than are to be found in any other Work: they are arranged under distinct Heads, and interspersed with much curious but relative Information. 3. The Description and Use of small Orreries or Planetaria, &c. 4. An Introduction to Practical Astronomy, by a Set of easy and entertaining Problems. *Second Edition,* 8vo. Price 10s. 6d. in Boards, illustrated with twenty-one Plates.

IV. AN INTRODUCTION TO PRACTICAL ASTRONOMY, or the Use of the Quadrant and Equatorial, being extracted from the preceding Work. Sewed, with two Plates, 2s. 6d.

V. AN APPENDIX to the GEOMETRICAL AND GRAPHICAL ESSAYS, containing the following Table by Mr. John Gale, viz. a Table of the Northings, Southings, Eastings, and Westings, to every Degree and fifteenth Minute of the Quadrant, Radius from 1 to 100, with all the intermediate Numbers, computed to three Places of Decimals.

In the Press,

AND WILL BE PUBLISHED IN JUNE NEXT,

ESSAYS

ON THE

MICROSCOPE,

In 4to, with many Plates. The Second Edition, corrected and improved.

Also, in the Press,

AND SPEEDILY WILL BE PUBLISHED,

LECTURES

IN NATURAL AND EXPERIMENTAL

PHILOSOPHY,

In Five Volumes, 8vo. The Second Edition, with many Plates, considerable Alterations and Improvements; containing more complete Explanations of the Instruments, Machines, &c. and the Description of many others not inserted in the former Edition, by W. JONES.

A TABLE

OF

CONTENTS.

ADDENDA.

GEOMETRICAL

AND

GRAPHICAL

ESSAYS.

NECESSARY DEFINITIONS, *and* FIRST PRINCIPLES.

GEOMETRY originally signified, according to the etymology of the name, the art of measuring the earth; but is now the science that treats of, and considers the properties of magnitude in general---In other words, extension and figure are the objects of geometry. It is a science, in which human reason has the most ample field, and can go deeper, and with more certainty, than in any other. It is divided into two parts, theoretical and practical.

Theoretical geometry considers and treats of first principles abstractedly. Practical geometry applies these considerations to the purposes of life. By practical geometry many operations are performed of the utmost importance to society and the arts. "The effects thereof are extended through the principal operations of human skill: it conducts the soldier in the field, the seaman on the ocean: it gives strength to the fortress, and elegance to the palace."

The invention of geometry has been, by all the most eminent writers on the science, attributed to the Egyptians; and, that to the frequent inundations of the river Nile upon the country, we owe the rise of this sublime branch of human knowledge; the land-marks and boundaries being in this way destroyed, the previous knowledge of the figure and dimensions was the only method of ascertaining individual property again. But, surely, it is not necessary to gratify learned curiosity by such accounts as these; for geometry is an art that must have grown with man; it is, in a great measure, natural to the human mind; we were born spectators of the universe, which is the kingdom of geometry, and are continually obliged to judge of heights, measure distances, ascertain the figure, and estimate the bulk of bodies.

The first definition in geometry is a *point*, which is considered by geometricians, as that which has no parts or magnitude.

A *line* is length without breadth.

A *strait line* is that which lies evenly between its extreme points or ends.

A *superficies* is that which has only length and breadth.

A *plane angle* is an opening, or corner, made by two strait lines meeting one another.

When a strait line AB, *fig.* 1, *plate* 4, standing upon another C D, makes angles A B C, A B D, on each side equal to one another; each of these angles is called a *right angle*; and the line A B is said to be *perpendicular* to the line C D.

It is usual to express an angle by three letters, that placed at the angular point being always in the middle; as B is the angle of ABC.

An *obtuse angle* is that which is greater than a right angle.

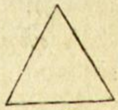
An *acute angle* is that which is less than a right angle.

A line A B, *fig.* 2, *plate* 4, cutting another line C D in E, will make the opposite angles equal, namely, the angle A E C equal to B E D, and A E D equal to B E C.

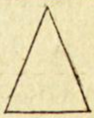
A line A B, *fig.* 3, *plate* 4, standing any-way upon another C D, makes two angles CBA, ABD, which, taken together, are equal to two right angles.

A *plane triangle* is a figure bounded by three right lines.

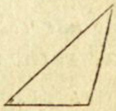
An *equilateral triangle* is that which has three equal sides.



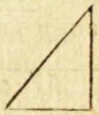
An *isosceles triangle* is that which has only two equal sides.



A *scalene triangle* is that which has all its sides unequal.

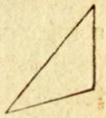


A *right-angled triangle* is that which has one right angle.



In a right-angled triangle, the side opposite to the right angle is called the *hypothenuse*.

An *oblique-angled triangle* is that which has no right angle.



In the same triangle, opposite to the greater side is the greater angle; and opposite to the greater angle is the greater side.

If any side of a plane triangle be produced, the outward angle will be equal to both the inward remote angles.

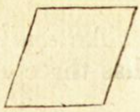
The three angles of any plane triangle taken together, are equal to two right angles.

Parallel lines are those which have no inclination towards each other, or which are every-where equidistant.

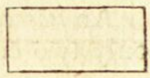
All plane figures, bounded by four right lines, are called *quadrangles*, or *quadrilaterals*.

A *square* is a quadrangle, whose sides are all equal, and its angles all right angles.

A *rhombus* is a quadrangle, whose sides are all equal, but its angles not right angles.

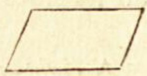


A *parallelogram* is a quadrangle, whose opposite sides are parallel.

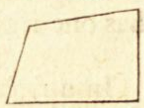


A *rectangle* is a parallelogram, whose angles are all right angles.

A *rhomboid* is a parallelogram, whose angles are not right angles.



All other four-sided figures besides these, are called *trapeziums*.



A right line joining any two opposite angles of a four-sided figure, is called the *diagonal*.

All plane figures contained under more than four sides, are called *polygons*.

Polygons having five sides, are called *pentagons*; those having six sides, *hexagons*; with seven sides, *heptagons*; and so on.

A *regular polygon* is that whose angles and sides are all equal.

The *base* of any figure is that side on which it is supposed to stand, and the *altitude* is the perpendicular falling thereon from the opposite angle.

Parallelograms upon the same base, and between the same parallels, are equal.

Parallelograms having the same base, and equal altitudes, are equal.

Parallelograms having equal bases, and equal altitudes, are equal.

If a triangle and parallelogram have equal bases and equal altitudes, the triangle is half the parallelogram.

A *circle* is a plane figure, bounded by a curve line called the *circumference*, every part whereof is equally distant from a point within the same figure, called the *center*.

Any part of the circumference of a circle is called an *arch*.

Any right line drawn from the center to the circumference of a circle, is called a *radius*.

All the radii of the same circle are equal.

The circumference of every circle is supposed to be divided into 360 equal parts, called *degrees*; each degree into 60 equal parts, called *minutes*, &c.

A *quadrant* of a circle will therefore contain 90 degrees, being a fourth part of 360.

Equal angles at the centers of all circles, will intercept equal numbers of degrees, minutes, &c. in their circumferences.

The *measure* of every plane angle is an arch of a circle, whose center is the angular point, and is said to be of so many degrees, minutes, &c. as are contained in its measuring arch.

All right angles, therefore, are of 90 degrees, or contain 90 degrees, because their measure is a quadrant.

The three angles of every plane triangle taken together, contain 180 degrees, being equal to two right angles.

In a right-angled plane triangle, the sum of its two acute angles is 90 degrees.

The *complement* of an arch, or of an angle, is its difference from a quadrant or a right angle.

The *supplement* of an arch, or of an angle, is its difference from a semicircle, or two right angles.

The magnitudes of arches and angles are determined by certain strait lines, appertaining to a circle, called chords, sines, tangents, &c.

The *chord* of an arch is a strait line, joining its extreme points.

A *diameter* is a chord passing through the center.

A *segment* is any part of a circle bounded by an arch and its chord.

A *sector* is any part of a circle bounded by an arch, and two radii drawn to its extremities.

The *sine* of an arch is a line drawn from either end of it, perpendicular to a diameter meeting the other end.

The *versed sine* of an arch is that part of the diameter intercepted between the sine and the end of the said arch.

The *tangent* of an arch is a line proceeding from either end, perpendicular to the radius joining it; its length is limited by a line drawn from the center, through the other end.

The *secant* of an arch is the line proceeding from the center, and limiting the tangent of the same arch.

The *co-sine* and *co-tangent*, &c. of any arch is the sine and tangent, &c. of its complement.

Thus in *fig.* 4, *plate* 4, F O is the chord of the arch F V O, and F R is the sine of the arches F V, F A D; R V, R D are the versed sines of the arches F V, F A D.

V T is the tangent of the arch F V, and its supplement.

C T is the secant of the arch F V.

A I is the co-tangent, and C I the co-secant of the arch F V.

The chord of 60°, the sine of 90°, the versed sine of 90°, the tangent of 45, and the secant of 0.0, are all equal to the radius.

It is obvious, that in making use of these lines, we must always use the same radius, otherwise there would be no settled proportions between them.

Whosoever considers the whole extent and depth of geometry, will find that the main design of all its speculations is mensuration. To this the Elements of Euclid are almost entirely devoted, and this has been the end of the most laboured geometrical disquisitions, of either the ancients or moderns.

Now the whole mensuration of figures may be reduced to the measure of triangles, which are always the half of a rectangle of the same base and altitude, and, consequently, their area is obtained by taking the half of the product of the base multiplied by the altitude.

By dividing a polygon into triangles, and taking the value of these, that of the polygon is obtained; by considering the circle as a polygon, with an infinite number of sides, we obtain the measure thereof to a sufficient degree of accuracy.

The theory of triangles is, as it were, the hinge upon which all geometrical knowledge turns.

All triangles are more or less similar, according as their angles are nearer to, or more remote from, equality.

The similitude is perfect, when all the angles of the one are equal respectively to those of the other; the sides are then also proportional.

The angles and the sides determine both the relative and absolute size, not only of triangles, but of all things.

Strictly speaking, angles only determine the relative size; equiangular triangles may be of very unequal magnitudes, yet perfectly similar.

But, when they are also equilateral, the one having its sides equal to the homologous sides of the other, they are not only similar and equiangled, but are equal in every respect.

The angles, therefore, determine the relative species of the triangle; the sides, its absolute size, and, consequently, that of every other figure, as all are resolvible into triangles.

Yet the essence of a triangle seems to consist much more in the angles than the sides; for the angles are the true, precise, and determined boundaries thereof: their equation is always fixed and limited to two right angles.

The sides have no fixed equation, but may be extended from the infinitely little, to the infinitely great, without the triangle changing its nature and kind.

It is in the theory of isoperimetrical figures* that we feel how efficacious angles are, and how inefficacious lines, to determine not only the kind, but the size of the triangles, and all kinds of figures,

* Isoperimetrical figures are such as have equal circumferences.

For, the lines still subsisting the same, we see how a square decreases, in proportion as it is changed into a more oblique rhomboid; and thus acquires more acute angles. The same observation holds good in all kinds of figures, whether plane or solid.

Of all isoperimetrical figures, the plane triangle and solid triangle, or pyramid, are the least capacious; and, amongst these, those have the least capacity, whose angles are most acute.

But curved surfaces, and curved bodies, and, among curves, the circle and sphere, are those whose capacity are the largest, being formed, if we may so speak, of the most obtuse angles.

The theory of geometry may, therefore, be reduced to the doctrine of angles, for it treats only of the boundaries of things, and by angles the ultimate bounds of all things are formed. It is the angles which give them their figure.

Angles are measured by the circle; to these we may add parallels, which, according to the signification of the term, are the source of all geometrical similitude and comparison.

The taking and measuring of angles is the chief operation in practical geometry, and of great use and extent in surveying, navigation, geography, astronomy, &c. and the instruments generally used for this purpose, are quadrants, sextants, theodolites, circumferentors, &c. as described in the following pages. It is necessary for the learner first to be acquainted with the names and uses of the drawing instruments; which are as follow.

OF MATHEMATICAL

DRAWING INSTRUMENTS.

Common Names of the principal Instruments, as represented in Plates 1, 2, *and* 3.

Plate 1, *fig.* A, is a pair of proportional compasses, without an adjusting screw.

B, a pair of best drawing compasses; *b*, the plain point with a joint; *c*, the ink point; *d*, the dotting point; *e*, the pencil or crayon point; P Q, additional pieces fitting into the place of the moveable point *b*, and to which the other parts are fitted.

F, a pair of bow compasses for ink; G, a ditto for a pencil; H, a pair of ditto with a plain point for stepping minute divisions; *h*, a screw to one of the legs thereof, which acts like the spring leg of the hair compasses.

L, the hair compasses; *n*, the screw that acts upon the spring leg.

I K, the drawing pen; I, the upper part; *k*, the protracting pin thereof; K, the lower, or pen part.

N, a pair of triangular compasses.

V, a pair of portable compasses which contains the ink and pencil points within its two legs.

O, the feeder and tracing point.

R, a pair of bisecting compasses, called wholes and halves.

S, a small protracting pin.

T, a knife, screw-driver, and key, in one piece.

Plate 2, *fig.* A, the common parallel rule.

B, the double barred ditto.

C, the improved double barred parallel rule.

D, the cross barred parallel rule. Of these rules, that figured at C is the most perfect.

E, Eckhardts, or the rolling parallel rule.

F G H, the rectangular parallel rule.

I K L, the protracting parallel rule.

M N O, Haywood's parallel rule.

Plate 3, *fig.* 1, the German parallel rule.

Fig. 2, a semicircular protractor; *fig.* 3, a rectangular ditto.

Fig. 4 and 5, the two faces of a sector.

Fig. 6, Jackson's parallel rule.

Fig. 7 and 8, two views of a pair of proportionable compasses, with an adjusting screw.

Fig. 9, a pair of sectoral compasses. In this instrument are combined the sector, beam elliptical, and calliper compasses; *fig.* 9, *a*, the square for ellipses; *b c*, the points to work therein; *d e*, the calliper points.

Fig. 10, a pair of beam compasses.

Fig. 11, Sisson's protracting scale.

Fig. 12, improved triangular compasses.

Fig. 13, a pair of small compasses with a beam and micrometer.

The strictness of geometrical demonstration admits of no other instruments, than a rule and a pair of compasses. But, in proportion as the practice of geometry was extended to the different arts, either connected with, or dependent upon it, new instruments became necessary, some to answer peculiar purposes, some to facilitate operation, and others to promote accuracy.

It is the business of this work to describe these instruments, and explain their various uses. In performing this task, a difficulty arose relative to the arrangement of the subject, whether each instrument, with its application, should be described separately, or whether the description should be introduced under those problems, for whose performance they were peculiarly designed. After some consideration, I determined to adopt neither rigidly, but to use either the one or the other, as they appeared to answer best the purposes of science.

As almost every artist, whose operations are connected with mathematical designing, furnishes himself with a case of drawing instruments suited to his peculiar purposes, they are fitted up in various modes, some containing more, others, fewer instruments. The smallest collection put into a case, consists of a plane scale, a pair of compasses with a moveable leg, and two spare points, which may be applied occasionally to the compasses; one of these points is to hold ink; the other, a porte crayon, for holding a piece of black-lead pencil.

What is called a full pocket case, contains the following instruments.

A pair of large compasses with a moveable point, an ink point, a pencil point, and one for dotting; either of these points may be inserted in the compasses, instead of the moveable leg.

A pair of plain compasses, somewhat smaller than those with the moveable leg.

A drawing pen, with a protracting pin in the upper part.

A pair of bow compasses.

A sector.

A plain scale.

A protractor.

A parallel rule.

A pencil.

The plain scale, the protractor, and parallel rule, are sometimes so constructed, as to form but one instrument; but it is a construction not to be recommended, as it injures the plain scale, and lessens the accuracy of the protractor. In a case with the best instruments, the protractor and plain scale are always combined. The instruments in most general use are those of six inches; instruments are seldom made longer, but often smaller. Those of six inches are, however, to be preferred, in general, before any other size; they will effect all that can be performed with the shorter ones, while, at the same time, they are better adapted to large work.

Large collections are called, *magazine cases of instruments*; these generally contain

A pair of six inch compasses with a moveable leg, an ink point, a dotting point, the crayon point, so contrived as to hold a whole pencil, two additional pieces to lengthen occasionally one leg of the compasses, and thereby enable them to measure greater extents, and describe circles of a larger radius.

A pair of hair compasses.

A pair of bow compasses.

A pair of triangular compasses.

A sector.

A parallel rule.

A protractor.

A pair of proportional compasses, either with or without an adjusting screw.

A pair of wholes and halves.

Two drawing pens, and a pointril.

A pair of small hair compasses, with a head similar to those of the bow compasses.

A knife, a file, key, and screw-driver for the compasses, in one piece.

A small set of fine water colours.

To these some of the following instruments are often added.

A pair of beam compasses.
A pair of gunners callipers.
A pair of elliptical compasses.
A pair of spiral ditto.
A pair of perspective compasses.
A pair of compasses with a micrometer screw.
A rule for drawing lines, tending to a center at a great distance.
A protractor and parallel rule, such as is represented at *fig.* I K L, *plate* 2.
One or more of the parallel rules represented, *plate* 2.
A pantographer.
A pair of sectoral compasses, forming, at the same time, a pair of beam and calliper compasses.

OF DRAWING COMPASSES.

Compasses are made either of silver or brass, but with steel points. The joints should always be framed of different substances; thus, one side, or part, should be of silver or brass, and the other of steel. The difference in the texture and pores of the two metals causes the parts to adhere less together, diminishes the wear, and promotes uniformity in their motion. The truth of the work is ascertained by the smoothness and equality of the motion at the joint, for all shake and irregularity is a certain sign of imperfection. The points should be of steel, so tempered, as neither to be easily bent or blunted; not too fine and tapering, and yet meeting closely when the compasses are shut.

As an instrument of art, compasses are so well known, that it would be superfluous to enumerate their various uses; suffice it then to say, that they are used to transfer small distances, measure given spaces, and describe arches and circles.

If the arch or circle is to be described obscurely, the steel points are best adapted to the purpose; if it is to be in ink or black lead, either the drawing pen, or crayon points are to be used.

To use a pair of compasses. Place the thumb and middle finger of the right hand in the opposite hollows in the shanks of the compasses, then press the compasses, and the legs will open a little way; this being done, push the innermost leg with the third finger, elevating, at the same time, the furthermost, with the nail of the middle finger, till the compasses are sufficiently opened to receive the middle and third finger; they may then be extended at pleasure, by pushing the furthermost leg outwards with the middle, or pressing it inwards with the fore finger. In describing circles, or arches, set one foot of the compasses on the center, and then roll the head of the compasses between the middle and fore finger, the other point pressing at the same time upon the paper. They should be held as upright as possible, and care should be taken not to press forcibly upon them, but rather to let them act by their own weight; the legs should never be so far extended, as to form an obtuse angle with the paper or plane, on which they are used.

The ink and crayon points have a joint just under that part, which fits into the compasses, by this they may be always so placed, as to be set nearly perpendicular to the paper; the end of the shank of the best compasses is framed so as to form a strong spring, to bind firmly the moveable points, and prevent them from shaking. This is found to be a more effectual method than that by a screw.

Fig. B, *plate* 1, represents a pair of the best compasses, with the plain point, *c*, the ink, *d*, the dotting, *e*, the crayon point.

In small cases, the crayon and ink points are joined by a middle piece, with a socket at each end to receive the points, which, by this means, only occupy one place in the case.

Two additional pieces, *fig.* P, Q, *plate* 1, are often applied to these compasses; these, by lengthening the leg *b*, enable them to strike larger circles, or measure greater extents, than they would otherwise perform, and that without the inconveniences attending longer compasses. When compasses are furnished with this additional piece, the moveable leg has a joint, as at *b*, that it may be placed perpendicular to the paper.

Of the hair compasses, fig. L, *plate* 1. They are so named, on account of a contrivance in the shank to set them with greater accuracy than can be effected by the motion of the joint alone. One of the steel points is fastened near the top of the compasses, and may be moved very gradually by turning the screw *n*, either backwards or forwards.

To use these compasses. 1. Place the leg, to which the screw is annexed, outermost; 2. Set the fixed leg on that point, from whence the extent is to be taken; 3. Open the compasses as nearly as possible to the required distance, and then make the points accurately coincide therewith by turning the screw.

Of the bow compasses, fig. F, *plate* 1. These are a small pair, usually with a point for ink; they are used to describe small arches or circles, which they do much more conveniently than large compasses, not only on account of their size, but also from the shape of the head, which rolls with great ease between the fingers. It is, for this reason, customary to put into magazine cases of instruments, *a small pair of hair compasses, fig.* H, *plate* 1, with a head similar to the bows; these are principally used for repeating divisions of a small but equal extent, a practice that has acquired the name of *stepping*.

Of the drawing pen and protracting pin, *fig.* I K, *plate* 1. The pen part of this instrument is used to draw strait lines; it consists of two blades with steel points fixed to a handle, the blades are so bent, that the ends of the steel points meet, and yet leave a sufficient cavity for the ink; the blades may be opened more or less by a screw, and, being properly set, will draw a line of any assigned thickness. One of the blades is framed with a joint, that the points may be separated, and thus cleaned more conveniently; a small quantity only of ink should be put at one time into the drawing pen, and this should be placed in the cavity, between the blades, by a common pen, or the feeder; the drawing pen acts better, if the feeder, or pen, by which the ink is inserted, be made to pass through the blades. To use the drawing pen, first feed it with ink, then regulate it to the thickness of the required line by the screw. In drawing lines, incline the pen a small degree, taking care, however, that the edges of both the blades touch the paper, keeping the pen close to the rule and in the same direction during the whole operation; the blades should always be wiped very clean, before the pen is put away.

These directions are equally applicable to the ink point of the compasses, only observing, that when an arch or circle is to be described, of more than an inch radius, the point should be so bent, that the blades of the pen may be nearly perpendicular to the paper, and both of them touch it at the same time.

The protracting pin k, is only a short piece of steel wire, with a very fine point, fixed at one end of the upper part of the handle of the drawing pen. It is used to mark the intersection of lines, or to set off divisions from the plotting scale, and protractor.

The feeder, fig. O, *plate* 1, is a thin flat piece of metal; it sometimes forms one end of a cap to fit on a pencil, or it is framed at the top of the tracing point, as in the figure. It serves to place the ink between the blades of the drawing pens, or to pass between them when the ink does not flow freely. *The tracing point*, or *pointrel*, is a pointed piece of steel fitted to a brass handle; it is used to draw obscure lines, or to trace the out-lines of a drawing or print, when an exact copy is required, an article that will be fully explained in the course of this work; it forms the bottom part of the feeder O.

Of triangular compasses. A pair of these are represented at *fig.* N, *plate* 1. They consist of a pair of compasses, to whose head a joint and socket is fitted for the reception of a third leg, which may be moved in almost every direction.

These compasses, though exceedingly useful, are but little known; they are very serviceable in copying all kinds of drawings, as from two fixed points they will always ascertain the exact position of a third point.

Fig. 12, *plate* 3, represents another kind, which has some advantages over the preceding. 1. That there are many situations so oblique, that the third point cannot be ascertained by the former, though it may by these. 2. It extends much further than the other, in proportion to its size. 3. The points are in all positions perpendicular to the paper.

Of wholes and halves, fig. R, *plate* 1. A name given to these compasses, because that when the longer legs are opened to any given line, the shorter ones will be opened to the half of that line; being always a bisection.

Fig. V, represents a new pair of very curious and portable compasses, which may be considered as a case of instruments in itself. The ink and pencil points slide into the legs by spring sockets at *a*; the ink, or pencil point, is readily placed, by only sliding either out of the socket, reversing it, and sliding in the plain point in its stead.

Proportional compasses. These compasses are of two kinds, one plain, represented *fig.* A, *plate* 1; the other with an adjusting screw, of which there are two views, one edgeways, *fig.* 8, *plate* 3, the other in the front, *fig.* 7, *plate* 3: the principle on which they both act is exactly the same; those with an adjusting screw are more easily set to any given division or line, and are also more firmly fixed, when adjusted.

There is a groove in each shank of these compasses, and the center is moveable, being constructed to slide with regularity in these grooves, and when properly placed, is fixed by a nut and screw; on one side of these grooves are placed two scales, one for lines, the other for circles. By the scale of lines, a right line may be divided into any number of equal parts expressed on the scale. By the scale for circles, a regular polygon may be inscribed in a circle, provided the sides do not exceed the numbers on the scale. Sometimes are added a scale for superficies and a scale for solids.

To divide a given line into a proposed number (11) *of equal parts.* 1. Shut the compasses. 2. Unscrew the milled nut, and move the slider until the line across it coincide with the 11th division on the scale. 3. Tighten the screw, that the slider may be immoveable. 4. Open the compasses, so that the longer points may take in exactly the given line, and the shorter will give you $\frac{1}{11}$th of that line.

To inscribe in a circle a regular polygon of 12 *sides*. 1. Shut the compasses. 2. Unscrew the milled nut, and set the division on the slider to coincide with the 12th division on the scale of circles. 3. Fasten the milled nut. 4. Open the compasses, so that the longer legs may take the radius, and the distance between the shorter legs will be the side of the required polygon.

To use the proportional compasses with an adjusting screw. The application being exactly the same as the simple one, we have nothing more to describe than the use and advantage of the adjusting screw. 1. Shut the legs close, slacken the screws of the nuts *g* and *f*; move the nuts and slider *k* to the division wanted, as near as can be readily done by the hand, and screw fast the nut *f*: then, by turning the adjuster *h*, the mark on the slider *k* may be brought exactly to the division: screw fast the nut *g*. 2. Open the compasses; gently lift the end *e* of the screw of the nut *f* out of the hole in the bottom of the nut *g*; move the beam round its pillar *a*, and slip the point *e* into the hole in the pin *n*, which is fixed to the under leg; slacken the screw of the nut *f*; take the given line between the longer points of the compasses, and screw fast the nut *f*: then may the shorter points of the compasses be used, without any danger of the legs changing their position; this being one of the inconveniences that attended the proportional compasses, before this ingenious contrivance.

Fig. 10, *plate* 3, represents a pair of *beam compasses*; they are used for taking off and transferring divisions from a diagonal or nonius scale, describing large arches, and bisecting lines or arches. It is the instrument upon which Mr. Bird principally depended, in dividing those instruments, whose accuracy has so much contributed to the progress of astronomy. These compasses consist of a long beam made of brass or wood, furnished with two brass boxes, the one fixed at the end, the other sliding along the beam, to any part of which it may be firmly fixed by the screw P. An adjusting screw and micrometer are adapted to the box A at the end of the beam; by these, the point connected therewith may be moved with extreme regularity and exactness, even less than the thousandth part of an inch.

Fig. 13, *plate* 3, is *a small pair of beam compasses,* with a micrometer and adjusting screw, for accurately ascertaining and laying down small distances.

Fig. 11, *plate* 3, represents *a scale of equal parts,* constructed by Mr. Sisson; that figured here contains two scales, one of three chains, the other of four chains in an inch, being those most frequently used; each of these is divided into 10 links, which are again subdivided by a nonius into single links; the index carries the protracting pin for setting off the lengths of the several station lines on the plan. By means of an instrument of this kind, the length of a station line may be laid down on paper with as much exactness as it can be measured on land.

OF PARALLEL RULES.

Parallel lines occur so continually in every species of mathematical drawing, that it is no wonder so many instruments have been contrived to delineate them with more expedition than could be effected by the general geometrical methods: of the various contrivances for this purpose, the following are those most approved.

1. *The common parallel rule, fig.* A, *plate* 2. This consists of two strait rules, which are connected together, and always maintained in a parallel position by the two equal and parallel bars, which move very freely on the center, or rivets, by which they are fastened to the strait rules.

2. *The double parallel rule, fig.* B, *plate* 2. This instrument is constructed exactly upon the same principles as the foregoing, but with this advantage, that in using it, the moveable rule may always be so placed, that its ends may be exactly over, or even with, the ends of the fixed rule, whereas, in the former kind, they are always shifting away from the ends of the fixed rule.

This instrument consists of two equal flat rules, and a middle piece; they are connected together by four brass bars, the ends of two bars are rivetted on the middle line of one of the strait rules; the ends of the other two bars are rivetted on the middle line of the other strait rule; the other ends of the brass bars are taken two and two, and rivetted on the middle piece, as is evident from the figure; it would be needless to observe, that the brass bars move freely on their rivets, as so many centers.

3. *Of the improved double parallel rule, fig.* C, *plate* 2. The motions of this rule are more regular than those of the preceding one, but with somewhat more friction; its construction is evident from the figure; it was contrived by the ingenious mechanic, Mr. Haywood.

4. *The cross barred parallel rule, fig.* D, *plate* 2. In this, two strait rules are joined by two brass bars, which cross each other, and turn on their inter-

section as on a center; one end of each bar moves on a center, the other slides in a groove, as the rules recede from each other.

As the four parallel rules above described, are all used in the same way, one problem will serve for them all; ex. gr. *a right line being given to draw a line parallel thereto by either of the foregoing instruments:*

Set the edge of the uppermost rule to the given line; press the edge of the lower rule tight to the paper with one hand, and, with the other, move the upper rule, till its edge coincides with the given point; and a line drawn along the edge through the point, is the line required.

5. *Of the rolling parallel rule.* This instrument was contrived by Mr. Eckhardt, and the simplicity of the construction does credit to the inventor; it must, however, be owned, that it requires some practice and attention to use it with success.

Fig. E. *plate* 2, represents this rule; it is a rectangular parallelogram of black ebony, with slips of ivory laid on the edges of the rule, and divided into inches and tenths. The rule is supported by two small wheels, which are connected together by a long axis, the wheels being exactly of the same size, and their rolling surfaces being parallel to the axis; when they are rolled backwards or forwards, the axis and rule will move in a direction parallel to themselves. The wheels are somewhat indented, to prevent their sliding on the paper; small ivory cylinders are sometimes affixed to the rollers, as in this figure; they are called rolling scales. The circumferences of these are so adjusted, that they indicate, with exactness, the parts of an inch moved through by the rule.

In rolling these rules, one hand only must be used, and the fingers should be placed nearly in the middle of the rule, that one end may not have a tendency to move faster than the other. The wheels only should touch the paper when the rule is moving, and the surface of the paper should be smooth and flat.

In using the rule with the rolling scales, to draw a line parallel to a given line at any *determined* distance, adjust the edge of the rule to the given line, and pressing the edge down, raise the wheels a little from the paper, and you may turn the cylinders round, to bring the first division to the index; then, if you move the rule towards you, look at the ivory cylinder on the left hand, and the numbers will shew in tenths of an inch, how much the rule moves. If you move the rule from you, then it will be shewn by the numbers on the right hand cylinder.

To raise a perpendicular from a given point on a given line. Adjust the edge of the rule to the line, placing any one of the divisions on the edge of the rule to the given point; then roll the rule to any distance, and make a dot or point on the paper, at the same division on the edge of the rule; through this point draw the perpendicular.

To let fall a perpendicular from any given point to a given line. Adjust the rule to the given line, and roll it to the given point; then, observing what division, or point, on the edge of the rule the given point comes to, roll the rule back again to the given line, and the division, or point, on the edge of the rule will shew the point on the given line, to which the perpendicular is to be drawn.

By this method of drawing perpendiculars, squares and parallelograms may be easily drawn of any dimensions.

To divide any given line into a number of equal parts. Draw a right line from either of the extreme points of the given line, making any angle with it. By means of the rolling scales, divide that line into as many inches, or parts of an inch, as will equal the number of parts into which the given line is to be divided. Join the last point of division, and the extreme point of the given line: to that line draw parallel lines through the other points of division, and they will divide the given line into equal parts.

6. *Of the square parallel rule.* The evident advantages of the T square and drawing board over every other kind of parallel rule, gave rise to a variety of contrivances to be used, when a drawing board was not at hand, or could not, on account of the size of the paper, be conveniently used; among these are, 1. *The square parallel rule.* 2. *The parallel rule and protractor*, both of which I contrived some years since, as substitutes to the T square. The square parallel rule, besides its use as a parallel rule, is peculiarly applicable to the mode of plotting recommended by Mr. Gale. Its use, as a rule for drawing parallel lines at given distances from each other, for raising perpendiculars, forming squares, rectangles, &c. is evident from a view of the figure alone; so that what has been already said of other rules, will be sufficient to explain how this may be used. It is also evident, that it will plot with as much accuracy as the beam, *fig.* 11, *plate* 3.

Fig. F G H, *plate* 2, represents this instrument; the two ends F G are lower than the rest of the rule, that weights may be laid on them to steady the rule, when both hands are wanted. The two rules are fastened together by the brass ends, the frame is made to slide regularly between the two rules, carrying at the same time, the rule H in a position at right angles to the edges of the rule E G.

There are slits in the frame a, b, with marks to coincide with the respective scales on the rules, while the frame is moving up or down: c is a point for pricking off with certainty divisions from the scales. The limb, or rule H, is made to take off, that other rules with different scales of equal parts may be applied; when taken off, this instrument has this further advantage, that if the distances, to which the parallels are to be drawn, exceed the limits of the rules, the square part, when taken off, may be used with any strait rule, by applying the perpendicular part against it.

7. *Fig.* I K L, *plate* 2, represents the *protracting parallel rule*; the uses of this in drawing parallel lines in different directions, are so evident from an inspection of the figure, as to render a particular description unnecessary. It answers all the purposes of the T square and bevil, and is peculiarly useful to surveyors for plotting and protracting, which will be seen, when we come to treat of those branches.

M N O is a parallel rule upon the same principle as the former: it was also contrived by Mr. Haywood. It is made either of wood edged with ivory, or of brass, and any scales of equal parts placed on it to the convenience of the purchaser. Each of the rules M N turns upon a center; its use as a parallel rule is evident from the figure, but it would require more pages than can be spared to describe all the uses of which it is capable; it forms the best kind of callipers, or guage; serves for laying down divisions and angles with peculiar accuracy; answers as a square, or bevil; indeed, scarce any artist can use it, without reaping considerable advantage from it, and finding uses peculiar to his own line of business.

8. *Of the German parallel rule, fig.* 1, *plate* 3. This was, probably, one of the first instruments invented to facilitate the drawing of parallel lines. It has, however, only been introduced within these few years among the English artists; and, as this introduction probably came from some German work, it has thence acquired its name. It consists of a square and a strait rule, the edge of the square is moved in use by one hand against the rule, which is kept steady by the other, the edge having been previously set to the given line: its use and construction is obvious from *fig.* 1, *plate* 3. Simple as it is in its principle, it has undergone some variations, two of which I shall mention; the one by Mr. Jackson, of Tottenham; the other by Mr. Marquois, of London.

Mr. Jackson's, fig. 6, *plate* 3, consists of two equal triangular pieces of brass, ivory, or wood, A B C, D E F, right angled at B and E; the edges A B and A C are divided into any convenient number of equal parts, the divisions in each equal; B C and D E are divided into the same number of equal parts as A C, one side of D E F may be divided as a protractor.

To draw a line G H, *fig.* V, *plate* 2, *parallel to a given line, through a point* P, *or at a given distance.*

Place the edge D E upon the given line I K, and let the instrument form a rectangle, then slide the upper piece till it come to the given point or distance, keeping the other steady with the left hand, and draw the line G H. By moving the piece equal distances by the scale on B C, any number of equidistant parallels may be obtained.

If the distance, *fig.* X, *plate* 2, between the given and required lines be considerable, place A B upon I K, and E F against A C, then slide the pieces alternately till D E comes to the required point; in this manner it is easy also to construct any square or rectangle, &c.

From any given point or angle P, *fig.* S, *plate* 2, *to let fall or raise a perpendicular on a given line.* Place either of the edges AC upon GP, and slide AB upon it, till it comes to the point P, and draw PH.

To divide a line into any proposed number of equal parts, fig. T, *plate* 2. Find the proposed number in the scale BC, and let it terminate at G, then place the rules in a rectangular form, and move the whole about the point G, till the side DE touches H; now move D one, two, or three divisions, according to the number and size of the required divisions, down BC, and make a dot at I, where DE cuts the line for the first part; then move one or more divisions as before, make a second dot, and thus proceed till the whole be completed.

Of Marquois's parallel scales. These consist of a right angled triangle, whose hypothenuse, or longest side, is three times the length of the shortest, and two rectangular scales. It is from this relative length of the hypothenuse that these scales derive their peculiar advantages, and it is this alone that renders them different from the common German parallel rule: for this we are much indebted to Mr. Marquois.

What has been already said of the German rule, applying equally to those of Mr. Marquois's, I shall proceed to explain their chief and peculiar excellence. On each edge of the rectangular rule are placed two scales, one close to the edge, the other within this. The outer scale, Mr. Marquois terms the artificial scale, the inner one, the natural scale: the divisions on the outer are always three times longer than those on the inner scale, as, to derive any advantage from this invention, they must always bear the same proportion to each other, that the shortest side of the right angled triangle does to the longest. The triangle has a line near the middle of it, to serve as an index, or pointer; when in use, this point should be placed so as to coincide with the O division of the scales; the numbers on the scales are reckoned both ways from this division; consequently, by confining the rule, and sliding the triangle either way, parallel lines may be drawn on either side of a given line, at any distance pointed out by the index on the triangle. The advantages of this contrivance are: 1. That the sight is greatly assisted, as the divisions on the outer scale are so much larger than those of the inner one, and yet answer the same purpose, for the edge of the right angled triangle only moves through one third of the space passed over by the index. 2. That it promotes accuracy, for all error in the setting of the index, or triangle, is diminished one third.

Mr. Marquois recommends the young student to procure two rules of about two feet long, having one of the edges divided into inches and tenths, and several triangles with their hypothenuse in different proportions to their respective perpendiculars. Thus, if you would make it answer for a scale of twenty to an inch, the hypothenuse must be twice the length of the perpendicular; if a scale of 30 be required, three times; of 40, four times; of 50, five times, and so on. Thus also for intermediate proportions; if a scale of 25 is wanted, the hypothenuse must be in the proportion of 25 to 2; if 35, of 7 to 42, &c. Or a triangle may be formed, in which the hypothenuse may be so set as to bear any required proportion with the perpendicular.

OF THE PROTRACTOR.

This is an instrument used to protract, or lay down an angle containing any number of degrees, or to find how many degrees are contained in any given angle. There are two kinds put into cases of mathematical drawing instruments; one in the form of a semicircle, the other in the form of a parallelogram. The circle is undoubtedly the only natural measure of angles; when a strait line is therefore used, the divisions thereon are derived from a circle, or its properties, and the strait line is made use of for some relative convenience: it is thus the parallelogram is often used as a protractor, instead of the semicircle, because it is in some cases more convenient, and that other scales, &c. may be placed upon it.

The semicircular protractor, fig. 2, *plate* 3, is divided into 180 equal parts or degrees, which are numbered at every tenth degree each way, for the conveniency of reckoning either from the right towards the left, or from the left towards the right; or the more easily to lay down an angle from either end of the line, beginning at each end with 10, 20, &c. and proceeding to 180 degrees. The edge is the diameter of the semicircle, and the mark in the middle points out the center. *Fig.* 3, *plate* 3, is a *protractor in the form of a parallelogram:* the divisions are here, as in the semicircular one, numbered both ways; the blank side represents the diameter of a circle. The side of the protractor to be applied to the paper is made flat, and that whereon the degrees are marked, is chamfered or sloped away to the edge, that an angle may be more easily measured, and the divisions set off with greater exactness.

Protractors of horn are, from their transparency, very convenient in measuring angles, and raising perpendiculars. When they are out of use, they should be kept in a book to prevent their warping.

Upon some protractors the numbers denoting the angle for regular polygons are laid down, to avoid the trouble of a reference to a table, or the operation of dividing; thus, the number 5, for a pentagon is set against 72°; the number 6, for a hexagon, against 60°; the number 7, for a heptagon, against $51\frac{1}{2}^\circ$; and so on.

Protractors for the purposes of surveying will be described in their proper place.

Application of the protractor to use. 1. *A number of degrees being given, to protract, or lay down an angle, whose measure shall be equal thereto.*

Thus, to lay down an angle of 60 degrees from the point A of the line AB, *fig.* 14, *plate* 3, apply the diameter of the protractor to the line AB, so that the center thereof may coincide exactly with the point A; then with a protracting pin make a fine dot at C against 60 upon the limb of the protractor; now remove the protractor, and draw a line from A through the point C, and the angle CAB contains the given number of degrees.

2. *To find the number of degrees contained in a given angle* BAC.

Place the center of the protractor upon the angular point A, and the fiducial edge, or diameter, exactly upon the line AB; then the degree upon the limb that is cut by the line CA will be the measure of the given angle, which, in the present instance, is found to be 60 degrees.

3. *From a given point* A, *in the line* AB, *to erect a perpendicular to that line.*

Apply the protractor to the line AB, so that the center may coincide with the point A, and the division marked 90 may be cut by the line A B, then a line DA drawn against the diameter of the protractor will be perpendicular to A B.

Further uses of this instrument will occur in most parts of this work, particularly its use in constructing polygons, which will be found under their respective heads. Indeed, the general use being explained, the particular application must be left to the practitioner, or this work would be unnecessarily swelled by a tedious detail and continual repetitions.

OF THE PLAIN SCALE.

The divisions laid down on the plain scale are of two kinds, the one having more immediate relation to the circle and its properties, the other being merely concerned with dividing strait lines.

It has been already observed, that though arches of a circle are the most natural measures of an angle, yet in many cases right lines were substituted, as being more convenient; for the comparison of one right line with another, is more natural and easy, than the comparison of a right line with a curve; hence it is usual to measure the quantities of angles not by the arch itself, which is described on the angular point, but by certain lines described about that arch. See definitions.

The lines laid down on the plain scales for the measuring of angles, or the protracting scales, are, 1. A line of *chords* marked CHO. 2. A line of *sines* marked SIN. of *tangents* marked TAN. of *semitangents* marked ST. and of *secants* marked SEC. this last is often upon the same line as the sines, because its gradations do not begin till the sines end.

There are two other scales, namely, the *rhumbs*, marked RU. and *long*, marked LON. Scales of latitude and hours are sometimes put upon the plain scale; but, as dialling is now but very little studied, they are only made to order.

The divisions used for measuring strait lines are called *scales of equal parts*, and are of various lengths for the convenience of delineating any figure of a large or smaller size, according to the fancy or purposes of the draughts-man. They are, indeed, nothing more than a measure in miniature for laying down upon paper, &c. any known measure, as chains, yards, feet, &c. each part on the scale answering to one foot, one yard, &c. and the plan will be larger or smaller, as the scale contains a smaller or a greater number of parts in an inch. Hence a variety of scales is useful to lay down lines of any required length, and of a convenient proportion with respect to the size of the drawing. If none of the scales happen to suit the purpose, recourse should be had to the *line of lines* on the sector; for, by the different openings of that instrument, a line of any length may be divided into as many equal parts as any person chuses.

Scales of equal parts are divided into two kinds, the one simple, the other diagonally divided.

Six of the simply divided scales are generally placed one above another upon the same rule; they are divided into as many equal parts as the length of the rule will admit of; the numbers placed on the right hand, shew how many parts in an inch each scale is divided into. The upper scale is sometimes shortened for the sake of introducing another, called the line of chords.

The first of the larger, or primary divisions, on every scale is subdivided into 10 equal parts, which small parts are those which give a name to the scale; thus it is called a scale of 20, when 20 of these divisions are equal to one inch. If, therefore, these lesser divisions be taken as units, and each represents one league, one mile, one chain, or one yard, &c. then will the larger divisions be so many tens; but if the subdivisions are supposed to be tens, the larger divisions will be hundreds.

To illustrate this, suppose it were required to set off from either of the scales of equal parts $\frac{36}{10}$, 36, or 360 parts, either miles or leagues. Set one foot of your compasses on 3, among the larger or primary divisions, and open the other point till it falls on the 6th subdivision, reckoning backwards or towards the left hand. Then will this extent represent $\frac{36}{10}$, 36, or 360 miles or leagues, &c. and bear the same proportion in the plan as the line measured does to the thing represented.

To adapt these scales to feet and inches, the first primary division is often duodecimally divided by an upper line; therefore, to lay down any number of feet and inches, as for instance, eight feet eight inches, extend the compasses from eight of the larger to eight of the upper small ones, and that distance laid down on the plan will represent eight feet eight inches.

Of the scale of equal parts diagonally divided. The use of this scale is the same as those already described. But by it a plane may be more accurately divided than by the former; for any one of the larger divisions may by this be subdivided into 100 equal parts; and, therefore, if the scale contains 10 of the larger divisions, any number under 1000 may be laid down with accuracy.

The diagonal scale is seldom placed on the same side of the rule with the other plotting scales. The first division of the diagonal scale, if it be a foot long, is generally an inch divided into 100 equal parts, and at the opposite end there is usually half an inch divided into 100 equal parts. If the scale be six inches long, one end has commonly half an inch, the other a quarter of an inch subdivided into 100 equal parts.

The nature of this scale will be better understood by considering its construction. For this purpose:

First. Draw eleven parallel lines at equal distances; divide the upper of these lines into such a number of equal parts, as the scale to be expressed is intended to contain; from each of these divisions draw perpendicular lines through the eleven parallels.

Secondly. Subdivide the first of these divisions into ten equal parts, both in the upper and lower lines.

Thirdly. Subdivide again each of these subdivisions, by drawing diagonal lines from the 10th below to the 9th above; from the 8th below to the 7th above; and so on, till from the first below to the 0 above: by these lines each of the small divisions is divided into ten parts, and, consequently, the whole first space into 100 equal parts; for, as each of the subdivisions is one tenth part of the whole first space or division, so each parallel above it is one tenth of such subdivision, and, consequently, one hundreth part of the whole first space; and if there be ten of the larger divisions, one thousandth part of the whole space.

If, therefore, the larger divisions be accounted as units, the first subdivisions will be tenth parts of an unit, and the second, marked by the diagonal upon the parallels, hundredth parts of the unit. But, if we suppose the larger divisions to be tens, the first subdivisions will be units, and the second tenths. If the larger are hundreds, then will the first be tens, and the second units.

The numbers, therefore, 576, 57,6, 5,76, are all expressible by the same extent of the compasses: thus, setting one foot in the number five of the larger divisions, extend the other along the sixth parallel to the seventh diagonal. For, if the five larger divisions be taken for 500, seven of the first subdivisions will be 70, which upon the sixth parallel, taking in six of the second subdivisions for units, makes the whole number 576. Or, if the five larger divisions be taken for five tens, or 50, seven of the first subdivisions will be seven units, and the six second subdivisions upon the sixth parallel, will be six tenths of an unit. Lastly, if the five larger divisions be only esteemed as five units, then will the seven first subdivisions be seven tenths, and the six second subdivisions be the six hundredth parts of an unit.

Of the use of the scales of equal parts. Though what I have already said on this head may be deemed sufficient, I shall not scruple to introduce a few more examples, in order to render the young practitioner more perfect in the management of an instrument, that will be continually occurring to him in practical geometry. He will have already observed, that by scales of equal parts lines may be laid down, or geometrical figures constructed, whose right-lined sides shall be in the same proportion as any given numbers.

Example 1. *To take off the number* 4,79 *from a diagonal scale.* Set one foot of the compasses on the point where the fourth vertical line cuts the seventh horizontal line, and extend the other foot to the point where the ninth diagonal cuts the seventh horizontal line.

Example 2. *To take off the number* 76,4. Observe the points where the sixth horizontal cuts the seventh vertical and fourth diagonal line, the extent between these points will represent the number 76,4.

In the first example each primary division is taken for one, in the second it is taken for ten.

Example 3. *To lay down a line of* 7,85 *chains by the diagonal scale.* Set one point of your compasses where the eighth parallel, counting upwards, cuts the seventh vertical line; and extend the other point to the intersection of the same eighth parallel with the fifth diagonal. Set off the extent 7,85 thus found on the line.

Example 4. *To measure by the diagonal scale a line that is already laid down.* Take the extent of the line in your compasses, place one foot on the first vertical line that will bring the other foot among the diagonals; move both feet upwards till one of them fall into the point where the diagonal from the nearest tenth cuts the same parallel as is cut by the other on the vertical line; then one foot shews the chain, and the other the hundredth parts or odd links. Thus, if one foot is on the eighth diagonal of the fourth parallel, while the other is on the same parallel, but coincides with the twelfth vertical, we have 12 chains, 48 links, or 12,48 chains.

Example 5. *Three adjacent parts of any right-lined triangle being given, to form the plan thereof. Thus, suppose the base of a triangle, fig.* 15, *plate* 3, 40 *chains, the angle* ABC *equal* 36 *deg. and angle* BAC *equal* 41 *deg.*

Draw the line AB, and from any of the scales of equal parts take off 40, and set it on the same line from A to B for the base of the triangle; at the points A, B, make the angle ABC equal to 36 degrees, and BAC to 41, and the triangle will be formed; then take in your compasses the length of the side AC, and apply it to the same scale, and you will find its length to be 24 chains; do the same by the side BC, and you will find it measure 27 chains, and the protractor will shew that the angle ACB contains 103 degrees.

Example 6. *Given the base* AB, *fig.* 16, *plate* 3, *of a triangle* 327 *yards, the angle* CAB 44°, 30′, *and the side* AC 208 *yards, to constitute the said triangle, and find the length of the other sides.*

Draw the line AB at pleasure, then take 327 parts from the scale, and lay it from A to B; set the center of the protractor to the point A, lay off 44° 30′, and by that mark draw AC; then take with the compasses from the same scale 208, and lay it from A to C, and join CB, and the parts of the triangle in the plan will bear the same proportion to each other as the real parts in the field do.

OF THE REMAINING LINES ON THE PLAIN SCALE.

OF THE PROTRACTING SCALES.

1. *Of the line of chords.* This line is used to set off an angle from a given point in any right line, or to measure the quantity of an angle already laid down.

Thus to draw a line AC, *fig.* 14, *plate* 3, that shall make with the line AB an angle, containing a given number of degrees, suppose 40 degrees.

Open your compasses to the extent of 60 degrees upon the line of chords, (which is always equal to the radius of the circle of projection,) and setting one foot in the angular point, with that extent describe an arch; then taking the extent of 40 degrees from the said chord line, set it off from the given line on the arch described; a right line drawn from the given point, through the point marked upon the arch, will form the required angle.

The degrees contained in an angle already laid down, are found nearly in the same manner; for instance, to measure the angle CAB. From the center describe an arch with the chord of 60 degrees, and the distance CB, measured on the chords, will give the number of degrees contained in the angle.

If the number of degrees are more than 90, they must be measured upon the chords at twice: thus, if 120 degrees were to be practised, 60 may be taken from the chords, and those degrees be laid off twice upon the arch. Degrees taken from the chords are always to be counted from the beginning of the scale.

Of the rhumb line. This is, in fact, a line of chords constructed to a quadrant divided into eight parts or points of the compass, in order to facilitate the work of the navigator in laying down a ship's course.

Thus, supposing the ship's course to be N N E, and it be required to lay down that angle. Draw the line AB, *fig.* 14, *plate* 3, to represent the meridian, and about the point A sweep an arch with the chord of 60 degrees; then take the extent to the second rhumb, from the line of rhumbs, and set it off on the arch from B to e, and draw the line A e, and the angle BAe will represent the ship's course. The second rhumb was taken, because NNE is the second point from the north, *Of the line of longitudes.* The line of longitudes is a line divided into sixty unequal parts, and so applied to the line of chords, as to shew by inspection, the number of equatorial miles contained in a degree on any parallel of latitude. The graduated line of chords is necessary, in order to shew the latitudes; the line of longitude shews the quantity of a degree on each parallel in sixtieth parts of an equatorial degree, that is, miles. The use of this line will be evident from the following example. A ship in latitude 44° 12′ N. sails E. 79 miles, required her difference of longitude. Opposite to 44¼, nearest equal to the latitude on the line of chords, stands 43 on the line of longitude, which is therefore the number of miles in a degree of longitude in that latitude. Whence as 43 : 60 : : 79 : 110 miles the difference of longitude.

The lines of tangents, semitangents, and secants serve to find the centers and poles of projected circles in the stereographical projection of the sphere.

The line of sines is principally used for the orthographic projection of the sphere; but as the application of these lines is the same as that of similar lines on the sector, we shall refer the reader to the explanation of those lines in our description of that instrument.

The lines of latitudes and hours are used conjointly, and serve very readily to mark the hour lines in the construction of dials; they are generally on the most complete sorts of scales and sectors; for the uses of which see treatises on dialling.

OF THE SECTOR.

Amidst the variety of mathematical instruments that have been contrived to facilitate the art of drawing, there is none so extensive in its use, or of such general application as the *sector*. It is an universal scale, uniting, as it were, angles and parallel lines, the rule and the compass, which are the only means that geometry makes use of for measuring, whether in speculation or practice. The real inventor of this valuable instrument is unknown; yet of so much merit has the invention appeared, that it was claimed by Galileo, and disputed by nations.

This instrument derives its name from the tenth definition of the third book of Euclid, where he defines the sector of a circle. It is formed of two equal rules, (*fig.* 4 and 5, *plate* 3,) AB, DB, called legs; these legs are moveable about the center C of a joint d e f, and will, consequently, by their different openings, represent every possible variety of plane angles. The distance of the extremities of these rules are the subtenses or chords, or the arches they describe.

Sectors are made of different sizes, but their length is usually denominated from the length of the legs when the sector is shut. Thus a sector of six inches, when the legs are close together, forms a rule of 12 inches when opened; and a foot sector is two feet long when opened to its greatest extent. In describing the lines usually placed on this instrument, I refer to those commonly laid down on the best six-inch brass sectors. But as the principles are the same in all, and the differences little more than in the number of subdivisions, it is to be presumed that no difficulty will occur in the application of what is here said to sectors of a larger radius.

Of this instrument, Dr. Priestley thus speaks in his Treatise on Perspective. " Besides the small sector in the common pocket cases of instruments, I would advise a person who proposes to learn to draw, to get another of one foot radius. Two sectors are in many cases exceedingly useful, if not absolutely necessary; and I would not advise a person to be sparing of expense in procuring a very good instrument, the uses of which are so various and important."

The scales, or lines graduated upon the faces of the instrument, and which are to be used as *sectoral lines*, proceed from the center; and are, 1. Two scales of equal parts, one on each leg, marked LIN. or L. each of these scales, from the great extensiveness of its use, is called the *line of lines*. 2. Two lines of *chords*, marked CHO. or C. 3. Two lines of *secants*, marked SEC. or S. A line of *polygons*, marked POL. Upon the other face, the sectoral lines are, 1. Two lines of sines, marked *sin.* or S. 2. Two lines of tangents, marked *tan.* 3. Between the lines of tangents and sines, there is another line of tangents to a lesser radius, to supply the defect of the former, and extending from 45° to 75°.

Each pair of these lines (except the line of polygons) is so adjusted as to make equal angles at the center, and consequently at whatever distance the sector be opened, the angles will be always respectively equal. That is, the distance between 10 and 10 on the line of lines, will be equal to 60 and 60 on the line of chords, 90 and 90 on the line of sines, and 45 and 45 on the line of tangents.

Besides the sectoral scales, there are others on each face, placed parallel to the outward edges, and used as those of the common plain scale. There are on the face, *fig.* 5, 1. A line of inches. 2. A line of latitudes. 3. A line of hours. 4. A line of inclination of meridians. 5. A line of chords. On the face, *fig.* 4, three logarithmic scales, namely, one of numbers, one of sines, and one of tangents; these are used when the sector is fully opened, the legs forming one line; their use will be explained when we treat of trigonometry.

To read and estimate the divisions on the sectoral lines. The value of the divisions on most of the lines are determined by the figures adjacent to them; these proceed by tens, which constitute the divisions of the first order, and are numbered accordingly; but the value of the divisions on the line of lines, that are distinguished by figures, is entirely arbitrary, and may represent any value that is given to them; hence the figures 1, 2, 3, 4, &c. may denote either 10, 20, 30, 40; or 100, 200, 300, 400, and so on.

The line of lines is divided into ten equal parts, numbered 1, 2, 3, to 10; these may be called divisions of the first order; each of these are again subdivided into 10 other equal parts, which may be called divisions of the second order; and each of these is divided into two equal parts, forming divisions of the third order.

The divisions on all the scales are contained between four parallel lines; those of the first order extend to the most distant; those of the third, to the least; those of the second to the intermediate parallel.

When the whole line of lines represents 100, the divisions of the first order, or those to which the figures are annexed, represent tens; those of the second order, units; those of the third order, the halves of these units. If the whole line represent ten, then the divisions of the first order are units; those of the second, tenths, and the thirds, twentieths.

In the line of tangents, the divisions to which the numbers are affixed, are the degrees expressed by those numbers. Every fifth degree is denoted by a line somewhat longer than the rest; between every number and each fifth degree, there are four divisions, longer than the intermediate adjacent ones, these are whole degrees; the shorter ones, or those of the third order, are 30 minutes.

From the center, to 60 degrees, *the line of sines* is divided like the line of tangents; from 60 to 70, it is divided only to every degree; from 70 to 80, to every two degrees; from 80 to 90, the division must be estimated by the eye.

The divisions on *the line of chords* are to be estimated in the same manner as the tangents.

The *lesser line of tangents* is graduated every two degrees from 45 to 50; but from 50 to 60, to every degree; from 60 to the end, to half degrees.

The *line of secants* from 0 to 10, is to be estimated by the eye; from 20 to 50 it is divided to every two degrees; from 50 to 60, to every degree; and from 60 to the end, to every half degree.

OF THE GENERAL LAW OR FOUNDATION OF SECTORAL LINES.

Let C A, C B, *fig.* 17, *plate* 3, represent a pair of sectoral lines, (ex. gr. those of the line of lines,) forming the angle A C B; divide each of these lines into four equal parts, in the points H, D, F; I, E, G; draw the lines H I, D E, F G, AB. Then because C A, C B, are equal, their sections are also equal, the triangles are equiangular, having a common angle at C, and equal angles at the base; and therefore, the sides about the equal angles will be proportional; for as C H to CA, so is H I to AB, and, therefore, as CA to C H, so is AB to H I, and, consequently, as C H to H I, so is C A to A B; and thence if C H be one fourth of C A, H I will be one fourth of A B, and so of all other sections.

Hence, as in all operations on the sectoral lines, there are two triangles, both isosceles aud equiangled; isosceles, because the pairs of sectoral lines are equal by construction; equiangled, because of the common angle at the center; the sides encompassing the equal angles are, therefore, proportional.

Hence also, if the lines C A, C B, represent the line of chords, sines, or tangents; that is, if C A, A B be the radius, and the line C F the chord, sine, or tangent of any proposed number of degrees, then the line F G will be the chord, sine, or tangent, of the same number of degrees, to the radius A B.

OF THE GENERAL MODE OF USING SECTORAL LINES.

It is necessary to explain, in this place, a few terms, either used by other writers in their description of the sector, or such as we may occasionally use ourselves.

The solution of questions on the sector is said to be *simple*, when the work is begun and ended on the same line; *compound*, when the operation begins on one line, and is finished on the other.

The operation varies also by the manner in which the compasses are applied to the sector. If a measure be taken on any of the sectoral lines, beginning at the center, it is called *a lateral distance*. But if the measure be taken from any point in one line, to its corresponding point on the line of the same denomination, on the other leg, it is called *a transverse* or *parallel distance*.

The divisions of each sectoral line are bounded by three parallel lines; the innermost of these is that on which the points of the compasses are to be placed, because this alone is the line which goes to the center, and is alone, therefore, the sectoral line.

We shall now proceed to give a few general instances of the manner of operating with the sector, and then proceed to practical geometry, exemplifying its use further in the progress of the work, as occasion offers.

Multiplication by the line of lines. Make the lateral distance of one of the factors the parallel distance of 10; then the parallel distance of the other factor is the product.

Example. Multiply 5 by 6, extend the compasses from the center of the sector to 5 on the primary divisions, and open the sector till this distance become the parallel distance from 10 to 10 on the same divisions; then the parallel distance from 6 to 6, extended from the center of the sector, shall reach to 3, which is now to be reckoned 30. At the same opening of the sector, the parallel distance of 7 shall reach from the center to 35, that of 8 shall reach from the center to 40, &c.

Division by the line of lines. Make the lateral distance of the dividend the parallel distance of the divisor, the parallel distance of 10 is the quotient. Thus, to divide 30 by 5, make the lateral distance of 30, viz. 3 on the primary divisions, the parallel distance of 5 of the same divisions; then the parallel distance of 10, extended from the center, shall reach to 6.

Proportion by the line of lines. Make the lateral distance of the second term the parallel distance of the first term; the parallel distance of the third term is the fourth proportional. *Example.* To find a fourth proportional to 8, 4, and 6, take the lateral distance of 4, and make it the parallel distance of 8; then the parallel distance of 6, extended from the center, shall reach to the fourth proportional 3.

In the same manner a third proportional is found to two numbers. Thus, to find a third proportional to 8 and 4, the sector remaining as in the former example, the parallel distance of 4, extended from the center, shall reach to the third proportional 2. In all these cases, if the number to be made a parallel distance be too great for the sector, some aliquot part of it is to be taken, and the answer multiplied by the number by which the first number was divided. Thus, if it were required to find a fourth proportional to 4, 8, and 6; because the lateral distance of the second term 8 cannot be made the parallel distance of the first term 4, take the lateral distance of 4, viz. the half of 8, and make it the parallel distance of the first term 4; then the parallel distance of the third term 6, shall reach from the center to 6, viz. the half of 12. Any other aliquot part of a number may be used in the same way. In like manner, if the number proposed be too small to be made the parallel distance, it may be multiplied by some number, and the answer is to be divided by the same number.

To protract angles by the line of chords. Case 1. When the given degrees are under 60. 1. With any radius AB, *fig.* 14, *plate* 3, on A as a center, describe the arch B G. 2. Make the same radius a transverse distance between 60 and 60 on the line of chords. 3. Take out the transverse distance of the given degrees, and lay this on the arch from B towards G, which will mark out the angular distance required.

Case 2. When the given degrees are more than 60. 1. Open the sector, and describe the arch as before. 2. Take $\frac{1}{2}$ or $\frac{1}{3}$ of the given degrees, and take the transverse distance of this $\frac{1}{2}$ or $\frac{1}{3}$, and lay it off from B towards G, twice, if the degrees were halved, three times if the third was used as a transverse distance.

Case 3. When the required angle is less than 6 degrees; suppose 3. 1. Open the sector to the given radius, and describe the arch as before. 2. Set off the radius from B to C. 3. Set off the chord of 57 degrees backward from C to f, which will give the arc f b of three degrees.

SOME USES OF THE SECTORAL SCALES OF SINES, TANGENTS, AND SECANTS.*

Given the radius of a circle, (suppose equal to two inches,) required the sine and tangent of 28° 30′ *to that radius.*

Solution. Open the sector so that the transverse distance of 90 and 90 on the sines, or of 45 and 45 on the tangents, may be equal to the given radius, viz. two inches; then will the transverse distance of 28° 30′, taken from the sines, be the length of that sine to the given radius; or if taken from the tangents, will be the length of that tangent to the given radius.

But if the secant of 28° 30′ *was required?*

Make the given radius, two inches, a transverse distance to 0 and 0, at the beginning of the line of secants; and then take the transverse distance of the degrees wanted, viz. 28° 30′.

* Robertson on Mathematical Instruments.

A tangent greater than 45° *(suppose* 60°*) is found thus.*

Make the given radius, suppose two inches, a transverse distance to 45 and 45 at the beginning of the scale of upper tangents; and then the required number 60° 00′ may be taken from this scale.

The scales of upper tangents and secants do not run quite to 76 degrees; and as the tangent and secant may be sometimes wanted to a greater number of degrees than can be introduced on the sector, they may be readily found by the help of the annexed table of the natural tangents and secants of the degrees above 75; the radius of the circle being unity.

Degrees.	Nat. Tangent.	Nat. Secant.
76	4,011	4,133
77	4,331	4,445
78	4,701	4,810
79	5,144	5,241
80	5,671	5,759
81	6,314	6,392
82	7,115	7,185
83	8,144	8,205
84	9,514	9,567
85	11,430	11,474
86	14,301	14,335
87	19,081	19,107
88	28,636	28,654
89	57,290	57,300

Measure the radius of the circle used upon any scale of equal parts. Multiply the tabular number by the parts in the radius, and the product will give the length of the tangent or secant sought, to be taken from the same scale of equal parts.

EXAMPLE. *Required the length of the tangent and secant of* 80 *degrees to a circle, whose radius, measured on a scale of* 25 *parts to an inch, is* $47\frac{1}{2}$ *of those parts?*

	tangent.	secant.
Against 80 degrees stands	5,671	5,759
The radius is	47,5	47,5
	28355	28795
	39697	40313
	22684	23036
	269,3725	273,5525

So the length of the tangent on the twenty-fifth scale will be $269\frac{1}{3}$ nearly. And that of the secant about $273\frac{1}{2}$.

Or thus. The tangent of any number of degrees may be taken from the sector at once; if the radius of the circle can be made a transverse distance to the complement of those degrees on the lower tangent.

EXAMPLE. *To find the tangent of* 78 *degrees to a radius of two inches.*

Make two inches a transverse distance to 12 degrees on the lower tangents; then the transverse distance of 45 degrees will be the tangent of 78 degrees.

In like manner the secant of any number of degrees may be taken from the sines, if the radius of the circle can be made a transverse distance to the co-sine of those degrees. Thus making two inches a transverse distance to the sine of twelve degrees; then the transverse distance of 90 and 90 will be the secant of 78 degrees.

From hence it will be easy to find the degrees answering to a given line, expressing the length of a tangent or secant, which is too long to be measured on those scales, when the sector is set to the given radius.

Thus, for a tangent, make the given line a transverse distance to 45 and 45 on the lower tangents; then take the given radius, and apply it to the lower tangents; and the degrees where it becomes a transverse distance is the co-tangent of the degrees answering to the given line.

And for a secant; make the given line a transverse distance to 90 and 90 on the sines. Then the degrees answering to the given radius, applied as a transverse distance on the sines, will be the co-sine of the degrees answering to the given secant line.

Given the length of the sine, tangent, or secant, of any degrees; to find the length of the radius to that sine, tangent, or secant.

Make the given length a transverse distance to its given degrees on its respective scale: then,

In the sines. The transverse distance of 90 and 90 will be the radius sought.

In the lower tangents. The transverse distance of 45 and 45, near the end of the sector, will be the radius sought.

In the upper tangents. The transverse distance of 45 and 45, taken towards the center of the sector on the line of upper tangents, will be the center sought.

In the secant. The transverse distance of 0 and 0, or the beginning of the secants, near the center of the sector, will be the radius sought.

Given the radius and any line representing a sine, tangent, or secant; to find the degrees corresponding to that line.

SOLUTION. Set the sector to the given radius, according as a sine, or tangent, or secant is concerned.

Take the given line between the compasses; apply the two feet transversely to the scale concerned, and slide the feet along till they both rest on like divisions on both legs; then will those divisions shew the degrees and parts corresponding to the given line.

To find the length of a versed sine to a given number of degrees, and a given radius.

Make the transverse distance of 90 and 90 on the sines, equal to the given radius.

Take the transverse distance of the sine complement of the given degrees.

If the given degrees are less than 90, the difference between the sine complement and the radius gives the versed sine.

If the given degrees are more than 90, the sum of the sine complement and the radius gives the versed sine.

To open the legs of the sector, so that the corresponding double scales of lines, chords, sines, and tangents, may make each a right angle.

On the lines, make the lateral distance 10, a distance between eight on one leg, and six on the other leg.

On the sines, make the lateral distance 90, a transverse distance from 45 to 45; or from 40 to 50; or from 30 to 60; or from the sine of any degrees to their complement.

Or on the sines, make the lateral distance of 45 a transverse distance between 30 and 30.

SELECT GEOMETRICAL PROBLEMS.

Science may suppose, and the mind conceive things as possible, and easy to be effected, in which art and practice often find insuperable difficulties. "Pure science has to do only with ideas; but in the application of its laws to the use of life, we are constrained to submit to the imperfections of matter and the influence of accident." Thus practical geometry shews how to perform what theory supposes; in the theory, however, it is sufficient to have only a right conception of the objects on which the demonstrations are founded; drawing or delineations being of no further use than to assist the imagination, and lessen the exertions of the mind. But in practical geometry, we not only consider the things as possible to be effected, but are to teach the ways, the instruments, and the operations by which they may be actually performed. It is not sufficient here to shew, that a right line may be drawn between two points, or a circle described about a center, and then demonstrate their properties; but we must actually delineate them, and exhibit the figures to the senses: and it will be found, that the drawing of a strait line, which occurs in all geometrical operations, and which in theory is conceived as easy to be effected, is in practice attended with considerable difficulties.

To draw a strait line between two points upon a plane, we lay a rule so that the strait edge thereof may just pass by the two points; then moving a fine pointed needle, or drawing pen, along this edge, we draw a line from one point to the other, which for common purposes is sufficiently exact; but where great accuracy is required, it will be found extremely difficult to lay the rule equally, with respect to both the points, so as not to be nearer to one point than the other. It is difficult also so to carry the needle or pen, that it shall neither incline more to one side than the other of the rule; and thirdly, it is very difficult to find a rule that shall be perfectly strait. If the two points be very far distant, it is almost impossible to draw the line with accuracy and exactness; a circular line may be described more easily, and more exactly, than a strait, or any other line, though even then many difficulties occur, when the circle is required to be of a large radius.

It is from a thorough consideration of these difficulties, that geometricians will not allow those lines to be geometrical, which in their description require the sliding of a point along the edge of a rule, as in the ellipse, and several other curve lines, whose properties have been as fully investigated, and as clearly demonstrated, as those of the circle.

From hence also we may deduce some of those maxims which have been introduced into practice by *Bird* and *Smeaton*, which will be seen in their proper place. And let no one consider these reflections as the effect of too scrupulous exactness, or as an unnecessary aim at precision; for as the foundation of all our knowledge in geography, navigation and astronomy, is built on observation, and all observations are made with instruments, it follows, that the truth of the observations, and the accuracy of the deductions therefrom, will principally depend on the exactness with which the instruments are made and divided; and that these sciences will advance in proportion as those are less difficult in their use, and more perfect in the performance of their respective operations.

There is scarce any thing which proves more clearly the distinction between mind and body, and the superiority of the one over the other, than a reflection on the rigid exactness of speculative geometry, and the inaccuracy of practice, that is not directed by theory on one hand, and its approximation to perfection on the other, when guided by a just theory.

In theory, most figures may be measured to an almost infinite exactness, yet nothing can be more inaccurate and gross, than the ordinary methods of mensuration; but an intelligent practice finds a medium, and corrects the imperfection of our mechanical organs, by the resources of the mind. If we were more perfect, there would be less room for the exertions of our mind, and our knowledge would be less.

If it had been easy to measure all things with exactness, we should have been ignorant of many curious properties in numbers, and been deprived of the advantages we derive from logarithms, sines, tangents, &c. If practice were perfect, it is doubtful whether we should have ever been in possession of theory.

We sometimes consider with a kind of envy, the mechanical perfection and exactness that is to be found in the works of some animals; but this perfection, which does honor to the Creator, does little to them; they are so perfect, only because they are beasts.

The imperfection of our organs is abundantly made up by the perfection of the mind, of which we are ourselves to be the artificers.

If any wish to see the difficulties of rendering practice as perfect as theory, and the wonderful resources of the mind, in order to attain this degree of perfection, let him consider the operations of *General Roy*, at Hounslow-heath; operations that cannot be too much considered, nor too much praised by every practitioner in the art of geometry. See *Philosoph. Trans.* vol. 80, et seq.

PROBLEM 1. *To erect a perpendicular at or near the end of a given right line,* C D, *fig.* 5, *plate* 4.

Method 1. On C, with the radius C D, describe a faint arc *e f* on D; with the same radius, cross *e f* at G, on G as a center; with the same radius, describe the arc D E F; set off the extent D G twice, that is from D to E, and from E to F. Join the points D and F by a right line, and it will be the perpendicular required.

Method 2, *fig.* 5. On any point G, with the radius D G, describe an arc F E D; then a rule laid on C and G, will cut this arc in F, a line joining the points F and D will be the required perpendicular.

Method 3. 1. From the point C, *fig.* 6, *plate* 4, with any radius describe the arc *r n m*, cutting the line A C in r. 2. From the point r, with the same radius, cross the arc in n, and from the point n, cross it in m. 3. From the points n and m, with the same, or any other radius, describe two arcs cutting each other in S. 4. Through the points S and C, draw the line S C, and it will be the perpendicular required.

Method 4. *By the line of lines on the sector, fig.* 7, *plate* 4. 1. Take the extent of the given line A C. 2. Open the sector, till this extent is a transverse distance between 8 and 8. 3. Take out the transverse distance between 6 and 6, and from A with that extent sweep a faint arc at B. 4. Take out the distance between 10 and 10, and with it from C, cross the former arc at B. 5. A line drawn through A and B, will be the perpendicular required; the numbers 6, 8, 10, are used as multiples of 3, 4, 5.

By this method, *a perpendicular may be easily and accurately erected on the ground.*

Method 5. Let A C, *fig.* 7, *plate* 4, be the given line, and A the given point. 1. At any point D, with the radius DA, describe the arc EAB. 2. With a rule on E and D, cross this arc at B. 3. Through A and B draw a right line, and it will be the required perpendicular.

PROBLEM 2. *To raise a perpendicular from the middle, or any other point* G, *of a given line* AB, *fig.* 8, *plate* 4.

1. On G, with any convenient distance within the limits of the line, mark or set off the points n and m. 2. From n and m with any radius greater than GA, describe two arcs intersecting at C. 3. Join CG by a line, and it will be the perpendicular required.

PROBLEM 3. *From a given point* C, *fig.* 8, *plate* 4, *out of a given line* AB, *to let fall a perpendicular.*

When the point is nearly opposite to the middle, of the line, this problem is the converse of the preceding one. Therefore, 1. From C, with any radius, describe the arc n m. 2. From n m, with the same, or any other radius, describe two arcs intersecting each other at S. 3. Through the points C S draw the line C S, which will be the required line.

When the point is nearly opposite to the end of the line, it is the converse of Method 5, Problem 1, *fig.* 7, *plate* 4.

1. Draw a faint line through B, and any convenient point E, of the line AC. 2. Bisect this line at D. 3. From D with the radius DE describe an arc cutting AC at A. 4. Through A and B draw the line AB, and it will be the perpendicular required.

Another method. 1. From A, *fig.* 9, *plate* 4, or any other point in AB, with the radius AC, describe the arc C D. 2. From any other point n, with the radius n C, describe another arc cutting the former in D. 3. Join the point C D by a line

C G D, and C G will be the perpendicular required.

PROBLEM 4. *Through a given point* C, *to draw a line parallel to a given strait line* AB, *fig.* 10, *plate* 4.

1. On any point D, (within the given line, or without it, and at a convenient distance from C,) describe an arc passing through C, and cutting the given line in A. 2. With the same radius describe another arc cutting AB at B. 3. Make BE equal to AC. 4. Draw a line CE through the point C and E, and it will be the required parallel.

This problem answers whether the required line is to be near to, or far from the given line; or whether the point D be situated on AB, or any where between it and the required line.

PROBLEM 5. *At the given point* D, *to make an angle equal to a given angle* A B C, *fig.* 12, *plate* 4.

1. From B, with any radius, describe the arc n m, cutting the legs B A B C, in the points n and m. 2. Draw the line D r, and from the point D, with the same radius as before, describe the arc r s. 3. Take the distance m n, and apply it to the arc r s, from r to s. 4. Through the points D and s draw the line D s, and the angle r D s will be equal to the angle m B n, or A B C, as required.

PROBLEM 6. *To extend with accuracy a short strait line to any assignable length*; *or, through two given points at a small distance from each other to draw a strait line.*

It frequently happens that a line as short as that between A and B, *fig.* 11, *plate* 4, is required to be extended to a considerable length, which is scarce attainable by the help of a rule alone; but may be performed by means of this problem, without error. Let the given line be AB, or the two points A and B; then from A as a center, describe an arch C B D; and from the point B, lay off B C equal to B D; and from C and D as centers, with any radius, describe two arcs intersecting at E. From the point A describe the arc F E G, making E F equal to E G; then from F and G as centers, describe two arcs intersecting at H, and so on; then a strait line from B drawn through E will pass in continuation through H, and in a similar manner the line may be extended to any assignable length.

OF THE DIVISION OF STRAIT LINES.

PROBLEM 7. *To bisect or divide a given strait line* AB *into two equal parts, fig.* 13, *plate* 4.

1. On A and B as centers, with any radius greater than half AB, describe arcs intersecting each other at C and D. 2. Draw the line C D, and the point F, where it cuts A B, will be the middle of the line.

If the line to be bisected be near the extreme edge of any plane, describe two pair of arcs of different radii above the given line, as at C and E; then a line C produced, will bisect AB in F.

By the line of lines on the sector. 1. Take AB in the compasses. 2. Open the sector till this extent is a transverse distance between 10 and 10. 3. The extent from 5 to 5 on the same line, set off from A or B, gives the half required: by this means a given line may be readily divided into 2, 4, 8, 16, 32, 64, 128, &c. equal parts.

PROBLEM 8. *To divide a given strait line* AB *into any number of equal parts, for instance, five.*

Method 1, *fig.* 14, *plate* 4. 1. Through A, one extremity of the line AB, draw AC, making any angle therewith. 2. Set off on this line from A to H as many equal parts of any length as AB is to be divided into. 3. Join H B. 4. Parallel to H B draw lines through the points D, E, F, G, and these will divide the line A B into the parts required.

Second method, fig. 15, *plate* 4. 1. Through B draw L D, forming any angle with AB. 2. Take any point D either above or below AB, and through D, draw D K parallel to AB. 3. On D set off five equal parts D F, F G, G H, H I, I K. 4. Through A and K draw A K, cutting B D in L. 5. Lines drawn through L, and the points F, G, H, I, K, will divide the line AB into the required number of parts.

Third method, fig. 17, *plate* 4. 1. From the ends of the line AB, draw two lines AC, BD, parallel to each other. 2. In each of these lines, beginning at A and B, set off as many equal parts less one, as AB is to be divided into, in the present instance four equal parts, AI, I K, K L, L M, on AC; and four, B E, E F, F G, G H, on B D. 3. Draw lines from M to E, from L to F, K to G, I to H, and AB will be divided into five equal parts.

Fourth method, fig. 16, *plate* 4. 1. Draw any two lines C E, D F parallel to each other. 2. Set off on each of these lines, beginning at C and D, any number of equal parts. 3. Join each point in C E with its opposite point in D F. 4. Take the extent of the given line in your compasses. 5. Set one foot of the compasses opened to this extent in D, and move the other about till it crosses N G in I. 6. Join D I, which being equal to AB, transfer the divisions of D I to AB, and it will be divided as required. H M is a line of a different length to be divided in the same number of parts.

The foregoing methods are introduced on account not only of their own peculiar advantages, but because they also are the foundation of several mechanical methods of division.

PROBLEM 9. *To cut off from a given line* AB *any odd part, as* $\frac{1}{3}$*d*, $\frac{1}{5}$*th*, $\frac{1}{6}$*th*, $\frac{1}{7}$*th*, *&c. of that line, fig.* 18, *plate* 4.

1. Draw through either end A, a line AC, forming any angle with AB. 2. Make AC equal to AB. 3. Through C and B draw the line CD. 4. Make BD equal to CB. 5. Bisect AC in a. 6. A rule on a and D will cut off a B equal $\frac{1}{3}$d of AB.

If it be required to divide AB into five equal parts, 1. Add unity to the given number, and halve it, $5+1=6$, $\frac{6}{2}=3$. 2. Divide AC into three parts; or, as AB is equal to AC, set off Ab equal Aa. 3. A rule on D, and b will cut off bB $\frac{1}{5}$th part of AB. 4. Divide Ab into four equal parts by two bisections, and AB will be divided into five equal parts.

To divide AB into seven equal parts, $7+1=8$, $\frac{8}{2}=4$. 1. Now divide AC into four parts, or bisect a C in c, and c C will be the $\frac{1}{4}$th of AC. 2. A rule on c and D cuts off c B $\frac{1}{7}$th of AB. 3. Bisect Ac, and the extent cB will divide each half into three equal parts, and consequently the whole line into seven equal parts.

To divide AB into nine equal parts, $9+1=10$ $\frac{10}{2}=5$. Here, 1. Make Ad equal to Ab, and d C will be $\frac{1}{5}$th of AC. 2. A rule on D and d cuts off dB $\frac{1}{9}$ of AC. 3. Bisect Ad. 4. Halve each of these bisections, and Ad is divided into four equal parts. 5. The extent dB will bisect each of these, and thus divide AB into nine equal parts.

If any odd number can be subdivided, as 9 by 3, then first divide the given line into three parts, and take the third as a new line, and find the third thereof as before, which gives the ninth part required.

Method 2. Let D B, *fig.* 19, *plate* 4, be the given line. 1. Make two equilateral triangles A D B, C D B, one on each side of the line D B. 2. Bisect AB in G. 3. Draw C G, which will cut off H B equal $\frac{1}{3}$d of D B. 4. Draw D F, and make G F equal to D G. 5. Draw H F which cuts off B h equal $\frac{1}{4}$ of AB or D B. 6. C h cuts D B in i one fifth part. 7. F i cuts AB in k equal $\frac{1}{6}$th of D B. 8. C k cuts D b in l equal $\frac{1}{7}$th of D B. 9. F l cuts AB in m equal $\frac{1}{8}$th of D B. 10. C m cuts D B at n equal $\frac{1}{9}$th part thereof.

Method 3. Let AB, *fig.* 12, *plate* 5, be the given line to be divided into its aliquot parts $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$. 1. On AB erect the square AB C D. 2. Draw the two diagonals AC, D B, which will cross each other at E. 3. Through E draw F E G parallel to AD, cutting AB in G. 4. Join D G, and the line will cut the diagonal AC at H. 5. Through H draw I H K parallel to AD. 6. Draw D K crossing AC in L. 7. And through L draw M L N parallel to AD, and so proceed as far as necessary. AG is $\frac{1}{2}$, AK $\frac{1}{3}$, AN $\frac{1}{4}$ of AB.*

Method 4. Let AB, *fig.* 13, *plate* 5, be the given line to be subdivided. 1. Through A and B draw C D, F E parallel to each other. 2. Make CA, AD, F B, F E equal to each other. 3. Draw C E, which shall divide AB into two equal parts at G. 4. Draw AE, DB, intersecting each other at H. 5. Draw C H intersecting AB at I, making AI $\frac{1}{3}$d of AB. 6. Draw D F cutting AE in K. 7. Join C K, which will cut AB in L, making AL equal $\frac{1}{4}$ of AB. 8. Then draw D g, cutting AE in M, and proceed as before.

* Hooke's Posthumous Works.

Corollary. Hence a given line may be accurately divided into any prime number whatsoever, by first cutting off the odd part, then dividing the remainder by continual bisections.

Problem 10. *An easy, simple, and very useful method of laying down a scale for dividing lines into any number of equal parts, or for reducing plans to any size less than the original.*

If the scale is for dividing lines into two equal parts, constitute a triangle, so that the hypothenuse may be twice the length of the perpendicular. Let it be three times for dividing them into three equal parts; four, for four parts, and so on: *fig.* 11, *plate* 5, represents a set of triangles so constituted.

To find the third of the line by this scale. 1. Take any line in your compasses, and set off this extent from A towards $\frac{1}{3}$, on the line marked one third; then close the compasses so as to strike an arc that shall touch the base AC, and this distance will be the $\frac{1}{3}$ of the given line. Similar to this is what is termed the angle of reduction, or proportion, described by some foreign writers, and which we shall introduce in its proper place.

Problem 11. *To divide by the sector a given strait line into any number of equal parts.*

Case 1. Where the given line is to be divided into a number of equal parts that may be obtained by a continual bisection.

In this case the operations are best performed by continual bisection; let it be required to divide AB, *fig.* 6, *plate* 5, into 16 equal parts. 1. Make AB a transverse distance between 10 and 10 on the line of lines. 2. Take out from thence the distance between 5 and 5, and set it from A or B to 8, and AB will be divided into two equal parts. 3. Make A 8 a transverse distance between 10 and

10, and 4 the transverse distance between 5 and 5, will bisect 8 A, and 8 B at 4 and 12; and thus AB is divided into four equal parts in the points 4, 8, and 12. 4. The extent A 4, put between 10 and 10, and then the distance between 5 and 5 applied from A to 2, from 4 to 6, from 8 to 10, and from 12 to 14, will bisect each of those parts, and divide the whole line into eight equal parts. 5. To bisect each of these, we might take the extent of A 2, and place it between 10 and 10 as before; but as the spaces are too small for that purpose, take three of them in the compasses, and open the sector at 10 and 10, so as to accord with this measure. 6. Take out the transverse measure between 5 and 5, and one foot of the compasses in A will give the point 3, in 4 will fall on 7 and 1, on 8 will give 5 and 11, on 12 gives 9 and 15, and on B will give 13. Thus we have, in a correct and easy manner divided AB into 16 equal parts by a continual bisection.

If it were required to bisect each of the foregoing divisions, it would be best to open the sector at 10 and 10, with the extent of five of the divisions already obtained; then take out the transverse distance between 5 and 5, and set it off from the other divisions, and they will thereby be bisected, and the line divided into 32 equal parts.

Case 2. When the given line cannot be divided by bisection.

Let the given line be AB, *fig.* 7, *plate* 4, to be divided into 14 equal parts, a number which is not a multiple of 2.

1. Take the extent AB, and open the sector to it on the terms 10 and 10, and the transverse distance of 5 and 5, set from A or B to 7, will divide AB into two equal parts, each of which are to be subdivided into 7, which may be done by dividing

A 7 into 6 and 1, or 4 and 3, which last is preferable to the first, as by it the operation may be finished with only two bisections.

2. Therefore open the sector in the terms 7 and 7, with the extent A 7; then take out the transverse distance between 4 and 4, this laid off from A, gives the point 4, from 7 gives 3 and 11, and from B gives 10.

3. Make A 4 a transverse distance at 10 and 10, then the transverse distance between 5 and 5 bisects A b, and 10 B in 2 and 12, and gives the point 6 and 8; then one foot in 3 gives 1 and 5, and from 11, 13 and 9: lastly, from 4 it gives 6, and from 10, 8; and thus the line AB is divided into 14 equal parts.

PROBLEM 12. *To make a scale of equal parts containing any given number in an inch.*

Example. To construct a scale of feet and inches in such a manner, that 25 of the smallest parts shall be equal to one inch, and 12 of them represent one foot.

By the line of lines on the sector. 1. Multiply the given numbers by 4, the products will be 100, and 48. 2. Take one inch between your compasses, and make it a transverse distance between 100 and 100, and the distance between 48 and 48 will be 12 of these 25 parts in an inch; this extent set off from A to 1, *fig.* 3, *plate* 5, from 1 to 2, &c. to 12 at B divides AB into a scale of 12 feet. 3. Set off one of these parts from A to a, to be subdivided into 12 parts to represent inches. 4. To this end divide this into three parts; thus take the extent A 2 of two of these parts, and make it a transverse distance between 9 and 9. 5. Set the distance between 6 and 6 from b to e, the same extent from 1 gives g, and from e gives n, thus dividing A a into three equal parts in the points n and g. 6. By two bisections each of these may be subdivided into four equal parts, and thus the whole space into 12 equal parts.

When a small number of divisions are required, as 1, 2, or 3, instead of taking the transverse distance near the center of the sector, the division will be more accurately performed by using the following method.

Thus, if three parts are required from A, of which the whole line contains 90, make AD, *fig.* 4, *plate* 5, a transverse distance between 90 and 90; then take the distance between 87 and 87, which set off from D to E backwards, and AE will contain the three desired parts.

Example 2. Supposing a scale of six inches to contain 140 poles, to open the sector so that it may answer for such a scale; divide 140 by 2, which gives 70, the half of 6 equal to 3; because 140 was too large to be set off on the line of lines. Make three inches a transverse distance between 70 and 70, and your sector becomes the required scale.

Example 3. To make a scale of seven inches that shall contain 180 fathom; $\frac{180}{2}=9 . \frac{7}{2}=3\frac{1}{2}$, therefore make $3\frac{1}{2}$ a transverse between 9 and 9, and you have the required scale.

OF PROPORTIONAL LINES.

PROBLEM 13. *To cut a given line* AD, *fig.* 14, *plate* 5, *into two unequal parts that shall have any given proportion, ex. gr. of* C *to* D.

1. Draw AG forming any angle with AD. 2. From A on AG set off AC equal to C, and CE equal to D. 3. Draw ED, and parallel to it CB, which will cut AD at B in the required proportion.

To divide by the sector the line AB, *fig.* 1, *plate* 5, in the proportion of 3 to 2. Now as 3 and 2 would fall near the center, multiply them by 2, thereby forming 6 and 4, which use instead of 3 and 2. As the parts are to be as 6 to 4, the whole line will be 10; therefore make AB a transverse distance between 10 and 10, and then the transverse distance between 6 and 6, set off from B to e, is $\frac{3}{5}$ths of AB; or the distance between 4 and 4 will give A e $\frac{2}{5}$ths of AB; therefore AB is cut in the proportion of 3 to 2.

Example 2. To cut AB, *fig.* 2, *plate* 5, in the proportion of 4 to 5; here we may use the numbers themselves; therefore with AB open the sector at 9 and 9, the sum of the two numbers; then the distance between 5 and 5 set off from B to c, or between 4 and 4, set off A to c, and it divides AB in the required proportion.

Note. If the numbers be too small, use their equimultiples; if too large, subdivide them.

Corollary. From this problem we obtain another mode of dividing a strait line into any number of equal parts.

PROBLEM 14. *To estimate the proportion between two or more given lines, as* AB, C D, E F, *fig.* 9, *plate* 5.

Make AB a transverse distance between 10 and 10, then take the extents severally of CD and EF, and carry them along the line of lines, till both points rest exactly upon the same number; in the first it will be found to be 85, in the second 67. Therefore AB is to C D as 100 to 85, to E F as 100 to 67, and of C D to E F as 85 to 67.

PROBLEM 15. *To find a third proportional to two given right lines* A *and* B, *fig.* 15, *plate* 5.

1. From the point D draw two right lines D E, D F, making any angle whatever. 2. In these

lines take DG equal to the first term A, and DC, DH, each equal to the second term B. 3. Join GH, and draw CF parallel thereto; then DF will be the third proportional required, that is, DG (A) to DC, (B,) so is DH (B) to DF.

By the sector. 1. Make AB, *fig.* 5, *plate* 5, a transverse distance between 100 and 100. 2. Find the transverse distance of EF, which suppose 50. 3. Make EF a transverse distance between 100 and 100. 4. Take the extent between 50 and 50, and it will be the third proportional CD required.

PROBLEM 16. *To find a fourth proportional to three given right lines* A, B, C, *fig.* 16, *plate* 5.

1. From the point *a* draw two right lines, making any angle whatever. 2. In these lines make ab equal to the first term A, ac equal to the second B, and ad equal to the third C. 3. Join bc, and draw de parallel thereto, and ae will be the fourth proportional required; that is, ab (A) is to ac (B,) so is ad (C) to ae.

By the sector. 1. Make the line A a transverse distance between 100 and 100. 2. Find the transverse measure of B, which is 60. 3. Make c the third line a transverse measure between 100 and 100. 4. The measure between 60 and 60 will be the fourth proportional.

PROBLEM 17. *To find a mean proportional between two given strait lines* A *and* B, *fig.* 17, *plate* 5.

1. Draw any right line, in which take CE equal to A, and EA equal to B. 2. Bisect AC in B, and with BA, or BC as a radius, describe the semicircle ADC. 3. From the point E draw EG perpendicular to AC, and it will be the mean proportional required.

By the sector. Join the lines together, (suppose them 40 and 90) and get the sum of them, 130; then find the half of this sum 65, and half the difference 25. Open the line of lines, so that they may be at right angles to each other; then take with the compasses the lateral distance 65, and apply one foot to the half difference 25, and the other foot will reach to 60, the mean proportional required; for 40 to 60, so is 60 to 90.

PROBLEM 18. *To cut a given line* AB *into extreme and mean proportion, fig.* 18, *plate* 5.

1. Extend AB to C. 2. At A erect a perpendicular AD, and make it equal to AB. 3. Set the half of AD or AB from A to F. 4. With the radius FD describe the arc DG, and AB will be divided into extreme and mean proportion. AG is the greater segment.

By the sector. Make AB a transverse distance between 60 and 60 of the line of chords. 2. Take out the transverse distance between the chord of 36, which set from A to G, gives the greatest segment.

Or make AB, a transverse distance between 54 and 54 of the line of sines, then is the distance between 30 and 30 the greater segment, and 18 and 18 the lesser segment.

PROBLEM 19. *To divide a given strait line in the same proportion as another given strait line is divided, fig.* 10, *plate* 5.

Let AB, or CD be two given strait lines, the first divided into 100, the second into 60 equal parts; it is required to divide EF into 100, and GH into 60 equal parts.

Make EF a transverse distance in the terms 100 and 100, then the transverse measure between 90 and 90 set from E to 90, and from F to 10; the measure between 80 and 80 set from E to 80, and from F to 20, and the measure between 70 and 70 set from E gives 70, from F 30. The distance between 60 and 60, gives 60 and 40; and lastly, the transverse measure between 50 and 50 bisects the given line in the point 50, and we shall, by five transverse extents, have divided the line EF into 10 equal parts, each of which are to be subdivided into 10 smaller divisions by problems 11 and 12.

To divide GH into 60 parts, as we have supposed CD to be divided, make GH a transverse distance in the terms of 60, then work as before.

PROBLEM 20. *To find the angular point of two given lines* AK, c P, *fig.* 22, *plate* 5, *which incline to each other without producing either of them.*

Through A draw at pleasure AN, yet so as not to cut c P too obliquely.

1. Draw the parallel lines Aν, Gε, Kδ. 2. Take any number of times the extents, AN, GO, KP, and set them on their respective lines, as from N to γ and ν, from o to β and ε, from P to α and δ, and a line through δ, ε, υ, and another through α, β, γ, will tend to the same point as the lines AB, c P.

Method 2. 1. Through AB and CD, *fig.* 21, *plate* 5, draw any two parallel lines, as GH and FE. 2. Set off the extent BD twice, from B to G, and D to H; and the extent AC twice, from A to F, and from C to E. 3. A line passing through F and G will intersect another line passing through E and H in I, the angular point required.

The extent FA, GB, may be multiplied or divided, so as to suit peculiar circumstances.

Corollary. Hence, if any two lines be given that tend to the same angular point, a third, or more lines may be drawn that shall tend to the same point, and yet pass through a given point.

Solution by the sector. *Case* 1. When the proposed point e is between the two given lines VL and a b, *fig*. 23, *plate* 5.

Through e draw a line a v, cutting a b at a, and VL at V, then from any other point b, in a b, the farther the better, draw b x parallel to a V, and cutting V L in x, make a v a transverse distance between 100 and 100, on the line of lines; take the extent e v, and find its transverse measure, which suppose to be 60; now make x b a transverse distance between 100 and 100, and take out the transverse distance of the terms 60, which set off from x to f, then a line drawn through the points e and f, shall tend to the same inaccessible point q with the given lines ab, V L.

Case 2. When the proposed point e is without the given lines ab and V L, *fig*. 24.

Through e draw any line e v, cutting ab in a, and V L in v; and from any other point b in ab draw x F parallel to e v, and cutting V L in x, make v e a transverse distance in the terms of 100; find the transverse measure of a v, which suppose 72; make x b a transverse measure of 72, and take out the distance between the terms of 100, which set off from x to f, and a line e f drawn through e and f, will tend to the same point with the line ab.

If the given or required lines fall so near each other, that neither of them can be measured on the terms of 100, then use any other number, as 80, 70, 60, &c. as a transverse measure, and work with that as you did with 100.

This problem is of considerable use in many geometrical operations, but particularly so in perspective; for we may consider V L as a vanishing line, and the other two lines as images on the picture; hence having any image given on the picture that tends to an inaccessible vanishing point, as many more images of lines tending to the same point as may be required, are readily drawn. This problem is more fully illustrated, and all the various cases investigated, in another part of this work.

PROBLEM 21. *Upon a given right line* AB, *fig.* 19, *plate* 5, *to make an equilateral triangle.*

1. From A and B, with a radius equal to AB, describe arcs cutting in C. 2. Draw AC and BC, and the figure ACB is the triangle required.

An isosceles triangle may be formed in the same manner.

PROBLEM 22. *To make a triangle, whose three sides shall be respectively equal to three given lines,* A, B, C, *fig.* 20, *plate* 5, *provided any two of them be greater than the third.*

1. Draw a line BC equal to the line B. 2. On B, with a radius equal to C, describe an arc at A. 3. On C, with a radius equal to A, describe another arc, cutting the former at A. 4. Draw the lines AC and AB, and the figure ABC will be the triangle required.

PROBLEM 23. *Upon a given line* AB, *fig.* 1, *plate* 6, *to describe a square.*

1. From the point B draw BD perpendicular, and equal to AB. 2. On A and D, with the radius AB, describe arcs cutting in C. 3. Draw AC and CD, and the figure ABCD is the required square.

PROBLEM 24. *To describe a rectangle or parallelogram, whose length and breadth shall be equal to two given lines* A *and* B, *fig.* 2, *plate* 6.

1. Draw CD equal to A, and make DE perpendicular thereto, and equal to B. 2. On the points E and C, with the radii A and B, describe arcs cutting in F. 3. Join CF and EF; then CDEF will be the rectangle required.

PROBLEM 25. *Upon a given line* AB, *to construct a rhombus, fig.* 3, *plate* 6.

1. On B, with the radius AB, describe an arc at D. 2. On A, with the same radius, describe

an arc at C. 3. On C, but still with the same radius, make the intersection D. 4. Draw the lines AC, DC, DB, and you have the required figure.

Having two given lines AB, AD, *and a given angle, to construct a rhomboides.*

Make the angle ACD equal to the given angle, and set off CD equal to AB, and AC equal to AD; then from A, with the distance AB, describe an arc at B; intersect this arc with the extent AD, set off from D; join AB, BD, and the figure is completed.

PROBLEM 26. *Having the diagonal* AD, *and four sides* AB, BD, DC, AC, *to construct a trapezium, fig.* 5, *plate* 6.

Draw an occult line AD, and make it equal to the given diagonal. Take AB in the compasses, and from A strike an arc at B; intersect this arc from D with the extent DB, and draw AB, DB; now with the other two lines AC, CD, and from A and D make an intersection at C; join DC, AC, and the figure is completed.

PROBLEM 27. *Having the four sides and one angle, to construct a trapezium, fig.* 5, *plate* 6.

Make the line AB equal to its given side, and at A make the angle CAB equal to the given angle, and AC equal to the given side AC; then with the extent BD from B, describe an arc at D, intersect this from C with the extent CD; join the several lines, and the figure is obtained.

PROBLEM 28. *To find the center of a circle, fig.* 6, *plate* 5.

1. Draw any chord AB, and bisect it with the chord CD. 2. Bisect CD by the chord EF, and their intersection o will be the center of the circle.

PROBLEM 29. *To describe the circumference of a circle through any three given points* A B C, *fig.* 7, *plate* 6, *provided they are not in a strait line.*

1. From the middle point B draw the chords BA and B C. 2. Bisect these chords with the perpendicular lines n O, n O. 3. From the point of intersection O, and radius O A, O B, and O C, you may describe the required circle A B C. By this problem a portion of the circumference of a circle may be finished, by assuming three points.

PROBLEM 30. *To draw a tangent to a given circle, that shall pass through a given point* A, *fig.* 8 *and* 9, *plate* 6.

Case 1. When the point A is in the circumference of the circle, *fig.* 8, *plate* 6.

1. From the center O, draw the radius O A. 2. Through the point A, draw C D perpendicular to O A, and it will be the required tangent.

Case 2. When the point A is without the circle, *fig.* 9, *plate* 6.

1. From the center O draw O A, and bisect it in n. 2. From the point n, with the radius n A, or n O, describe the semicircle A D O, cutting the given circle in D. 3. Through the points A and D, draw A B, the tangent required.

PROBLEM 31. *To cut off from a circle, fig.* 10, *plate* 6, *a segment containing any proposed angle,* ex. gr. 120°.

Let F be the point from whence it is required to draw a chord which shall contain an angle of 120°. 1. Through F draw F R a tangent to the circle. 2. From F draw F A, making an angle of 60 degrees, with the tangent F R, and F C A is the segment required.

PROBLEM 32. *On a given line* AB *to describe the segment of a circle capable of containing a given angle, fig.* 11, *plate* 6.

Draw AC and BC, making the angles ABC, BAC, each equal to the given angle. Draw AD perpendicular to AC, and BD perpendicular to BC; with the center D, and radius DA, or DB, describe the segment AEB, and any angle made in this segment will be equal to the given angle. A more easy solution of this problem will be given when we come to apply it to practice.

PROBLEM 33. *To describe an arc of a circle that shall contain any number of degrees, without compasses, or without finding the center of the circle, fig.* 12, *plate* 6.

Geometrically by finding points through which the arc is to pass, let AB be the given chord.

1. Draw AF, making any angle with BA. 2. At any point F, in AF, make the angle EFG, equal to the given angle. 3. Through B draw BE parallel to FG, and the intersection gives the point E, in the same manner as many points D, C, &c. may be found, as will be necessary to complete the arc.

This subject will be found fully investigated hereafter.

PROBLEM 34. *To inscribe a circle in a given triangle,* ABC, *fig* 13, *plate* 6.

1. Bisect the angles A and B with the lines AO and BO. 2. From the point of intersection O, let fall the perpendicular ON, and it will be the radius of the required circle.

PROBLEM 35. *To inscribe a pentagon, a hexagon, or a decagon, in a given circle, fig.* 15, *plate* 6.

1. Draw the diameters AB and CE at right angles to each other. 2. Bisect DB at G. 3. On G, with the radius GC, describe the arc CF,

4. Join C and F, and the line C F will be one side of the required pentagon.

The two sides D C, F D of the triangle F D C, enable us to inscribe a hexagon or decagon in the same circle; for D C is the side of the hexagon, D F that of the decagon.

PROBLEM 36. *To inscribe a square or an octagon in a given circle, fig.* 16, *plate* 6.

1. Draw the diameters A C, B D, at right angles to each other. 2. Draw the lines A D, B A, B C, C D, and you obtain the required square.

FOR THE OCTAGON.

Bisect the arc A B of the square in the point F, and the line A E being carried eight times round, will form the octagon required.

PROBLEM 37. *In a given circle to inscribe an equilateral triangle, an hexagon, or a dodecagon, fig.* 17, *plate* 6.

FOR THE EQUILATERAL TRIANGLE.

1. From any point A as a center, with a distance equal to the radius A O, describe the arc F O B. 2. Draw the line B F, and make B D equal to B F. 3. Join D F, and D B F will be the equilateral triangle required.

FOR THE HEXAGON.

Carry the radius A O six times round the circumference, and you obtain a hexagon.

FOR THE DODECAGON.

Bisect the arc A B of the hexagon in the point n, and the line A n being carried twelve times round the circumference, will form the required dodecagon.

PROBLEM 38. *Another method to inscribe a dodecagon in a circle, or to divide the circumference of a given circle into* 12 *equal parts, each of* 30 *degrees, fig.* 18, *plate* 6.

1. Draw the two diameters 1, 7, 4, 10, perpendicular to each other. 2. With the radius of the circle and on the four extremities 1, 4, 7, 10, as centers, describe arcs through the center of the circle; these arcs will cut the circumference in the points required, dividing it into 12 equal parts, at the points marked with the numbers.

PROBLEM 39. *To find the angles at the center, and the sides of a regular polygon.*

Divide 360 by the number of sides in the proposed polygon, thus $\frac{360}{5}$ gives 72, for the angle at the center of a pentagon. To find the angle formed by the sides, subtract the angle at the center from 180, and the remainder is the angle required; thus 72° from 180°, gives 108° for the angle of a pentagon.

A TABLE, *shewing the angles at the centers and circumferences of regular polygons, from three to twelve sides inclusive.*

Names.	Sides.	Angles at Center.	Angles at Cir.
Trigon	3	120° 00′	60° 00′
Square	4	90 00	90 00
Pentagon	5	72 00	108 00
Hexagon	6	60 00	120 00
Heptagon	7	51 $25\frac{5}{7}$	128 $34\frac{2}{7}$
Octagon	8	45 00	135 00
Nonagon	9	40 00	140 00
Decagon	10	36 00	144 00
Endecagon	11	32 $43\frac{7}{11}$	147 $16\frac{4}{11}$
Dodecagon	12	30 00	150 00

This table is constructed by dividing 360, the degrees in a circumference, by the number of sides in each polygon, and the quotients are the angles at the centers; the angle at the center subtracted from 180 degrees, leaves the angle at the circumference.

PROBLEM 40. *In a given circle to inscribe any regular polygon, fig.* 14, *plate* 6.

1. At the center c make an angle equal to the center of the polygon, as contained in the preceding table, and join the angular points AB. 2. The distance AB will be one side of the polygon, which being carried round the circumference, the proper number of times will complete the figure.

Another method, which approximates very nearly the truth, fig. 19, *plate* 6.

1. Divide the diameter AB into as many equal parts as the figure has sides. 2. From the center O raise the perpendicular Om. 3. Make mn equal to three fourths of Om. 4. From n draw nC, through the second division of the diameter. 5. Join the points AC, and the line AC will be the side of the required polygon, in this instance a pentagon.

PROBLEM 41. *About any given triangle* ABC, *to circumscribe a circle, fig.* 21, *plate* 6.

1. Bisect any two sides AB, BC, by the perpendiculars mo, no. 2. From the point of intersection o, with the distance OA or OB, describe the required circle.

PROBLEM 42. *About a given circle to circumscribe a pentagon, fig.* 20, *plate* 6.

1. Inscribe a pentagon within the circle, 2. Through the middle of each side draw the lines OA, OB, OC, OD, and OE. 3. Through the point n draw the tangent AB, meeting OA and OB in A and B. 4. Through the points A and m,

draw the line A m C, meeting OC in C. 5. In like manner draw the lines C D, D E, E B, and A B C D E will be the pentagon required.

In the same manner you may about a given circle circumscribe any polygon.

PROBLEM 43. *About a given square to circumscribe a circle, fig. 22, plate 6.*

1. Draw the two diagonals AD, B C, intersecting each other at O. 2. From O, with the distance OA or O B, describe the circle AB C D, which will circumscribe the square.

PROBLEM 44. *On a given line* AB *to make a regular hexagon, fig. 24, plate 6.*

1. On AB make the equilateral triangle AO B. 2. From the point O, with the distance OA or O B, describe the circle AB C D E F. 3. Carry AB six times round the circumference, and it will form the required hexagon.

PROBLEM 45. *On a given line* AB *to form a regular polygon of any proposed number of sides, fig. 14, plate 6.*

1. Make the angles CAB, C BA each equal to half the angles at the circumference; see the preceding table. 2. From the point of intersection c, with the distance CA, describe a circle. 3. Apply the chord AB to the circumference the proposed number of times, and it will form the required polygon.

PROBLEM 46. *On a given line* AB, *fig. 25, plate 6, to form a regular octagon.*

1. On the extremities of the given line AB erect the indefinite perpendiculars AF and B E. 2. Produce AB both ways to s and w, and bisect the angle nAs and oBw by the lines AH, B C. 3. Make AH and B C each equal to AB, and draw the line H C. 4. Make ov equal to on, and through v draw G D parallel to H C. 5. Draw

H G and C D parallel to A F and B E, and make c E equal to c D. 6. Through E draw E F, parallel to A B, and join the points G F and D E, and ABCDEFGH will be the octagon required.

PROBLEM 47. *On a given right line* AB, *fig.* 26, *plate* 6, *to describe a regular pentagon.*

1. Make Bm perpendicular, and equal to AB. 2. Bisect AB in n. 3. On n, with distance n m, cross AB produced in O. 4. On A and B, with radius AO, describe arcs intersecting at D. 5. On D, with radius AB, describe the arc E C, and on A and B, with the same extent, intersect this arc at E and C. 6. Join AE, E D, D C, C B, and you complete the figure.

PROBLEM 48. *Upon a given right line* AB, *fig.* 27, *plate* 6, *to describe a triangle similar to the triangle* C D E.

1. At the end A of the given line AB, make an angle F A B, equal to the angle E C D. 2. At B make the angle A B F equal to the angle C D E. 3. Draw the two sides AF, B F, and A B F will be the required triangle.

PROBLEM 49. *To describe a polygon similar to a given polygon* ABCDEF, *one of its sides* ab *being given, fig.* 28, *plate* 6.

1. Draw AC, AD, AF. 2. Set off ab on AB, from a to r. 3. Draw r g parallel to BE, meeting AF in g. 4. Through the point g draw gh, parallel to F C, meeting A C in h. 5. Through the point h draw the parallel hi. 6. Through i draw i k parallel to E D, and the figure A r g h i k will be similar to the figure ABCDEF.

PROBLEM 50. *To reduce a figure by a scale, fig.* 28 *and* 29, *plate* 6.

1. Measure each side of the figure AB C D E with the scale G H. 2. Make ab as many parts of a smaller scale K L, as AB was of the larger.

3. bc as many of K L, as B C of G H, and ac of K L, A C, &c. by which means the figure will be reduced to a smaller one.

OF THE TRANSFORMATION AND REDUCTION OF FIGURES.

PROBLEM 51. *To change a triangle into another of equal extent, but different height, fig.* 1, 2, 3, 4, *plate* 7.

Let A B C be the given triangle, D a point at the given height.

Case 1. Where the point D, *fig.* 1 and 2, *plate* 7, is either in one of the sides, or in the prolongation of a side. 1. Draw a line from D to the opposite angle C. 2. Draw a line A E parallel thereto from A, the summit of the given triangle. 3. Join D E, and B D E is the required triangle.

Case 2. When the point D, *fig.* 3 and 4, *plate* 7, is neither in one of the sides, nor in the prolongation thereof. 1. Draw an indefinite line B D a, from B through the point D. 2. Draw from A, the summit of the given triangle, a line A a, parallel to the base B C, and cutting the line B D in a. 3. Join a C, and the triangle B a C is equal to the triangle B A C; and the point D being in the same line with B a. 4. By the preceding case, find a triangle from D, equal to B a C; i. e. join D C, draw a E parallel thereto, then join D E, and B D E is the required triangle.

Corollary. If it be required to change the triangle B A C into an equal triangle, of which the height and angle B D E are given: 1. Draw the indefinite line B D A, making the required angle with B C. 2. Take on B D a a point D at the given height; and, 3. Construct the triangle by the foregoing rules.

PROBLEM 52. *To make an isosceles triangle* AEB, *fig.* 5, *plate* 7, *equal to the scalene triangle* ACB.

1. Bisect the base in D. 2. Erect the perpendicular DE. 3. Draw CE parallel to AB. 4. Draw AE, EB, and AEB is the required triangle.

PROBLEM 53. *To make an equilateral triangle equal to a given scalene triangle* ABC, *fig.* 7, *plate* 7.

1. On the base AB make an equilateral triangle ABD. 2. Prolong BD towards E. 3. Draw CE parallel to AB. 4. Bisect DE at I, on DI describe the semicircle DFE. 5. Draw BF, the mean proportional between BE, BD. 6. With BF from B, describe the arc FGH; with the same radius from G, intersect this arc at H, draw BH, GH, and BGH is the triangle required.

Corollary. If you want an equilateral triangle equal to a rectangle, or to an isosceles triangle; find a scalene triangle respectively equal to each, and then work by the foregoing problem.

PROBLEM 54. *To reduce a rectilinear figure* ABCDE, *fig.* 8 and 9, *plate* 7, *to another equal to it, but with one side less.*

1. Join the extremities E, C, of two sides DE, DC, of the same angle D. 2. From D draw a line DF parallel to EC. 3. Draw EF, and you obtain a new polygon ABFE, equal to ABCDE, but with one side less.

Corollary. Hence every rectilinear figure may be reduced to a triangle, by reducing it successively to a figure with one side less, until it is brought to one with only three sides.

For example; let it be required to reduce the polygon ABCDEF, *fig.* 10 and 11, *plate* 7, into a triangle IAH, with its summit at A, in the circumference of the polygon, and its base on the base thereof prolonged.

1. Draw the diagonal D F. 2. Draw E G parallel to DF. 3. Draw F G, which gives us a new polygon, ABCGF, with one side less. 4. To reduce ABCGF, draw AG, and parallel thereto F H; then join AH, and you obtain a polygon ABCH, equal to the preceding one ABCGF. 5. The polygon ABCH having a side AH, which may serve for a side of the triangle, you have only to reduce the part AB C, by drawing AC, and parallel thereto B I; join AI, and you obtain the required triangle I A H.

N. B. In figure 10 the point A is taken at one of the angular points of the given polygon; in figure 11 it is in one of the sides, in which case there is one reduction more to be made, than when it is at the angular point.

Corollary. As a triangle may be changed into another of any given height, and with the angle at the base equal to a given angle; if it be required to reduce a polygon to a triangle of a given height, and the angle at the base also given, you must first reduce it into a triangle by this problem, and then change that triangle into one, with the data, as given by the problem 51.

Corollary. If the given figure is a parallelogram, *fig.* 12, *plate* 7, draw the diagonal E C, and D F parallel thereto; join E F, and the triangle E B F is equal to the parallelogram EBCD.

OF THE ADDITION OF FIGURES.

1. If the figures to be added are triangles of the same height as AMB, BNC, COD, DPE, *fig.* 14, *plate* 7, make a line AE equal to the sum of their bases, and constitute a triangle AME thereon, whose height is equal to the given height, and AME will be equal to the given triangles.

2. If the given figures are triangles of different heights, or different polygons, they must first be reduced to triangles of the same height, and then these may be added together.

3. If the triangle, into which they are to be summed up, is to be of a given height, and with a given angle at the base, they must first be reduced into one triangle, and then that changed into another by the preceding rules.

4. The triangle obtained may be changed into a parallelogram by the last corollary.

MULTIPLICATION OF FIGURES.

1. To multiply AMB, *fig.* 13, *plate* 7, by a given number, for example, by 4; or more accurately, to find a triangle that shall be quadruple the triangle AMB. Lengthen the base AE, so that it may be four times AB; join ME, and the triangle AME will be quadruple the triangle AMB,

2. By reducing any figure to a triangle, we may obtain a triangle which may be multiplied in the same manner.

SUBTRACTION OF FIGURES.

1. If the two triangles BAC, dac, *fig.* 15, *plate* 7, are of the same height, take from the base BC of the first a part DC, equal to the base dc of the other, and join AD; then will the triangle ABD be the difference between the two triangles.

If the two triangles be not of the same height, they must be reduced to it by the preceding rules, and then the difference may be found as above; or if a polygon is to be taken from another, and a triangle found equal to the remainder, it may be easily effected, by reducing them to triangles of the same height.

2. A triangle may be taken from a polygon by drawing a line within the polygon, from a given point F on one of its sides. To effect this, let us suppose the triangle, to be taken from the polygon ABCDE, *fig.* 16, *plate* 7, has been changed into a triangle M O P, *fig.* 26, whose height above its base M P is equal to that of the given point F, above AB of *fig.* 16; this done, on AB (prolonged if necessary) lay off AG equal to O P, join F G, and the triangle AFG is equal to the triangle M O P. There are, however, three cases in the solution of this problem, which we shall therefore notice by themselves.

If the base M P does not exceed AB, *fig.* 16, *plate* 7, the point G will fall thereon, and the problem will be solved.

But if the base M P exceeds the base AB, G will be found upon AB prolonged, *fig.* 17 and 18, *plate* 7; join F B, and draw G H parallel thereto; from the situation of this point arise the other two cases.

Case 1. When the point H, *fig.* 17, *plate* 7, is found on the side BC, contiguous to the side AB, join F H, and the quadrilateral figure FABH is equal to the triangle M O P.

Case 2. When H, *fig.* 18, *plate* 7, meets B C prolonged, from F draw F C and H I parallel thereto; then join F I, and the pentagon FAB C I is equal to the triangle M O P.

DIVISION OF RECTILINEAR FIGURES.

1. To divide the triangle AM E, *fig.* 13, *plate* 7, into four equal parts; divide the base into four equal parts by the points B, C, D; draw M D, M C, M B, and the triangle will be divided into four equal parts.

2. If the triangle AME, *fig.* 19, *plate* 7, is to be divided into four equal parts from a point m, in one of its sides, change it into another Ame, with its summit at m, and then divide it into four equal parts as before; if the lines of division are contained in the triangle AME, the problem is solved; but if some of the lines, as mD, terminate without the triangle, join mE, and draw dD parallel thereto; join md, and the quadrilateral mCEd is equal to CmD $\frac{1}{4}$th of AME, and the triangle is divided into four equal parts.

3. To divide the polygon ABCDEF, *fig.* 20, *plate* 7, into a given number of equal parts, ex. gr. four, from a point G, situated in the side AF; 1. Change the polygon into a triangle AGM, whose summit is at G. 2. Divide this triangle into as many equal triangles AGH, HGI, IGK, KGM, as the polygon is required to be divided into. 3. Subtract from the polygon a part equal to the triangle AGH; then a part equal to the triangle AGI, and afterwards a part equal to the triangle AGK, and the lines GH, GR, GO, drawn from the point G, to make these subtractions, will divide the polygon into four equal parts, all which will be sufficiently evident from consulting the figure.

PROBLEM 55. *Three points, fig.* 21 *to* 25, *plate* 7, N, O, A, *being given, arranged in any manner on a strait line, to find two other points*, B, b, *in the same line so situated, that as*

$$NO : AB :: AB : NB$$
$$NO : Ab :: Ab : NB$$

Make $AP = \frac{NO}{4.}$, and $PL = \frac{NO}{4}$, placing them one after the other, so that $AL = \frac{NO}{2}$.

Observing, 1. That in the two figures 21 and 22, A is placed between N and O; and that AP is taken on AO, prolonged if necessary.

2. In *fig*. 23, where the point O is situated between N and A, AP should be put on the side opposite to AO.

3. In *fig*. 24, where the point N is situated between A and O, if AP be smaller than AN, it must be taken on the side opposite to AN.

4. In *fig*. 25, where the point N is also placed between A and O, if AP be greater than AN, it must be placed on AN prolonged.

Now by problem 17 make MN in all the five figures a mean proportional between NO, NP, and carry this line from L to B, and from L to b. and NO will be : AB :: AB : NB
NO : Ab :: Ab : NB.

PROBLEM 56. *Two lines* EF, GH, *fig*. 27, 28, 29, *plate* 7, *intersecting each other at* A, *being given, to draw from* C *a third line* BD, *which shall form with the other two a triangle* DAB *equal to a given triangle* X.

1. From C draw CN parallel to EF. 2. Change the triangle X into another CNO, whose summit is at the point C. 3. Find on GH a point B, so that NO : AB :: AB : NB, and from this point B, draw the line CB, and DAB shall be the required triangle.

Scholium. This problem may be used to cut off one rectilinear figure from another, by drawing a line from a given point.

Thus, if from a point C, without or within the triangle EAH, *fig*. 30, a right line is required to be drawn, that shall cut off a part DAB, equal to the triangle X, *fig*. 30; it is evident this may be effected by the preceding problem.

If it be required to draw a line B D from a point c, which shall cut off from the quadrilateral figure E F G H,, a portion D F G B, *fig.* 31, equal to the triangle Z; if you are sure, that the right line BD will cut the two opposite sides EF, GH, prolong E F, G H, till they meet; then form a triangle Z, equal to the two triangles Z and F A G; and then take Z from A E H by a line B D from the point C, which is effected by the preceding problem.

If it be required to take from a polygon Y, a part D F I H B equal to a triangle X; and that the line BD is to cut the two sides E F, G H; prolong these sides till they meet in A; then make a triangle Z, equal to the triangle X, and the figure A F I H; and then retrench from the triangle E A G the triangle D A B equal to Z, by a line B D, from a given point C.

As all rectilinear figures may be reduced to triangles, we may, by this problem, take one rectilinear figure from another by a strait line drawn from a given point.

PROBLEM 57. *To make a triangle equal to any given quadrilateral figure* ABCD, *fig.* 33, *plate* 7.

1. Draw the diagonal B D. 2. Draw CE parallel thereto, intersecting AD produced in E. 3. Join AC, and ACE is the required triangle.

PROBLEM 58. *To make a rectangle, or parallelogram equal to a given triangle* ACE, *fig.* 33, *plate* 7.

1. Bisect the base AE in D. 2. Through C draw CB parallel to AD. 3. Draw CD, BA, parallel to each other, and either perpendicular to A E, or making any angle with it. And the rectangle or parallelogram ABCD will be equal to the given triangle.

PROBLEM 59. *To make a triangle equal to a given circle, fig.* 34, *plate* 7.*

Draw the radius OB, and tangent AB perpendicular thereto; make AB equal to three times the diameter of the circle, and $\frac{1}{7}$ more; join AO, and the triangle AOB will be nearly equal the given circle.

PROBLEM 60. *To make a square equal to a given rectangle,* ABCD, *fig.* 35, *plate* 7.

Produce one side AB, till BL be equal to the other BC. 2. Bisect AL in O. 3. With the distance AO, describe the semicircle LFA. 4. Produce BC to F. 5. On BF make the square BFGH, which is equal to the rectangle ABCD.

ADDITION AND SUBTRACTION OF SIMILAR FIGURES.

PROBLEM 61. *To make a square equal to the sum of any number of squares taken together,* ex. gr. *equal to three given squares, whose sides are equal to the lines* ABC, *fig.* 36, *plate* 7.

1. Draw the indefinite lines ED, DF, at right angles to each other. 2. Make DG equal to A, and DH equal to B. 3. Join G and H, and GH will be the side of a square, equal the two squares whose sides are A and B. 4. Make DL equal GH, DK equal C, and join KL; then will KL be the side of a square equal the three given squares. Or, after the same manner may a square be constructed equal to any number of given squares.

PROBLEM 62. *To describe a figure equal to the sum of any given number of similar figures, fig.* 36, *plate* 7.

This problem is similar to the foregoing: 1. Form a right angle. 2. Set off thereon two homologous sides of the given figures, as from D to G, and from

* Strictly, this can only be solved but by an approximation; the area, or squaring of the circle is yet a desiderata in mathematics. See *Hutton's Mathematical and Philosophical Dictionary*, 2 Vols. 4to. 1796. EDIT.

D to H. 3. Draw G H, and thereon describe a figure similar to one of the given ones, and it will be equal to their sum. In the same manner you may go on, adding a greater number of similar figures together.

If the similar figures be circles, take the radii or diameters for the homologous lines.

Problem 63. *To make a square equal to the difference of two given squares, whose sides are* A B, C D, *fig.* 37, *plate* 7.

1. On one end B of the shortest line raise a perpendicular B F. 2. With the extent C D from A, cut B F in F, and B F will be the side of the required square.

In the same manner the difference between any two similar figures may be found.

Problem 64. *To make a figure which shall be similar to, and contain a given figure, a certain number of times. Let* M N *be an homologous side of the given figure, fig.* 38, *plate* 7.

1. Draw the indefinite line B Z. 2. At any point D, raise D A perpendicular to B Z. 3. Make B D equal to M N, and B C as many times a multiple of B D, as the required figure is to be of the given one. 4. Bisect B C, and describe the semicircle B A C. 5. Draw A C, B C. 6. Make A E equal M N. 7. Draw E F parallel to B C, and E F will be the homologous side of the required figure.

Problem 65. *To reduce a complex figure from one scale to another, mechanically, by means of squares. Fig.* 39, *plate* 7.

1. Divide the given figure by cross lines into as many squares as may be thought necessary. 2. Divide another paper into the same number of squares, either greater, equal, or less, as required. 3. Draw in every square what is contained in the

correspondent square of the given figure, and you will obtain a copy tolerably exact.

PROBLEM 66. *To enlarge a map or plan, and make it twice, three, four, or five, &c. times larger than the original, fig. 12, plate 8.*

1. Draw the indefinite line ab. 2. Raise a perpendicular at a. 3. Divide the original plan into squares by the preceding problem. 4. Take the side of one of the squares, which set off from a to d, and on the perpendicular from a to e, finish the square a e f d, which is equal to one of the squares of the proposed plan. 5. Take the diagonal d e, set it off from a to g, and from a to l; complete the square a l n g, and it will be double the square a e f d. To find one three times greater, take d g, and with that extent form the square a m o h, which will be the square required. With d h you may form a square that will contain the given one a e f d four times. The line d l gives a square five times larger than the original square.

PROBLEM 67. *To reduce a map $\frac{1}{2}$, $\frac{1}{3}$d, $\frac{1}{4}$th, $\frac{1}{5}$th, &c. of the original, fig. 5, plate 8.*

1. Divide the given plan into squares by problem 65. 2. Draw a line, on which set off from A to B one side of one of these squares. 3. Divide this line into two equal parts at F, and on F as a center, with FA or FB, describe the semicircle AHB.

4. To obtain $\frac{1}{2}$ the given square, at F erect the perpendicular FH, and draw the right line AH, which will be the side of the required square. 5. For $\frac{1}{3}$d, divide AB into three parts; take one of these parts, set it off from A to C, at C raise the perpendicular CI, through I draw AI, and it will be the side of the required square. 6. For $\frac{1}{4}$th, divide AB into four equal parts, set off one of these from B to E, at E make EG perpendicular to AB,

join B G, and it will be the side of a square $\frac{1}{4}$th of the given one.

PROBLEM 68. *To make a map or plan in proportion to a given one,* e. gr. *as three to five, fig.* 6, *plate* 8.

The original plan being divided into squares, 1. draw AM equal to the side of one of these squares. 2. Divide AM into five equal parts. 3. At the third divison raise the perpendicular CD, and draw AD, which will be the side of the required square.

PROBLEM 69. *To reduce figures by the angle of reduction. Let* a b *be the given side on which it is required to construct a figure similar to* ACDE, *fig.* 1, 2, 3, *plate* 8.

1. Form an angle LMN at pleasure, and set off the side AB from M to I. 2. From I, with ab, cut ML in K. 3. Draw the line IK, and several lines parallel to, and on both sides of it. The angle LMN is called the angle of proportion or reduction. 4. Draw the diagonal lines BC, AD, AE, BD. 5. Take the distance BC, and set it off from M towards L on ML. 6. Measure its corresponding line KI. 7. From b describe the arc n o. 8. Now take AC, set it off on ML, and find its correspondent line fg. 9. From a, with the radius fg, cut the former arc no in c, and thus proceed till you have completed the figure.

PROBLEM 70. *To enlarge a figure by the angle of reduction. Let* abcde *be the given figure, and* AB *the given side, fig.* 3, 1, *and* 4, *plate* 8.

1. Form, as in the preceding problem, the angle LMN, by setting off ab from M to H, and from H with AB, cutting ML in I. 2. Draw HI, and parallels to, and on both sides of it. 3. Take the diagonal bc, set it off from M towards L, and take off its corresponding line qr. 4. With

qr as a radius on B describe the arc mn. 5. Take the same correspondent line to ac, and on A cut mn in c, and so on for the other sides.

CURIOUS PROBLEMS ON THE DIVISION OF LINES AND CIRCLES.

PROBLEM 71. *To cut off from any given arc of a circle a third, a fifth, a seventh, &c. odd parts, and thence to divide that arc into any number of equal parts, fig. 7, plate 8.*

Example 1. To divide the arc AKB into three equal parts, CA being the radius, and C the center of the arc.

Bisect AB in K, draw the two radii CK, CB, and the chord AB; produce AB at pleasure, and make BL equal AB; bisect AC at G; then a rule on G and L will cut CB in E, and BE will be $\frac{1}{3}$d, and CE $\frac{2}{3}$ds of the radius CB; on CB with CE describe the arc Eed; lastly, set off the extent Ee or De from B to a, and from a to b, and the arc AKB will be divided into three equal parts.

Corollary. Hence having a sextant, quadrant, &c. accurately divided, $\frac{1}{2}$ the chord of any arc set off upon any other arc of $\frac{1}{2}$ that radius will cut off an arc similar to the first, and containing the same number of degrees.

Also $\frac{1}{3}$d, $\frac{1}{4}$th, $\frac{1}{5}$th, &c. of a larger chord will constantly cut similar arcs on a circle whose radius is $\frac{1}{3}$d, $\frac{1}{4}$th, $\frac{1}{5}$th, &c. of the radius of the first arc.

Example 2. Let it be required to divide the arc AKB into five equal parts, or to find the $\frac{1}{5}$th part of the arc AB.

Having bisected the given arc AB in K, and drawn the three radii CA, CK, CB, and having found the fifth part of IB of the radius CB, with radius CI describe the arc InM, which will be bisected in n, by the line C K; then take the extent I n, or its equal M n, and set it off twice from A to B; that is, first, from A to d, and from d to o, and oB will be $\frac{1}{5}$th of the arc AB. Again, set off the same extent from B to m, and from m to c, and the arc AB will be accurately divided into five equal parts.

Example 3. To divide the given arc AB into seven equal parts. AB being bisected as before, and the radii CA, CK, CB, drawn, find by problem 9 the seventh part of P B of the radius C B, and with the radius C P describe the arc P r N; then set off the extent P r twice from A to 3, and from 3 to 6, and 6 B will be the seventh part of the given arc AB; the compasses being kept to the same opening P r, set it from B to 4, from 4 to 1; then the extent A 1 will bisect 1, 3 into 2, and 4, 6 into 5; and thus divide the given arc into seven equal parts.

PROBLEM 72. *To inscribe a regular heptagon in a circle, fig.* 8, *plate* 8.

In *fig.* 8, let the arc B D be $\frac{1}{8}$th part of the given circle, and AB the radius of the circle. Divide AB into eight equal parts, then on center A, with radius AC, describe the arc C E, bisect the arc B D in a, and set off this arc Ba from C to b, and from b to c; then through A and c draw Ace, cutting B D in e, and Be will be $\frac{1}{7}$th part of the given circle.

Corollary 1. Hence we have a method of finding the seventh part of any given angle; for, if from the extremities of the given arc radii be drawn to the center, and one of these be divided into eight equal parts, and seven of these parts be taken, and another arc described therewith, the greater arc will be to the lesser, as 8 to 7, and so of any other proportion.

Corollary 2. If an arc be described with the radius, it will be equal to the $\frac{1}{8}$th part of a circle whose radius is AB, and to the seventh part of a circle whose radius is AC, and to the sixth part of a circle whose radius is AL, &c.

Corollary 3. Hence also a pentagon may be derived from a hexagon, *fig.* 9, *plate* 8. Let the given circle be ABCDEF, in which a pentagon is to be inscribed; with the radius AC set off AF equal $\frac{1}{6}$th of the circle, divide AC into six equal parts; then c G will be five of these parts; with radius C G describe the arc G H, bisect AF in q, and make G P and P H each equal to A q; then through C and H draw the semi-diameter C w, cutting the given circle in w, join A w, and it will be one side of the required pentagon.

Corollary 4. Hence as radius divides a circle into six equal parts, each equal 60 degrees, twice radius gives 120 degrees, or the third part of the circumference.

Once radius gives 60 degrees, and that arc bisected gives 30 degrees, which, added to 60, divides the circumference into four equal parts; whence we divide it into two, three, four, five, six equal parts; the preceding corollary divides into five equal parts, the arc of a quadrant bisected divides it into eight equal parts. By problem 71 we obtain the seventh part of a circle, and by this method divide it into any number of equal parts, even a prime number; for the odd unit may be cut off by the preceding problem, and the remaining part be subdivided by continual bisection, till another prime number arises to be cut off in the same manner.

PROBLEM 73. *To divide a given right line, or an arc of a circle, into any number of equal parts by the help of a pair of beam, or other compasses, the* *distance of whose points shall not be nearer to each other than the given line, fig.* 11, *plate* 8.

From this problem, published by Clavius, the Jesuit, in 1611, in a treatise on the construction of a dialling instrument, it is presumed that he was the original inventor of that species of division, called *Nonius*, and which by many modern mathematicians has been called the scale of *Vernier*.

Let AB be the given line, or circular arch, to be divided into a number of equal parts. Produce them at pleasure; then take the extent AB, and set it off on the prolonged line, as many times as the given line is to be divided into smaller parts, BC, CD, DE, EF, FG. Then divide the whole line AG into as many equal parts as are required in AB, as GH, HI, IK, KL, LA, each of which contains the given line, and one of those parts into which the given line is to be divided. For AG is to AL as AF to AB; in other words, AL is contained five times in AG, as AB in AF; therefore, since AG contains AF, and $\frac{1}{5}$th of AF, AL will contain AB, and $\frac{1}{5}$th of AB; therefore BL is the $\frac{1}{5}$th of AB. Then as GH contains AB plus, FH, which is $\frac{1}{5}$ths of AB, EI will be $\frac{2}{5}$ths of AB, DK $\frac{3}{5}$th, CL $\frac{4}{5}$ths. Therefore, if we set off the interval GH from F and H, we obtain two parts at L and I, set off from the points near E, gives three parts between DK, from the four points at D and K, gives four parts at CL, and the next setting off one more of these parts; so that, lastly, the extent GH set off from the points between C and L, divides the given line as required.

To divide a given line AC, *or arc of any circle, into any number of equal parts, suppose* 30, *fig.* 11, *plate* 8.

1. Divide it into any number of equal parts less than 30, yet so that they may be aliquot parts

of 30; as for example, AG is divided into six equal parts, AB, BC, CD, DE, EF, FG, each of which are to be subdivided into five equal parts. 2. Divide the first part AB into five parts, by means of the interval AL or GH, as taught in this problem. If, now, one foot of the compasses be put into the point A, (the extent AL remaining between them unaltered) and then into the point next to A, and so on to the next succeeding point, the whole line AG will be divided by the other foot of the compasses into 30 equal parts.

Or if the right line, or arc, be first divided into five equal parts, each of these must be subdivided into six parts, which may be effected by bisecting each part, and then dividing the halves into three parts.

Or it may be still better to bisect three of the first five parts, and then to divide four of these into three, which being set off from every point, will complete the division required.

Corollary. It frequently happens that so many small divisions are required, that, notwithstanding their limited number, they can be hardly taken between the points of the compasses without error; in this case use the following method.

If the whole number of smaller parts can be subdivided, take so many of the small parts in the given line, as each is to be subdivided into, yet so that they may together make up the whole of the given line. For if the first of these parts be cut into as many smaller parts as the proposed number requires every one of them to contain, and the same is also done in the remaining parts, we shall obtain the given number of smaller parts.

If 84 parts are to be taken in the proposed line, first bisect it, and each half will contain 42; bisect these again, and you have four parts, each of which is to contain 21; and these, divided into three, give 12 parts, each of which is to contain seven parts; subdivide these into seven each.

But if the proposed number of small parts cannot be thus subdivided, it will be necessary to take a number a little less or greater, that will be capable of subdivision; for if the superfluous parts are rejected, or those wanting, added, we shall obtain the proposed number of parts. Thus if 74 parts are to be cut from any given line of 80 parts; 1. Bisect the given line, and each ½ will contain 40. 2. Bisect these again, and you have four parts to contain 20 each. 3. Each of these bisected, you have eight parts to contain 10 each. 4. Bisect these, and you obtain 16 parts, each to be divided into five parts. 6. Reject six of the parts, and the remainder is the 74 parts proposed.

Or if 72 parts be proposed; divide the line into 24 equal parts, and each of these into three parts, and you obtain 72; to which adding two, you will obtain the number of 74.

PROBLEM 74. *To cut off from the circumference of any given arc of a circle any number of degrees and minutes, fig. 3, plate 9.*

1. Let it be proposed to cut off from any arc 57 degrees; with the radius of the given arc, or circle, describe a separate arc as AB, and having set off the radius from A to C, bisect AC in E, then AE and EC will be each of them an arc of 30 degrees. Make CB equal to AE, and AB will be a quadrant, or 90 degrees, and will also be divided into three equal parts; next, divide each of these into three by the preceding rules, and the quadrant will be divided into nine equal parts, each containing 10 degrees. Lastly, divide the first of these into 10 degrees, then set one foot of the compasses into the seventh single degree, and extend the

other to the 50th, and the distance between the points of the compasses will contain 57 degrees, which transfer to the given arc. Or at two operations; first, take 50 degrees, and then from the first 10 take seven, and you have 57 degrees.

2. Let it be required to cut off from any given arc of a circle 45 degrees, 53 minutes, *fig.* 3, *plate* 9.

Divide the arc of 53 degrees of the quadrant AB, whose radius is AC (or rather its equal arc FG) into 60 equal parts, first into 5, and then one of these into three; or first into three, and one of these into five. Again, one of these bisected, and this bisection again bisected, gives the 60th part of an arc of 53 degrees.

For the 5th part of the arc FG is FH, containing 12 parts; its third part is FI, containing four parts; the $\frac{1}{2}$ of FI is N, which contains two parts; and FN again bisected in K, leaves FK the 60th part of the arc FG; consequently, FK comprehends 53 minutes; therefore, add the arc FK to 45 degrees, and the arc AF will contain 45 degrees 53 minutes.

Corollary. If we describe a separate arc LM with the radius AC, and set off thereon the extent LM of 61 degrees of the given arc AB, and divide LM into 60 equal parts; thus, first into two, then both of these into three, and then the first of these three into 10 by the former rules, which gives the 60th part of the arc LM. And as one division of the arc LM contains by construction one degree of the quadrant AB, and one sixtieth part of a degree more, that is one minute, therefore two divisions of LM contain two degrees, and two minutes over; three divisions exceed three degrees by three minutes, and so of all the rest.

Whence if one division of L M be set off from any degree on AB, it will add one minute to that degree; two adds two minutes; three, three minutes, and so on.

When the division is so small that the compasses will hardly take it in without error, take two, three, four, or more of the parts on L, set them off from as many degrees back from the degree intended, and you will obtain the degree and minute required.

PROBLEM 75. *To divide a circle into any uneven number of equal parts.*

Example 1. Let it be required to divide a circle into $346\frac{2}{3}$ equal parts.

Reduce the whole into 3d parts, which gives us 1040; find the greatest multiple of 3 less than 1040, which may be bisected; this number will be found in a double geometrical progression, whose first term is 3, as in the margin; 768 the ninth number, is the number sought, as in the margin. Subtract 768 from 1040, the remainder is 272, then find how many degrees and minutes this remainder contains by the rule of three. As 1040 is to 360 degrees, so is 272 to $94^\circ\ 9'\ 23''$. Now set off $94^\circ\ 9'\ 23''$ upon the circle to be divided, and divide the remaining part of that circle by continual bisections, till you come to the number 3, which will be one of the required divisions of the 346 equal parts, by which dividing the arc of $94^\circ\ 9'\ 23''$ you will have the whole circle divided into $346\frac{2}{3}$ equal parts; for there will be 256 divisions in the greatest arc, and $90\frac{2}{3}$ in the other.

3	2
6	2
12	2
24	2
48	2
96	2
192	2
384	2
768	2

Example 2. Let it be required to divide a circle into 179 equal parts. Find the greatest number not exceeding 179, which may be continually bisected to unity, which you will find to be 128. Subtract 128 from 179, the remainder is 51; then find what part of the circle this remainder will occupy by the following proportion, as 179 is to 360, so is 51 to $102^\circ\ 34'\ 11''$; set off from the circle an arc of $102^\circ\ 34'\ 11''$, and divide the remaining part of the circle by continual bisections, seven of which will be unity in this example; by which means this part of the circle will be divided into 128 equal parts, and the remaining 51 may be obtained by using as many of the former bisections as the space will contain, so that the whole circumference will be divided into 179 equal parts.

Example 3. Let it be required to divide a circle into $29\frac{1}{2}$ equal parts, to represent the days of the moon's age.

Reduce the given number into halves, which gives 59 parts; seek the greatest number, not exceeding 59, which may be continually bisected to unity, which you will find to be 32. Subtract this from 59, the remainder is 27; and find, as before, the angle equal to the remainder by this proportion, as 59 is to 360, so is 27 to $164^\circ\ 44'\ 44''$; set off $164^\circ\ 44'\ 44''$, divide the remaining part of the circle by continual bisections, which will divide this portion into 32 parts, and from that, the rest into $27\frac{1}{2}$, making $29\frac{1}{2}$ parts, as required.

Example 4. Let it be required to divide a circle into $365^\circ\ 5'\ 49''$ equal parts, the length of a tropical year.

Reduce the whole into minutes, which will be 525949; then seek the greatest multiple of 1440, the minutes in a solar day, that may be halved, and is at the same time less than 525949; this you will find to be 368640, which subtracted from 525949, leaves 157309. To find the number of degrees that is to contain this number, use the following proportion; as 525949 is to 157309 multiplied by 360, so is 157309 to 107° 40′ 27″ 49‴.

Now set off an angle of 107° 40′ 27″ 49‴ upon the circle to be divided, and divide the remaining part of that circle by continual bisections, till you come to the number 1440, which in this case is unity, or one natural solar day; by which, dividing the arc of 107° 40′ 27″ 49‴, the whole circle will be divided into 365° 5′ 49″; for there will be 256 divisions, or days, in the greater arc, and 109° 5′ 49″ in the lesser arc.

Example 5. Let it be required to divide a circle into 365¼ equal parts; which is the quantity of a Julian year.

Reduce the whole into four parts, which gives us 1461; 1024 is the greatest multiple of 2, less than 1461; when subtracted from 1461, we have for a remainder 437. Then by the following proportion, as 1461 is to 437 × 360, so is 407 to 107° 40′ 46″ 49‴, the degrees to be occupied by this remainder.

Set off an angle of 107° 40′ 47″ upon the circle to be divided, and divide the remainder by continual bisections, until you arrive at unity, by which dividing the arc, you will have the whole circle divided into 365¼ parts.

PROBLEM 76. *To divide a quadrant, or circle, into degrees, fig.* 10, *plate* 8.

Let AB be the quadrant, C the center thereof. With the radius AC describe the two arcs AD, BE, and the quadrant will be divided into three equal parts, each equal to 30 degrees; then divide each of these into five parts, by the preceding rules, and the quadrant is divided into 15 equal parts; bisect these parts, and then subdivide as already directed, and the quadrant will be divided into 90 degrees. Other methods will be soon explained more at large.

PROBLEM 77. *To find what part any smaller line or arc, is of a greater, as for example, any angle is of a semicircle.*

Take the smaller with a pair of compasses, and with this opening step the greater. With the remainder, or surplus, step one of the former steps; with the remainder of this, step one the last steps, setting down the number of steps each time. About five times will measure angles to five seconds.

Then to find the fraction, expressing what part of the whole the smaller part is,

Suppose the number of steps each time to be
e. d. c. b. a.
9. 7. 8. 2. 5. Then $5 \times 2 + 1 = 11$, and $11 \times 8 + 5 = 93$, and $93 \times 7 + 11 = 662$, and $662 \times 9 + 93 = 6051$; so that $\frac{662}{6051}$ is the fraction required.

If we call the number of steps a b c d e beginning at the last, the rule may run thus: multiply a by b, and add 1; multiply that sum by c, and add a; multiply this sum by d, and add the preceding sum; multiply this sum by e, and add the preceding sum; then the two last sums are the terms of the fraction.

d. c. b. a.

Example 1. Suppose the steps are 3. 5. 1. 9. then $9 \times 1 + 1 = 10$, $10 \times 5 + 9 = 59$, and $59 \times 3 + 10 = 187$; hence the terms are $\frac{59}{187}$. Now $180^\circ \times 59 = 10620$, this divided by 187 gives 56°, with a remainder of 148, $148 \times 60 = 8880$, $\frac{8880}{187}$, gives 47, &c. so that the measure required is $56^\circ\ 47'\ 29''$.

Example 3. Take a semicircle three inches radius and let the angle be 2 in 1; then the steps will be 4, 2, 1, 2, 3, 2. the answer $41^\circ\ 10'\ 50''\ 9'''$.

The whole circle, continued and stepped with the same opening, gave 8. 1. 3. 2. 3. for the same, yet the answers agreed to the tenth of a second.

By the same method any given line may be measured, and the proportion it bears to any other strait line found. Or it will give the exact value of any strait line, ex. gr. the opening of a pair of compasses by stepping any known given line with it, *and this much nearer than the eye can discern, by comparing it with any other line, as a foot, a yard, &c.*

This method will be found more accurate than by scales, or even tables of sines, tangents, &c. because the measure of a chord cannot be so nicely determined by the eye with extreme exactness.

There may be some apparent difficulty attending the rule when put in practice, it being impossible to assign any example which another person can repeat with perfect accuracy, on account of the inequality in the scales, by which the same steps, or line, will be measured by different persons. There will, therefore, be always some small variation in the answer; it is however, demonstrably true, that the answer given by the problem is most accurately the measure of the given angle, although you can never delineate another angle, or line, exactly equal to the given one, first measured by way of example, and this arising from the inequality of our various scales, our inattention in measuring, and the imperfection of our eyes. Hence, though to all appearance two angles may appear perfectly equal to each other, this method will give the true measure of each, and assign the minutest difference between them.

Figure 1, *plate* 9, will illustrate clearly this method; thus, to measure the angle A C B, take A B between your compasses, and step B a, a b, b c, there will be c D over.

Take c D, and with it step A e, e f, and you will have f B over; with this opening step A g, g h, and you will have h e over, and so on.

PROBLEM 78. *To divide a large quadrant or circle.*

We shall here give the principal methods used by instrument-makers, before the publication of Mr. *Bird's* method by the Board of Longitude, leaving it to artists to judge of their respective merits, and to use them separately, or combine them together, as occasion may require; avoiding a minute detail of particulars, as that will be found when we come to describe Mr. *Bird's* method. It will be necessary, however, previously to mention a few circumstances, which, though in common use, had not been described until Mr. *Bird's* and Mr. *Ludlam's* comment thereon were published.

"In all mathematical instruments, divided by hand, and not by an engine, or pattern, the circles, or lines, which bound the divisions are not those which are actually divided by the compasses."

"A faint circle is drawn very near the bounding circle; it is this that is originally divided. It has been termed the *primitive circle.*"

"The divisions made upon this circle are faint arcs, struck with the beam compasses; fine points, or conical holes, are made by the prick punch, or pointing tool, at the points where these arcs cross the primitive circle; these are called *original points.*"

"The *visible divisions* are transferred from the original points to the space between the bounding circles, and are cut by the beam compasses; they are therefore always arcs of a circle, though so short, as not to be distinguished from strait lines."

Method 1. The faint or primitive arc is first struck; the exact measure of the radius thereof is then obtained upon a standard scale with a nonius division of 1000 parts of an inch, which if the radius exceed 10 inches, may be obtained to five places of figures. This measure is the chord of 60. The other chords necessary to be laid off are computed by the subjoined proportion,* and then taken off from the standard scale to be laid down on the quadrant.

Set off the chord of 60°, then add to it the chord of 30, and you obtain the 90th degree.

Mr. *Bird*, to obtain 90°, bisects the chord of 60°, and then sets off the same chord from 30 to 90°, and not of 30 from 60° to 90°. Some of the advantages that arise from this method are these; for whether the chord of 30 be taken accurately or not from the scale of equal parts, yet the arc of 60 will be truly bisected, (see remarks on bisection hereafter) and if the radius unaltered be set off from the point of bisection, it will give 90 true; but if the chord 30, as taken from the scale, be laid off from 60 to 90, then an error in that chord will make an equal error in the place of 90°.

Sixty degrees is divided into three parts by setting off the computed chord of 20 degrees, and the whole quadrant is divided to every 10 degrees, by setting off the same extent from the other points.

Thirty degrees, bisected by the computed chord of 15, gives 15°, which stepped from the points

* As the radius is to the given angle, so is the measure of the radius to half the required chord.

already found, divides the quadrant to every fifth degree.

The computed chord of 6° being laid off, divides 30 degrees into five parts; and set off from the other divisions, subdivides the quadrant into single degrees.

Thus with five extents of the beam compasses, and none of them less than six degrees, the quadrant is divided into 90 degrees.

Fifteen degrees bisected, gives 7° 30′, which set off from the other divisions, divides the quadrant into half degrees.

The chord of 6° 40′ divides 20° into three parts, and set off from the rest of the divisions, divides the whole instrument to every ten minutes.

The chord of 10° 5′ divides the degrees into 12 parts, each equal to five minutes of a degree.

Method 2. The chords are here supposed to be computed as before, and taken off from the nonius scale.

1. Radius bisected divides the quadrant into three parts, each equal to 30 degrees.

2. The chord of 10° gives nine parts, each equal to 10 degrees.

3. Thirty degrees bisected and set off, gives 18 parts, each equal to five degrees.

4. Thirty degrees into five, by the chord of 6°; then set off as before gives 90 parts, each equal to 1 degree.

5. The chord of 6° 40′ gives 270 parts, each equal to 20 minutes.

6. The chord of 7° 30′ gives 540 parts, each equal to 10 minutes.

7. The chord of 7° 45′ gives 1080 parts, each equal to five minutes.

Or *Method* 3. The computed chords supposed,

1. 60	into $\frac{1}{2}$	gives		3 parts, equal	30°
2. 30	$\frac{1}{2}$	equal	15	6 - - - - - - - -	15°
3. 15	3	equal	5	18 - - - - - - - -	5°
4. 20	5	equal	6	90 - - - - - - - -	1°
5. 20	3	equal	6° 40′	270 - - - - - - - -	20′
6. 15	$\frac{1}{2}$	equal	7° 30′	540 - - - - - - - -	10′
7. 15, 30	$\frac{1}{2}$	equal	7° 45′	1080 - - - - - - - -	5′

Thus may the practitioner vary his numbers for any division whatsoever, and yet preserve a sufficient extent between the points of his compasses.

If the quadrant be divided as above, to every 15 degrees, and then the computed arc of 16 degrees set off, this arc may be divided by continual bisection into single degrees. If from the arc of 45°, 2° 20′ be taken, or 1° 10′ from 22° 30′, we may obtain every fifth minute by continual bisection. If to the arc of 7° 10′ be added the arc of 62 minutes, the arc of every single minute may be had by bisection.

Of Mr. Bird's method of dividing.
Fig. 2, *plate* 9.

In 1767 the Commissioners of Longitude proposed an handsome reward to Mr. *Bird*, on condition, among other things, that he should publish an account of his method of dividing astronomical instruments; which was accordingly done: and a tract, describing his method of dividing, was written by him, and published by order of the Commissioners of Longitude in the same year; some defects in this publication were supplied by the Rev. Mr. *Ludlam*, one of the gentlemen who attended Mr. *Bird* to be instructed by him in his method of dividing, in consequence of the Board's agreement with him. Mr. *Ludlam's* tract was published in 1787, in 4to.

I shall use my endeavours to render this method still clearer to the practitioner, by combining and arranging the subject of both tracts.

Mr. Bird's method.

1. One of the first requisites is a scale of inches, each inch being subdivided into 10 equal parts.

2. Contiguous to this line of inches, there must be a nonius, in which 10.1 inches is divided into 100 equal parts, thus shewing the 1000ndth part of an inch. By the assistance of a magnifying glass of one inch focus, the 3000ndth part may be estimated.

3. Six beam compasses are necessary, furnished with magnifying glasses of not more than one inch focus. The longest beam is to measure the radius or chord of 60; the second for the chord of 42.40; the third for the chord of 30; the fourth for 10.20; the fifth for 4.40; the sixth for the chord of 15 degrees.

4. Compute the chords by the rules given, and take their computed length from the scale in the different beam compasses.

5. Let these operations be performed in the evening, and let the scale and the different beam compasses be laid upon the instrument to be divided, and remain there till the next morning.

6. The next morning, before sun-rise, examine the compasses by the scale, and rectify them, if they are either lengthened or shortened by any change in the temperature of the air.

7. The quadrant and scale being of the same temperature, describe the faint arc *b d*, or primitive circle; then with the compasses that are set

to the radius and with a fine prick punch, make a point at *a*, which is to be the o point of the quadrant; see *fig*. 2, *plate* 9.

8. With the same beam compasses unaltered lay off from a to c the chord of 60°, making a fine point at c.

9. Bisect the arc a c with the chord of 30°.

10. Then from the point c, with the beam compasses containing 60, mark the point r, which is that of 90 degrees.

11. Next, with the beam compasses containing 15°, bisect the arc c r in n, which gives 75°.

12. Lay off from n towards r the chord of 10° 20′, and from r towards n the chord of 4° 40′; these two ought to meet exactly at the point g of 85° 20′.

13. Now as in large instruments each degree is generally subdivided into 12 equal parts, of five minutes each, we shall find that 85° 20′ contains 10.24 such parts, because 20′ equal 4 of these parts, and 85 × 12 makes 1020; now 1024 is a number divisible by continual bisection.

The last computed chord was 42° 40′, with which a g was bisected in o, and a o, o g, were bisected by trials. Though Mr. *Bird* seems to have used this method himself, still he thinks it more adviseable to take the computed chord of 21° 20′, and by it find the point g; then proceed by continual bisections till you have 1024 parts. Thus the arc 85° 20′, by ten bisections, will give us the arcs 42° 40′, 21° 80′, 10° 40′, 5° 20′, 2° 40′, 1° 20′, 40′, 20′, 10′, 5′.

14. To fill up the space between g and r, 85° 20′, and 90°, which is 4° 40′, or 4 × 12 + 8 equal to 56 divisions; the chord of 64 divisions was laid off from g towards d, and divided like the rest by continual bisections, as was also from a towards b.

If the work is well performed, you will again find the points 30, 45, 60, 75, and 90, without any sensible difference. It is evident that these arcs, as well as those of 15°, are multiples of the arc of 5′; for one degree contains 12 arcs of 5′ each, of which 15° contains 180; the arc of 30° contains 360; the arc of 60°, 720; that of 75°, 900; and, therefore, 90° contains 1080.

Mr. *Graham*, in 1725 applied to the quadrant divided into 90°, or rather into 1080 parts of five minutes each, another quadrant, which he divided into 96 equal parts, subdividing each of these into 16 equal parts, forming in all 1536. This arc is a severe check upon the divisions of the other; but *Bird* says, that if his instructions be strictly followed, the coincidence between them will be surprising, and their difference from the truth exceedingly small.

The arc of 96° is to be divided first into three equal parts, in the same manner as the arc of 90°; each third contains 512 divisions, which number is divisible continually by 2, and gives 16 in each 96th part of the whole.

The next step is to cut the linear divisions from the points obtained by the foregoing rules. For this purpose a pair of beam compasses is to be used, both of whose points are conical and very sharp. Draw a tangent to the arc b d, suppose at e, it will intersect the arc x y in q, this will be the distance between the points of the beam compasses to cut the divisions nearly at right angles to the arc. The point of the beam compasses next the right hand is to be placed in the point r, the other point to fall freely into the arc x y; then pressing gently upon the screw head which fastens the socket, cut the divisions with the point towards the right hand, proceeding thus till you have finished all the divisions of the limb.

FOR THE NONIUS.

1. Chuse any part of the arc where there is a coincidence of the 90 and 96 arcs; for example, at c, the point of 60°. Draw the faint arc s t and i k, which may be continued to any length towards A; upon these the nonius divisions are to be divided in points. The original points for the nonius of the 90th arc are to be made upon the arc s t; the original points for the nonius of the 96th arc are to be made upon the arc i k.

Because 90 is to 96, as 15 to 16, there will be a coincidence at 15° and 16pts, 30° and 32pts, 45° and 48pts; 60° and 64pts, 90° and 96pts.

2. Draw a tangent line to the primitive circle as before, intersecting the arc d, which gives the distance of the points of the beam compasses, with which the nonius of the 90th arc must be cut.

3. Let us suppose then that the nonius is to subdivide the divisions of the limb to half a minute, which is effected by making 10 divisions of the nonius equal to 11 divisions of the limb; measure the radius of the arc, and compute the chord of 16, or rather 32 of the nonius division, which may easily be obtained by the following proportion; if 10 divisions of the nonius plate make 55 minutes of a degree, what will 32 of those divisions make? the answer is 2° 56′, the chord of which must be computed and taken from the scale of equal parts; but as different subdivisions by the nonius may be required, let *n* be the number of nonius divisions, *m* the number of minutes taken in by the nonius b, 16, 32 or 64, and X the arc sought; then as n : m :: b : X.

4. Lay off with the beam compasses, having the length of the tangent 90 between the points, the point q from e, q being a point in the arc s t, and e an original point in the primitive circle, and the chord of 32 from q towards the left hand, (the chord of 32 being the chord which subtends 32 divisions on the nonius plate, or the chord of 2° 56′; this chord to be computed from the radius with which the faint arc s t was struck, and taken off the scale of equal parts,) and divide by continual bisections; ten of those divisions, counting from q to the left, will be the required points.

The nonius belonging to the 96th arc is subject to no difficulty, as the number should always be 16, 32, &c. that the extremes may be laid off from the divisions of the limb without computation. To be more particular, the length of the tangent line, or radius, with which the divisions of the nonius of the 96 arc are to be cut, must be found in the way before directed for the nonius of the 90 arc; the ark i k standing instead of the arc s t. Having the tangental distance between the points of the compasses from one of the original points in the primitive 96, lay off a point on the faint arc i k towards the left hand; count from that point on the primitive 96 circle 17 points to the left hand, and lay off from thence another point on the faint arc i k; the distance between those two points in the faint arc i k is to be subdivided by bisections into 16 parts, and those parts pointed; from these points the visible divisions of the nonius are to be cut.

Mr. *Ludlam* thinks, that instead of laying the extent of the nonius single, it would be better to lay it off double or quadruple; thus, instead of 17 points to the left hand, count 34 or 68, and that both ways, to the right and to the left, and lay off a point from each extreme on the faint arc i k; subdivide the whole between these extreme points by continual bisections, till you get 16 points together on the left hand side of the middle, answering to the extent of the nonius. No more of the subdivisions are to be completed than are necessary to obtain the middle portion of 16 points as before.

The nonius points obtained, *the next process* is to transfer them on the nonius plate, which plate is chamfered on both edges; on the inner edge is the nonius for the 90 arc, on the outer edge is the nonius for the 96 arc, in the middle between the two chamfers is a flat part parallel to the under surface of the nonius plate; upon the flat part the faint line of the next operation, is to be drawn. *To find the place where the nonius is to begin upon the chamfered edge of the nonius plate*, measure the distance of the center of the quadrant from the axis of the telescope; this distance from the axis of the telescope at the eye end, will be the place for the first division of the nonius; then draw a faint line from the center on the flat part of the nonius plate.

Fasten the nonius plate to the arc with two pair of hand vices; then with one point of the beam compasses in the center of the quadrant, and the other at the middle of the nonius plate, draw a faint arc from end to end; where this arc cuts the faint line before-mentioned, make a fine point; from this point lay off on each side another point, which may be at any distance in the arc, only care must be taken that they be equally distant from the middle point; from the two last make a faint intersection as near as possible to either of the chamfered edges of the nonius plate; through this intersection the first division of the nonius must be cut.

Mr. Bird's method of dividing his scale of equal parts.

Let us suppose that we have 90 inches to divide into 900 equal parts, take the third of this number, or 300; now, the first power of 2 above this is 512; therefore, take $\frac{512}{10}$ths of an inch in one pair of beam compasses, $\frac{256}{10}$ in another, $\frac{128}{10}$ in a third, and $\frac{64}{10}$ in a fourth; then lay the scale from which these measures were taken, the scale to be divided, and the beam compasses near together, in a room facing the north; let them lie there the whole night; the next morning correct your compasses, and lay off $\frac{512}{10}$ three times; then with the compasses $\frac{256}{10}$, $\frac{128}{10}$, $\frac{64}{10}$, bisect these three spaces as expeditiously as possible; the space 64 is so small that there is no danger from any partial or unequal expansion, therefore the remainder may be finished by continual bisections. The linear divisions are to be cut from the points with the beam compasses as before described.

The nonius of this scale is $\frac{101}{10}$ths of an inch long, which is to be divided into 100 equal parts, as 100 is to 101, so is 256 : 258,56 tenths of an inch, the integer being $\frac{1}{10}$. Suppose the scale to be numbered at every inch from left to right; then make a fine point exactly against $\frac{1}{10}$, to the left of o, from this lay off 258,56 to the right hand, which divide after the common method.

If you are not furnished with a scale long enough to lay off 258,56, then set off $\frac{250}{10}$, and add 8,56, from a diagonal scale.

Of Mr. Bird's pointing tool, and method of pointing.

The pointing tool consisted of a steel wire $\frac{1}{10}$ inch diameter, inserted into a brass wire $\frac{1}{6}$ inch diameter, the brass part $2\frac{1}{2}$ long, the steel part stood out $\frac{3}{4}$, whole length $3\frac{1}{4}$ inches. The angle of the conical point about 20 or 25 degrees, somewhat above a steel temper; the top of the brass part was rounded off, to receive the pressure of the finger; the steel point should be first turned, hardened, and tempered, and then whetted on the oil-stone, by turning the pointril round, and at the same time drawing it along the oil-stone, not against, but from the point; this will make a sharp point, and also a kind of very fine teeth along the slant side of the cone, and give it the nature of a very fine countersink.

In striking the primitive circle by the beam compasses, the cutting point raises up the metal a little on each side the arc; the metal so thrown up forms what is called the bur. When an arc is struck across the primitive circle, this bur will be in some measure thrown down; but if that circle be struck again ever so lightly, the bur will be raised up again; the arcs struck across the primitive circle have also their bur. Two such rasures or trenches across each other, will of course have four salient, or prominent angles within; and as the sides of the trenches slope, so do also the lines which terminate the four solid angles. You may therefore, *feel* what you cannot *see*; when the conical points bear against all four solid angles, they will guide, and keep the point of the tool in the center of decussation, while keeping the tool upright, pressing it gently with one hand, and turning it round with the other, you make a conical hole, into which you can at any time put the point of the beam compasses, and feel, as well as see, when it is lodged there.

RULES OR MAXIMS LAID DOWN BY MR. BIRD.

1. The points of the beam compasses should never be brought nearer together than two or three inches, except near the end of the line or arc to be divided; and there spring dividers with round moveable points had best be used.

2. The prick-punch, used to mark the points, should be very sharp and round, the conical point being formed to a very acute angle; the point to be made by it ought not to exceed the one thousandth of an inch. When lines, or divisions, are to be traced from these points, a magnifying glass of ½ an inch focus must be used, which will render the impression or scratch made by the beam compasses sufficiently visible; and if the impression be not too faint, feeling will contribute, as well as seeing, towards making the points properly.

3. The method of finding the principal points by computing the chords, is preferable to other methods; as by taking up much less time, there is much less risk of any error from the expansion of the instrument, or beam compasses.

4. To avoid all possible error from expansion, Mr. *Bird* never admitted more than one person, and him only as an assistant; nor suffered any fire in the room, till the principal points were laid down.

Mr. *Bird* guards, by this method, against any inequality that might possibly happen among the original points, by first setting out a few capital points, distributed equally through the arc, leaving the intervals to be filled up afterwards; he could by this method check the distant divisions with respect to each other, and shorten the time of the most essential operations.

5. Great care is to be observed in pointing intersections, which is more difficult than in pointing from a single line, made by one point of the compasses. For in bisections, the place to be pointed is laid off from the right to the left, and from the left to the right. If any error arises from an alteration of the compasses, it will be shewn double; even if the chord be taken a little too long, or too short, it will not occasion any inequality, provided the point be made in the middle, between the two short lines traced by the compasses.

Now, as Mr. *Ludlam* observes, if the bisecting chord be taken exactly, the two fore-mentioned faint arcs will intersect each other in the primitive circle, otherwise the intersection will fall above or below it. In either case, the eye, assisted by a magnifier, can accurately distinguish on the primitive circle the middle between these two arcs, and a point may be made by the pointing tool.

In small portions of the primitive circles, the two faint arcs will intersect in so acute an angle, that they will run into one another, and form as it were a single line; yet even here, though the bisecting chord be not exact, if the intersection be pointed as before, the point will fall in the middle of the portion to be bisected.

If, in the course of bisecting, you meet with a hole already made with the pointril, the point of the compasses should fall exactly into that hole, both from the right and left hand, and you may readily feel what you cannot see, whether it fit or no; if it fits, the point of the compasses will have a firm bearing against the bottom of the conical hole, and strike a solid blow against it; if it does not exactly coincide with the center of the hole, the slant part of the point will slide down the slant side of the hole, drawing, or pushing the other point of the compasses from its place.

Mr. *Bird*'s method of transferring the divisions by the beam compasses from the original points, is founded on this maxim, *that a right line cannot be cut upon brass, so as accurately to pass through two given points*; but that a circle may be described from any center to pass with accuracy through a given point. It is exceeding difficult, in the first place, to fix the rule accurately, and keep it firmly to the two points; and, secondly, supposing it could be held properly, yet, as the very point of the knife which enters the metal and ploughs it out, cannot bear against the rule, but some other part above that point will bear against it; it follows, that if the knife be held in a different situation to or from the rule, it will throw the cutting point out or in; besides, any hardness or inequality of the metal will turn the knife out of its course, for the rule does not oppose the knife in departing from it, and the force of the hand cannot hold it to it. For these reasons it is almost impossible to draw a knife a second time against the rule, and cut within the same line as before.

On the other hand, an arc of a circle may always be described by the beam compasses so as to pass through a given point, provided both points of the compasses be conical. Let one point of the compasses be set in the given point or conical hole in the brass plain; make the other point, whatever be its distance, the central, or still point; with the former point cut the arc, and it will be sure to pass through the given point in the brass plain, and the operation may be repeated safely, and the stroke be strengthened by degrees, as the moving point is not likely to be shifted out of its direction, nor the cutting point to be broken.

The visible divisions on a large quadrant are always the arcs of a circle, though so short as not to be distinguished from strait lines; they should be perpendicular to the arc that bounds them, and therefore the still or central point of the beam compasses must be somewhere in the tangent to that arc; the bounding circle of the visible divisions, and the primitive circle should be very near each other, that the arc forming the visible divisions may be as to sense perpendicular to both circles, and each visible division shew the original point from which it was cut.

Another maxim of Mr. *Bird*'s, attributed to Mr. *Graham*, *That it is possible practically to bisect an arc, or right line, but not to trisect, quinquisect, &c.* The advantages to be obtained by bisection have been already seen; we have now to shew the objections against trisecting, &c.

1. That as the points of trisection in the primitive circle must be made by pressing the point of the beam compasses down into the metal, the least extuberance, or hard particle, will cause a deviation in the first impression of a taper point, and force the point of the compasses out of its place; when a point is made by the pointing tool, the tool is kept turning round while it is pressed down, and therefore drills a conical hole.

2. Much less force is necessary to make a scratch or faint arc, than a hole by a pressure downwards of the point of the compasses.

3. So much time must be spent in trials, that a partial expansion would probably take place; and, perhaps, many false marks, or holes made, which might occasion considerable error.

Another maxim of Mr. *Bird*'s was this, that *stepping* was liable to great uncertainties, and not to be trusted; that is, if the chord of 16° was assumed, and laid down five times in succession, by turning the compasses over upon the primitive circle, yet the arcs so marked would not, in his opinion be equal.

DESCRIPTION OF MR. BIRD'S SCALE OF EQUAL PARTS.

It consists of a scale of inches, each divided into tenths, and numbered at every inch from the left to the right, thus, 0, 1, 2, 3, &c. in the order of the natural numbers. The nonius scale is below this, but contiguous to it, so that one common line terminates the bottoms of the divisions on the scale of inches, and the tops of the divisions on the nonius; this nonius scale contains in length 101 tenths of an inch, this length is divided into 100 equal parts, or visible divisions; the left hand division of this scale is set off from a point $\frac{1}{10}$ of an inch to the left of 0, on the scale of inches; therefore, the right hand end of the scale reaches to, and coincides with the 10th inch on the scale of inches. Every tenth division on this nonius scale is figured from the right to the left, thus, 100. 90. 80. 70. 60. 50. 40. 30. 20. 10. 0. and thus 0 on the nonius coincides with 10 on the inches; and 100 on the nonius falls against the first subdivision (of tenths) to the left hand of 0 on the inches; and these two, viz. the first and last, are the only two strokes that do coincide in the two scales.

To take off any given number of inches, decimals, and millesimals of an inch; for example, 42,764, observe, that one point of the beam compass must stand in a (pointed) division on the nonius, and the other point of the compasses in a pointed division on the scale of inches.

The left hand point of the compasses must stand in that division on the nonius which expresses the number of millesimal parts; this, in our example, is 64.

To find where the other point must stand in the scale of inches and tenths, add 10 to the given number of inches and tenths (exclusive of the two millesimal figures;) from this sum subtract the two millesimal figures, considered now as units and tenths, and the remainder will shew in what division, on the scale of inches, the other point of the compasses must stand; thus, in our example, add 10 to 42,7, and the sum is 52,7; from this subtract 6,4 and the remainder is 46,3. Set then one point of the compasses in the 64th division on the nonius, and the other point in 46,3 on the scale of inches, and the two points will comprehend between them 42,764 inches.

This will be plain, if we consider that the junction of the two scales is at the 10th inch on the scale of inches; therefore, the compasses will comprehend 36,3 inches on the scale of inches; but it will likewise comprehend 64 divisions on the nonius scale. Each of these divisions is one tenth and one millesimal part of an inch; therefore, 64 divisions is 64 tenths, and 64 millesimal parts, or 6,464 inches; to 6,464 inches taken on the nonius, add 36,3 inches taken on the scale of inches, and the whole length is 42,764 inches; and thus the whole is taken from two scales, viz. inches and the nonius; each subdivision in the former is $\frac{1}{10}$ of an inch, each subdivision in the latter is $\frac{1}{10}+\frac{1}{1000}$th of an inch. By taking a proper number of each sort of subdivisions (the lesser and the greater) the length sought is obtained.

The business of taking a given length will be expedited, and carried on with far less danger of injuring the scale, if the proposed length be first of all taken, *nearly*, on the scale of inches only, guessing the millesimal parts; thus, in our case, we ought to take off from the scale of inches 42,7, and above half a tenth more; for then if one point be set in the proper division on the nonius, the other point will fall so near the proper division on the scale of inches, as to point it out; and the point of the compasses may be brought, by the regulating screw, to fall exactly into the true division.

If, when the points of the compasses are set, they do not comprehend an integral number of millesimal parts, they will not precisely fall into any two divisions, but will either exceed, or fall short; let the exact distance of the points of the compasses be 42,7645; if, as before, the left hand point be set in the 64th division of the nonius, then the right hand point will exceed 46,3 among the inches; if the left hand point be carried one division more to the left, and stand in 65 of the nonius, then the right hand point will fall short of 46,2 in the scale of inches; the excess in the former case being equal to the defect in the latter. By observing whether the difference be equal, or as great again in one case as the other, we may estimate to $\frac{1}{3}$d part of a millesimal. See Mr. *Bird*'s Tract, p. 2.

It may be asked, why should the nonius scale commence at the 10th inch; why not at 0, and so the nonius scale lay wholly on the left hand of the scale of inches? and, in this case, both scales might be in one right line, and not one under the other; but, in such a case, a less distance than 10 inches could not always be found upon the scale, as appears from the rule before given. The number 10 must not, in this case, be added to the inches and tenths, and then the subtraction before directed would not always be possible.

Yet, upon this principle, a scale in one continued line may be constructed for laying off inches, tenths, and hundredths of an inch, for any length above one inch; at the head of the scale of inches, to the left hand of 0, and in the same line, set off eleven tenths of an inch (or the multiple,) which subdivide in Mr. *Bird's* way, into ten equal parts. Such a compound scale would be far more exact than the common diagonal scale; for the divisions being pointed, you may feel far more nicely than you can see, when the points of the beam compasses are set to the exact distance. But to return to Mr. *Bird's* Tract.

The nature of Mr. *Bird's* scale being known, there will be no difficulty in understanding his directions how to divide it. A scale of this kind is far preferable to any diagonal scale; not only on account of the extreme difficulty of drawing the diagonals exactly, but also because there is no check upon the errors in that scale; here the uniform manner in which the strokes of one scale separate from those of the other, is some evidence of the truth of both; but Mr. *Bird's* method of assuming a much longer line than what is absolutely necessary for the scale, subdividing the whole by a continual bisection, and pointing the divisions as before explained, and guarding against partial expansions of the metal, is sure to render the divisions perfectly equal. The want of such a scale of equal parts (owing, perhaps, to their ignorance of constructing it) is one reason why Mr. *Bird's* method of dividing is not in so great estimation among mathematical instrument makers, as it justly deserves.

AN OBSERVATION, OR METHOD OF GRADUATION, OF MR. SMEATON'S.

As it is my intention to collect in this place whatever is valuable on this subject, I cannot refrain from inserting the following remark of Mr. *Smeaton*'s, though it militates strongly against one of Mr. *Bird*'s maxims. He advises us to compute from the measured radius the chord of 16 degrees only, and to take it from an excellent plain scale, and lay it off five times in succession from the primary point of 0 given, this would give 80 degrees; then to bisect each of these arcs, and to lay off one of them beyond the 80th, which would give the 88th degree; then proceed by bisection, till you come to an arc of two degrees, which laid off from the 88th degree, will give the 90 degrees; then proceed again by bisection, till you have reduced the degrees into quarters, or every fifteen minutes. Here Mr. *Smeaton* would stop, being apprehensive that divisions, when over close, cannot be accurately obtained even by bisection.

If it were necessary to have subdivisions upon the limb equivalent to five minutes, he advises us to compute the chord of 21° 20′ only, and to lay it off four times from the primary point; the last would give 85° 20′, and then to supply the remainder from the bisected divisions as they rise, *not from other computed chords*.

Mr. *Bird* asserts, that after he had proceeded by the bisections from the arc of 85° 20′, the several points of 30. 60. 75. 60. *fell in without sensible inequality*, and so indeed they might, though they were not equally true in their places; for whatever error was in them would be communicated to all connected with, or taking their departure from them. Every heterogeneous mixture should be avoided.

It is not the same thing whether you *twice* take a measure as *nearly* as you can, and lay it off separately, or lay off two openings of the compasses in succession *unaltered*; for though the same opening, carefully taken off from the same scale a second time, will doubtless fall into the holes made by the first, without sensible error; yet, as the sloping sides of the conical cavities made by the first points, will conduct the points themselves to the center, there may be an error, which, though insensible to the sight, would have been avoided by the more simple process of laying off the opening twice, without altering the compasses.

As the whole of the 90 arc may now be divided by bisection, it is equally unexceptionable with the 96 arc; and, consequently, if another arc of 90, upon a different radius, was laid down, they would be real checks upon each other.

Mr. *Ramsden*, in laying down the original divisions on his dividing engine, divided his circle first into five parts, and each of these into three; these parts were then bisected four times; but being apprehensive some error might arise from quinquisection, and trisection, in order to examine the accuracy of the divisions, he described another circle $\frac{1}{10}$ inch within the former, by continual bisections, but found no sensible difference between the two sets of divisions. It appears also, that Mr. *Bird*, notwithstanding all his objections to, and declamations against the practice of stepping, sometimes used it himself.

OF THE NONIUS DIVISIONS.

It will be necessary to give the young practitioner some account of the nature and use of that admirable contrivance commonly called a nonius, by which the divisions on the limbs of instruments are subdivided.

The nonius depends on this simple circumstance, that if any line be divided into equal parts, the length of each part will be greater, the fewer divisions there are in the original; on the contrary, the length of each division will be less in proportion, as the divisions are more numerous.

Thus, let us suppose the limb of Hadley's quadrant divided to every 20 minutes, which are the smallest divisions on the quadrant; the two extreme strokes on the nonius contain seven degrees, or 21 of the afore-mentioned small divisions, but that it is divided only into 20 parts; each of these parts will be longer than those on the arc, in the proportion of 21 to 20; that is to say, they will be one-twentieth part, or one minute longer than the divisions on the arc; consequently, if the first, or index division of the nonius, be set precisely opposite to any degree, the relative position of the nonius and the arc must be altered one minute before the next division on the nonius will coincide with the next division on the arc, the second division will require a change of two minutes; the third, of three minutes, and so on, till the 20th stroke on the nonius arrive at the next 20 minutes on the arc; the index division will then have moved exactly 20 minutes from the division whence it set out, and the intermediate divisions of each minute have been regularly pointed out by the divisions of the nonius.

To render this still plainer, we must observe that the index, or counting division of the nonius, is distinguished by the mark 0, which is placed on the extreme right hand division; the numbers running regularly on thus, 20, 15, 10, 5, 0.

The index division points out the entire degrees and odd 20 minutes, subtended by the objects observed; but the intermediate divisions are shewn by the other strokes of the nonius; thus, look among the strokes of the nonius for one that stands directly opposite to, or perfectly coincident with some one division on the limb; this division reckoned on the nonius, shews the number of minutes to be added to what is pointed out by the index division.

To illustrate this subject, let us suppose two cases. The first, when the index division perfectly coincides with a division on the limb of the quadrant: here there is no difficulty, for at whatsoever division it is, that division indicates the required angle. If the index divisions stand at 40 degrees, 40 degrees is the measure of the required angle. If it coincide with the next division beyond 40 on the right hand, 40 degrees 20 minutes is the angle. If with the second division beyond 40, then 40 degrees 40 minutes is the angle, and so in every other instance.

The second case is, when the index line does not coincide with any division on the limb. We are, in this instance, to look for a division on the nonius that shall stand directly opposite to one on the limb, and that division gives us the odd minutes, to be added to those pointed out by the index division: thus, suppose the index division does not coincide with 40 degrees, but that the next division to it is the first coincident division, then is the required angle 40 degrees 1 minute.

If it had been the second division, the angle would have been 40 degrees 2 minutes, and so on to 20 minutes, when the index division coincides with the first 20 minutes from 40 degrees. Again, let us suppose the index division to stand between 30 degrees, and 30 degrees 20 minutes, and that the 16th division on the nonius coincides exactly with a division on the limb, then the angle is 30 degrees 16 minutes. Further, let the index division stand between 35 degrees 20 minutes, and 35 degrees 40 minutes, and at the same time the 12th division on the nonius stand directly opposite to a division on the arc, then the angle will be 35 degrees 32 minutes.

A GENERAL RULE FOR KNOWING THE VALUE OF EACH DIVISION, ON ANY NONIUS WHATSOEVER.

1. Find the value of each of the divisions, or subdivisions, of the limb to which the nonius is applied. 2. Divide the quantity of minutes or seconds thus found, by the number of divisions on the nonius, and the quotient will give the value of the nonius division.

Thus, suppose each subdivision of the limb be 30 minutes, and that the nonius has 15 divisions, then $\frac{30}{15}$ gives two minutes for the value of the nonius. If the nonius has 10 divisions, it would give three minutes; if the limb be divided to every 12 minutes, and the nonius to 24 parts, then 12 minutes, or 720 seconds divided by 24, gives 30 seconds for the required value.

OF INSTRUMENTS FOR DESCRIBING CIRCLES OF EVERY POSSIBLE MAGNITUDE.

As there are many cases where arcs are required to be drawn of a radius too large for any ordinary compasses, Mr. *Heywood* and myself contrived several instruments for this purpose; the most perfect of these is delineated at *fig*. 5, *plate* 11. It is an instrument that must give great satisfaction to every one who uses it, as it is so extensive in its nature, being capable of describing arcs from an infinite radius, or a strait line, to those of two or three inches diameter. When it was first contrived, both Mr. *Heywood* and myself were ignorant of what had been done by that ever to be celebrated mechanician, Dr. *Hooke*.

Since the invention thereof, I have received some very valuable communications from different gentlemen, who saw and admired the simplicity of its construction; among others, from Mr. *Nicholson*, author of several very valuable works; Dr. *Rotherham*, Earl *Stanhope*, and *J. Priestley*, Esq. of Bradford, Yorkshire; the last gentleman has favoured me with so complete an investigation of the subject, and a description of so many admirable contrivances to answer the purpose of the artist, that any thing I could say would be altogether superfluous; I shall, therefore, be very brief in my description of the instrument, represented *fig*. 5, *plate* 11, that I may not keep the reader from Mr. *Priestley*'s valuable essay, subjoining Dr. *Hooke*'s account of his own contrivance to that of ours. Much is always to be gained from an attention to this great man; and I am sure my reader will think his time well employed in perusing the short extract I shall here insert.

The branches A and B, *fig.* 5, *plate* 11, carry two independent equal wheels C, D. The pencil, or point E, is in a line drawn between the center of the axis of the branches, and equidistant from each; a weight is to be placed over the pencil when in use. When all the wheels have their axes in one line, and the instrument is moved in rotation, it will describe an infinitely small circle; in this case the instrument will overset. When the two wheels C, D, have their horizontal axes parallel to each other, a right line, or infinitely large circle will be described; when these axes are inclined to each other, a circle of finite magnitude will be described.

The distance between one axis and the center, (or pencil,) being taken as unity, or the common radius, the numbers 1, 2, 3, 4, &c. being sought for in the natural tangents, will give arcs of inclination for setting the nonii, and at which circles of the radii of the said numbers, multiplied into the common radius, will be described.

			° ′
	0.1		5.43
	0.2		11.19
	0.3		16.42
	0.4		21.48
	0.5		26.34
	0.6	is the radius of a	30.58
The common	0.7	circle made by	35.
radius	0.8	the rollers when	38.40
multiplied	0.9	inclined at these	41.59
by	1.0	angles:	45.
	2.0		63.26
	3.0		71.34
	4.0		75.58
	5.0		78.42
	6.0		80.32

			° ′
The common radius multiplied by	7.0	is the radius of a circle made by the rollers when inclined at these angles:	81.52
	8.0		82.53
	9.0		83.40
	10.0		84.17
	20.0		87. 8
	30.0		88. 5
	40.0		88.34

Extracts from Dr. *Hooke, on the Difficulty, &c. of Drawing Arcs of Great Circles.* "This thing, says he, is so difficult, that it is almost impossible, especially where exactness is required, as I was sufficiently satisfied by the difficulties that occurred in striking a part of the arc of a circle of 60 feet for the radius, for the gage of a tool for grinding telescope glasses of that length; whereby it was found, that the beam compasses made with all care and circumspection imaginable, and used with as great care, would not perform the operation; nor by the way, an angular compass, such as described by *Guido Ubaldus*, by *Clavius*, and by *Blagrave*, &c.

"The Royal Society met; I discoursed of my instrument to draw a great circle, and produced an instrument I had provided for that purpose; and therewith, by the direction of a wire about 100 feet long, I shewed how to draw a circle of that radius, which gave great satisfaction, &c. Again, at the last meeting I endeavoured to explain the difficulties there are in making considerable discoveries either in nature or art; and yet, when they are discovered, they often seem so obvious and plain, that it seems more difficult to give a reason why they were not sooner discovered, than how they came to be detected now: how easy it was, we now think, to find out a method of printing letters, and yet, except what may have happened in *China*, there is no specimen or history of any thing of that kind done in this part of the world. How obvious was the vibration of pendulous bodies? and yet, we do not find that it was made use of to divide the spaces of time, till *Galileo* discovered its isochronous motion, and thought of that proper motion for it, &c. And though it may be difficult enough to find a way before it be shewn, every one will be ready enough to say when done, that it is easy to do, and was obvious to be thought of and invented."

To illustrate this, the Doctor produced an instrument somewhat similar to that described, *fig.* 5, *plate* 11, as appears from the journal of the Royal Society, where it is said, that Dr. *Hooke* produced an instrument capable of describing very large circles, by the help of two rolling circles, or truckles in the two ends of a rule, made so as to be turned in their sockets to any assigned angle. In another place he had extended his views relative to this instrument, that he had contrived it to draw the arc of a circle to a center at a considerable distance, where the center cannot be approached, as from the top of a pole set up in the midst of a wood, or from the spindle of a vane at the top of a tower, or from a point on the other side of a river; in all which cases the center cannot be conveniently approached, otherwise than by the sight. This he performed by two telescopes, so placed at the truckles, as thereby to see through both of them the given center, and by thus directing them to the center, to set the truckles to their true inclination, so as to describe by their motion, any part of such a circle as shall be desired.

METHODS OF DESCRIBING ARCS OF CIRCLES OF LARGE MAGNITUDE. BY J. PRIESTLEY, ESQ. OF BRADFORD, YORKSHIRE.

In the projection of the sphere, perspective and architecture, as well as in many other branches of practical mathematics, it is often required to draw arcs of circles, whose radii are too great to admit the use of common, or even beam compasses; and to draw lines tending to a given point, whose situation is too distant to be brought upon the plan. The following essay is intended to furnish some methods, and describe a few instruments that may assist the artist in the performance of both these problems.

OF FINDING POINTS IN, AND DESCRIBING ARCS OF LARGE CIRCLES.

The methods and instruments I shall propose for this purpose, will chiefly depend on the following propositions, which I shall premise as principles.

Principle 1. The angles in the same segment of a circle, are equal one to another.

Let A C D B, *fig*. 1, *plate* 10, be the segment of a circle; the angles formed by lines drawn from the extremities A and B, of the base of the segment, to any points C and D in its arc, as the angles A C B, A D B, are equal.

This is the 31st proposition of *Euclid*'s third book of the Elements of Geometry.

Principle 2. If upon the ends A B, *fig*. 2, *plate* 10, of a right line A B as an axis, two circles or rollers C D and E F be firmly fixed, so that the said line shall pass through the centers, and at right angles to the plains of the circles; and the whole be suffered to roll upon a plain without sliding :

1. If the rollers C D and E F be equal in diameter, the lines, described upon the plain by their circumferences, will be parallel right lines; and the axis AB, and every line DF, drawn between contemporary points of contact of the rollers and plain, will be parallel among themselves.

2. If the rollers CD and EF be unequal, then lines formed by their circumferences upon the plain will be concentric circles; and the axis AB, and also the lines D F, will, in every situation, tend to the common center of those circles.

Principle 3. If there be two equal circles or rollers A and B, *fig.* 3, *plate* 10, each separately fixed to its own axis, moveable on pivots; and these axes placed in a proper frame, so as to be in the same plain, and to maintain the situation given them with respect to each other; and if the apparatus be rolled upon a plain without sliding:

1. If the axes C D and E F, be placed in a parallel situation, the circumferences of the rollers A and B will trace upon the plain strait lines; which will be at right angles to the axes C D and E F.

2. If the axes C D and E F, continuing as before in the same plain, be inclined to each other, so as if produced to meet in some point G, the rollers A and B will describe in their motions upon the plain arcs of the same, or of concentric circles, whose center is a point H, in that plain perpendicularly under the point of intersection G of the two axes.

I shall not stop to demonstrate the truth of the two last principles, it will easily appear on seeing the operations performed.

OF THE SIMPLE BEVEL.

In the performance of some of the following problems, an instrument not unlike *fig.* 4, *plate* 10, will be found useful. It consists of two rulers, moveable on a common center, like a carpenter's rule, with a contrivance to keep them fixed at any required angle. The center C must move on a very fine axis, so as to lie in a line with the fiducial edges C B, C D of the rulers, and project as little as possible before them. The fiducial edges of the legs represent the sides of any given angle, and their intersection or center C its angular point.

A more complete instrument of this kind, adapted to various uses, will be described hereafter.

N. B. A pin fixed in the lower rule, passes through a semicircular groove in the upper, and has a nut A which screws upon it, in order to fix the rulers or legs, when placed at the desired angle.

PROBLEM 1. *Given the three points* A, B *and* C, *supposed to be in the circumference of a circle too large to be described by a pair of compasses; to find any number of other points in that circumference.*

This may be performed various ways. As for example, *fig.* 5, *plate* 10.

1. Join AC, which bisect with the line F M G at right angles; from B, draw BD parallel to AC, cutting FG in E; and making E D=E B D will be a point in the same circumference, in which are A, B and C.

By joining AB, and bisecting it at right angles with I K; and from C drawing C a parallel to AB, cutting I K in L, and making L a=L C, a will be another of the required points.

Continuing to draw from the point, last found, lines alternately parallel to AC and AB; those lines will be cut at right angles by F G and I K respectively; and by making the parts equal on each side of F G and I K, they become chords of the circle, in which are the original points A, B, and C, and, of consequence, determine a series of points on each side of the circumference.

It is plain from the construction, (which is too evident to require a formal demonstration,) that the arcs AD, A a, C c, &c. intercepted between the points A and D, A and a, C and c, &c. are equal to the arc B C, and to one another.

In like manner, joining B C, and bisecting it at right angles with P Q; drawing A c′ parallel to B C, and making R c′=A R, (c′) is another of the required points; and, from (c′) the point last found, drawing c′a′ parallel to C A, and making a′ N=N c′, (a′) is another point in the same circumference; and the arcs comprehended between C c′ and A a′ are equal to that between AB. Hence, by means of the perpendiculars P Q and F G, any number of points in the circumference of the circle, passing through the given ones, A, B and C may be found, whose distance is equal to AB, in the same manner, as points at the distance of B C were found by the help of the perpendiculars I K and F G.

Again, if A, C and (c) or A, C and (c′) be taken as the three given points, multiples of the arc AC may be found in the same manner as those of the arc AB were found as above described.

2. Another method of performing this problem, is as follows, *fig.* 6, *plate* 10. Produce C B and CA; and with a convenient radius on C, describe the arc D E; on which set off the parts F G, G E, &c. each equal to D F; draw C G, C E, &c. con-

tinued out beyond G and E if necessary; take the distance AB, and with one foot of the compasses in A, strike an arc to cut CG produced in H; and H is a point in the circumference of the circle that passes through the given points ABC; with the same opening AB, and center H, strike an arc to cut CE produced in I, which will be another of the required points, and the process may be continued as far as is necessary.

The reason of this construction is obvious; for since the angles, BCA, ACH, HCI, &c. are equal, they must intercept equal arcs BA, AH, HI of the circumference.

If it were required to find a number of points K, L, &c. on the other side, whose distances were equal to BC, lay down a number of angles CAK, KAL, &c. each equal to BAC, and make the distances CK, KL, &c. each equal to BC.

BY THE BEVEL.

This problem is much easier solved by the help of the bevel above described, as follows. See *fig.* 5.

Bring the center of the bevel to the middle B, of the three given points A, B and C, and holding it there, open or shut the instrument till the fiducial edges of the legs lie upon the other two points, and fix them there, by means of the screw A, (*fig.* 4); this is called *setting* the bevel to the given points. Then removing the center of the bevel, to any part between B and A or C, the legs of it being at the same time kept upon A and C, that center will describe (or be always found in) the arc which passes through the given points, and will, by that means, ascertain as many others as may be required within the limits of A and C.

In order to find points without those limits, proceed thus: the bevel being *set* above described, bring the center to C, and mark the distance C B upon the *left* leg; remove the center to B, and mark the distance B A on the same leg; then placing the center on A, bring the right leg upon B, and the first mark will fall upon (a) a point in the circumference of the circle, passing through A, B and C, whose distance from A is equal to the distance B C. Removing the center of the bevel to the point (a) last found, and bringing the right leg to A, the second mark will find another point (a″) in the same circumference, whose distance a a″ is equal A B. Proceeding in this manner, any number of points may be found, whose distances on the circumference are alternately B C and B A.

In the same manner, making similar marks on the *right* leg, points on the other side, as at (c′) and (c″) are found, whose distances C c′, c′ c″, are equal to B A, B C respectively.

It is almost unnecessary to add, that intermediate points between any of the above are given by the bevel, in the same manner as between the original points.

Problem 2. *Fig.* 7, *plate* 10. *Three points,* A, B *and* C, *being given, as in the last problem, to find a fourth point* D, *situated in the circumference of the circle passing through* A, B *and* C, *and at a given number of degrees distant from any of these points*; A *for instance.*

Make the angles A B D, and A C D, each equal to one half of the angle, which contains the given number of degrees, and the intersection of the lines B D, C D gives the point D required.

For, an angle at the circumference being equal to half that at the center, the arc A D will contain twice the number of degrees contained by either of the angles A B D or A C D.

PROBLEM 3. *Fig.* 8, *plate* 10. *Given three points, as in the former problems, to draw a line from any of them, tending to the center of the circle, which passes through them all.*

Let A, B and C be the given points, and let it be required to draw A D, so as, if continued, it would pass through the center of the circle containing A, B and C.

Make the angle B A D equal to the complement of the angle B C A, and A D is the line required.

For, supposing A E a tangent to the point A, then is E A D a right angle, and E A B = B C A; whence, B A D = right angle, less the ∠ B C A, or the complement of B C A.

Corollary 1. A D being drawn, lines from B and C, or any other points in the same circle, are easily found; thus, make A B G = B A D, which gives B G; then make B C F = C B G, which gives C F; or C F may be had without the intervention of B G, by making A C F = C A D.

Corollary 2. A tangent to the circle, at any of the points (A for instance), is thus found.

Make B A E = B C A, and the line A E will touch the circle at A.

By the bevel. Set the bevel to the three given points A, B and C, (*fig.* 8,) lay the center on A, and the right leg to the point C; and the other leg will give the tangent A G′. Draw A D perpendicular to A G′ for the line required.

For B A E being = B C A, the ∠ E A C is the supplement to ∠ A B C, or that to which the bevel is set; hence, when one leg is applied to C, and the center brought to A, the direction of the other leg must be in that of the tangent G E.

PROBLEM 4. *Fig.* 9, *plate* 10. *Three points being given, as in the former problems, to draw from a given fourth point a line tending to the center of a circle passing through the three first points.*

Let the three points, through which the circle is supposed to pass, be A, B and C, and the given fourth point D; it is required to draw through D a line Dd tending to the center of the said circle.

From A and B, the two points nearest D, draw by the last problem, the lines Aa, Bb, tending to the said center; join AB, and from any point E, taking in Bb, (the farther from B the better) draw EF parallel to Aa, cutting AB in F; join AD and BD, and draw FG parallel to AD, cutting DB in G; join GE, and through D parallel thereto, draw Dd for the line required.

For, (continuing Dd and Bb till they meet in O,) since Aa and Bb, if produced, would meet in the center, and FE is parallel to Aa, we have BF : BA :: BE : radius; also, since AD and FG are parallel, BF : BA :: BG : BD; therefore BG : BD :: BE : radius; but from the parallel lines Dd and GE, we have BG : BD :: BE : BO; hence BO is the radius of the circle passing through A, B and C.

By the bevel. On D with radius DA describe an arc AK; *set* the bevel to the three given points A, B and C, and bring its center (always keeping the legs on A and C) to fall on the arc AK, as at H; on A and H severally, with any convenient radius, strike two arcs crossing each other at I; and the required line Dd will pass through the points I and D.

For a line drawn from A to H will be a common chord to the circles AHK and ABC; and the line ID bisecting it at right angles, must pass through both their centers.

PROBLEM 5. *Fig.* 9, *plate* 10. *Three points being given, as before, together with a fourth point, to find two other points, such, that a circle passing through them and the fourth point, shall be concentric to that passing through three given points.*

Let A, B and C be the three given points, and D the fourth point; it is required to find two other points, as N and P, such, that a circle passing through N, D and P shall have the same center with that passing through A, B and C.

The geometrical construction being performed as directed by the last problem, continue E G to L, making E L = E B; and through B and L draw B L M, cutting D d produced in M; make A N and B P severally equal to M D, and N and P are the points required.

For, since L E is parallel to M O, we have B E : L E :: B O : M O; but B E = L E by the construction; therefore, M O = B O = radius of the circle passing through A, B and C, and M is in the circumference of that circle. Also, N, D and P being points of the radii, equally distant from A, M and B respectively, they will be in the circumference of a circle concentric to that passing through A, M and B, or A, B and C.

By the bevel. Draw A a and C c tending to the center, by problem 3; set the bevel to the three given points A, B and C; bring the center of the bevel to D, and move it upon that point till its legs cut off equal parts A N, C Q of the lines A a and C c; and N and Q will be the points required.

For, supposing lines drawn from A to C, and from N to Q, the segments A B C and N D Q will be similar ones; and consequently, the angles contained in them will be equal.

PROBLEM 6. *Fig.* 10, *plate* 10. *Three points,* A, B *and* C, *lying in the circumference of a circle,* *being given as before: and a fourth point* D, *to find another point* F, *such, that a circle passing through* F *and* D *shall touch the other passing through* A, B *and* C, *at any of these points*; *as for instance*, B.

Draw B E a tangent to the arc A B C, by problem 3, corollary 2; and join B D; draw B F, making the angle D B F = E B D, with the distance B D; on D strike an arc to cut B F in F; and F is the point sought.

Since D F = D B, the $\angle$ D F B = D B F; but D B F = D B E by construction; therefore, D F B = D B E, and E B is a tangent to the arc B D F at B; but E B is also a tangent to the arc A B C (by construction) at the same point; hence, the arc B D F touches A B C as required.

PROBLEM 7. *Fig.* 13, *plate* 10. *Two lines tending to a distant point being given, and also a point in one of them*; *to find two other points, (one of which must be in the other given line,) such, that a circle passing through those three points, may have its center at the point of intersection of the given lines.*

Let the given lines be A B and C D, and E the given point in one of them; it is required to find two other points, as I and H, one of which (I) shall be in the other line, such, that a circle H I E passing through the three points, shall have its center at O, where the given lines, if produced, would meet.

From E, the given point, draw E H, crossing A B at right angles in F; make F H = F E, and H is one of the required points. From any point D in C D, the farther from E the better, draw G D parallel to A B, and make the angle H E I equal to half the angle G D E; and E I will cut A B in I, the other required point.

For, since E H crosses A B at right angles, and H F is equal to F E, I H will be equal to I E, and the ∠ H E I = ∠ E H I; also, since G D is parallel to A B, the ∠ G D E = ∠ F O E = double ∠ I H E = double ∠ H E I; but H E I = half ∠ G D E by construction; hence, the points E, I and H are in the circle whose center is O.

By the bevel. Draw E H at right angles to A B, and make F H = F E as before; set the bevel to the angle G D O, and keeping its legs on the points H and E, bring its center to the line A B, which will give the point I.

PROBLEM 8. *Fig.* 13, *plate* 10. *Two lines tending to a distant point being given, to find the distance of that point.*

Let A B and C D be the two given lines, tending to a distant point O; and let it be required to find the distance of that point, from any point (E for instance) in either of the given lines.

From E draw E F perpendicular to A B; and from D (a point taken any where in C D, the farther from E the better) draw D G parallel to A B. On a scale of inches and parts measure the lengths of G E, E D and E F separately; then say, as the length of G E is to E D, so is E F to E O, the distance sought.

For the triangles E G D and E F O are similar, and from thence the rule is manifest.

OF INSTRUMENTS FOR DRAWING ARCS OF LARGE CIRCLES, AND LINES TENDING TO A DISTANT POINT.

I shall now proceed to give some idea of a few instruments for these purposes, whose *rationale* depends on the principles laid down in the beginning of this essay.

1. AN IMPROVED BEVEL.

Fig. 12, *plate* 10, is a sketch of an instrument grounded upon principle 1, p. 134, by which the arcs of circles of any radius, without the limits attainable by a common pair of compasses, may be described.

It consists of a ruler AB, composed of two pieces rivetted together near C, the center, or axis, and of a triangular part C F E D. The axis is a hollow socket, fixed to the triangular part, about which another socket, fixed to the arm C B of the ruler AB, turns. These sockets are open in the front, for part of their length upwards, as represented in the section at I, in order that the point of a tracer or pen, fitted to slide in the socket, may be more easily seen.

The triangular part is furnished with a graduated arc D E, by which, and the vernier at B, the angle D C B may be determined to a minute. A groove is made in this arc, by which, and by the nut and screw at B, or some similar contrivance, the ruler AB may be fixed in any required position.

A scale of radii is put on the arm C B, by which the instrument may be *set* to describe arcs of given circles, not less than 20 inches in diameter. In order to *set* the instrument to any given radius, the number expressing it in inches on C B is brought to cut a fine line drawn on C D, parallel, and near to the fiducial edge of it, and the arms fastened in that position by the screw at B.

Two heavy pieces of lead or brass, G, G, made in form of the sector of a circle, the angular parts being of steel and wrought to a true upright edge, as shewn at H, are used with this instrument, whose arms are made to bear against those edges when the arcs are drawn. The under sides of these sectors are furnished with fine short points, to prevent them from sliding.

The fiducial edges of the arms C A and C D are each divided from the center C into 200 equal parts.

The instrument might be furnished with small castors, like the pentagraph; but little buttons fixed on its underside, near A, E and D, will enable it to slide with sufficient ease.

SOME INSTANCES OF ITS USE.

1. *To describe an arc, which shall pass through three given points.*

Place the sectors G, G, with their angular edges over the two extreme points; apply the arms of the bevel to them, and bring at the same time its center C (that is, the point of the tracer, or pen, put into the socket) to the third point, and there fix the arm C B; then, bringing the tracer to the left hand sector, slide the bevel, keeping the arms constantly bearing against the two sectors, till it comes to the right hand sector, by which the required arc will be described by the motion of its center C.

If the arc be wanted in some part of the drawing *without* the given points, find, by problem 1, p. 136, other points in those parts where the arc is required. By this means a given arc may be lengthened as far as is requisite.

2. *To describe an arc of a given radius, not less than* 10 *inches.*

Fix the arm C B so that the part of its edge, corresponding to the given radius, always reckoned in inches, may lie over the fine line drawn on C D for that purpose: bring the center to the point through which the arc is required to pass, and dispose the bevel in the direction it is intended to be drawn; place the sectors G, G, exactly to the divisions 100 on each arm, and strike the arc as above described.

3. *The bevel being set to strike arcs of a given radius, as directed in the last paragraph, to draw other arcs whose radii shall have a given proportion to that of the first arc.*

Suppose the bevel to be *set* for describing arcs of 50 inches radius, and it be required to draw arcs of 60 inches radius, with the bevel so *set*.

Say, as 50, the radius to which the bevel is *set*, is to 60, the radius of the arcs required; so is the constant number 100 to 120, the number on the arms CA and CD, to which the sectors must be placed, in order to describe arcs of 60 inches radius.

N. B. When it is said that the bevel is *set* to draw arcs of a particular radius, it is always understood that the sectors G, G, are to be placed at No. 100 on CA and CD, when those arcs are drawn.

4. *An arc* ACB *(fig.* 11, *plate* 10*) being given, to draw other arcs concentric thereto, which shall pass through given points, as* P′ *for instance.*

Through the extremities A and B of the given arc draw lines AP, BP tending to its center, by problem 3, p. 140. Take the nearest distance of the given point P from the arc, and set it from A to P, and from B to P. Hold the center of the level on C, (any point near the middle of the given arc) and bring its arms to pass through A and B at the same time, and there fix them. Place the sectors to the points P and P, and with the bevel, set as before directed, draw an arc, which will pass through P′, the given point, and be concentric to the given arc ACB.

5. *Through a point* A, *(fig.* 14, *plate* 10*) in the given line* AB, *to strike an arc of a given radius, and whose center shall lie in that line, produced if necessary.*

Set the bevel to the given radius, as above described, *(Method* 2.*)*

Through A, at right angles to AB, draw CD; lay the center of the bevel, *set* as above, on A, and the arm CA, on the line AC, and draw a line AE along the edge CD of the other arm. Divide the angle DAE into two equal parts by the line AF, place the bevel so, that its center being at A, the arm CD shall lie on AF; while in this situation, place the sectors at No. 100 on each arm, and then strike the arc.

6. *An arc being given, to find the length of its radius.*

Place the center of the bevel on the middle of the arc, and open or shut the arms, till No. 100 on CA and CD fall upon the arc on each side the center; the radius will be found on CB (in inches) at that point of it, where it is cut by the line drawn on CD.

If the extent of the arc be not equal to that between the two Numb. 100, make use of the Numb. 50, in which case the radius found on CB will be double of that sought; or the arc may be lengthened, by problem 1, till it be of an extent sufficient to admit the two Numbers 100.

Many more instances of the use of this instrument might be given; but from what has been already done, and an attentive perusal of the foregoing problems, the principle of them may be easily conceived.

2. THE OBLIQUE RULER.

An instrument for drawing lines that are parallel, is called a *parallel* ruler; one for drawing lines tending to a point, as such lines are oblique to each other, may, by analogy, be called an *oblique* ruler.

Fig. 17, *plate* 10, represents a simple contrivance for this purpose; it consists of a cylindrical or prismatical tube AB, to one end of which is fixed the roller A; into this tube there slides another C B of six or eight flat sides. The tubes slide stiffly, so as to remain in the position in which they are placed. Upon the end C, screw different rollers, all of them something smaller than A.

In order to describe arcs, a drawing pen E, and a tracer may be put on the pin D, and are retained there by a screw G; the pen is furnished with a moveable arm E F, having a small ball of brass F at the end, whose use is to cause the pen to press with due force upon the paper, the degree of which can be regulated by placing the arm in different positions.

The ruler AB being set to any given line, by rolling it along other lines may be drawn, all of which will tend to some *one* point in the given line, or a continuation of it, whose distance will be greater, as the distance between the rollers A and C is increased; and as the diameter of C approaches that of A; all which is evident from principle 2, page 134.

It also appears from the said principle, that during the motion of the ruler, any point in its axis will accurately describe the arc of a circle, having the said point of intersection for its center; and, consequently, the pen or tracer, put on the pin D, will describe such arcs.

The rollers, as C, which screw upon the end of the inner tube are numbered, 1, 2, 3, &c. and as many scales are drawn on that tube as there are rollers, one belonging to each, and numbered accordingly. These scales shew the distance in inches, of the center or point of intersection, reckoned from the middle of the pin D, (agreeing to the point of the pen or tracer;) thus,

No. 1, will describe circles, or serve for drawing lines tending to a point, whose radius or distance from D, is from 1200 inches to 600 inches, according as the tube is drawn out.

No. 2	-	from 600 inches	to	300 inches
3	-	300	-	150
4	-	150	-	80
5	-	80	-	40
6	-	40	-	20
7	-	20	-	10

If it should be required to extend the radius or distance farther than 1200 inches, by using another ruler, it might be carried to 2400 inches; but lines in any common sized drawing, which tend to a point above 100 feet distance, may be esteemed as parallel.

3. ANOTHER RULER OF THE SAME KIND.

Fig. 18, *plate* 10. This is nothing more than the last instrument applied to a flat ruler, in the manner the rolling parallel rulers are made.

C D is an hexagonal axis, moveable on pivots in the heads A and F fixed upon a flat ruler; on this axis the smaller roller B, is made to slide through one half of its length; the larger roller A, is screwed on the other end of the axis, and can be changed occasionally for others of different diameters. Scales adapted to each of the rollers at A, are either put on the flat sides of the axis from C to E, or drawn on the corresponding part of the flat ruler; and the scales and rulers distinguished by the same number: at F is a screw to raise or lower the end C of the axis, till the ruler goes parallel to the paper on which the drawing is made; and at G there is a socket, to which a drawing pen and tracer is adapted for describing arcs.

In using these instruments, the fingers should be placed about the middle part between the rollers; and the ruler drawn, or pushed at right angles to its length. The tube AB, *fig.* 14, and one, or both of the edges of the flat ruler, *fig.* 18, are divided into inches and tenths.

4. THE CYCLOGRAPH.

This instrument is constructed upon the third principle mentioned in page 135, of this Essay.

Fig. 16, *plate* 10, is composed of five rulers; four of them DE, DF, GE and GF, forming a trapezium, are moveable on the joints D, E, F and G; the fifth ruler DI, passes under the joint D, and through a socket carrying the opposite joint G. The distances from the center of the joint D, to that of the joints E and F, are exactly equal, as are the distances from G to the same joints. The rulers DE and DF pass beyond the joints E and F, where a roller is fixed to each; the rollers are fixed upon their axes, which move freely, but steadily on pivots, so as to admit of no shake by which the inclination of the axes can be varied. The ruler ID passing beyond the joint D, carries a third roller A, like the others, whose axis lies precisely in the direction of that ruler; the axes of B and C extend to K and L.

A scale is put on the ruler D I, from H to G, shewing, by the position of the socket G thereon, the length of the radius of the arc in inches, that would be described by the end I, in that position of the trapezium. When the socket G is brought to the end of the scale near I, the axes of the two rollers B and C, the ruler D I, and the axis of the roller A, are precisely parallel; and in this position, the end I, or any other point in D I, will describe strait lines at right angles to D I; but on sliding the socket G towards H, an inclination is given to the axes of B and C, so as to tend to some point in the line I D, continued beyond D, whose distance from I is shewn by the scale.

A proper socket, for holding a pen or tracer, is made to put on the end I, for the purpose of describing arcs; and another is made for fixing on any part of the ruler D I, for the more convenient description of concentric arcs, where a number are wanted.

It is plain from this description, that the middle ruler D I in this instrument, is a true *oblique* ruler, by which lines may be drawn tending to a point, whose distance from I is shewn by the position of the socket G on the scale; and the instrument is made sufficiently large, so as to answer this purpose as well as the other.

5. A DIFFERENT CONSTRUCTION OF THE SAME INSTRUMENT.

In *Fig.* 16, *plate* 10, the part, intended to be used in drawing lines, lies within the trapezium, which is made large on that account; but this is not necessary; and *fig.* 15, *plate* 10, will give an idea of a like instrument, where the trapezium may be made much smaller, and consequently less cumbersome.

DBEC represents such a trapezium, rollers, socket, and scale as above described, but much smaller. Here the ruler ED is continued a sufficient length beyond D, as to A, where the third roller is fixed; a pen or tracer may be fitted to the end E, or made to slide between D and A, for the purpose of drawing arcs.

METHODS OF DESCRIBING AN ELLIPSE, AND SOME OTHER CURVES.

To describe an ellipse, the transverse and conjugate axes being given.

Let AB be the given transverse, and CD the conjugate axis, *fig.* 13, *plate* 13.

Method 1. By the line of sines on the sector, open the sector with the extent AG of the semi-transverse axis in the terms of 90 and 90; take out the transverse distance of 70 and 70, 60 and 60, and so for every tenth sine, and set them off from G to A, and from G to B; then draw lines through these points perpendicular to AB. Make GC a transverse distance between 90 and 90, and set off each tenth sine from G towards C, and from G towards D, and through these points draw lines parallel to AB, which will intersect the perpendiculars to AB in the points A, a, b, c, d, e, f, g, h, i, k, l, m, n, o, p, q, B, for half the ellipse, through which points and the intersections of the other half, a curve being drawn with a steady hand, will complete the ellipse.

Method 2. With the elliptical compasses, *fig.* 3, *plate* 11, apply the transverse axis of the elliptical compasses to the line AB, and discharge the screws of both the sliders; set the beam over the transverse axis AB, and slide it backwards and forwards until the pencil or ink point coincide with the point A, and tighten the screw of that slider which moves on the conjugate axis; now turn the beam, so as to lay over the conjugate axis C D, and make the pencil or ink point coincide with the point C, and then fix the screw, which is over the slider of the transverse axis of the compasses; the compasses being thus adjusted, move the ink point gently from A, through C to B, and it will describe the semi-ellipse A C B; reverse the elliptical compasses, and describe the other semi-ellipse B D A. These compasses were contrived by my *Father* in 1748; they are superior to the trammel which describes the whole ellipse, as these will describe an ellipse of any excentricity, which the others will not.

Through any given point F *to describe an ellipse, the transverse axis* AB *being given.*

Apply the transverse axis of the elliptical compasses to the given line AB, and adjust it to the point A; fix the conjugate screw, and turn the beam to F, sliding it till it coincide therewith, and proceed as in the preceding problem.

Fig. 2, *plate* 11, represents another kind of elliptical apparatus, acting upon the principle of the oval lathes; the paper is fixed upon the board AB, the pencil C is set to the transverse diameter by sliding it on the bar D E, and is adjusted to the conjugate diameter by the screw G; by turning the board AB, an ellipse will be described by the pencil. *Fig.* 2, A, *plate* 11, is the trammel, in which the pins on the under side of the board AB, move for the description of the ellipse.

Ellipses are described in a very pleasing manner by the *geometric pen*, *fig.* 1, *plate* 11; this part of that instrument is frequently made separate.

To describe a parabola, whose parameter shall be equal to a given line. *Fig.* 17, *plate* 13.

Draw a line to represent the axis, in which make AB equal to half the given parameter. Open the sector, so that AB may be the transverse distance between 90 and 90 on the line of sines, and set off every tenth sine from A towards B; and through the points thus found, draw lines at right angles to the axis AB. Make the lines A a, 10 b, 20 c, 30 d, 40 e, &c. respectively equal to the chords of 90°, 80°, 70°, 60°, 50°, &c. to the radius AB, and the points a b c d e, &c. will be in the parabolic curve: for greater exactness, intermediate points may be obtained from the intermediate degrees; and a curve drawn through these points and the vertex B, will be the parabola required: if the whole curve be wanted, the same operation must be performed on the other side of the axis.

As the chords on the sector run no further than 60°, those of 70, 80 and 90, may be found by taking the transverse distance of the sines of 35°, 40°, 45°, to the radius AB, and applying those distances twice along the lines, 20 c, 10 b, &c.

Fig. 4, *plate* 11, is an instrument for describing a parabola; the figure will render its use sufficiently evident to every geometrician. A B C D is a wooden frame, whose sides AC, B D are parallel to each other; E F G H is a square frame of brass or wood, sliding against the sides AC, B D of the exterior frame; H a socket sliding on the bar E F of the interior frame, and carrying the pencil I; K a fixed point in the board, (the situation of which may be varied occasionally); E a K is a thread equal in length to E F, one end thereof is fixed at E, the other to the piece K, going over the pencil at a. Bring the frame, so that the pencil may be in a line with the point K; then slide it in the exterior frame, and the pencil will describe one part of a parabola. If the frame E F G H be turned about, so that E F may be on the other side of the point K, the remaining part of the parabola may be completed.

To describe an hyperbola, the vertex A, *and asymptotes* B H, B I *being given. Fig.* 18, *pl.* 13.

Draw AI, AC, parallel to the asymptotes. Make AC a transverse distance to 45, and 45, on the upper tangents of the sector, and apply from B as many of these tangents taken transversely as may be thought convenient; as B D 50°, D E 55°, and so on; and through these points draw D d, E c, &c. parallel to AC.

Make AC a transverse distance between 45 and 45 of the lower tangents, and take the transverse distance of the cotangents before used, and lay them on those parallel lines; thus making D d equal 40°, E C to 35°, E F to 80°, &c. and these points will be in the hyperbolic curve, and a line drawn through them will be the hyperbola required.

To assist the hand in drawing curves through a number of points, artists make use of what is termed *the bow*, consisting of a spring of hard wood, or steel, so adapted to a firm strait rule, that it may be bent more or less by three screws passing through the strait rule.

A set of spirals cut out in brass, are extremely convenient for the same purpose; for there are few curve lines of a short extent, to which some part of these will not apply.

Fig. 6, *plate* 11, represents an instrument for drawing spirals; A the foot by which it is affixed to the paper, B the pencil, *a*, *b*, *c*, *d* a running line going over the cone G and cylinder H, the ends being fastened to the pin *e*. On turning the frame K N O, the thread carries the pencil progressively from the cone to the cylinder, and thus describes a spiral. The size of the spiral may be varied, by placing the thread in different grooves, by putting it on the furthermost cone, or by putting on a larger cone.

OF THE GEOMETRIC PEN.

The geometric pen is an instrument in which, by a circular motion, a right line, a circle, an ellipse, and a great variety of geometrical figures, may be described.

This curious instrument was invented and described by *John Baptist Suardi*, in a work entitled *Nuovo Istromenti per la Descrizzione di diverse Curve Antichi e Moderne*, &c.

Though several writers have taken notice of the curves arising from the compound motion of two circles, one moving round the other, yet no one seems to have realized the principle, and reduced it to practice, before *J. B. Suardi*. It has lately been happily introduced into the steam engine by Messieurs *Watt* and *Bolton*; a proof, among many others, not only of the use of these speculations, but of the advantages to be derived from the higher parts of the mathematics, in the hands of an ingenious mechanic. There never was, perhaps, any instrument which delineates so many curves as the geometric pen; the author enumerates 1273, as possible to be described by it in the simple form, and with the few wheels appropriated to it for the present work.

Fig 1, *plate* 11, represents the *geometric pen*; A, B, C, the stand by which it is supported; the legs A, B, C, are contrived to fold one within the other, for the convenience of packing.

A strong axis D is fitted to the top of the frame; to the lower part of this axis any of the wheels (as i) may be adapted; when screwed to it they are immoveable.

E G is an arm contrived to turn round upon the main axis D; two sliding boxes are fitted to this arm; to these boxes any of the wheels belonging to the geometric pen may be fixed, and then moved so that the wheels may take into each other, and the immoveable wheel i; it is evident, that by making the arm E G revolve round the axis D, these wheels will be made to revolve also, and that the number of their revolutions will depend on the proportion between the teeth.

f g is an arm carrying the pencil; this arm slides backwards and forwards in the box c d, in order that the distance of the pencil from the center of the wheel h may be easily varied; the box c d is fitted to the axis of the wheel h, and turns round with it, carrying the arm f g along with it; it is evident, therefore, that the revolutions will be fewer or greater, in proportion to the difference between the numbers of the teeth in the wheels h and i; this bar and socket are easily removed for changing the wheels.

When two wheels only are used, the bar f g moves in the same direction with the bar E G; but if another wheel is introduced between them, they move in contrary directions.

The number of teeth in the wheels, and consequently, the relative velocity of the epicycle, or arm f g, may be varied in infinitum.

The numbers we have used are, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96.

The construction and application of this instrument is so evident from the figure, that nothing more need be pointed out than the combinations by which the figures here delineated may be produced.

To render the description as concise as possible, I shall in future describe the arm E G by the letter A, and f g by the letter B.

To describe *fig*. 1, *plate* 12. The radius of A must be to that of B, as 10 to 5 nearly, their velocities, or the numbers of teeth in the wheels, to be equal, the motion to be in the same direction.

If the length of B be varied, the looped figure, delineated at *fig*. 12, will be produced.

A circle may be described by equal wheels, and any radius, but the bars, must move in contrary directions.

To describe the two level figures, see *fig*. 11, *plate* 12. Let the radius of A to B be as 10 to $3\frac{1}{2}$, the velocities as 1 to 2, the motion in the same direction.

To describe by this CIRCULAR MOTION, A STRAIT LINE AND AN ELLIPSE. For a strait line, equal radii, the velocity as 1 to 2, the motion in a contrary direction; the same data will give a variety of ellipses, only the radii must be unequal; the ellipses may be described in any direction; see *fig*. 10, *plate* 13.

Fig. 13, *plate* 12, with seven leaves, is to be formed when the radii are as 7 to 2, velocity as 2 to 3, motion in contrary directions.

The six triangular figures, seen at *fig*. 2, 4, 6, 8, 9, 10, are all produced by the same wheels, by only varying the length of the arm B, the velocity should be as 1 to 3, the arms are to move in contrary directions.

Fig. 3, *plate* 12, with eight leaves, is formed by equal radii, velocities as 5 to 8, A and B to move the same way; if an intermediate wheel is added, and thus a motion produced in a contrary direction, the pencil will delineate *fig.* 16, *plate* 12.

The ten-leaved figure, *fig.* 15, *plate* 12, is produced by equal radii, velocity as 3 to 10, directions of the motions contrary to each other.

Hitherto the velocity of the epicycle has been the greatest; in the three following figures the curves are produced when the velocity of the epicycle is less than that of the primum mobile.

For *fig.* 7, the radius of A to B to be as 2 to 1, the velocity as 3 to 2; to be moved the same way.

For *fig.* 14, the radius of A, somewhat less than the diameter given to B, the velocity as 3 to 1; to be moved in a contrary direction.

For *fig.* 5, equal radii, velocity as 3 to 1; moved the same way. These instances are sufficient to shew how much may be performed by this instrument; with a few additional pieces, it may be made to describe a cycloid, with a circular base, spirals, and particularly the spiral of Archimedes, &c.

OF THE DIVISION OF LAND.

To know how to divide land into any number of equal, or unequal parts, according to any assigned proportion, and to make proper allowances for the different qualities of the land to be divided, form a material and useful branch of surveying.

In dividing of land, numerous cases arise; in some it is to be divided by lines parallel to each other, and to a given fence, or road; sometimes, they are to intersect a given line; the division is often to be made according to the particular directions of the parties concerned. In a subject which has been treated on so often, novelty is, perhaps, not to be desired, and scarcely expected. No considerable improvement has been made in this branch of surveying since the time of *Speidell*. Mr. *Talbot*, whom we shall chiefly follow, has arranged the subject better than those who preceded him, and added thereto two or three problems; his work is well worth the surveyor's perusal. Some problems also in the foregoing part of this work should be considered in this place.

PROBLEM 1. *To divide a triangle in a given ratio, by right lines drawn from any angle to the opposite side thereof.*

1. Divide the opposite side in the proposed ratio.

2. Draw lines from the several points of division to the given angle, and then divide the triangle as required.

Thus, to divide the triangle ABC, *fig.* 13, *plate* 8, containing $26\frac{1}{2}$ acres, into three parts, in proportion to the numbers 40, 20, 10, the lines of division to proceed from the angle C to AB, whose length is 28 chains; now, as the ratio of 40, 20, 10, is the same as 4, 2, 1, whose sum is 7, divide AB into seven equal parts; draw C a at four of these parts, C b at six of them, and the triangle is divided as required.

Arithmetically. As 7, the sum of the ratios, is to AB 28 chains; so is 4, 2, 1, to 16, 8, and 4 chains respectively; therefore A a = 16, A b = 8, and b B = 4 chains.

To know how many acres in each part, say, as the sum of the ratios is to the whole quantity of land, so is each ratio to the quantity of acres;

$$\text{as}\begin{cases} 7 : 26{,}5 :: 4 : 15{,}142857 \\ 7 : 26{,}5 :: 2 : 7{,}571428 \\ 7 : 26{,}5 :: 1 : 3{,}785714 \end{cases} \begin{matrix} = \text{triangle } ACa \\ = \text{triangle } aCb \\ = \text{triangle } bCB.\text{*} \end{matrix}$$

PROBLEM 2. *To divide a triangular field into any number of parts, and in any given proportion, from a given point in one of the sides.*

1. Divide the triangle into the given proportion, from the angle opposite the given point. 2. Reduce this triangle by problem 51, so as to pass through the given point.

Thus, to divide the field A B C, *fig.* 14, *plate* 8, of seven acres, into two parts, in the proportion of 2 to 5, for two different tenants, from a pond b, in B C, but so that both may have the benefit of the pond.

1. Divide B C into seven equal parts, make B a = 5, then C a = 2, draw A a, and the field is divided in the given ratio. 2. To reduce this to the point b, draw A b and a c parallel thereto, join c b, and it will be the required dividing line.

Operation in the field. Divide B C in the ratio required, and set up a mark at the point *a*, and also at the pond *b*; at A, with the theodolite, or other instrument, measure the angle b A a; at *a* lay off the same angle A a c, which will give the point *c* in the side A c, from whence the fence must go to the pond.

2. To divide A B C, *fig.* 15, *plate* 8, into three equal parts from the pond c. 1. Divide A B into three equal parts, A a, a b, b B, and C a C b will divide it, as required, from the angle A; reduce these as above directed to c d and c e, and they will be the true dividing lines.

* Talbot's Complete Art of Land Measuring.

PROBLEM 3. *To divide a triangular field in any required ratio, by lines drawn parallel to one side, and cutting the others.*

Let ABC, *fig.* 16, *plate* 8, be the given triangle to be divided into three equal parts, by lines parallel to AB, and cutting AC, BC.

Rule 1. Divide one of the sides that is to be cut by the parallel lines, into the given ratio. 2. Find a mean proportional between this side, and the first division next the parallel side. 3. Draw a line parallel to the given side through the mean proportional. 4. Proceed in the same manner with the remaining triangle.

Example 1. Divide BC into three equal parts BD, DP, PC. 2. Find a mean proportional between BC and DC. 3. Make CG equal to this mean proportional, and draw GH parallel to AB. Proceed in the same manner with the remaining triangle CHG, dividing GC into two equal parts at I, finding a mean proportional between CG and CI; and then making CL equal to this mean proportional, and drawing LM parallel to AB, the triangle will be divided as required.

A square, or rectangle, a rhombus, or rhomboides, may be divided into any given ratio, by lines cutting two opposite parallel sides, by dividing the sides into the proposed ratio, and joining the points of division.

PROBLEM 4. *To divide a right-lined figure into any proposed ratio, by lines proceeding from one angle.*

PROBLEM 5. *To divide a right-lined figure into any proposed ratio, by lines proceeding from a given point in one of the sides.*

PROBLEM 6. *To divide a right-lined figure into any proposed ratio, by right lines proceeding from a given point within the said figure or field.*

It would be needless to enter into a detail of the mode of performing the three foregoing problems, as the subject has been already sufficiently treated of in pages 84, 85, &c.

PROBLEM 7.* "*It is required to divide any given quantity of ground into any given number of parts, and in proportion as any given numbers.*"

Rule. "Divide the given piece after the rule of Fellowship, by dividing the whole content by the sum of the numbers expressing the proportions of the several shares, and multiplying the quotient severally by the said proportional numbers for the respective shares required."

Example. It is required to divide 300 acres of land among A, B, C and D, whose claims upon it are respectively in proportion as the numbers 1, 3, 6, 10, or whose estates may be supposed 100l. 300l. 600l. and 1000l. per annum.

The sum of these proportional numbers is 20, by which dividing 300 acres, the quotient is 15 acres, which being multiplied by each of the numbers 1, 3, 6, 10, we obtain for the several shares as follows:

		a.	*r.*	*p.*	
A's share	=	15	0	00	
B's share	=	45	0	00	
C's share	=	90	0	00	
D's share	=	150	0	00	
Sum	=	300	0	00	the proof.

But this is upon supposition that the land is all of an equal value.

Now let us suppose the land, to be laid off for each person's share, is of the following different

* Problem 7, an erroneous rule given by a late writer, introduced here to prevent the practitioner being led into error.—From Talbot's Complete Art of Land Measuring, Problem 1, page 265, and Appendix, page 410.

values per acre, viz. A's = 5s. B's = 8s. C's = 12s. D's = 15s. an acre; whose sum, 40s. divided by 4, their number, quotes 10s. for the mean value per acre. And, according to a late author, we must augment or diminish each share as follows:

	s.		*s.*		*a.*		*a.*	
as	5	:	10	::	15	:	30	A's share
	8	:	10	::	45	:	56,3	B's share
	12	:	10	::	90	:	75	C's share
	15	:	10	::	150	:	100	D's share

sum of shares = 261,3 acres,'

which is less than 300, by more than 38 acres, and shews the rule to be absolutely false; for when each person's share is laid out as above, there remain 38 acres unapplied; I suppose for the use of the surveyor.

But suppose we change the value of each person's land, and call A's 15s. B's 12s. C's 8s. and D's 5s. then we shall have

	s.		*s.*		*a.*		*a.*	
as	15	:	10	::	15	:	10	A's share
	12	:	10	::	45	:	37,5	B's share
	8	:	10	::	90	:	112,5	C's share
	5	:	10	::	150	:	300	D's share

sum of the shares 460 acres:

how must the surveyor manage here, as he must make 460 acres of 300; for here D's share only takes the whole 300, where is he to find the 160 acres for the other three shares? But enough of this; see it truly and methodically performed in the next problem.

PROBLEM 8. *It is required to divide any given quantity of land among any given number of persons, in proportion to their several estates, and the value of the land that falls to each person's share.*

Rule. Divide the yearly value of each person's estate by the value per acre of the land that is allotted for his share, and take the sum of the quotients, by which divide the whole given quantity of land, and this quotient will be a common multiplier, by which multiply each particular quotient, and the product will be each particular share of the land.

Or say, as the sum of all the quotients is to the whole quantity of land, so is each particular quotient to its proportional share of the land.

Example. Let 300 acres of land be divided among A, B, C, D, whose estates are 100l. 300l. 600l. and 1000l. respectively per annum; and the value of the land allotted to each is 5, 8, 12, and 15 shillings an acre, as in the example to the last problem.

Then $\frac{100}{5} = 20$, $\frac{300}{8} = 37{,}5$ $\frac{600}{12} = 50$, and $\frac{1000}{15} = 66{,}666$, and the sum of these quotients is $174{,}166 \therefore \frac{300}{174{,}166} = 1{,}72248$, the common multiplier:

$$\text{then}\begin{cases} 1{,}72248 \times 20 & = 34{,}45 \quad \text{A's share} \\ 1{,}72248 \times 37{,}5 & = 64{,}593 \quad \text{B's share} \\ 1{,}72248 \times 50 & = 86{,}124 \quad \text{C's share} \\ 1{,}72248 \times 66{,}666 & = 114{,}832 \quad \text{D's share} \end{cases}$$

(*acres.*)

sum of the shares $= 299{,}999$ acres, or 300 very nearly, and is a proof of the whole.

Let us now change the values of land as in the last problem, and see what each will have for his share. Suppose A's 15, B's 12, C's 8, and D's 5 shillings an acre; then $\frac{100}{15}=6,666$, $\frac{300}{12}=25$, $\frac{600}{8}=75$, $\frac{1000}{5}=200$, and the sum of these quotients is 306,666, therefore $\frac{306}{306,666}=0,97826$ the common multiplier:

$$\text{then}\begin{cases}0,97826\times 6,666 = & 6,5217 \text{ A's share}\\ 0,97826\times 25, = & 24,4565 \text{ B's share}\\ 0,97826\times 75, = & 73,3695 \text{ C's share}\\ 0,97826\times 200, = & 195,6520 \text{ D's share}\end{cases}$$

(*acres.*)

the sum of the shares = 299,9997 acres, which proves the whole to be right.

Example 2. Let 500 acres be divided among six persons, whose estates are as follow; viz. A's 40l. B's 20l. C's 10l. D's 100l. E's 400l. and F's 1000l. per annum, and the value of the land most convenient for each is A's, B's and C's, each 7s. D's 10's. E's 15s. and F's 12 s. an acre; now each estate divided by the value of his share of the land, will stand thus:

	s.		*l.*			
A	7	)	40	(	5,71428	quotients;
B	7	)	20	(	2,85714	
C	7	)	10	(	1,42857	
D	10	)	100	(	10,	
E	15	)	400	(	26,66666	
F	12	)	1000	(	83,33333	

sum of the quotients=129,99998, or 130.

Now $\frac{500}{130}=3,846153$, the common multiplier; by which, multiplying each quotient, we shall have for each share as follows, viz.

	acres.
A =	21,9778
B =	10,9918
C =	5,4941
D =	38,4615
E =	102,5641
F =	320,5127

their sum is = 500,0020, and thus proves the whole.

N. B. If any single share should contain land of several different values, then use the means to divide his estate by.

Also if there be different quantities as well as values, find what each quantity is worth at its value, and add their sums together; then say, as the sum of the quantities is to this sum; so is one acre to its mean value to be made use of.

Now having found each person's share, they may be laid out in any form required, by the directions and problems given in the next section. But when several shares contain land of the same value, it is best to lay out their sum in the most convenient form, and then subdivide it.

For dividing of commons, &c. Surveyors generally measure the land of different value in separate parcels, and find the separate value thereof, which added in different sums, gives the whole content, and whole value.

By problem 1, they find each man's proportional share of the whole value, and then lay out for each person, a quantity of land equal in value to his share; this they effect by first laying out a quantity by guess, and then casting it up and finding its value; and if such value be equal to his share of the whole value, the dividing line is right; if otherwise, they shift the dividing line a little, till by trial they find a quantity just equal in value to the value of the required share.

If any single share contains land of several different values, each is measured separately, and their several values found; and if the sum of them be equal to the value sought, the division is right; if not, it must be altered till it is so.

OF LAYING OUT ANY GIVEN QUANTITY OF LAND.

As the quantity of land is generally given in acres, roods, and perches, it is necessary, first, to reduce them to square links, which may be performed by the following rule.

To reduce acres, roods, and perches into square links.

Rule 1. To the acres annex five cyphers on the right hand, and the whole will be links. 2. Place five cyphers to the right of the roods, and divide this by 4, the quotient will be links. 3. Place four cyphers on the right hand of the perches, divide this by 16, the quotient will be links. 4. These sums added together, give the sum of square links in the given quantity.

PROBLEM 1. *To lay out a piece of land, containing any given number of acres, in form of a square.*

This is no other than to determine the side of a square that shall contain any desired number of acres; reduce, therefore, the given number of acres to square links, and the square root thereof will be the side of the square required.

PROBLEM 2. *To lay out any desired quantity of land in form of a parallelogram, having either its base or altitude given.*

Divide the content, or area, by the given base, and the quotient is the altitude; if divided by the altitude, the quotient is the base; by the same rule a rectangle may be laid out.

PROBLEM 3. *To lay out any desired quantity of land in form of a parallelogram, whose base shall be 2, 3, 4, &c. times greater than its altitude.*

Divide the area by the number of times the base is to be greater than the altitude, and extract the square root of the quotient; this square root will be the required altitude, which being multiplied by the number of times that the base is to be greater than the altitude, will give the length of the required base.

PROBLEM 4. *To lay out a given quantity of land in form of a triangle, having either the base or the perpendicular given.*

Divide the area by half the given base, if the base be given; or by half the given perpendicular, if the perpendicular be given; and the quotient will be the perpendicular, or base required.

PROBLEM 5. *To lay out any given quantity of land in a regular polygon.*

1. Find in the following table, the area of a polygon of the same name with that required, the side of which is 1. 2. Divide the proposed area by that found in the table. 3. Extract the square root of the quotient, and the root is the side of the polygon required.

No. sides	Names.	Areas.
3	Triangle	0,433
4	Square	1,
5	Pentagon	1,72
6	Hexagon	2,598
7	Heptagon	3,634
8	Octagon	4,828
9	Nonagon	6,182
10	Decagon	7,694
11	Undecagon	9,365
12	Duodecagon	11,196

PROBLEM 6. *To lay out any quantity of land in a circle.*

1. Divide the area by ,7854. 2. Extract the square root of the quotient, for the diameter required.

OF PLAIN TRIGONOMETRY.

Plain trigonometry is the art of measuring and computing the sides of plain triangles, or of such whose sides are right lines.

As this work is not intended to teach the elements of the mathematics, it will be sufficient for me just to point out a few of the principles, and give the rules of plain trigonometry, for those cases that occur in surveying. In most of those cases, it is required to find lines or angles, whose actual admeasurement is difficult or impracticable; they are discovered by the relation they bear to other given lines or angles, a calculation being instituted for that purpose; and as the comparison of one right line with another right line, is more convenient and easy, than the comparison of a right line to a curve; it has been found advantageous to measure the quantities of angles, not by the arc itself, which is described on the angular point, but by certain lines described about that arc.

If any three parts* of a plain triangle be given, any required part may be found both by construction and calculation.

If two angles of a plain triangle are known in degrees, minutes, &c. the third angle is found by subtracting their sum from 180 degrees.

In a right-angled plain triangle, if either acute angle (in degrees) be taken from 90 degrees, the remainder will express the other acute angle.

When the sine of an obtuse angle is required, subtract such obtuse angle from 180 degrees, and take the sine of the remainder, or supplement.

If two sides of a triangle are equal, a line bisecting the contained angle, will be perpendicular to the remaining side, and divide it equally.

Before the required side of a triangle can be found by calculation, its opposite angle must first be given, or found.

The required part of a triangle must be the last term of four proportionals, written in order under one another, whereof the three first are given or known.

In four proportional quantities, either of them may be made the last term; thus, let A, B, C, D, be proportional quantities.

As first to second, so is third to fourth, A:B::C:D.
As second to first, so is fourth to third, B:A::D:C.
As third to fourth, so is first to second, C:D::A:B.
As fourth to third, so is second to first, D:C::B:A.

Against the three first terms of every proportion, or stating, must be written their respective values taken from the proper tables.

* This is imperfectly stated by several writers. One of the given parts must be a side. A triangle consists of six parts, *viz.* three sides and three angles. EDIT.

If the value of the first term be taken from the sum of the second and third, the remainder will be the value of the fourth term or thing required; because the addition and subtraction of logarithms, corresponds with the multiplication and division of natural numbers.

If to the complement of the first value, be added the second and third values, the sum, rejecting the borrowed index, will be the tabular number expressing the thing required: this method is generally used when radius is not one of the proportionals.

The complement of any logarithm, sine, or tangent, in the common tables, is its difference from the radius 10.000.000, or its double 20.000.000.

CANONS FOR TRIGONOMETRICAL CALCULATION.

1. The following proportion is to be used when two angles of a triangle, and a side opposite to one of them, is given to find the other side.

As the sine of the angle opposite the given side, is to the sine of the angle opposite the required side; so is the given side to the required side.

2. When two sides and an angle opposite to one of them is given, to find another angle; use the following rule:

As the side opposite the given angle, is to the side opposite the required angle; so is the sine of the given angle, to the sine of the required angle.

The memory will be assisted in the foregoing cases, by observing that when a *side* is wanted, the proportion must begin with an *angle*; and when an *angle* is wanted, it must begin with a *side*.

3. When two sides of a triangle and the included angle are given, to find the other angles and side.

As the sum of the two given sides is to their difference; so is the tangent of half the sum of the two unknown angles, to the tangent of half their difference.

Half the difference thus found, added to half their sum, gives the greater of the two angles, which is the angle opposite the greatest side. If the third side is wanted, it may be found by solution 1.

4. The following steps and proportions are to be used when the three sides of a triangle are given, and the angles required.

Let ABC, *plate* 9, *fig.* 30, be the triangle; make the longest side AB the base; from C the angle opposite to the base, let fall the perpendicular CD on AB, this will divide the base into two segments AD, BD.

The difference between the two segments is found by the following proportion:

As the base AB, or sum of the two segments, is to the sum of the other sides (AC+BC); so is the difference of the sides (AC—BC), to the difference of the segments of the base (AD—DB).

Half the difference of the segments thus found, added to the half of AB, gives the greater segment AD, or subtracted, leaves the less DB.

In the triangle ADC, find the angle ACD by solution 2; for the two sides AD and AC are known, and the right angle at D is opposite to one of them.

The complement of ACD, gives the angle A.

Then in the triangle ABC, you have two sides AB, BC, and the angle at A, opposite to one of them, to find the angles C and B.

OF THE LOGARITHMIC SCALES ON THE SECTOR.

There are three of these lines usually put on the sector, they are often termed the Gunter's lines, and are made use of for readily working proportions; when used, the sector is to be quite opened like a strait rule.

If the 1 at the beginning of the scale, or at the left hand of the first interval, be taken for unity, then the 1 in the middle, or that which is at the end of the first interval and beginning of the second will express the number 10; and the ten at the end of the right-hand of the second interval or end of the scale, will represent the number 100. If the first is 10, the middle is 100, and the last 1000; the primary and intermediate divisions in each interval, are to be estimated according to the value set on their extremities.

In working proportions with these lines, attention must be paid to the terms, whether arithmetical or trigonometrical, that the first and third term may be of the same name, and the second and fourth of the same name. To work a proportion, take the extent on its proper line, from the first term to the third in your compasses, and applying one point of the compasses to the second, the other applied to the right or left, according as the fourth term is to be more or less than the second, will reach to the fourth.

Example 1. If 4 yards of cloth cost 18 shillings, what will 32 yards cost? This is solved by the line of numbers; take in your compasses the distance between 4 and 32, then apply one foot thereof on the same line at 18, and the other will reach 144, the shillings required.

Example 2. As radius to the hypothenuse 120, so is the sine of the angle opposite the base 30° $17'$ to the base. In this example, radius, or the sine of 90, and the sine of 30° $17'$ taken from the line of sines, and one foot being then applied to 120 on the line of numbers, and the other foot on the left will reach to $60\frac{1}{2}$ the length of the required base. The foot was applied to the left, because the legs of a right-angled triangle are less than the hypothenuse.

Example 3. As the cosine of the latitude 51° $30'$, (equal the sine of 38° $30'$) is to radius, so is the sine of the sun's declination 20° $14'$, to the sine of the sun's amplitude. Take the distance between the sines of 38° $30'$ and 20° $14'$ in your compasses; set one foot on the radius, or sine of 90°, and the other will reach to $330\frac{3}{4}^\circ$, the sun's amplitude required.

CURIOUS AND USEFUL TRIGONOMETRICAL PROBLEMS.

The following problems, though of the greatest use, and sometimes of absolute necessity to the surveyor, are not to be found in any of the common treatises on surveying. The maritime surveyor can scarce proceed without the knowledge of them; nor can a kingdom, province, or county be accurately surveyed, unless the surveyor is well acquainted with the use and application of them. Indeed, no man should attempt to survey a county, or a sea coast, who is not master of these problems. The second problem, which is peculiarly useful for determining the exact situation of sands, or rocks, within sight of three places upon land, whose distances are well known, was first proposed by Mr. *Townly*, and solved by Mr. *Collins*, Philosophical Transactions, No. 69. There is "no problem more useful in surveying, than that by which we find a *station*, by observed angles of three or more objects, whose reciprocal distances are known; but distance, and bearing from the place of observation are unknown.

"Previous to the resolution of these problems, another problem for the easy finding the segment of a circle, capable of containing a given angle, is necessary, as will be clear from the following observation.

"Two objects can *only* be seen under the *same* angle, from some part of a circle passing through those objects, and the place of observation.

"If the angle under which those objects appear, be less than 90°, the place of observation will be somewhere in the greater segment, and those objects will be seen under the same angle from every part of the segment.

"If the angle, under which those objects are seen, be more than 90°, the place of observation will be somewhere in the lesser segment, and those objects will be seen under the same angle from every part of that segment*." Hence, from the situation of three known objects, we are able to determine the station point with accuracy.

Problem. *To describe on a given line* B C, *fig.* 29, *plate* 9, *a segment of a circle, capable of containing a given angle.*

Method 1. Bisect B C in A. 2. Through the point of bisection, draw the indefinite right line D E perpendicular to B C. 3. Upon B C, at the point C, constitute the angles DCB, FCB, GCB,

* Dalrymple's Essay on Nautical Surveying, in which another mode of solving this problem is given.

H C B, respectively equal to the difference of the angles of the intended segments and 90 degrees: the angle to be formed on the same side with the segment, if the angle be less than 90; but on the opposite, if the angle is to be greater than 90 degrees. 4. The points D, F, G, H, where the angular lines C D, C F, C G, C H, &c. intersect the line D E, will be the centers of the intended segments.

Thus, if the intended segment is to contain an angle of 120°, constitute on B C, at C, (on the opposite side to which you intend the segment to be described,) the angle D C B equal to 30°, the difference between 90° and 120°; then on center D, and radius D C, describe the segment C, 120, B, in every part of which, the two-points C and B will subtend an angle of 120 degrees.

If I want the segment to contain 80 degrees at o, on B C make an angle B C G, equal 10 degrees, and on the same side of B C as the intended segment; then on G, with radius G C, describe segment C 80 B, in every part of which C and B will subtend an angle of 80 degrees.

Method 2. *By the sector.* Bisect B C as before, and draw the indefinite line D E, make A C radius, and with that extent open the sector at 45 on the line of tangents, and set off on the line D E, the tangent of the difference between the observed angle and 90 degrees, on the same side as the intended segment, if the observed angle is less than 90; on the contrary side if more than 90 degrees.

If the angle is of 90 degrees, A is the center of the circle, and A C the radius.

The intersection of the line D E with any circle, is the center of a segment, corresponding to half the angle of the segment in the first circle. Hence, if the difference between the observed angle and 90°, be more than the scale of tangents contains, find the center to double the angle observed, and the point where the circle cuts the indefinite line, will be the required center.

PROBLEM 1. *To determine the position of a point, from whence three points or a triangle can be discovered, whose distances are known.*

The point is either without, or within the given triangle, or in the direction of two points of the triangle.

CASE 1. *When the three given objects form a triangle, and the point or station whose position is required, is without the triangle.*

Example. Suppose I want to determine the position of a rock D, *fig.* 27, *plate* 9, from the shore; the distances of the three points A, C, B, or rather the three sides AC, CB, AB, of the triangle A, B, C, being given.

In the first place, the angles ADC, CDB, must be measured by an Hadley's sextant or theodolite; then the situation of the point D may be readily found, either by calculation or construction.

By construction. Method 1. On AC, *fig.* 28, *plate* 9, describe by the preceding problem, a circle capable of containing an angle equal to the angle ADC; on CB, a segment containing an angle equal to the angle CDB; and the point of intersection D is the place required.

Another method. Make the angle EBA, *fig.* 39, *plate* 9, equal to the angle ADE, and the angle BAE equal to the angle EDB. Through A, B, and the intersection E, describe a circle AEBD; through E, C, draw EC, and produce it to intersect the circle at D; join AD, BD, and the distances AD, CD, BD, will be the required distances.

Calculation. In the triangle ABC, are given the three sides, to find the angle BAC. In the triangle A E B, are given the angle BAE, the angles A B E, A E B, and the side AB, to find A E and B E.

In the triangle A E D, we have the side A E, and the angles A E D, A D E, and consequently D F A, to find the sides A D.

The angle A D E, added to the angle A E C, and then taken from 180°, gives the angle D A E. The angle C A E, taken from the angle D A E, gives the angle C A D, and hence D C. Lastly, the angle A E C, taken from A E B, gives D E B, and consequently, in the triangle D E B, we have E B, the angle D E B, and the angle E D B, to find B D.

In this method, when the angle B D C is less than that of B A C, the point C will be above the point E; but the calculation is so similar to the foregoing, as to require no particular explanation.

When the points E and C fall too nearly together, to produce E C towards D with certainty, the first method of construction is the most accurate.

CASE 2. *When the given place or station* D, *fig.* 38, *plate* 9, *is without the triangle made by the three given objects* A B C, *but in a line with one of the sides produced.*

Measure the angle A D B, then the problem may be easily resolved, either by construction or calculation.

By construction. Subtract the measured angle A D B from the angle C A B, and you obtain the angle A B D; then at B, on the side B A, draw the angle A B D, and it will meet the produced side C A at D; and D A, D C, D B, will be the required distances.

By calculation. In the triangle A B D, the angle D is obtained by observation, the $\angle$ B A D is the supplement of the angle C A B to 180°: two angles of the triangle being thus known, the third is also known; we have, consequently, in the triangle A B D, three angles and one side given to find the length of the other two sides, which are readily obtained by the preceding canons.

CASE 3. *When the station point is in one of the sides of the given triangle, fig.* 1, *plate* 13.

By construction. 1. Measure the angle B D C. 2. Make the angle B A E equal to the observed angle. 3. Draw C D parallel to E A, and D is the station point required.

By calculation. Find the angle B in the triangle A B C, then the angles B and B D C being known, we obtain D C B; and, consequently, as sin. angle B D C to B C, so is sin. angle D C B to B D.

CASE 4. *When the three given places are in a strait line, fig.* 2, *plate* 13.

Example. Being at sea, near a strait shore, I observed three objects, A, B, C, which were truly laid down on my chart; I wished to lay down the place of a sunken rock D; for this purpose the angles A D B, B D C, were observed with Hadley's quadrant.

By construction. Method 1. On A B, *fig.* 3, *plate* 13, describe the segment of a circle, capable of containing the observed angle A D B. On B C describe the segment of a circle, capable of containing the angle B D C; the point D will be at the intersection of the arcs, and by joining D A, D B, D C, you obtain the required distances.

Method 2. Make the angle A C E, *fig.* 4, *plate* 13, equal to A D B, and the angle E A C equal to B D C; and from the point of intersection E, through B, draw a line to E D, to intersect the arc A D C; join A, D; and D, C, and D A, D B, D C, are the required distances.

By calculation. 1. In the triangle CAE, *fig.* 4, *plate* 13, we have all the angles, and the side AC, to find AE. 2. In the triangle ABE, AB, AE, and the included angle are given, to find the angles AEB, ABE. 3. In the triangle BDC, the angle BDC and DCB (=ABE) are given, and consequently the angle DCB, and the side BC; hence it is easy to obtain DB.

CASE 5. *When the station falls within the triangle, formed by the three given objects, fig.* 19, *plate* 13.

Let ABC represent three towers, whose distance from each other is known; to find the distance from the tower D, measure the angles ADC, BDC, ADB.

Construction. On two of the given sides AC, AB, *fig.* 20 *plate* 13, describe segments of circles capable of containing the given angles, and the point D of their intersection will be the required place.

Another method. Or, we may proceed as in some of the foregoing cases; making the angle ABE, *fig.* 21, *plate* 13, equal to the angle ADE, and BAE equal BDE; and describe a circle through the three points A, B, E, and join E, C, by the line EC; and the point D, where BC intersects the circle E, A, D, B, E, will be the required station.

CASE 6. *When the station point* D, *fig.* 7, *plate* 13, *falls without the triangle* ABC, *but the point* C *falls towards* D.

Thus, let ABC, *fig.* 7, *plate* 13, represent three towers, whose respective distances from each other are known; required their distance from the point D. Measure the angles ADC, BDC, and to prove the truth of the observations, measure also ADB.

Then, *by construction, method* 1. On AC, *fig.* 8, *plate* 13, describe a circle capable of containing the angle BDC, and on AB, one capable of containing the angle A D B, and the point of intersection will be the place required.

Or, it may be constructed by method 2, case 1, which gives us the point D, *fig.* 9, *plate* 13, compared with *fig.* 39, *plate* 9. The calculation is upon principles so exactly like those in method 2, case 1, that a further detail would be superfluous.

The point of station D *found instrumentally.* The point D may be readily laid down on a draught, by drawing on a loose transparent paper indefinite right lines DA, D B, D C, at angles equal to those observed; which being placed on the draught so as each line may pass over, or coincide with, its respective object, the angular point D will then coincide with the place of observation. Or,

Provide a graduated semicircle of brass, about six inches in diameter, having three radii with chamfered edges, each about 20 inches long, (or as long as it may be judged the distance of the stations of the three given objects may require) one of which radii to be a continuation of the diameter that passes through the beginning of the degrees of the semicircle, but immoveably fixed to it, the other two moveable round the center, so as to be set and screwed fast to the semicircle at any angle. In the center let there be a small socket, or hole, to admit a pin for marking the central point on the draught. When the sloped edges of the two moveable radii are set and screwed fast to the semicircle, at the respective degrees and minutes of the two observed angles, and the whole instrument moved on the draught until the edges of the three radii are made to lie along the three stasimetric points, each touching its respective point, the center of the semicircle will then be in the point of station D; which may be marked on the draught, through the socket, with a pin. Such an instrument as this may be called a *station-pointer*; and would prove convenient for finding the point of station readily and accurately, except when the given objects were near; when the breadth of the arc, and of the radii, and of the brass about the center of the semicircle, might hinder the points from being seen, or the radii so placed as to comprehend a very small angle between them.

The three succeeding problems may occur at sea, in finding the distances and position of the rocks, sands, &c. from the shore, in many cases of maritime surveying; they are also very serviceable in making a map of a country, from a series of triangles derived from one or more measured bases.

PROBLEM 1. *Given the distance of two objects* AB, *fig.* 5, *plate* 9, *and the angles* ADB, BDC, BCA, *to find the distance of the two stations* D, C, *from the objects* A, B.

By construction. Assume d c any number at pleasure, and make the angles b d c, a d c, &c. respectively equal to the angles B D C, A D C, &c. and join a b; it is plain that this figure must be similar to that required; therefore draw AB, *fig.* 4, *plate* 9, equal the given distance, and make ABC equal to a b c, BAC to b a c, and so on respectively; join the points, and you have the distances required.

By calculation. in the triangle a d c, we have d c, a d c, and a c d, to find a d, a c; in b c d, we have in like manner the three angles, and d c, to find d b, d c.

In the triangle a d b, we have a d, b d, and the angle a d b, to find a b. Hence by the nature of similar figures, as a b to AB :: d c : DC :: a d : AD :: b d : BD :: b c : BC.

PROBLEM 2. *The distances of three objects* A, B, C, *from each other, and the angles* ADC, CDE, C E D, C E B, *fig.* 6, *plate* 9, *being given, to find the sides* AD, D C, D E, E C, *and* E B.

Assume any line d c, at pleasure, make the angle c d e equal the angle C D E, and angle c e d equal to the angle C E D; also the angle c d a, equal to the angle C D A, and the angle c e b equal to the angle C E B; produce a d, b e to intersect each other at f, and join c f.

It is evident that the figures c d f e, C D F E, are similar; therefore, on A C, *fig.* 7, *plate* 9, describe a segment of a circle, containing an angle A F C equal to a f c; and on C B a segment capable of containing an angle C F B, equal the angle c f b; from the point of intersection F draw F A, F B, F C; make the angle F C D equal the angle f c d, and F C E equal the angle f c e, which completes the construction; then by assuming d e equal to any number, the rest may be found as before.

This method fails when A D is parallel to B E, *fig.* 8, *plate* 9; therefore, having described the segments A D C, B C, draw C F, to cut off a segment equal to the angle C D F, and the right line C G, to cut off a segment equal to the angle C E G; G F will be in the right line D E; therefore, join G F, and produce the line each way, till it intersects the segments, and the points D, E, will be the stations required.

PROBLEM 3. *Four points* B, C, D, F, *fig.* 9, *plate* 9, *or the four sides of a quadrilateral figure, with its angles being given, and the angles* B A C, B A E, A E D, D E F, *known by observation, to find the station point* A E, *and, consequently, the length of the lines* A B, A C, E D, E F.

By construction. 1. On B C describe the segment of a circle, to contain an angle equal to B A C. 2. From C draw the chord C M, so that the angle B C M may be equal to the supplement of the angle B A E. 3. On D F describe the segment of a circle capable of containing an angle equal to D E F; join M N, cutting the two circles at A and E, the required points.

By calculation. In the triangle B C M, the angle B C M, (the supplement of B A E,) and the angle B M C, (equal B A C,) and the side B C are given, whence it is easy to find M C. In the same manner, D N in the triangle D N F may be found; but the angle M C D (equal angle B C D, less angle B C M) is known with the legs M C, C D; consequently, M D, and the angle M D C, will be readily found.

The angle M D N (equal angle C D F, less C D M, less F D N,) and M D, D N, are known; whence we find M M, and the angles D M N, D N M.

The angle C M A, (equal D M C added to D M N,) the angle M A C, (equal M A B added to B A C,) and the side M C are given; therefore, by calculation, M A, and A C will also be known.

In the triangle E D N, the side D N, and the angles E and N are given; whence we find E N, E D, and, consequently, A E equal M N, less M A, less E N.

And in the triangle A B C, the angle A, with its sides B C, A C, are known; hence A B, and angle B C A, are found.

In the triangle E F D, the angle E, with the sides E D, D F, being known, E F and the angle E D F will be found.

Lastly, in the triangle A C D, the angle A C D, (equal B C D, less B C A,) and A C, C D, are given; hence A D is found, as in the same manner E C in the triangle E C D.

Note. That in this problem, and in problems 1 and 2, if the two stations fall in a right line with either of the given objects, the problem is indeterminate. As to the other cases of this problem, they fall in with what has already been said.

The solution of this problem is general, and may be used for the two preceding ones: for suppose C D the same point in the last figure, it gives the solution of problem 2; but if B, C, be supposed the same points D F, you obtain the solution of problem 1.

PROBLEM 4. *Having the distance and magnetic bearings of two points* A *and* B, *fig.* 10, *plate* 9, *protracted at any station* S, *not very oblique to* AB, *to find its distance from these points by the needle.*

At S, with a good magnetic needle, take the bearings of A and B in degrees, and parts of a degree; then from these points, draw out their respective bearings in the opposite direction towards S; that is, if A bears exactly north, draw a line from the point A exactly south; if it bears east, 10 or 20 degrees southward, draw the line west, 10 or 20 degrees northward; and so for any other bearing, draw the opposite bearing of B in the same manner, and S, the intersection of these two points, will be the point of station, and SA, SB, the distances required.

This is an easy method of finding the distance of any station from two places, whose distances have been accurately determined before, and will be found very convenient in the course of a survey, and on many occasions sufficiently exact, provided the places are not too remote from the station, nor the intersection of the bearings too oblique. If the needle be good, a distance of 20 miles is not too far, when the angle subtended by the two places, is not less than 50 degrees, or more than 140.

PROBLEM 5. *To reduce angles to the center of the station.*

In surveys of kingdoms, provinces, counties, &c. where signals, churches, &c. at a distance are used for points of observation, it very often happens that the instrument cannot be placed exactly at the center of the signal or mark of observation; consequently, the angle observed will be either greater, less, or equal to that which would have have been found at the center. This problem shews how to reduce them to the center; the correction seldom amounts to more than a few seconds, and is, therefore, seldom considered, unless where great accuracy is required.

The observer may be considered in three different positions with respect to the center, and the objects; for he is either in a line with the center, and one of these objects, or in an intermediate one, that is, a line from this center to the observer produced, would pass between the objects; or he is, lastly, in an oblique direction, so that a line from the center to him would pass without the objects.

In the first position, *fig.* 11, *plate* 9, where the observer is at o, between the center and one of the objects, the exterior angle m o n is greater than the angle m e n, at the center, by the angle e m o; therefore, taking e m o from the observed angle, you have that at the center.

If the observer is at a, *fig.* 11, *plate* 9, the exterior angle m a n is greater than that of m e n at the center, by the value of n; therefore, taking this from the observed angle, you obtain the angle at the center. But if the observer is further from the objects than the center, as at i, the observed angle m i n is less than that at the center m e n, by the angle m; therefore, by adding m to the observed angle, you obtain the angle m e n at the center. In the same manner, if the observer is at u, we should add the angle n to the observed m u n, in order to have the angle m e n at the center.

Case 2. When the observer is at o, *fig.* 12, *plate* 9, draw a o, and the exterior angle d exceeds the angle u at the center by the angle m, and the exterior angle e exceeds the angle at the center a, by the angle n; therefore, m o n exceeds the angle at the center, by the value of the two angles m and n: these, therefore, must be subtracted from it to obtain the central angle. On the contrary, if the observer is at a, the two angles m and n must be added to the observed angle.

Case 3. *Fig.* 13, *plate* 9. When the observer is at o, having measured the angles m o n, m o e, the angle i is exterior to the two triangles m o i, n e i; therefore, to render m e n, equal to m i n, we must add the angle n; and to render the exterior angle m i n, equal to the observed angle m o n, we must take away the angle m; therefore, adding m to the observed angle, and subtracting n from the total, we obtain the central angle m e n.

From what has been said, it is clear, that in the first case, you are to add or subtract from the observed angles, that of the angles m or n, which is not in the direction of the observer.

In the second position, you have either to subtract or add the two angles m e n.

In the third position, you add to the observed angle, that of the two m or n, which is of the same side with the observer, and subtract the other.

To know the position of the observer, care must be taken to measure the distance of the instrument from the center, and the angle this center makes with the objects.

An inspection of the figures is sufficient to shew how the value of the angles m n may be obtained. Thus, in the triangle m o e, we have the angle at o, the distance o e, and the distances e m, o m, (which are considered as equal,) given.

OF THE REDUCTION OF TRIANGLES, FROM ONE PLAIN TO ANOTHER*.

After the reduction of the observed angles to the center of each respective station, it is generally necessary to reduce the parts of one, or of several triangles to the same level.

Case 1. Let us suppose the three points A, P, E, *fig.* 15, *plate* 9, to be equally distant from the center of the earth, and that the point R is higher than these points by the distance or quantity R E; now it is required to reduce the triangle A P R to that A P E.

By the following rule, you may reduce the angles R A P, R P A, which have their summits in the plain of reduction, to the angle E P A, E A P.

Rule The cosine of the reduced angle is equal to the cosine of the observed angle, divided by the cosine of the angle of elevation.

These two angles being known, the third E is consequently known; we shall, however, give a rule for finding A E P, independent of the other two.

Rule. The cosine of the reduced angle is equal to the cosine of the observed angle, lessened by the rectangle of the sines of the angles of elevation, divided by the rectangle of the cosine of the same angles.

* Traité de Trigonometrie, par Cagnoli.

The reduction of the sides can be no difficulty.

Case 2, *Fig.* 17, *plate* 9. Let A R r be the triangle to be reduced to the plain A E e, the points E, e, of the vertical lines R E, r e, being supposed equally distant from the center of the earth.

Prolong the plain A E e to P, that is, till it meets the line R r produced to P; and the value of E A e will be found by the following formulæ.

1. Tangent $\frac{1}{2}$ (P A R + P A r) = tangent $\frac{1}{2}$ R A r $\times$ $\frac{\text{tangent } \frac{1}{2} (\text{RAE} + \text{rAe})}{\text{tangent } \frac{1}{2} (\text{RAE} - \text{rAe})}$. Knowing the half sum and half difference of P A R and P A r, we obtain the value of each of the angles; the value of P A E and P A e, may be then obtained by the first of the two preceding rules, and the difference between them is the angle sought.

Let C, *fig.* 16, *plate* 9, be the center of the earth, let A B be the side of a triangle reduced to a common horizon by the preceding methods; if it be required to reduce this to the plain D E, as these planes are parallel, the angles will remain the same; therefore, the sides only are to be reduced, the mode of performing which is evident from the figure.

Method of referring a series of triangles to a meridian line, and another line perpendicular to it.

This method will be found somewhat similar to one used by Mr. *Gale*, and described at length in the article of surveying; it is a mode that should be adopted wherever extreme accuracy is required, for whatever care is taken to protract a series of triangles, the protractor, the points of the compasses, the thickness of the line, the inequality of the paper, &c. will produce in the fixing of the points of a triangle an error, which, though small at first, will have its influence on those that succeed, and become very sensible, in proportion as the number of triangles is augmented. This multiplication of errors is avoided by the following problem.

Let AB, *fig.* 14, *plate* 9, be the meridian, CD the perpendicular, and the triangles o a d, d a e, d e g, e g i, g i l, those that have been observed; from the point o, (which is always supposed to be on a meridian, or whose relation to a meridian is known) observe the angle B o a, to know how much the point a declines from the meridian.

In the right-angled triangle o B a, we have the angle B o a, and the right angle, and consequently, the angle o a B, together with the side o a, to find O B, and B a.

For the point d, add the angle B o a to the observed angle a o d, for the ∠ d o b, or its equal o d m, and the complement is the angle m o d, whence as before, to find o m and m d.

For the point G, add the angles m d o, o d a, a d e, and e d g, which subtract from 360, to obtain the angle g d r, of the right-angled triangle g d r; hence we also readily, as in the preceding triangles, obtain r g = m t, which added to m o, gives t o the distance from the meridian. Then we obtain r d, from which taking d m, you obtain r m, equal g t, the distance from the perpendicular.

For the point e, take the right angle r d f, from the two angles r d g, g d e, and the remainder is the angle f d e of the right-angled triangle d f e; hence we obtain f e d and d f, which added to d b, gives b f, equal to x e, the distance from the meridian; from the same right-angled triangle we obtain f e, which added to f n, equal to d m, gives e n, the distance from the perpendicular.

For the point i, add together the angles r g d, d g e, e g i, and from the sum subtract the right angle r g h, and you obtain the angle g h i of the right-angled triangle h g i, and consequently the angle i; hence also we get h i, equal t p, which added to t o, gives o p distance from the meridian, and g h, from which subtracting g t, we obtain t h, equal p i distance from the perpendicular.

For the point l, the angle g h i, added to the angle l g i, gives the angle l g k of the right-angled triangle g k l, and of course the angle g l k, whence we obtain k l or t y, which added to t o, gives o y distance from the meridian; hence we also obtain g k, which taken from g t, gives k t, equal l y distance from the perpendicular.

If, before the operation, no fixed meridian was given, one may be assumed as near as possible from the point o; for the error in its position will not at all influence the respective position of the triangles.

To the mathematical student, who may have a desire to proceed on a complete course of the mathematical sciences, the following eminent authors are recommended.

Algebra, by Simpson, 8vo. 1755; Maclaurin, 8vo. 1771; Bonnycastle's Introduction, 12mo. 1796.

Geometry. Euclid's Elements, by Dr. Simson, 8vo. 1791; Elements, by Simpson, 8vo. 1768, and Emerson, 8vo. 1763.

Trigonometry, by Simpson, 8vo. 1765; Emerson, 8vo. 1788; Traité, by Cagnoli, 4to. Paris, 1786; and a Treatise now in the press, 8vo. by T. Keith.

Conic Sections, by Hamilton, 4to. 1758; Hutton, 8vo, 1787; Elements, by Vince, 8vo. 1781, and Newton, 8vo. 1794.

Fluxions, by Simpson, 8vo. 1776, and Maclaurin, 4to. 2 vols. 1742; Introduction, by Rowe, 8vo. 1767, and Emerson, 8vo, 1768.

Logarithmic Tables, by Taylor, to every second of the quadrant, 4to, 1792, and Hutton, 8vo. 1794.

Mensuration, by Hutton, 8vo, 1788.

Mathematical Dictionary, by Hutton, in 2 vols. 4to. 1796.

To these may be added, *Emerson's Mathematical Works*, in 10 vols. 8vo. and a *course* now publishing at Cambridge by Vince and Wood, in --- vols. 8vo.

EDIT.

OF

SURVEYING.

THE practice of surveying may be considered as consisting of four parts. 1. Measuring strait lines. 2. Finding the position of strait lines with respect to each other. 3. Laying down, or planning upon paper these positions and measures. 4. Obtaining the superficial measure of the land to be surveyed.

We may, therefore, define *land surveying* to be the art which teaches us to find how many times any customary measure is contained in a given piece of ground, and to exhibit the true boundaries thereof in a plan or map.

A station line is a strait line, whose length is accurately ascertained by a chain, and the bearing determined by some graduated instrument.

An offset is the distance of any angular point in the boundary from the station line, measured by a line perpendicular thereto.

The curvature of the earth within the limits of an ordinary survey, is so inconsiderable, that its surface may be safely considered by the land surveyor as *a plain.* In a large extent, as a province, or a kingdom, the curvature of the earth's surface becomes very considerable, and due allowance must be made for it.

All plains, how many sides soever they consist of, may be reduced into triangles, and may therefore be considered as composed thereof; and, consequently, what is required in surveying, are such instruments as will measure the length of a side, and the quantity of an angle of a triangle.

GENERAL RULES.

A few general observations, or hints, can only be expected in this place; for, after all that we can say, the surveyor must depend on his own judgment for contriving his work, and his own skill in discriminating, among various methods, that which is best.

The first business of the surveyor is to take such a general view of the ground to be surveyed, as will fix a map thereof in his mind, and thence determine the situations for his station lines, and the places where his instruments may be used to the greatest advantage.

Having settled the plan of operations, his next business is to examine his instruments, and see that they are all in proper order, and accurately adjusted. He should measure carefully his chain, and if there be any errors therein, correct them; prepare staves, marks, &c. for distinguishing the several stations.

The fewer stations that can be made use of, the less will be the labour of the survey; it will also be more accurate, less liable to mistakes while in the field, or errors when plotting the work at home.

The station lines should always be as long as possible, where it can be done without rendering the offsets too large; where great accuracy is required, these lines should be repeatedly measured, the first great point being the careful mensuration of your station lines; the second, to determine the situation of places adjoining to them. For every station line is the basis of the succeeding operations, and fixes the situation of the different parts.

The surveyor should so contrive his plan, as to avoid the multiplication of small errors, and particularly those that by communication will extend themselves through the whole operation. If the estate be large, or if it be subdivided into a great number of fields, it would be improper to survey the fields singly, and then put them together, nor could a survey be accurately made by taking all the angles and boundaries that inclose it. If possible, fix upon, for station points, two or more eminent situations in the estate, from whence the principal parts may be seen; let these be as far distant from each other as possible; the work will be more accurate when but few of these stations are made use of, and the station lines will be more convenient, if they are situated near the boundaries of the estate.

Marks should be erected at the intersection of all hedges with the station line, in order to know where to measure from, when the fields are surveyed. All necessary angles between the main stations should be taken, carefully measuring in a right line the distance from each station, noting down, while measuring, those distances where the lines meet a hedge, a ditch, &c If any remarkable object be situated near the station line, its perpendicular distance therefrom should be ascertained; in the same manner, all offsets from the ends of hedges, ponds, houses, &c. from the main station line should be obtained, care being taken that all observations from the station line, as the measure of angles, &c. be always made from points in the station line.

When the main stations, and every thing adjoining to them, have been found, then the estate may be subdivided into two or three parts by new station lines, fixing the stations where the best views can be obtained; these station lines must be accurately measured, and the places where they intersect hedges be exactly ascertained, and all the necessary offsets determined.

This effected, proceed to survey the adjoining fields, by observing the angles that the sides make with the station lines at their intersections therewith; the distances of each corner of the field from these intersections, and that of all necessary offsets.

Every thing that could be determined from these stations being found, assume more internal stations, and thus continue to divide and subdivide, till at last you obtain single fields, repeating the same operations, as well for the inner as for the exterior work, till all be finished.

Every operation performed, and every observation made, is to be carefully noted down, as the data for fixing the situations upon the plan. The work should be closed as often as convenient, and in as few lines as possible; what is performed in one day should be carefully laid down every night, in order not only to discover the regular process of the work, but to find whether any circumstance has been neglected, or any error committed, noticing in the field-book, how one field lies by another, that they may not be displaced in the draft.

If an estate be so situated, that the whole cannot be surveyed together, because one part cannot be seen from the other, divide it into three or four parts, and survey them separately, as if they were lands belonging to different persons, and at last join them together.

As it is thus necessary to lay down the work as you proceed, it will be proper to find the most convenient scale for this purpose: to obtain this, measure the whole length of the estate in chains, then consider how many inches long your plan is to be, and from these conditions, you will ascertain how many chains you have in an inch, and thence choose your scale.

In order that the surveyor may prove his work, and see daily that it goes on right, let him choose some conspicuous object that may be seen from all, or greater part of the estate he is surveying, and then measure with accuracy the angle this object makes, with two of the most convenient stations in the first round, entering them in the field-book or sketch where they were taken; when you plot your first round, you will find the true situation of this object by the intersection of the angles. Measure the angle this object makes, with one of your station lines in the second, third, &c. rounds: these angles, when plotted one day after another, will intersect each other in the place of the object, if the work be right; otherwise some mistake has been committed, which must be corrected before the work is carried any further.

Fields plotted from measured lines only, are always plotted nearest the truth, when those lines form at their meeting, angles nearly approaching to right angles.

OF THE MOST ADVANTAGEOUS CIRCUMSTANCES FOR A SERIES OF TRIANGLES.

The three angles of every triangle should always, if possible, be measured.

As it is impossible to avoid some degree of error in taking of angles, we should be careful so to order our operations, that this error may have the least possible influence on the sides, the exact measure of which is the end of the operations.

Now, in a right-lined triangle, it is necessary to have at least one side measured mediately or immediately; the choice of the base is therefore the fundamental operation; the determinations will be most accurate to find one side, when the base is equal to the side required; to find two sides, an equilateral triangle is most advantageous.

In general, when the base cannot be equal to the side or sides sought, it should be as long as possible, and the angles at the base should be equal.

In any particular case, where only two angles of a triangle can be actually observed, they should be each of them as near as possible to 45°; at any rate their sum should not differ much from 90°, for the less the computed angle differs from 90°; the less chance there will be of any considerable error in the intersection.

DESCRIPTION OF THE VARIOUS INSTRUMENTS USED IN SURVEYING, AND THE METHOD OF APPLYING THEM TO PRACTICE, AND EXAMINING THEIR ADJUSTMENTS.

The variety of instruments that are now made use of in surveying is so great, and the improvements that have been made within these few years are so numerous, that a particular description of each is become necessary, that by seeing their respective merits or defects, the purchaser may be enabled to avail himself of the one, and avoid the other, and be also enabled to select those that are best adapted to his purposes.

The accuracy of geometrical and trigonometrical mensuration, depends in a great degree on the exactness and perfection of the instruments made use of; if these are defective in construction, or difficult in use, the surveyor will either be subject to error, or embarrassed with continual obstacles. If the adjustments, by which they are to be rendered fit for observation, be troublesome and inconvenient, they will be taken upon trust, and the instrument will be used without examination, and thus subject the surveyor to errors, that he can neither account for, nor correct.

In the present state of science, it may be laid down as a maxim, that every instrument should be so contrived, that the observer may easily examine and rectify the principal parts; for however careful the instrument-maker may be, however perfect the execution thereof, it is not possible that any instrument should long remain accurately fixed in the position in which it came out of the maker's hands, and therefore the principal parts should be moveable, to be rectified occasionally by the observer.

AN ENUMERATION OF INSTRUMENTS NECESSARY FOR A SURVEYOR;

Fewer or more of which will be wanted, according to the extent of his work, and the accuracy required.

A case of good pocket instruments.
A pair of beam compasses.
A set of feather-edged plotting scales.
Three or four parallel rules, either those of *fig.* A, B and C, *plate* 2, or *fig.* F G H, *plate* 2.
A pair of proportionable compasses.
A pair of triangular ditto.
A pantagraph.
A cross staff.
A circumferentor.
An Hadley's sextant.
An artificial horizon.
A theodolite.
A surveying compass,

Measuring chains, and measuring tapes.
King's surveying quadrant.
A perambulator, or measuring wheel.
A spirit level with telescope.
Station staves, used with the level.
A protractor, with or without a nonius.

To be added for county and marine surveying:

An astronomical quadrant, or circular instrument.
A good refracting and reflecting telescope.
A copying glass.

For marine surveying.

A station pointer.
An azimuth compass.
One or two boat compasses.

Besides these, a number of measuring rods, iron pins, or arrows, &c. will be found very convenient, and two or three offset staves, which are strait pieces of wood, six feet seven inches long, and about an inch and a quarter square; they should be accurately divided into ten equal parts, each of which will be equal to one link. These are used for measuring offsets, and to examine and adjust the chain.

Five or six staves of about five feet in length, and one inch and an half in diameter, the upper part painted white, the lower end shod with iron, to be struck into the ground as marks.

Twenty or more iron arrows, ten of which are always wanted to use with the chain, to count the number of links, and preserve the direction of the chain, so that the distance measured may be *really* in a strait line.

The pocket measuring tapes, in leather boxes, are often very convenient and useful. They are made to the different lengths of one, two, three, four poles, or sixty-six feet and 100 feet; divided, on one side into feet and inches, and on the other into links of the chain. Instead of the latter, are sometimes placed the centesimals of a yard, or three feet into 100 equal parts.

OF THE INSTRUMENTS USED IN MEASURING STRAIT LINES.

OF THE CHAIN.

The length of a strait line must be found mechanically by the chain, previous to ascertaining any distance by trigonometry: on the exactness of this mensuration the truth of the operations will depend. The surveyor, therefore, cannot be too careful in guarding against, rectifying, or making allowances for every possible error; and the chain should be examined previous and subsequent to every operation.

For the chain, however useful and necessary, is not infallible, it is liable to many errors. 1. In itself. 2. In the method of using it. 3. In the uncertainty of pitching the arrows; so that the surveyor, who wishes to obtain an accurate survey, will depend as little as possible upon it, using it only where absolutely necessary as a basis, and then with every possible precaution.

If the chain be stretched too tight, the rings will give, the arrow incline, and the measured base will appear shorter than it really is; on the other hand, if it be not drawn sufficiently tight, the measure obtained will be too long. I have been informed by an accurate and very intelligent surveyor, that when the chain has been much used, he has generally found it necessary to shorten it every second or third day. Chains made of strong wire are preferred.

Gunter's chain is the measure universally adopted in this kingdom for the purpose of land surveying, being exceedingly well adapted for the mensuration of land, and affording very expeditious methods of casting up what is measured. It is sixty-six feet, or four poles in length, and is divided into 100 links, each link with the rings between them is 7.92 inches long, every tenth link is pointed out by pieces of brass of different shapes, for the more readily counting of the odd links.

The English acre is 4840 square yards, and Gunter's chain is 22 yards in length, and divided into 100 links; and the square chain, or 22 multiplied by 22, gives 484, exactly the tenth part of an acre; and ten chains squared are equal to one acre; consequently, as the chain is divided into 100 links, every superficial chain contains 100 multiplied by 100, that is 10.000 square links; and 10 superficial chains, or one acre, contains 100.000 square links.

If, therefore, the content of a field, cast up in square links, be divided by 100.000, or (which is the same thing) if from the content we cut off the five last figures, the remaining figure towards the left hand gives the content in acres, and consequently the number of acres at first sight; the remaining decimal fraction, multiplied by 4, gives the roods, and the decimal part of this last product multiplied by 40, gives the poles or perches.

Thus, if a field contains 16,54321 square links, we see immediately that it contains 16 acres. 54321 multiplied by 4, gives 2,17284, or 2 roods and 17284 parts; these, multiplied by 40, produce 6,91360, or 6 poles, 91360 parts.

Directions for using the chain. Marks are first to be set up at the places whose distances are to be obtained; the place where you begin may be called your first station; and the station to which you measure, the second station. Two persons are to hold the chain, one at each end; the foremost, or chain leader, must be provided with nine arrows, one of which is to be put down perpendicularly at the end of the chain when stretched out, and to be afterwards taken up by the follower, by way of keeping an account of the number of chains. When the arrows have been all put down, the leader must wait till the follower brings him the arrows, then proceeding onwards as before, but without leaving an arrow at the tenth extention of the chain. In order to keep an account of the number of times which the arrows are thus exchanged, they should each tie a knot on a string, carried for that purpose, and which may be fastened to the button, or button-hole of the coat; they should also call out the number of those exchanges, that the surveyor may have a check on them.

It is very necessary that the chain bearers should proceed in a strait line; to this end, the second, and all the succeeding arrows, should always be so placed, that the next foregoing one may be in a line with it, the place measured from, and that to which you are advancing; it is a very good method to set up a staff at every ten chains, as well for the purpose of a guide to preserve the rectilinear direction, as to prevent mistakes.

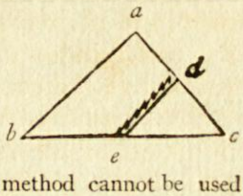
All distances of offsets from the chain line to any boundary which are less than a chain, are most conveniently measured by the offset staff; the measure must always be obtained in a direction perpendicular to the chain.

The several problems that may be solved by the chain alone, will be found in that part of the work, which treats of practical geometry on the ground.*

DESCRIPTION AND USE OF THE SURVEYING QUADRANT, FOR ADJUSTING AND REGULATING THE MEASURES OBTAINED BY THE CHAIN WHEN USED ON HILLY GROUND, INVENTED BY R. KING, SURVEYOR.

There are two circumstances to be considered in the measuring of lines in an inclined situation: the *first* regards the plotting, or laying down the measures on paper; the *second*, the area, or superficial content of the land. With respect to the first, it is evident that the oblique lines will be longer than the horizontal ones, or base; if, therefore, the plan be laid down according to such measures, all the other parts thereof would be thereby pushed out of their true situations; hence it becomes necessary to reduce the hypothenusal

* The best method of surveying by the chain, and now generally used by the more skilful surveyors, I judge, a sketch of here will be acceptable to many readers. It consists of forming the estates into triangles, and applying lines within them parallel and contiguous to every fence and line to be laid down, with offsets from these lines when necessary. The peculiar advantage of this method is, that, after three lines are measured and laid down, every other line proves itself upon application. Thus, if the triangle *a b c* be laid down, and the points *d* and *e* given in the sides, when the line *d e* has been measured for the purpose of taking a fence contiguous to it, it will prove itself when laid down, from the two extremities being given. This method cannot be used in woods, where the principal lines could not be observed, or in surveying roads, or very detached parts of estates; in such cases recourse must be had to the theodolite, or other angular instrument. EDIT.



lines to horizontal, which is easily effected by Mr. *King*'s quadrant.

With respect to the area, there is a difference among surveyors; some contending that it should be made according to the hypothenusal; others, according to the horizontal lines: but, as all have agreed to the necessity of the reduction for the first purpose, we need not enter minutely into their reasons here; for, even if we admit that in some cases more may be grown on the hypothenusal plain than the horizontal, even then the area should be given according to both suppositions, as the hilly and uneven ground requires more labour in the working.

The quadrant AB, *fig.* 1, *plate* 14, is fitted to a wooden square, which slides upon an offset staff, and may be fixed at any height by means of the screw C, which draws in the diagonal of the staff, thus embracing the four sides, and keeping the limb of the square perpendicular to the staff; the staff should be pointed with iron to prevent wear; when the staff is fixed in the ground on the station line, the square answers the purpose of a cross staff, and may, if desired, have sights fitted to it.

The quadrant is three inches radius, of brass, is furnished with a spirit level, and is fastened to the limb D E of the square, by the screw G.

When the several lines on the limb of the quadrant have their first division coincident with their respective index divisions, the axis of the level is parallel to the staff.

The first line next the edge of the quadrant is numbered from right to left, and is divided into 100 parts, which shews the number of links in the horizontal line, which are completed in 100 links on the hypothenusal line, and in proportion for any lesser number.

The second, or middlemost line, shews the number of links the chain is to be drawn forward, to render the hypothenusal measure the same as the horizontal.

The third, or uppermost line, gives the perpendicular height, when the horizontal line is equal to 100.

To use the quadrant. Lay the staff along the chain line on the ground, so that the plain of the quadrant may be upright, then move the quadrant till the bubble stands in the middle, and on the several lines you will have, 1. The horizontal length gone forward in that chain. 2. The links to be drawn forward to complete the horizontal chain. 3. The perpendicular height or descent made in going forward one horizontal chain.

The two first lines are of the utmost importance in surveying land, which cannot possibly be planned with any degree of accuracy without having the horizontal line, and this is not to be obtained by any instrument in use, without much loss of time to the surveyor. For with this, he has only to lay his staff on the ground, and set the quadrant till the bubble is in the middle of the space, which is very soon performed, and he saves by it more time in plotting his survey, than he can lose in the field; for as he completes the horizontal chain as he goes forwards, *the offsets are always in their right places*, and the field-book being kept by horizontal measure, his lines are always sure to close.

If the superficial content by the hypothenusal measure be required for any particular purpose, he has that likewise by entering in the margin of his field-book the links drawn forward in each chain, having thus the hypothenusal and horizontal length of every line.

The third line, which is the perpendicular height, may be used with success in finding the height of timber; thus measure with a tape of 100 feet, the surface of the ground from the root of the tree, and find, by the second line, how much the tape is to be drawn forward to complete the distance of 100 horizontal feet; and the line of perpendiculars shews how many feet the foot of the tree is above or below the place where the 100 feet distance is completed.

Then inverting the quadrant by means of sights fixed on the staff, place the staff in such a position, as to point to that part of the tree whose height you want; and slide the quadrant till the bubble stands level, you will have on the line of perpendiculars on the quadrant, the height of that part of the tree above the level of the place where you are; to which add or subtract the perpendicular height of the place from the foot of the tree, and you obtain the height required.

OF THE PERAMBULATOR, OR IMPROVED MEASURING WHEEL; THE WAY-WISER, AND THE PEDOMETER.

Fig. 6, *plate* 17, represents the perambulator, which consists of a wheel of wood A, shod or lined with iron to prevent the wear; a short axis is fixed to this wheel, which communicates, by a long pinion rod in one of the sides of the carriage B, motion to the wheel-work C, included in the box part of the instrument.

In this instrument, the circumference of the wheel A, is eight feet three inches, or half a pole; one revolution of this wheel turns a single-threaded worm once round; the worm takes into a wheel of 80 teeth, and turns it once round in 80 revolutions; on the socket of this wheel is fixed an index, which makes one revolution in 40 poles, or one furlong: on the axis of this worm is fixed another worm with a single thread, that takes into a wheel of 40 teeth; on the axis of this wheel is another worm with a single thread, turning about a wheel of 160 teeth, whose socket carries an index that makes one revolution in 80 furlongs, or 10 miles. On the dial plate, see *fig.* 7, there are three graduated circles, the outermost is divided into 220 parts, or the yards in a furlong; the next into 40 parts, the number of poles in a furlong; the third into 80 parts, the number of furlongs in 10 miles, every mile being distinguished by its proper Roman figure.

This wheel is much *superior* to those hitherto made, 1. Because the worms and wheels act without shake, and, as they have only very light indices to carry, move with little or no friction, and are, therefore, not liable to wear or be soon out of order; which is not the case with the general number of those that are made, in which there is a long train of wheels and pinions, and consequently much shake and friction. 2. The divisions on the graduated circles are at a much greater distance, and may therefore be subdivided into feet, if required. 3. The measure shewn by the indices is far more accurate, as there is no shake nor any loss of time in the action of one part or another. The instrument is sometimes made with a double wheel for steadiness when using, and also with a bell connected to the wheel-work, to strike the number of miles gone over.

This instrument is very useful for measuring roads, commons, and every thing where expedition is required; one objection is however made to it, namely, that it gives a measure somewhat too long by entering into hollows, and going over small hills. This is certainly the case; the measuring wheel is not an infallible mode of ascertaining the horizontal distance between any two places; but then it may with propriety be asked whether any other method is less fallible? whether, upon the whole, and in the circumstances to which the measuring wheel is usually appropriated, the chain is not equally uncertain, and the measure obtained from it as liable to error, as that from the wheel.

The *way-wiser* is a similar kind of instrument, but generally applied to carriages for measuring the roads or distance travelled. The best method for constructing such a one is represented in *fig.* 8, *plate* 17. A piece of plate iron A is screwed to the inside nave of the wheel; this being of a curvilineal shape, in every revolution of the coach-wheel B it pushes against the sliding bar C, which, at the other end, withinside of the brass box of wheel-work D, is cut with teeth, and thereby communicates motion to the wheel-work in the box. The bar is re-acted upon by a spring in the box, so as to drive it out again for the fresh impulse from the iron piece on the nave, at every revolution. As the wheels of carriages differ in size, the wheel-work is calculated to register the number of revolutions, and shew by three indices on the dial plate to the amount of 20,000. In any distance, or journey performed, the length of feet and inches in the circumference of the wheel must be first accurately measured, and that multiplied by the number shewn on the dial of the way-wiser gives the distance run.

By means of rods, universal joint, &c. it is often made to act within the carriage, so that the person may at any moment, without the trouble of getting out, see the number of the revolutions of the wheel. If the instrument is to be always applied to one wheel, a table may easily be constructed to shew the distance in miles and its parts by inspection only.

The *pedometer* is exactly the same kind of instrument as the way-wiser. The box containing the wheels is made of the size of a watch case, and goes into the fob, or breeches pocket; and, by means of a string and hook fastened to the waistband or at the knee, the number of steps a man takes in his regular paces are registered, from the action of the string upon the internal wheel-work, at every step, to the amount of 30,000. It is necessary to ascertain the distance walked, that the average length of one pace be previously known, and that multiplied by the number of steps registered on the dial plate.

OF THE SURVEYING CROSS, *fig.* 2, *plate* 14.

The cross consists of two pair of sights, placed at right angles to each other: these sights are sometimes pierced out in the circumference of a thick tube of brass about 2½ inches diameter, see *fig.* 3, *plate* 14. Sometimes it consists of four sights strongly fixed upon a brass cross; this is, when in use, screwed on a staff having a sharp point at the bottom to stick in the ground; one of this kind is represented at *fig.* 2, *plate* 14. The four sights screw off to make the instrument convenient for the pocket, and the staff which is about 4½ or five feet in length (for both the crosses) unscrews into three parts to go into a portmanteau, &c.

The surveying cross is a very useful instrument for placing of offsets, or even for measuring small pieces of ground; its accuracy depends on the sights being exactly at right angles to each other. It may be proved by looking at one object through two of the sights, and observing at the same time, without moving the instrument, another object through the other two sights; then turning the cross upon the staff, look at the same objects through the opposite sights; if they are accurately in the direction of the sights, the instrument is correct.*

It is usual, in order to ascertain a crooked line by offsets, first to measure a base or station line in the longest direction of the piece of ground, and while measuring, to find by the cross the places where perpendiculars would fall from the several corners and bends of the boundary; this is done by trials, fixing the instrument so, that by one pair of sights both ends of the line may be seen; and by the other pair, the corresponding bend or corner; then measuring the length of the said perpendicular. To be more particular, let A, h, i, k, l, m, *fig*. 35, *plate* 9, be a crooked hedge or river; measure a strait line, as AB, along the side of the foregoing line, and while measuring, observe when you are opposite to any bend or corner of the hedge, as at c, d, e; from thence measure the perpendicular offsets, as at c h, d i, &c. with the offset staff, if they are not too long; if so, with the chain. The situation of the offsets are readily found, as above directed, by the cross, or *King*'s

* I have made some additions to the box cross staff, which have been found useful and convenient for the pocket, where great accuracy is not required. See *fig*. 6. A compass and needle at the top A to give the bearings, aud a moveable graduated base at B, by rack-work and pinion C, to give an angle to 5′ of a degree by the nonius divided on the box above. Thus the surveyor may have a small theodolite, circumferentor, and cross staff all in one instrument. EDIT.

surveying quadrant; they are to be registered in the field-book.

Of surveying with the chain and cross. What has been denominated by many writers, surveying by the *chain only*, is in fact surveying by the cross and chain; for it is necessary to use the cross, or optical square, for determining their perpendicular lines, so that all that has been said, even by these men, in favour of the *chain alone*, is founded in fallacy. To survey the triangular field A B C, *fig.* 22, *plate* 9, by the chain and cross: 1. Set up marks at the corners of the field. 2. Beginning, suppose at A, measure on in a right line till you are arrived near the point D, where a perpendicular will fall from the angle, let the chain lie in the direction or line AB. 3. Fix the cross over AB, so as to see through one pair of sights the mark at A or B, and through the other, the mark at C; if it does not coincide at C with the mark, the cross must be moved backwards or forwards, till by trials one pair of the sights exactly coincide with the mark at C, and the other with A or B. 4. Observe how many chains and links the point D is from A, suppose 3. 0. 3. which must be entered in the field-book. 5. Measure the perpendicular D C, 643. 7. finish the measure of the base line, and the work is done. *This mode is used at present by many surveyors, probably because there is no check whereby to discover their errors, which must be very great, if the survey is of any extent.*

To plot this, make AB equal 11. 41. AD equal to 3. 0. 3. on the point D erect the perpendicular D C, and make it equal 6. 43. then draw AC, B C, and the triangle is formed.

OF THE OPTICAL SQUARE, *fig.* 4, *plate* 14.

This instrument has the two principal glasses of *Hadley*'s quadrant, was contrived by my father; it is in most, if not in all respects, superior to the common surveying cross, because it requires no staff, may be used in the hand, and is of course of great use to a military officer. It consists of two plain mirrors, so disposed, that an object seen by reflexion from both, will appear to coincide with another object seen by *direct vision* whenever the two objects subtend a right angle from the center of the instrument, and serves therefore to raise or let fall perpendiculars on the ground, as a square does on paper, of which we shall give some examples. Its application to the purposes of surveying will be evident from these, and what has been already said concerning the cross.

Fig. 4, *plate* 14, is a representation of the instrument without its cover, in order to render the construction more evident. There is a cover with a slit or sight for viewing the objects; the object seen directly, always coincides with the object seen by reflexion, when they are at right angles to each other*.

From a given point in a given line, to raise a perpendicular. 1. The observer is to stand with this instrument over the given point, causing a person to stand with a mark, or fixing one at some convenient distance on the given line. 2. An assistant must be placed at a convenient distance, with a mark somewhere near the line in which it is supposed the perpendicular will fall; then if on looking at one of the objects, the other be seen in

* See the description of a considerable improvement upon it, after the description of the Hadley's sextant, *plate* 19. EDIT.

a line with it, the place where the mark of the assistant is fixed is the required point.

From a given point over a given line to let fall a perpendicular. Every strait line is limited and determined by two points, through which it is supposed to pass; so in a field the line is determined by two fixed objects, as steeples, trees, marks erected for the purpose, &c. For the present operation, the two objects that determine must be on one side the point where the perpendicular falls; or, in other words, the observer must not be between the objects, he must place himself over the line, in which he will always be when the two objects coincide; he must move himself backwards on this line, till the mark, from whence the perpendicular is to be let fall, seen by direct vision, coincides with one of the objects which determine the given line seen by reflexion, and the instrument will be over the required point.

To measure inaccessible distances by the optical square.

Required the distance from the steeple A, *fig.* 20, *plate* 9, to B. Let the observer stand with his instrument at B, and direct an assistant to move about C with a staff as a mark, until he sees it coincide by direct vision with the object at A; let him fix the staff there; then let the observer walk along the line until A and B coincide in the instrument, and B D will be perpendicular to A C; measure the three lines B C, D C, B D, and then the following proportion will give the required distance, for as D C is to D B, so is B C to A B.

Second method. 1. Make B C, *fig.* 21, *plate* 9, perpendicular to B A. 2. Divide B C into four equal parts. 3. Make C D perpendicular to C B. 4. Bring F E A into one line, and the distance from C to F will be $\frac{1}{3}$ of the distance from B to A.

OF THE CIRCUMFERENTOR, *fig.* 1, *plate* 15.

This instrument consists of a compass box, a magnetic needle, and two plain sights, perpendicular to the meridian line in the box, by which the bearings of objects are taken from one station to another. It is not much used in England where land is valuable; but in America where land is not so dear, and where it is necessary to survey large tracts of ground, overstocked with wood, in a little time, and where the surveyor must take a multitude of angles, in which the sight of the two lines forming the angle may be hindered by underwood, the circumferentor is chiefly used.

The circumferentor, see *fig.* 1, *plate* 15, consists of a brass arm, about 14 or 15 inches long, with sights at each end, and in the middle thereof a circular box, with a glass cover, of about 5½ inches diameter; within the box is a brass graduated circle, the upper surface divided into 360 degrees, and numbered 10. 20. 30. to 360; every tenth degree is cut down on the inner edge of the circle. The bottom of the box is divided into four parts or quadrants, each of which is subdivided into 90 degrees numbered from the meridian, or north and south points, each way to the east and west points; in the middle of the box is placed a steel pin finely pointed, called the center pin, on which is placed a magnetic needle, the quality of which is such, that, allowing for the difference between the astronomic and magnetic meridians, however the instrument may be moved about, the bearing or angle, which any line makes with the magnetic meridian, is at once shewn by the needle.

At each end of the brass rule, and perpendicular thereto, sights are fixed; in each sight there is a large and small aperture, or slit, one over the other, these are alternate; that is, if the aperture be uppermost in one sight, it will be lowest in the other, and so of the small ones; a fine piece of sewing silk, or a horse hair runs through the middle of the large slit. Under the compass box is a socket to fit on the pin of the staff; the instrument may be turned round on this pin, or fixed in any situation by the milled screw; it may also be readily fixed in an horizontal direction by the ball and socket of the staff, moving for this purpose the box, till the ends of the needle are equidistant from the bottom, and traverse or play with freedom.

Occasional variations in the construction of this instrument, are, 1. In the sights, which are sometimes made to turn down upon an hinge, in order to lessen the bulk of the instrument, and render it more convenient for carriage; sometimes they are made to slide on and off with a dovetail; sometimes to fit on with a screw and two steady pins. 2. In the box, which in some instruments has a brass cover, and very often a spring is placed within the box to throw the needle off the center pin, and press the cap close against the glass, to preserve the point of the center pin from being blunted by the continual friction of the cap of the needle. 3. In the needle itself, which is made of different forms. 4. A further variation, and for the best, will be noticed under the account of the improved circumferentor.

The surveying compass represented at *fig.* 3, *plate* 15, is a species of circumferentor, which has hitherto been only applied to military purposes; it consists of a square box, within which there is a brass circle divided into 360 degrees; in the center of the box is a pin to carry a magnetic needle; a telescope is fixed to one side of the box, in such a manner as to be parallel to the north and south line; the telescope has a vertical motion for viewing objects in an inclined plain; at the bottom of the box is a socket to receive a stick or staff for supporting the instrument.

To use the circumferentor. Let ABC, *fig.* 19, *plate* 9, be the angle to be measured. 1. The instrument being fixed on the staff, place its center over the point B. 2. Set it horizontal, by moving the ball in its socket till the needle is parallel to the bottom of the compass box. 3. Turn that end of the compass box, on which the N. or fleur de lis is engraved, next the eye. 4. Look along BA, and observe at what degree the needle stands, suppose 30. 5. Turn the instrument round upon the pin of the ball and socket, till you can see the object C, and suppose the needle now to stand at 125. 6. Take the former number of observed degrees from the latter, and the remainder 95 is the required angle.

If, in two observations to find the measure of an angle, the needle points in the first on one side 360°, and in the second on the other, add what one wants of 360°, to what the other is past 360°, and the sum is the required angle.

This general idea of the use of the circumferentor, it is presumed, will be sufficient for the present; it will be more particularly treated of hereafter.

When in the use of the circumferentor, you look through the upper sights, from the ending of the station to the beginning, it is called a *back sight*; but when you look through the lower slit from the beginning of the station towards the end, it is termed the *fore sight*. A theodolite, or any instrument which is not set by the needle, must be fixed in its place, by taking back and fore sights at every station, for it is by the foregoing station that it is set parallel; but as the needle preserves its parallelism throughout the whole survey, whosoever works by the circumferentor, need take no more than one sight at every station.

There is, indeed, a difference between the magnetic and astronomic or true meridian, which is called the *variation of the needle*. This variation is different at different places, and is also different at different times; this difference in the variation is called the variation of the variation; but the increase and decrease thereof, both with respect to time and place, proceeds by such very small increments or decrements, as to be altogether insignificant and insensible, within the small limits of an ordinary survey, and the short time required for the performance thereof.*

OF THE IMPROVED CIRCUMFERENTOR.

The excellency and defects of the preceding instrument both originate in the needle; from the regular direction thereof, arise all its advantages; the unsteadiness of the needle, the difficulty of ascertaining with exactness the point at which it settles, are some of its principal defects. In this improved construction these are obviated, as will

* The present variation, or, more properly, the *declination* of the needle, is near 22° W. of the north at London; or two points in general may be allowed on an instrument to the E. to fix the just meridian. Its *inclination*, or dip, was about 72° of the north pole below the horizon in the year 1775. The inclination, as well as the declination of the needle, is found to be continually varying; and, from the observations and hypotheses hitherto made, not to develope any law by which its position can be determined for any future time. EDIT.

be evident from the following description. One of these instruments is represented at *fig.* 2, *plate* 15.

A pin of about three quarters of an inch diameter goes through the middle of the box, and forms as it were a vertical *axis*, on which the instrument may be turned round horizontally; on this axis *an index* AB is fastened, moving in the inside of the box, having a nonius on the outer end to cut, and subdivide the degrees on the graduated circle. By the help of this index, angles may be taken with much greater accuracy than by the needle alone; and, as an angle may be ascertained by the index with or without the needle, it of course removes the difficulties, which would otherwise arise if the needle should at any time happen to be acted upon, or drawn out of its ordinary position by extraneous matter; there is a pin beneath, whereby the index may be fastened temporarily to the bottom of the box, and a screw, as usual, to fix the whole occasionally to the pin of the ball and socket, so that the body of the instrument, and the index, may be either turned round together, or the one turned round, and the other remain fixed, as occasion shall require. A further improvement is that of *preventing all horizontal motion of the ball in the socket*; the ball has a motion in the socket every possible way, and every one of these possible motions is necessary, *except the horizontal one*, which is here totally destroyed, and every other possible motion left perfectly free.*

* The instrument is made to turn into a vertical position, and by the addition of a spirit level to take altitudes and depressions. The index AB has been found to interfere too much with the free play of the needle. In the year 1791 I contrived an external nonius piece *a*, *fig.* 6, to move against and round the graduated circle *b*, either with or without rack-work or pinion. The circle and compass plate are fixed, and the nonius piece and outside rim and sight carried round together when in use. This has been generally approved of. EDIT.

GENERAL IDEA OF THE USE OF THESE INSTRUMENTS.

For this purpose, let *fig.* 1, *plate* 18, represent a field to be surveyed. 1. Set up the circumferentor at any corner, as at A, and therewith take the course or bearing, or the angle that such a line makes with the magnetic meridian shewn by the needle, of the side AB, and measure the length thereof with the chain.

If the circumferentor be a common one, having no index in the box, the course or bearing is taken by simply turning the sights in a direct line from A to B, and when the needle settles, it will point out on the graduated limb the course or number of degrees which the line bears from the magnetic meridian.

But if the circumferentor has an index in the box, it is thus used. 1. Bring the index to the north point on the graduated limb, and fix it there, by fastening the body of the instrument and the under part together by the pin for that purpose, and turn the instrument about so that the needle shall settle at the same point; then fasten the under part of the instrument to the ball and socket, and taking the pin out, turn the sights in a direct line from A towards B, so will the course and bearing be pointed out on the graduated circle, both by the needle and by the index. This done, fasten the body of the instrument to the under part again, and having set the instrument up at B, turn the sights in a direct line back from B to A, and there fasten the under part of the instrument to the ball and socket; then take out the pin which fastens the body of the instrument to the under part, and turn the sights in a direct line towards C, and proceed in the same manner all round the survey; so will the courses or bearings of the several lines be pointed out both by the needle and the index, unless the needle should happen to be drawn out of its course by extraneous matter; but, in this case, the index will not only shew the course or bearing, but will likewise shew how much the needle is so drawn aside. After this long digression to explain more minutely the use of the instruments, we may proceed. 2. Set the circumferentor up at B, take the course and bearing of B C, and measure the length thereof, and so proceed with the sides C D, D E, E F, F G, G A, all the way round to the place of beginning, noting the several courses or bearings, and the lengths of the several sides in a field-book, which let us suppose to be as the following:

			°	′		
1.	A B	North	7		West	21. 00.
2.	B C	North	55	15	East	18. 20.
3.	C D	South	62	30	East	14. 40.
4.	D E	South	40		West	11.
5.	E F	South	4	15	East	14.
6.	F G	North	73	45	West	12. 40.
7.	G A	South	52		West	9. 17.

N. B. By north 7° west, is meant seven degrees to the westward, or left hand, of the north, as shewn by the needle; by north 55° 15′ east, fifty-five degrees fifteen minutes to the eastward, or right hand of the north, as shewn by the needle.

In like manner by south 62° 30′ east, is meant sixty-two degrees and thirty minutes to the eastward, or left hand of the south; and by south 40° west, forty degrees to the westward, or right hand of the south.

The 21 chains, 18 chains 20 links, &c. are the lengths or distances of the respective sides, as measured by the chain.

Fig. 4, *plate* 15, represents a small circumferentor, or theodolite; it is a kind that was much used by General *Roy*, for delineating the smaller parts of a survey. The diameter is 4 inches. It is better to have the sight pieces double, as shewn in *fig.* 2.

OF THE COMMON THEODOLITE.

The error to which an instrument is liable, where the whole dependance is placed on the needle, soon rendered some other invention necessary to measure angles with accuracy; among these, the common theodolite, with four plain sights, took the lead, being simple in construction, and easy in use.

The common theodolite is represented *fig.* 5, *plate* 14; it consists of a brass graduated circle, a moveable index AB; on the top of the index is a compass with a magnetic needle, the compass box is covered with a glass, two sights, C, D, are fixed to the index, one at each end, perpendicular to the plain of the instrument. There are two more sights E F, which are fitted to the graduated circle at the points of 360° and 180°; they all take on and off for the conveniency of packing. In each sight there is, as in the circumferentor, a large and a small aperture placed alternately, the large aperture in one sight being always opposed to the narrow aperture in the other; underneath the brass circle, and in the center thereof, is a sprang to fit on the pin of the ball and socket, which fixes on a three-legged staff.

The circle is divided into degrees, which are all numbered one way to 360°, usually from the left to the right, supposing yourself at the center of the instrument; on the end of the index is a nonius division, by which the degrees on the limb are subdivided to five minutes; the divisions on the ring of the compass box are numbered in a contrary direction to those of the limb.

As much of geometrical mensuration depends on the accuracy of the instrument, it behoves every surveyor to examine them carefully; different methods will be pointed out in this work, according to the nature of the respective instruments. In that under consideration, the index should move regularly when in use; the theodolite should always be placed truly horizontal, otherwise the angles measured by it will not be true; of this position you may judge with sufficient accuracy by the needle, for if this be originally well balanced, it will not be parallel to the compass plate, unless the instrument be horizontal; two bubbles, or spirit levels, are sometimes placed in a compass box at right angles to each other, in order to level the instrument, but it appears to me much better to depend on the needle: 1. Because the bubbles, from their size, are seldom accurate. 2. Because the operator cannot readily adjust them, or ascertain when they indicate a true level.

To examine the instrument; on an extensive plain set three marks to form a triangle; with your theodolite take the three angles of this triangle, and if these, when added together, make 180°, you may be certain of the justness of your instrument.

To examine the needle; observe accurately where the needle settles, and then remove it from that situation, by placing a piece of steel or magnet near it; if it afterwards settles at the same point, it is so far right, and you may judge it to be perfectly so, if it settles properly in all situations of the box. If in any situation of the box a deviation is observed, the error is most probably occasioned by some particles of steel in the metal, of which the compass box is made.

To examine the graduations; set the index division of the nonius to the beginning of each degree of the theodolite, and if the last division of the nonius always terminates precisely, at each application, with its respective degree, then the divisions are accurate.

Cautions in the use of the instrument. 1. Spread the legs that support the theodolite rather wide, and thrust them firmly into the ground, that they may neither yield, nor give unequally during the observation. 2. Set the instrument horizontal. 3. Screw the ball firmly in its socket, that, in turning the index, the theodolite may not vary from the objects to which it is directed. 4. Where accuracy is required, the angles should always be taken twice over, oftner where great accuracy is material, and the mean of the observation must be taken for the true angle.

To measure an angle with the theodolite. Let AB, BC, *fig.* 19, *plate* 9, represent two station lines; place the theodolite over the angular point, and direct the fixed sights along one of the lines, till you see through the sights the mark A; at this screw the instrument fast; then turn the moveable index, till through its sights you see the other mark C; then the degrees cut by the index upon the graduated limb, or ring of the instrument, shew the quantity of the angle.

The fixed sights are always to be directed to the last station, and those on the index to the next. If the beginning of the degrees are towards the surveyor, when the fixed sights are directed to an object, and the figured or N. point towards him in directing the index, then that end of the index towards the surveyor will point out the angle, and the south end of the needle the bearing; the application of the instrument to various cases that may occur in surveying, will be evident from what we shall say on that subject in the course of this work.

OF THE COMMON PLAIN TABLE, *fig.* 1, *plate* 17.

The tabular part of this instrument is usually made of two well-seasoned boards, forming a parallelogram of about 15 inches long, and 12 inches broad; the size is occasionally varied to suit the intentions of the operator.

The aforesaid parallelogram is framed with a ledge on each side to support a box frame, which frame confines the paper on the table, and keeps it close thereto; the frame is therefore so contrived, that it may be taken off and put on at pleasure, either side upwards. Each side of the frame is graduated; one side is usually divided into scales of equal parts, for drawing lines parallel or perpendicular to the edges of the table, and also for more conveniently shifting the paper; the other face, or side of the frame, is divided into 360°, from a brass center in the middle of the table, in order that angles may be measured as with a theodolite; on the same face of the frame, and on two of the edges, are graduated 180°; the center of these degrees is exactly in the middle between the two ends, and about $\frac{1}{4}$th part of the breadth from one of the sides.

A magnetic needle and compass box, covered with a glass and spring ring, slides in a dovetail on the under side of the table, and is fixed there by a finger screw; it serves to point out the direction, and be a check upon the sights.

There is also a brass index somewhat larger than the diagonal of the table, at each end of which a sight is fixed; the vertical hair, and the middle of the edge of the index, are in the same plain; this edge is chamfered, and is usually called the fiducial edge of the index. Scales of different parts in an inch are usually laid down on one side of the index.

Under the table is a sprang to fit on the pin of the ball and socket, by which it is placed upon a three-legged staff.

To place the paper on the table. Take a sheet of paper that will cover it, and wet it to make it expand, then spread it flat upon the table, pressing down the frame upon the edges to stretch it, and keep it in a fixed situation; when the paper is dry it will by contracting become smooth and flat.

To shift the paper on the plain table. When the paper on the table is full, and there is occasion for more, draw a line in any manner through the farthest point of the last station line, to which the work can be conveniently laid down; then take off the sheet of paper, and fix another on the table; draw a line upon it in a part most convenient for the rest of the work; then fold, or cut the old sheet of paper by the line drawn on it, apply the edge to the line on the new sheet, and, as they lie in that position, continue the last station line upon the new paper, placing upon it the rest of the measures, beginning where the old sheet left off, and so on from sheet to sheet.

To fasten all the sheets of paper together, and thus form one rough plan, join the aforesaid lines accurately together, in the same manner as when the lines were transferred from the old sheets to the new one. But if the joining lines upon the old and new sheets have not the same inclination to the side of the table, the needle will not point to the original degree when the table is rectified. If the needle therefore should respect the same degree of the compass the easiest way of drawing the line in the same position is to draw them both parallel to the same sides of the table, by means of the scales of equal parts on the two sides.

To use the plain table. Fix it at a convenient part of the ground, and make a point on the paper to represent that part of the ground.

Run a fine steel pin or needle through this point into the table, against which you must apply the fiducial edge of the index, moving it round till you perceive some remarkable object, or mark set up for that purpose. Then draw a line from the station point, along the fiducial edge of the index.

Now set the sights to another mark, or object, and draw that station line, and so proceed till you have obtained as many angular lines as are necessary from this station.

The next requisite, is the measure or distance from the station to as many objects as may be necessary by the chain, taking at the same time the offsets to the required corners or crooked parts of the hedges; setting off all the measures upon their respective lines upon the table.

Now remove the table to some other station, whose distance from the foregoing was previously measured; then lay down the objects which appear from thence, and continue these operations till your work is finished, measuring such lines as are necessary, and determining as many as you can by intersecting lines of direction, drawn from different stations.

It seems to be the universal opinion of the best surveyors, that the plain table is not an instrument to be trusted to in large surveys, or on hilly situations; that it can only be used to advantage in planning the ground plot of buildings, gardens, or a few small parcels of land nearly on a level.

Mr. *Gardiner*, whose authority as a surveyor is inferior to no one, asserts, that the plain table surveyors, when they find their work not to close right, do often close it wrong, not only to save time and labour, but the acknowledgement of an error; which they are not sure they can amend.

In uneven ground, where the table cannot in all stations be set horizontal, or uniformly in any one place, it is impossible the work should be true in all parts.

The contraction and expansion of the paper according to the state of moisture in the air, is a source of many errors in plotting; for between a dewy morning and the heat of the sun at noon, there is a great difference, which may in some degree be allowed for in small work, but cannot be remedied in surveys of considerable extent.

OF THE IMPROVED PLAIN TABLE, *fig*. 2, *plate* 17.

To remedy some of the inconveniences, and correct some of the errors to which the common plain table is liable, that which we are now going to describe has been constructed. It is usually called *Beighton's plain table*, though differing in many respects from that described by him in the *Philosophical Transactions*.

It is a plain board, 16 inches square, with a frame of box or brass round the edge, for the purpose of being graduated. On the sides, AB, CD, are two grooves and holdfasts for confining firmly, or easily removing the paper; they are disengaged by turning the screws under the table from the right towards the left, and drawn down and made to press on the paper by turning the screws the contrary way. When the holdfasts are screwed down, their surface is even with that of the table. There are two pincers under the table, to hold that part of the paper, which in some cases lies beyond the table, and prevent its flapping about with the wind.

The compass box is made to fit either side of the table, and is fixed by two screws, and does not, when fixed, project above one inch and an half from the side of the table.

There is an index with a semicircle, and telescopic sight, E F G; it is sometimes so constructed, as to answer the purpose of a parallel rule. The figure renders the whole so evident, that a greater detail would be superfluous.

The papers, or charts for this table, are to be either of fine thin pasteboard, fine paper pasted on cartridge paper, or two papers pasted together, cut as square as possible, and of such a length that they may slide in easily, between the upright parts, and under the flat part of the holders.

Any one of these charts may be put into the table at any of the four sides, be fixed, taken out, and changed at pleasure; any two of them may be joined together on the table, by making each of them meet exactly at the middle, whilst near one half of each will hang over the sides of the table; or, by doubling them both ways through the middle, four of them may be put on at one time meeting in the center of the table. For this purpose, each chart is always to be crossed quite through the middle; by these means the great trouble and inaccuracy in shifting the papers is removed.

The charts thus used, are readily laid together by corresponding numbers on their edges, and thus make up the whole map in one view; and, being in squares, are portable, easily copied, enlarged, or contracted.

The line of sight in viewing objects may, if that method be preferred, be always over the center of the table, and the station lines drawn parallel to those measured on the land. Underneath the table is a sprang to fit on the socket of a staff, with parallel plates and adjusting screws.

Mr. *Searle* contrived a plain table, whose size (which renders it convenient, while it multiplies every error) is only five inches square, and consists of two parts, the table and the frame; the frame, as usual, to tighten the paper observed upon. In the center of the table is a screw, on which the index sight turns; this screw is tightened after taking an observation.

THEODOLITES WITH TELESCOPIC SIGHTS.

In proportion as science advances, we find ourselves standing upon higher ground, and are enabled to see further, and distinguish objects better than those that went before us: thus the great advances in dividing of instruments have rendered observers more accurate, and more attentive to the necessary adjustments of their instruments. Instruments are not now considered as perfect, unless they are so constructed, that the person who uses them may either correct, or allow for the errors to which they are liable.

Theodolites with telescopic sights are, without doubt, the most accurate, commodious, and universal instruments for the purposes of surveying, and have been recommended as such by the most expert practitioners and best writers on the subject, as *Gardiner*, *Hammond*, *Cunn*, *Stone*, *Wyld*, *Waddington*, &c.

The leading requisites in a good theodolite are, 1. That the parts be firmly connected, so that they may always preserve the same figure. 2. The circles must be truly centered and accurately graduated. 3. The extremity of the line of sight should describe a true circle.

Fig 1, *plate* 16, represents a theodolite of the second best kind; the principal parts are, 1. A telescope and its level, C, C, D. 2. The vertical arc, B B. 3. The horizontal limb and compass, A A. 4. The staff with its parallel plates, E. The limb A A is generally made about seven inches in diameter.

An attentive view of the instrument, or drawing, compared with what has been said before, will shew that its perfect adjustment consists in the following particulars.

1. The horizontal circle A A must be truly level. 2. The plain of the vertical circle B B must be truly perpendicular to the horizon. 3. The line of sight, or line of collimation,* must be exactly in the center of the circles on which the telescope turns. 4. The level must be parallel to the line of collimation.

Of the telescope CC. Telescopic sights not only enable the operator to distinguish objects better, but direct the sight with much greater accuracy than is attainable with plain sights; hence also we can make use of much finer subdivisions. The telescope, generally applied to the best instruments, is of the achromatic kind, in order to obtain a larger field, and greater degree of magnifying power. In the focus of the eye-glass are two

* The line of collimation is the line of vision, cut by the intersecting point of the cross hairs in the telescope, answering to the visual line, by which we directly point at objects with plain sights.

very fine hairs, or wires, at right angles to each other, whose intersection is in the plain of the vertical arc. The object glass may be moved to different distances from the eye glasses, by turning the milled nut *a*, and may by this means be accommodated to the eye of the observer, and the distance of the object. The screws for moving and adjusting the cross hairs are sunk a little within the eye tube, and at about one inch from the eye end: there are four of these screws, two of which are exactly opposite to each other, and at right angles to the other two. By easing one of the screws, and tightening the opposite one, the wire connected with it may be moved in opposite directions. On the outside of the telescope are two metal rings, which are ground perfectly true; these rings are to lay on the supporters c, c, called Y's, which are fixed to the vertical arc.

Of the vertical arc B B. This arc is firmly fixed to a long axis which is at right angles to the plain of the arc. This axis is sustained by, and moveable on the two supporters, which are fixed firmly to the horizontal plate: on the upper part of the vertical arc are the two Y's for holding the telescope; the inner sides of these Y's are so framed, as to be tangents to the cylindric rings of the telescope, and therefore bear only on one part. The telescope is confined to the Y's by two loops, which turn on a joint, and may therefore be readily opened and turned back, when the two pins are taken out.

One side of the vertical arc is graduated to every half degree, which are subdivided to every minute of a degree by the nonius. It is numbered each way from 0 to 90°, towards the eye end, for angles of altitude; from 0 to 50°, towards the object end, for angles of depression. *On the other* *side* of the vertical arc are two ranges of divisions, the lowermost for taking the upright height of timber in 100th parts of the distance the instrument is placed at from the tree at the time of observation. The uppermost circle is for reducing hypothenusal lines to horizontal, or to shew the difference between the hypothenuse and base of a right-angled triangle, always supposing the hypothenuse to consist of 100 equal parts; consequently, it gives by inspection the number of links to be deducted from each chain's length, in measuring up or down any ascent or descent, in order to reduce it to a true horizontal distance, similar to those on *King*'s quadrant, p. 205.

The vertical arc is cut with teeth, or a rack, and may be moved regularly and with ease, by turning the milled nut b; there is sometimes placed about the nonius a steady pin, by which it may be fixed when at the o, or zero point of the divisions.

Of the compass. The compass is fixed to the upper horizontal plate; the ring of the compass is divided into 360°, which are numbered in a direction contrary to those on the horizontal limb. The bottom of the box is divided into four parts, or quadrants, each of which is subdivided to every 10 degrees, numbered from the meridian, or north and south points each way to the east and west points. In the middle of the box is a steel pin finely pointed, on which is placed the magnetic needle; there is a wire trigger for throwing the needle off its point when not in use.

Of the horizontal limb AA. This limb consists of two plates, one moveable on the other; the outside edge of the upper plate is chamfered, to serve as an index to the degrees on the lower. The upper plate, together with the compass, vertical arc, and telescope, are easily turned round by a pinion fixed to the screw c; d is a nut for fixing the index to any part of the limb, and thereby making it so secure, that there is no danger of its being moved out of its place, while the instrument is removed from one station to another. The horizontal limb is divided to half degrees, and numbered from the right hand towards the left, 10, 20, 30, &c. to 360; the divisions are subdivided by the nonius scale to every minute of a degree.

On the upper plate, opposite to the nonius, are a few divisions similar to those on the vertical arc, giving the 100th parts for measuring the diameter of trees, buildings, &c.

The whole instrument fits on the conical ferril of a strong brass headed staff, with three substantial wooden legs; the top, or head of the staff, consists of two brass plates E, parallel to each other; four screws pass through the upper plate, and rest on the lower plate; by the action of these screws the situation of the plate may be varied, so as to set the horizontal limb truly level, or in a plain parallel to the horizon; for this purpose, a strong pin is fixed to the underside of the plate, this pin is connected with a ball that fits into a socket in the lower plate; the axis of the pin and ball are so framed, as to be always perpendicular to the plate, and, consequently, to the horizontal limb.

To adjust the theodolite. As so much of surveying depends on the accuracy of the instruments, it is absolutely necessary that the surveyor should be very expert in their adjustments, without which he cannot expect the instruments will properly answer the purposes they were designed for, or that his surveys will have the requisite exactness.

The necessary adjustments to the theodolite, which we have just described, are, 1. That the line of sight, or collimation, be exactly in the center of the cylindric rings round the telescope, and which lie in the Y's. 2. That the level be parallel to this line, or the axis of the above-mentioned rings. 3. The horizontal limb must be so set, that when the vertical arc is at zero, and the upper part moved round, the bubble of the level will remain in the middle of the open space.

Previous to the adjustments, place the instrument upon the staff, and set the legs thereof firmly upon the ground, and at about three feet from each other, so that the telescope may be at a proper height for the eye, and that two of the screws on the staff that are opposite to each other may be nearly in the direction of some conspicuous and distant object.

To adjust the line of collimation. Having set up the theodolite agreeable to the foregoing direction, direct the telescope to some distant object, placing it so that the horizontal hair, or wire, may exactly coincide with some well defined part of the object; turn the telescope, that is, so that the tube of the spirit level D may be uppermost, and observe whether the horizontal hair still coincides with the object; if it does, the hair is in its right position; if not, correct half the difference by moving the hair, or wire, which motion is effected by easing one of the screws in the eye tube, and tightening the other; then turn the telescope round to its former position, with the tube of the spirit level lowermost, and make the hair coincide with the object, by moving the vertical arc; reverse the telescope again, and if the hair does not coincide with the same part of the object, you must repeat the foregoing operation, till in both positions it perfectly coincides with the same part of the object.

The precise situation of the horizontal hair being thus ascertained, adjust the vertical hair in the same manner, laying it for this purpose in an horizontal position: the spirit tube will, during the adjustment of the vertical hair, be at right angles to its former position. When the two wires are thus adjusted, their intersection will coincide exactly with the same point of the object, while the telescope is turned quite round; and the hairs are not properly adjusted, till this is effected.

Adjustment of the level. To render the level parallel to the line of collimation, place the vertical arc over one pair of the staff screws, then raise one of the screws, and depress the other, till the bubble of the level is stationary in the middle of the glass; now take the telescope out of the Y's, and turn it end for end, that is, let the eye end lay where the object end was placed; and if, when in this situation, the bubble remains in the middle as before, the level is well adjusted; if it does not, that end to which the bubble runs is too high; the position thereof must be corrected by turning with a screw-driver one or both of the screws which pass through the end of the tube, till the bubble has moved half the distance it ought to come to reach the middle, and cause it to move the other half by turning the staff screws. Return the telescope to its former position, and if the adjustments have been well made, the bubble will remain in the middle; if otherwise, the process of altering the level and the staff screws, with the reversing, must be repeated, till it bears this proof of its accuracy. In some instruments there is s provision for raising or lowering the Y's a small degree, in order more conveniently to make the bubble continue in its place when the vertical arc is at o, and the horizontal limb turn round.

To adjust the level of the horizontal limb. Place the level so that it may be in a line with two of the staff screws, then adjust it, or cause the bubble to become stationary in the middle of the open space by means of these screws. Turn the horizontal limb half round, and if the bubble remains in the middle as before, the level is well adjusted; if not, correct half the error by the screws at the end of the level, and the other half by the staff screws. Now return the horizontal limb to its former position, and if it remains in the middle, the errors are corrected; if not, the process of altering must be pursued till the error is annihilated. See this adjustment in the description of *Ramsden's* theodolite.

When the bubble is adjusted, the horizontal limb may always be levelled by means of the staff screws.

OF THE THEODOLITE, AS IMPROVED BY RAMSDEN, *fig.* 2, *plate* 16.

Among the improvements the instruments of science have received from Mr. *Ramsden*, and the perfection with which he has constructed them, we are to rank those of the theodolite; in the present instance, he has happily combined elegance and neatness of form, with accuracy of construction; and the surveyor will contemplate with pleasure this instrument, and the various methods by which the parts concur to give the most accurate result.

The principal parts of this instrument are however so similar to the foregoing, that a description thereof must, in some degree, be a repetition of what has been already described, and requires less detail here.

F F represents the horizontal limb, of six, seven, or eight inches in diameter, but generally of seven inches, so called, because when in use, it ought always to be placed parallel to the horizon. It consists, like the former, of two plates, the edges of these two are chamfered, so that the divisions and the nonius are in the same plain, which is oblique to the plain of the instrument. The limb is divided into half degrees, and subdivided by the nonius to every minute; it is numbered to 360° from the north towards the east: besides these, the tangents to 100 of the radius are laid down thereon.

The upper plate is moved by turning the pinion G: on this plate are placed, at right angles to each other, two spirit levels for adjusting more accurately the horizontal limb.

N O P is a solid piece fitted on the upper horizontal plate, by means of three capstan headed screws, passing through three similar screws. By the action of these, the vertical arc may be set perpendicular to the horizontal limb, or be made to move in a vertical plain. On this solid piece, are fixed two stout supports, to carry the axis of the vertical arc, which arc is moveable by the pinion E. On the upper part of the vertical arc, are the Y's and loops to support and confine the telescope; the Y's are tangents to the cylindric rings of the telescope, which rings are turned, and then ground as true as possible, and are prevented from moving backwards or forwards, by means of two shoulders. The telescope is achromatic, and about twelve inches in length, and may be adjusted to the eye of the observer, or the distance of the object, by turning the milled nut B. The hairs are adjusted by the screws in the eye tube at A. Under the telescope is fixed a spirit level C, the distance of whose ends from the telescope may be regulated by the screws c, c.

Beneath the horizontal limb there is a second or auxiliary telescope, which has both an horizontal and vertical motion: it is moved horizontally by the milled screw H, and when directed to any object, is fixed in its situation by another milled screw; it moves vertically on the axis; there is an adjustment to this axis, to make the line of collimation move in a vertical plain. By the horizontal motion, this telescope is easily set to what is called the backset stations; the under telescope keeping in view the back object, while the upper one is directed to the fore object. Underneath the lower telescope is a clip to fasten occasionally the main axis; this clip is tightened by the finger screw L, and when tightened, a small motion of the adjusting screw K will move the telescope a few degrees, in order to set it with great accuracy. Beneath these is the staff, the nature of which will be sufficiently evident from what was said thereon in the description of the last theodolite, or by inspection of the figure.

To adjust the levels of the horizontal plate. 1. Place the instrument on its staff, with the legs thereof at such a distance from each other, as will give the instrument a firm footing on the ground. 2. Set the nonius to 360, and move the instrument round, till one of the levels is either in a right line with two of the screws of the parallel plates, or else parallel to such a line. 3. By means of the two last mentioned screws, cause the bubble in the level to become stationary in the middle of the glass. 4. Turn the horizontal limb by the milled nut half round, or till the nonius is at 180, and if the bubble remains in the middle as before, the level is adjusted; if it does not, correct the position of the level, by turning one or both the screws which pass through its ends, till the bubble has moved half the distance it ought to come, to reach the middle, and cause it to move the other half by turning the screws of the parallel plates. 5. Return the horizontal limb to its former position, and if the adjustments have been well made, the bubble will remain in the middle; if otherwise, the process of altering must be repeated till it bears this proof of accuracy. 6. Now regulate the screws of the staff head, so that the bubble remain in the middle while the limb is turned quite round. 7. Adjust the other level by its own proper screws, to agree with that already adjusted.

To adjust the level under the telescope. 1. The horizontal plate being levelled, set the index of the nonius of the vertical arc to o, pull out the two pins, and open the loops which confine the telescope. 2. Adjust the bubble by its own screws. 3. Reverse the level, so that its right hand end may now be placed to the left; if the bubble continues to occupy the middle of the glass it is in its right position; if not, correct one half of the error by the capstan screws under the plate, and the other half by the screws under the level. 4. Reverse the level, and correct, if there is any occasion, continuing the operation till the error vanishes, and the bubble stands in the middle in both positions.

To adjust the line of collimation. 1. Direct the telescope, so that the horizontal wire may coincide with some well defined part of a remote object. 2. Turn the telescope so that the bubble may be uppermost; if the wire does not coincide with the same part of the object as before, correct half the difference by moving the vertical circle, and the other half by moving the wire, which is effected by the screws in the eye tube of the telescope; and so on repeatedly, till the difference wholly disappears. Lastly, adjust the vertical wire in the same manner; when the two wires are properly adjusted, their intersection will coincide exactly with the same point of an object, while the telescope is turned quite round.

VARIATIONS IN THE CONSTRUCTION OF THEODOLITES WITH TELESCOPIC SIGHTS.

To accommodate those who may not wish to go to the price of the foregoing instruments, others have been made with less work, in order to be afforded at a lower price; one of these is represented at *fig.* 5, *plate* 15. It is clear from the figure, that the difference consists principally in the solidity of the parts, and in there being no rack-work to give motion to the vertical arc and horizontal limb. The mode of using it is the same with the other, and the adjustment for the line of collimation, and the level under the telescope, is perfectly similar to the same adjustments in the instruments already described, a further description is therefore unnecessary.

A larger kind is also made without rack-work, similar to *fig.* 1, *plate* 16.

A small one, about four inches in diameter, was invented by the late Mr. *Benj. Martin*, in which the telescope, about six inches in length, with a level, has no vertical motion, but the horizontal motion is given by a pinion; and it may be turned into a vertical position to take angles of altitudes or depressions. The divisions by the nonius are to five minutes.

THE FOLLOWING ACCOUNT OF THEODOLITES IS ADDED BY THE EDITOR.

It generally happens, that the observer has occasion to take the vertical and horizontal angles at the same time, by portable as well as by larger theodolites; the following is, therefore, recommended as the most complete and portable instrument hitherto made, and is in truth almost the best theodolite in miniature. Its construction renders it somewhat more expensive than those before described. It is the one that the late author alluded to in a note, page 315 of the former edition of this book, but had not time to describe it.

*Fig.*7, *pl.*14, is a representation of the instrument. The graduated limb and index plate A, A, are about four inches in diameter, and move by rack and pinion B; it reads off by means of the nonius to three minutes of a degree; If the observer should not object to very fine divisions, it may be to two minutes of a degree. The achromatic telescope C is about six inches in length, and contains a small spirit bubble at C, partly sunk into the tube; it turns upon a long axis, and is moved very accurately by rack and pinion on the arc at D. This arc is necessarily of a short length, but admits about 30 degrees motion on each side of 0, for altitudes or depressions. The staff, which from one piece opens into a tripod, is about five feet in length, and has the parallel plates of adjustment at the top. A small screw from these screws into a socket under the limb A, and by an external rim of metal, the horizontal motion only of the theodolite is produced, when the plates are properly set by the screws. The telescope rests upon a cradle, and by opening the two semicircles *a, a,* it may be reversed, in order to adjust the spirit level, or prove its truth to the axis of the telescope. The adjustments of this little instrument, being in all respects similar to, and made, as in those just described, they will be evident to the reader, and quite unnecessary to repeat here.

The instrument, exclusive of its staff, packs into a pocket mahogany case, of 6 inches in length, 4¼ inches in breadth, and 3¾ inches in depth.

In surveys of very great extent and importance, or in great trigonometrical operations, a larger instrument is required, in order that the subdivisions may be greater in number, or the angle taken more accurately, to five, two, or even one second of a degree. Several plans have been suggested, but I do not at present see any better principle to adopt than that of the great one by Mr. *Ramsden* hereafter to be described. A proportionate reduction of its size, as well as simplifying its machinery and movements, necessary only for the grand purpose that it was applied to, would accommodate the practitioner with as complete an instrument as he could desire. The diameter of the horizontal circle I would recommend to be from about 15 to 20 inches, and the other parts in proportion. The price, according to the workmanship, would be from about 60 to 120 guineas, stand, cases, &c. included.

In the preceding impression of this work, the ingenious author, now deceased, made the frontispiece plate a representation of a new theodolite of his own contrivance, the adjustments of which he thought to be more perfect than those of any other, and annexed the description and mode of its adjustments to his preface. Future trials, however, on his own part, as well as by the hands of others, gave me reason to conclude that it was not answerable to the intended improvements; although more complicated and costly than that represented on *fig.* 2, *plate* 16, yet it was less susceptible of accuracy, and not so simple and easy in the adjustments; I have, therefore, thought proper to dispense with it here, and substitute a short description of the largest, most accurate and elegant theodolite ever made.

It is hardly necessary to acquaint the intelligent reader, that the theodolite is a kind of general angular instrument, not useful merely to ascertain angles for the surveyor, but also for many purposes in practical astronomy, and other sciences that have trigonometry as their fundamental basis. Some years ago it was found necessary to institute a course of trigonometrical operations in this country and in France, in order to determine with precision the distance between the Royal Observatories of Greenwich and Paris. The late General *Roy* was deputed as the chief manager in this country. A very accurate theodolite to take angles, and other instruments, were essentially necessary; and the General was fortunate enough to obtain the best articles and assistance that was ever afforded in any mathematical undertaking whatever.

The frontispiece to this book gives a general view of the theodolite, the reader must not expect from this a complete representation of all the minuter parts. In the General's account in the *Philosophical Transactions*, containing 26 quarto pages, and six large plates, replete with explanatory figures, he confined himself only to the describing of the principal parts; and the limits of this work will only admit of a summary description to convey some idea of its plan, and to shew the ingenious operator the superior utility and accuracy of the instrument. If a further and more particular knowledge be desired, it will be best obtained by a reference to page 135, vol. 80 of the *Transactions* before cited. A A a brass circle three feet diameter; B the principal, or transit achromatic telescope of 36 inches focal length, and 2½ inches aperture, admitting its adjustment by inversion on its supports, as performed by the transits in fixed observatories; C a small lanthorn fixed to an horizontal bar for giving light to the axis of the telescope upon an illuminator that reflects light on the wire in nocturnal observations; D a semicircle of six inches radius, attached to the axis of the transit. Each degree being divided into two parts, or 30′, and one revolution of the micrometer head moving the wire in the field of the microscope at *a* three minutes: therefore 10 revolutions produce 30′, which are shewn by a scale of 10 notches in the upper part of the field of the microscope, each notch corresponding to three minutes, or 180 seconds, and the head being divided into three minutes, and each minute into 12 parts; therefore 12 parts is equal to five seconds. When the angles of altitude and depression to be determined, are very small, they are measured by the motion of an horizontal wire in the focus of the eye glass of the telescope at *a*. Two spirit levels are used to this telescope; one to level the axis, making the long conical axis of the instrument truly vertical, not shewn in the plate, and the other level E is suspended on a rod attached to the telescope, and serves to make it horizontal when vertical angles are to be taken, The vertical bar F extending across the top of the axis, is supported by two braces G G that come from the cone, ⅓d above the plain of the instrument.

The great divided circle is attached by 10 brass conical tubes, or radii, to a large vertical conical axis H, of 24 inches in height, called the exterior axis. Within the base of this hollow axis, a cast steel collar is strongly driven; and on its top is inserted a thick bell-metal plate, with sloping cheeks, which by means of five screws can be raised or depressed a little.

The instrument rests on three feet, one of which is shewn at I, united at the center by a strong round plate of bell-metal, upon which rises another vertical hollow cone, going into the other, H, and is called the interior axis; a cast steel pivot in its top, with sloping cheeks, passes through the bell-metal plate at the top of the exterior axis, being ground to fit one another. The bell-metal base of this interior axis is also ground to fit the steel collar of that without it. When put together, the circle is to be lifted up by laying hold of its radii, and the exterior placed upon the interior axis, the cheeks at top, adjusted to their proper bearings, will then turn round smoothly and steadily, and free from any central shake; the great circle, exterior axis, and upper telescope, therefore, are moveable, independent of the lower parts.

The feet of the mahogany stand K, form a square of about three feet four inches at bottom, and by the separation of the legs, make an octagon at the top or the first plain; in the center of which is an opening, nine inches in diameter. On the top of this lies another mahogany octagonal plain, of rather greater dimensions than the former, with a circular curb about ½ an inch within the plain of its sides. This hath in its center an open conical brass socket, three inches in diameter; and on four of its opposite sides there are fixed four screws, acting against pieces of brass on the top of the stand. The plain, with every thing upon it, may, therefore, be moved in four opposite directions, until the plummet L, is brought to coincide with the station points underneath, in order to level the stand. The third or uppermost plain of mahogany is part of the instrument, being connected by screws, and carrying the handles whereby it is lifted up for use. In the middle of this bottom to the instrument there is another conical brass socket, ¾ inches in diameter, that turns easily on that in the center of the octagon underneath. In the cover of this socket is an hole concentric to the instrument, to admit the thread or wire to pass, which suspends the plummet at L. There is a small box with a winch handle at M, that serves occasionally to raise or lower the plummet. To the three feet there are screws, such as at N, for levelling the instrument; and also three blocks of box wood, and three brass conical rollers under the feet screws, fixed to the lower surface of the mahogany, to give the whole a perfectly easy motion; O, O, O, are three of the four screws attached to the octagonal plain, for accurately centering the instrument by the plummet. P and Q represent two positions of screws to give a circular motion to the entire machine, but these having been found to act by jerks from the great weight, another apparatus or clamp, see *fig.* 2, was adjusted, attached to the curbs, consisting of a brass cock fixed to, and projecting from the curb of the instrument; the cock being acted upon by two screws working in opposite directions, and which are clamped to the curb of the octagon.

The curb upon which the feet of the instrument rest, carries the mahogany balustrade R R, fitted to receive a mahogany cover, that guards the whole instrument. In this cover are four small openings, one for each of the vertical microscopes S, S, one for the clamp of the circle, and one for the socket of the *Hook*'s joint. This cover secures the circles and its cones from dirt, and serves conveniently for laying any thing upon, that may be wanted near at hand; and particularly lanthorns used at night, for reading off the divisions on the limb of the instrument.

There is a lower telescope T, lying exactly under the center of the instrument, and directed through one of the openings on the balustrade, and used only for terrestrial objects, requires but a small elevation, and has an axis of 17 inches in length, supported by the braces attached to the feet. It is moved by rack-work, by turning the pinion at V. There is a small horizontal motion that can be given to the right hand of the axis of the end of this telescope. The whole instrument being nicely levelled, the upper telescope at zero, and likewise on its object, the lower telescope by help of this adjustment is brought accurately to the same object, from the point of commencements from which the angles are to be measured.

There are three flat arms, one of which is represented at U, fixed by screws to the edge of the bell-metal plate. These arms are also braced to the feet of the instrument, rising as they project outwards towards the circumference of the circle, going beyond it about 1½ inches. One arm, lying directly over one of the feet, is that to which is attached wheels and screws moved by *Hook's* joint, not seen in the figure, and also a clamp to the circle. It is this that produces the accurate motion of the circle. The other two arms, one of which also lies over a foot, and the other directly opposite to it, become the diameter of the circle, having their extremities terminated on a kind of blunted triangular figure, forming the bases of pedestals, whereon stand the vertical microscopes S, S. The arms, braces, base, &c. are every where pierced, in order to lessen weight without diminishing strength.

The angles are not read off in this instrument by a nonius as common to others, but with *microscopes*, and which form the most essential part of the instrument. But a short account of them can be given here, an adequate idea can only be obtained by a reference to the *Philosophical Transactions*, page 145 and 149. The horizontal microscope for the vertical angles has been already mentioned. The two vertical ones S, S, are used for reading off the divisions on the opposite sides of the circle immediately under them. Each microscope contains two slides, one over the other, their contiguous surfaces in the foci of the eye-glasses. The upper one is a very thin brass plate, at its lower surface is attached a fixed wire, having no other motion than what is necessary for adjustment, by the left hand screw to its proper dot, as hereafter to be explained. The other slide is of steel of one entire piece, directly under the former, of sufficient thickness to permit a micrometer screw of about 72 threads in an inch to be formed of it. To its upper surface is fixed the immoveable wire, which changes its place by the motion of the micrometer head. This head is divided into 60 equal parts, each of which represents one second or angular motion of the telescope. This steel slide is attached by a chain to the spring of a watch coiled up within a small barrel adjacent to it in the frame. By this no time whatever is lost, the smallest motion of the head being instantly shewn by a proportionable motion of the wire in the field of the microscope.

Each microscope is supported between its pillars, and can be a little raised or depressed in respect to the plain of the circle by two levers. By this motion distinctness is obtained of the wires, and by the motion of the proper screw of the abject lense, which follows that given to the whole microscope, the scale is so adjusted, that 15 revolutions of the head shall move the wire over 15′, or one grand division of the limb, equal to 900″, each degree on the circle being only divided into four parts. To effect this, at the same time the fixed wire must bisect the dot on a gold tongue, the moveable wire must also bisect the dot at 180° on the limb, as well as a first notch in the magnified scale at the bottom of the plate. In this adjustment there is another circumstance to be attended to, viz. that 60 on the micrometer head should stand nearly vertical, so as to be conveniently seen; a few seconds of inclination are of no consequence, because the dart, or index, being brought to that position, whatever it may be, must always remain there, any derangement of the instrument excepted. But if, when the wires coincide with their respective dots, and the first notch 60 on the micrometer head should happen to be underneath, or so far from the vertex side as to be seen with difficulty, then the gold tongue is to be moved a little by capstan head screws, which act against each other on the opposite extremities of the axis. Thus, by repeated trials the purpose will be effected, viz. the 60, to which the dart is to be set, will stand in a place easily seen. It is not to be expected that each microscope will give just 900 seconds for the run of 15 minutes; without loss of time this cannot be done; besides, two observers will adjust the microscopes differently. After several trials of the runs in measuring 15 minutes in the different parts of the limb, one microscope gave 896″, while the other gave at a medium 901″; in a year afterwards, the former gave 900″, while the latter gave 894″. These differences were allowed for in the estimation of angles for computation.

The gold tongue mentioned is extremely thin, and goes close to the surface of the circle. This contrivance of a tongue with a dot was to guard against any error from any accidental motion given to the instrument between the observations, and if any, it immediately detected them. This was also a severe check upon the divisions of the instrument. General *Roy* observes, that it rarely happens that two observers, reading off with the opposite microscopes, differ more than half a second from each other at the first reading; and judges, that in favourable weather for repeating the observation with the telescope, a wonderful degree of accuracy in the measure of the angles may be obtained.*

For the auxiliary apparatus, such as the 100 *feet steel chain*, *portable scaffold*, *tripod ladder*, *common flag-staff*, *tripod for white lights*, *portable crane*, *&c.* the reader will see the account of in the Transactions before cited. The horizontal angles taken by the instrument as regulated by the General, since deceased, are to the tenth of a second.

* The weight of the whole instrument was about 200 pounds, and the price, as I have been informed, about 350 guineas. By the completion of the measurements and the necessary calculations, the difference of the two meridians made 9′ 20″, as before fixed by Dr. *Maskelyne*.

A second instrument has since been made, and is now using by Col. *Williams*, Cap. *Mudge*, and Mr. *Dalby*, from whose skill and ingenuity it is expected a very *accurate* survey of this country will be made. In this instrument, the great circle is divided to 10 minutes, improvements made in the microscopes, &c. by Mr. *Ramsden*. See *Philosophical Transactions* for 1795.

DESCRIPTION, USE,

AND METHOD OF ADJUSTING

HADLEY'S QUADRANT.*

"At the appointed time, when it pleased the Supreme Dispenser of every good gift to restore light to a bewildered world, and more particularly to manifest his wisdom in the simplicity, as well as in the grandeur of his works, he opened the glorious scene with the revival of sound astronomy." This observation of an excellent philosopher and physician † is verified in every instance of the progress of science; in each of which we may trace some of the steps of that vast plan of Divine Providence to which all things are converging, namely, the bringing all his creatures to a state of truth, goodness, and consequent happiness; an end worthy of the best and wisest of beings, and which we may perceive to be gradually effecting, by the advancement of knowledge, the diffusion of liberty, and the removal of error, that truth and virtue may at last shine forth in all the beauty of their native colours.

It is thus that the discovery of the compass gave rise to the present art of navigation; and when this art grew of more importance to mankind, Divine Providence blessed them with the invention of *Hadley's quadrant*, and in our own day and our own time has further improved both it and the art

* This account of Hadley's quadrant, &c. is extracted from a small tract I published thereon sometime since, 8vo.

† Sir John Pringle's Six Discourses to the Royal Society.

of navigation, by the present method of finding the longitude, which enables the mariner to ascertain with certainty his situation on the unvaried face of the ocean.

Hadley's quadrant or sextant is the only known instrument, on which the mariner can depend for determining with accuracy and precision his latitude and longitude. It is to the use of this instrument that navigation is indebted for the very great and rapid advances it has made within these few years. It is easy to manage, and of extensive use, requiring no peculiar steadiness of hand, nor any such fixed basis as is necessary to other astronomical instruments. It is not the science of navigation only which is indebted to this instrument, but its uses are so extensive in astronomy, that it may, accompanied with an artificial horizon, with propriety be called a *portable observatory*, and in this work we shall exemplify its application to surveying.

Mankind are ever desirous of knowing to whom they are indebted for any peculiar or useful discovery; it is the tribute of gratitude, and a reward to merit. In the present instance there is no difficulty in giving the information; the respective claims of the inventors are easily decided. The first thought originated with the celebrated Dr. *Hooke*, it was completed by Sir *Isaac Newton*, and published by Mr. *Hadley*.

Notwithstanding, however, the manifest superiority of this instrument over those that were in use at the time of its publication, it was many years before the sailors could be persuaded to adopt it, and lay aside their imperfect and inaccurate instruments: so great is the difficulty to remove prejudice, and emancipate the mind from the slavery of opinion.

No instrument has undergone, since the original invention, more changes than the quadrant of *Hadley*; of the various alterations, many have had no better foundation than the conceit and caprice of the makers, who by these attempts have often rendered the instrument more complicated in construction and more difficult in use, than it was in its original state.

ESSENTIAL PROPERTIES OF HADLEY'S QUADRANT.

It is not my intention under this head to enumerate all the advantages of this instrument; but barely to point out one or two of those essential properties which distinguish it from every other instrument of the kind, and rank it among one of the greatest improvements in the practice of navigation.

It is an essential property of this instrument, derived from the laws of reflection, that half degrees on the arc answer to whole ones in the angles measured: hence an octant, or the eighth part of a circle, or 45 degrees on the arc, serves to measure 90 degrees; and sextants will measure an angular distance of 120 degrees, though the arc of the instrument is no more than 60 degrees.* It is from this property that foreigners term that instrument an octant, which we usually call a quadrant, and which in effect it is. This property reduces indeed considerably the bulk of the instrument; but at the same time it calls for the utmost accuracy in the divisions, as every error on the arc is doubled in the observation.

* For a concise explanation of the theory, &c. see my pamphlet on this instrument, 8vo. EDIT.

Another essential, and indeed an invaluable property of this instrument, whereby it is rendered peculiarly advantageous in marine observations, is, that it does not require any peculiar steadiness of the hand, nor is liable to be disturbed by the ship's motion; for, provided the mariner can see distinctly the two objects in the field of his instrument, no motion nor vacillation of the ship will hinder his observation.

Thirdly, the errors to which it is liable are easily discovered, and readily rectified, while the application and use of it is facile and plain.

The principal requisites in a good sextant or quadrant, are, 1. That it be strong, and so constructed as not to bend across the plain. 2. That it be accurately divided. 3. That the surfaces of the glasses be perfectly plain and parallel to each other. 4. That the index turn upon a long axis. 5. That the motion be free and easy in every part, and yet without the least shake or jerk.

DESCRIPTION OF HADLEY'S QUADRANT.

Fig. 1, *plate* 19, represents a quadrant, or octant, of the common construction. The following parts are those which require the particular attention of the observer.

I. B C the arc.
II. A D the index, a b the nonius scale.
III. E the index-glass.
IV. F the fore horizon-glass.
V. G the back horizon-glass.
VI. K the dark glasses or screens.
VII. H I the vanes or sights.
VIII. The arc B C is called the *limb* or *quadrantal arc*; the arc c d lying from o, towards the right is called the arc of excess.

OF THE QUADRANT.

The quadrant consists of an arc B C, firmly attached to two radii, or bars, AB, AC, which are strengthened and bound together by the two braces L M.

Of the index. The index D is a flat bar of brass, that turns on the center of the octant; at the lower end of the index there is an oblong opening, to one side of this opening a nonius scale is fixed to subdivide the divisions of the arc; at the bottom or end of the index there is a piece of brass, which bends under the arc, carrying a spring to make the nonius scale lie close to the divisions; it is also furnished with a screw to fix the index in any desired position.

The best instruments have an adjusting screw fitted to the index, that it may be moved more slowly, and with greater regularity and accuracy than by the hand. It is proper however to observe, that the index must be previously fixed near its right position by the above-mentioned screw, before the adjusting screw is put in motion. See B C, *fig.* 4.

The circular arcs on the arc of the quadrant are drawn from the center on which the index turns: the smallest excentricity in the axis of the index would be productive of considerable errors.

The position of the index on the arc, after an observation, points out the number of degrees and minutes contained in the observed angle.

Of the index-glass E. Upon the index, and near its axis, is fixed a plain speculum, or mirror of glass, quicksilvered. It is set in a brass frame, and is placed so that the face of it is perpendicular to the plain of the instrument; this mirror being fixed to the index, moves along with it, and has its direction changed by the motion thereof.

This glass is designed to receive the image of the sun, or any other object, and reflect it upon either of the two horizon glasses F and G, according to the nature of the observation.

The brass frame with the glass is fixed to the index by the screw c; the other screw serves to replace it in a perpendicular position, if by any accident it has been deranged, as will be seen hereafter.

The index glass is often divided into two parts, the one silvered, the other black, with a small screen in front. A single black surface has indeed some advantages; but if the glasses be well selected, there is little danger to be apprehended of error, from a want of parallelism; more is to be feared from the surfaces not being flat.

Of the horizon glasses F, G. On the radius AB of the octant, are two small speculums. The surface of the upper one is parallel to the index glass, when the counting division of the index is at o on the arc; but the surface of the lower one is perpendicular to the index glass, when the index is at o degrees on the arc: these mirrors receive the reflected rays from the object, and transmit them to the observer.

The horizon glasses are not entirely quicksilvered; the upper one F, is only silvered on its lower part, or that half next the quadrant, the other half being transparent, and the back part of the frame is cut away, that nothing may impede the sight through the unsilvered part of the glass. The edge of the foil of this glass is nearly parallel to the plane of the instrument, and ought to be very sharp, and without a flaw.

The other horizon glass G is silvered at both ends; in the middle there is a transparent slit, through which the horizon, or other object, may be seen.

Each of these glasses is set in a brass frame, to which there is an axis; this axis passes through the wood-work, and is fitted to a lever on the under side of the quadrant; by this lever the glass may be turned a few degrees on its axis, in order to set it parallel to the index glass. The lever has a contrivance to turn it slowly, and a button to fix t. To set the glasses perpendicular to the plane of the quadrant, there are two sunk screws, one before and one behind each glass; these screws pass through the plate, on which the frame is fixed, into another plate, so that by loosening one, and tightening the other of these screws, the direction of the frame, with its mirror, may be altered, and thus be set perpendicular to the plane of the instrument. For the lever, &c. see *fig.* 13.

Of the shades, or dark glasses, K. There are two red or dark glasses, and one green one; they are used to prevent the bright rays of the sun, or the glare of the moon, from hurting the eye at the time of observation. They are each of them set in a brass frame, which turns on a center, so that they may be used separately, or together, as the brightness of the sun may require. The green glass may be used also alone, if the sun be very faint; it is also used for taking the altitude of the moon, and in ascertaining her distance from a fixed star.

When these glasses are used for the fore observation, they are fixed as at K in *fig.* 1; when used for the back observation, they are removed to N.

Of the two sight vanes H, I. Each of these vanes is a perforated piece of brass, designed to direct the sight parallel to the plane of the quadrant.

That which is fixed at I is used for the fore, the other for the back observation.

The vane I has two holes, one exactly at the height of the quicksilvered edge of the horizon glass, the other somewhat higher, to direct the sight to the middle of the transparent part of the mirror, for those objects which are bright enough to be reflected from the unsilvered part of the mirror.

Of the divisions on the limb of the quadrant, and of the nonius on the index. For a description of these divisions, see page 127.

Directions to hold the instrument. It is recommended to support the weight of the instrument by the right hand, and reserve the left to govern the index. Place the thumb of the right hand against the edge of the quadrant, under the swelling part on which the fore sight I stands, extending the fingers across the back of the quadrant, so as to lay hold on the opposite edge, placing the fore finger above, and the other fingers below the swelling part, or near the fore horizon glass; thus you may support the instrument conveniently, in a vertical position, by the right hand only; by resting the thumb of the left hand against the side, or the fingers against the middle bar, you may move the index gradually either way.

In the back observation, the instrument should be supported by the left hand, and the index be governed by the right.

Of the axis of vision, or line of sight. Of the two objects which are made to coincide by this instrument, the one is seen directly by a ray passing through, the other, by a ray reflected from, the same point of the horizon glass to the eye. This ray is called the visual ray; but when it is considered merely as a line drawn from the middle of the horizon glass to the eye-hole of the sight vane, it is called the axis of vision.

The axis of a tube, or telescope, used to direct the sight, is also called the axis of vision.

The quadrant, if it be held as before directed, may be easily turned round between the fingers and thumb, and thus nearly on a line parallel to the axis of vision; thus the plane of the quadrant will pass through the two objects when an observation is made, a circumstance absolutely necessary, and which is more readily effected when the instrument is furnished with a telescope; within the telescope are two parallel wires, which by turning the eye glass tube may be brought parallel to the plane of the quadrant, so that by bringing the object to the middle between them, you are certain of having the axis of vision parallel to the plane of the quadrant.

OF THE NECESSARY ADJUSTMENTS.

It is a peculiar excellence of Hadley's quadrant, that the errors to which it is liable are easily detected, and soon rectified; the observer may, therefore, if he will be attentive, always put his instrument in a state fit for accurate observation. The importance of this instrument to navigation is self-evident; yet much of this importance depends on the accuracy with which it is made, and the necessary attention of the observer; and one would hardly think it possible that any observer would, to save the trifling sum of one or two guineas, prefer an imperfect instrument to one that was rightly constructed and accurately made; or that any consideration should induce him to neglect the adjustments of an instrument, on whose truth he is so highly interested. But such is the nature of man! he is too apt to be lavish on baubles, and penurious in matters of consequence; active about trifles, indolent where his welfare and happiness are concerned. The adjustments for the fore observation are, 1. To set the fore horizon glass parallel to the index glass; this adjustment is of the utmost importance, and should always be made previous to actual observation. The second is, to see that the plane of this glass is perpendicular to the plane of the quadrant. 3. To see that the index glass is perpendicular to the plane of the instrument.

To adjust the fore horizon glass. This rectification is deemed of such importance, that it is usual to speak of it as if it included all the rest, and to call it ADJUSTING THE INSTRUMENT. It is so to place the horizon glass, that the index may shew upon the arc the true angle between the objects: for this purpose, set the index line of the nonius exactly against o on the limb, and fix it there by the screw at the under side. Now look through the sight I at the edge of the sea, or some very distant well-defined small object. The edge of the sea will be seen directly through the unsilvered part of the glass, but by reflection in the silvered part. If the horizon in the silvered part exactly meets, and forms one continued line with that seen through the unsilvered part, then is the instrument said to be adjusted, and the horizon glass to be parallel to the index glass. But if the horizons do not coincide, then loosen the milled nut on the under side of the quadrant, and turn the horizon glass on its axis, by means of the adjusting lever, till you have made them perfectly coincide; then fix the lever firmly in the situation thus obtained, by tightening the milled nut. This adjustment ought to be repeated before and after every astronomical observation.

So important is this rectification, that experienced observers, and those who are desirous of being very accurate, will not be content with the preceding mode of adjustment, but adopt another method, which is usually called finding the index error; a method preferable to the foregoing both for ease and accuracy.

To find the index error. Instead of fixing the index at o, and moving the horizon glass, till the image of a distant object coincides with the same object seen directly; *let the horizon glass remain fixed*, and move the index till the *image and object* coincide; then observe whether the index division on the nonius agrees with the o line on the arc; if it does not, the number by which they differ is a quantity to be added to the observation, if the index line is beyond the o on the limb; but if the index line of the nonius stands between o and 90 degrees, then this error is to be subtracted from the observation.*

We have already observed, that the part of the arc beyond o, towards the right hand, is called the arc of excess; and that the nonius, when at that part, must be read the contrary way, or, which is the same thing, you may read them off in the usual way, and then their complement to 20 min. will be the real number of degrees and minutes to be added to the observation.

* This adjustment may be made more accurately, or the error better found, by using the sun instead of the horizon; but this method requires another set of dark glasses, to darken the direct rays of the sun; such a set is applied to the best instruments, and this method of adjustment is explained in the following description of the sextant.

To make the index glass and fore horizon glass perpendicular to the plane of the instrument. Though these adjustments are neither so necessary nor important as the preceding one; yet, as after being once performed, they do not require to be repeated for a considerable time, and as they add to the accuracy of observation, they ought not to be neglected; and further, a knowledge of them enables the mariner to examine and form a proper judgment of his instrument.

To adjust the index glass. This adjustment consists in setting the plane of the index glass perpendicular to that of the instrument.

Method 1. By means of the two adjusting tools, represented at *fig.* 2 and 3, which are two wooden frames, with two lines on each, exactly at the same distance from the bottom.

Place the quadrant in an horizontal position on a table, put the index about the middle of the arc, turn back the dark glasses, place one of the above-mentioned tools near one end of the arc, the other at the opposite end, the side with the lines towards the index glass; then look down the index glass, directing the sight parallel to the plane of the instrument, you will see one of the tools by direct vision, the other by reflection in the mirror; by moving the index a little, they may be brought exactly together. If the lines coincide, the mirror is rightly fixed; if not, it must be restored to its proper situation by loosening the screw c, and tightening the screw d; or, *vice versa*, by tightening the screw c, and releasing the screw d.

Method 2. Hold the quadrant in an horizontal position, with the index glass close to the eye; look nearly in a right line down the glass, and in such a manner; that you may see the arc of the quadrant by direct view, and by reflection at the same time. If they join in one direct line, and the arc seen by reflection forms an exact plane with the arc seen by direct view, the glass is perpendicular to the plane of the quadrant; if not, the error must be rectified by altering the position of the screws behind the frame, as directed above.

To ascertain whether the fore horizon glass be perpendicular to the plane of the instrument. Having adjusted the index and horizon glasses agreeable to the foregoing directions, set the index division of the nonius exactly against o on the limb; hold the plane of the quadrant parallel to the horizon, and observe the image of any distant object at land, or at sea the horizon itself; if the image of the horizon at the edge of the silvered part coincide with the object seen directly, the glass is perpendicular to the plane of the instrument. If it fall above or below, it must be adjusted. If the image seen by reflection be higher than the object itself seen directly, release the fore screw and tighten the back screw; and, *vice versa*, if the image seen by reflection be lower, release the back screw and screw up the fore one; and thus proceed till both are of an equal height, and that by moving the index you can make the image and the object appear as one.

Or, adjust the fore horizon glass as directed in page 262; then incline the quadrant on one side as much as possible, provided the horizon continues to be seen in both parts of the glass. If, when the instrument is thus inclined, the edge of the sea continues to form one unbroken line, the quadrant is perfectly adjusted; but if the reflected horizon be separated from that seen by direct vision, the speculum is not perpendicular to the plane of the quadrant. And if the observer is inclined to the right, with the face of the quadrant upwards, and the reflected sea appears higher than the real sea, you must slacken the screw before the horizon glass, and tighten that which is behind it; but if the reflected sea appears lower, the contrary must be performed.

Care must be always taken in these adjustments to loosen one screw before the other is screwed up, and to leave the adjusting screws tight, or so as to draw with a moderate force against each other.

This adjustment may be also made by the sun, moon, or star; in this case the quadrant may be held in a vertical position; if the image seen by reflection appears to the right or left of the object seen directly, then the glass must be adjusted as before by the two screws.

OF THE ADJUSTMENTS FOR THE BACK OBSERVATION.

The back observation is so called, because the back is turned upon one of the two objects whose angular distance is to be measured.

The adjustment consists in making the reflected image of the object behind the observer coincide with another seen directly before him, at the same time that the index division of the nonius is directly against the o divison of the arc.

The method, therefore, of adjusting it consists in measuring the distance of two objects nearly 180 degrees apart from each other; the arc passing through each object must be measured in both its parts, and if the sum of the parts be 360 degrees, the speculums are adjusted; but, if not, the axis of the horizon glass must be moved till this sum is obtained.

Set the index as far behind o as twice the dip* of the horizon amounts to; then look at the horizon through the slit near G, and at the same time the opposite edge of the sea will appear by reflection inverted, or upside down. By moving the lever of the axis, if necessary, the edges may be made to coincide, and the quadrant is adjusted.

There is but one position in which the quadrant can be held with the limb downwards, without causing the reflected horizon to cross the part seen by direct vision.

If, on trial, this position be found to be that in which the plane of the quadrant is perpendicular to the horizon, no farther adjustment is necessary than the fore-mentioned one; but if the horizons cross each other when the quadrant is held upright, observe which part of the reflected horizon is lowest.

If the right-hand part be lowest, the sunk screw which is before the horizon glass must be tightened after slackening that which is behind the glass; but if the right hand is highest, the contrary must be performed: this adjustment is, however, of much less importance than the preceding, as it does not so much affect the angle measured.

INCONVENIENCIES AND INACCURACIES OF THE BACK OBSERVATION.

The occasions on which the back observation is to be used are, when the altitude of the sun or a star is to be taken, and the fore horizon is broken by adjacent shores; or when the angular distance between the moon and sun, or a star, exceeds 90

* This is according to the height of the observer's eye above the sec. See *Robertson's Navigation.*

degrees, and is required to be measured for obtaining the longitude at sea: but there are objections to its use in both cases; for if a known land lie a few miles to the north or southward of a ship, the latitude may be known from its bearing and distance, without having recourse to observation: and again, if the distance of the land in miles exceed the number of minutes in the dip, as is almost always the case in coasting along an open shore, the horizon will not be broken, and the fore observation may be used; and lastly, if the land be too near to use the fore observation, its extreme points will in general be so far asunder, as to prevent the adjustment, by taking away the back horizon. In the case of measuring the angular distances of the heavenly bodies, the very great accuracy required in these observations, makes it a matter of importance that the adjustments should be well made, and frequently examined into. But the quantity of the dip is varied by the pitching and rolling of the ship; and this variation, which is perceptible in the measuring altitude by the fore observation, is doubled in the adjustment for the back observation, and amounts to several minutes. It is likewise exceeding difficult, in a ship under way, to hold the quadrant for any length of time, so that the two horizons do not cross each other, and in the night the edge of the sea cannot be accurately distinguished. All these circumstances concur to render the adjustment uncertain; the fore observation is subject to none of these inconveniencies.*

* Nicholson's Navigator's Assistant.

DESCRIPTION, USE,

AND METHOD OF ADJUSTING

HADLEY's SEXTANT.

As the taking the angular distances of the moon and the sun, or a star, is one of the best and most accurate methods of discovering the longitude, it was necessary to enlarge the arc of the octant to the sixth part of a circle; but as the observations for determining the longitude must be made with the utmost accuracy, the framing of the instrument was also altered, that it might be rendered more adequate to the solution of this important problem. Hence arose the present construction of the sextant, in the description of which, it is presumed that the foregoing pages have been read, as otherwise we should be obliged to repeat the same observations.

Sextants are mostly executed (some trifling variations excepted) on two plans; in the one, all the adjustments are left to the observer: he has it in his power to examine and rectify every part of his instrument. This mode is founded on this general principle, that the parts of no instrument can be so fixed as to remain accurately in the same position they had when first put out of the maker's hands; and that therefore the principal parts should be made moveable, that their positions may be examined and rectified from time to time by the observer.

In the second construction, the principal adjustment, or that of the horizon glass for the index error, is rejected; and this rejection is grounded upon two reasons;

1. That from the nature of the adjustment it frequently happens that a sextant will alter even during the time of an observation, without any apparent cause whatever, or without being sensible at what period of the observation such alteration took place, and consequently the observer is unable to allow for the error of adjustment.

2. That this adjustment is not in itself sufficiently exact, it being impossible to adjust a sextant with the same accuracy by the coincidence of two images of an object, as by the contact of the limbs thereof; and hence experienced and accurate observers have always directed the index error to be found and allowed for, which renders the adjustment of the horizon glass in this direction useless; for it is easy to place it nearly parallel to the index glass, when the instrument is made, and then to fix it *firmly* in that position by screws. The utility of this method is confirmed by experience: many sextants, whose indices had been determined previous to their being carried out to India, have been found to remain the same at their return.

Notwithstanding the probable certainty of the horizon glass remaining permanently in its situation, the observer ought from time to time to examine the index error of his instrument, to see if it remains the same; or make the proper allowances for it, if any alteration should have taken place.

One material point in the formation of a sextant is so to construct it, that it may support its own weight, and not be liable to bend by any inclination of the plane of the instrument, as every flexure would alter the relative position of the mirrors, on which the determination of an angle depends.

Besides the errors now mentioned, to which the sextant or quadrant, &c. is liable, there is another which seems inseparable from the construction and materials of the instrument, and have been noticed by several skilful observers. It arises from the bending and elasticity of the index, and the resistance it meets with in turning round its center.

To obviate this error, let the observer be careful always to finish his observations, by moving the index in the same direction which was used in setting it to o for adjusting, or in finding the index error.

The direction of the motion is indeed indifferent; but as the common practice in observing is to finish the observations by a motion of the index in that direction which increases the angle, that is, in the fore observation from o towards 90, it would be well if the observer would adopt it as a general rule, to finish the motion of the index, by pushing it from him, or turning the screw in that direction which carries it farther from him. By finishing the motion of the index, we mean that the last motion of the index should be for some minutes of a degree at least in the required direction.

DESCRIPTION OF THE SEXTANT.

Fig. 4, *plate* 19, represents the sextant so framed, as not to be liable to bend. The arc AA, is divided into 120°, each degree is divided into three parts, of course equal to 20 minutes, which are again subdivided by the nonius into every half minute, or 30 seconds: see the nature of the nonius, and the general rule for estimating the value thereof, in the preceding part of these Essays. Every second division or minute on the nonius, is cut longer than the intermediate ones. The nonius is numbered at every fifth of these longer divisions, from the right towards the left, with 5, 10, 15, and 20, the first division towards the right hand being to be considered as the index division.

In order to observe with accuracy, and make the images come precisely in contact, an adjusting screw B is added to the index, which may be moved with greater accuracy than it can by hand; but this screw does not act until the index is fixed by the finger screw C. Care should be taken not to force the adjusting screw when it arrives at either extremity of its adjustment. When the index is to be moved any considerable quantity, the screw C at the back of the sextant must be loosened; but when the index is brought nearly to the division required, this back screw should be tightened, and then the index may be moved gradually by the adjusting screw. A small shade is sometimes fixed to that part of the index where the nonius is divided, this being covered with white paper, reflects the light strongly upon the divisions.

There are four tinged glasses at D, each of which is set in a separate frame turning on a center: they are used to screen and save the eye from the brightness of the solar image, and the glare of the moon, and may be used separately, or together, as occasion requires.

There are three more such glasses placed behind the horizon glass at E, to weaken the rays of the sun or moon, when they are viewed directly through the horizon glass. The paler glass is sometimes used in observing altitudes at sea, to take off the strong glare of the horizon.

The frame of the index glass I, is firmly fixed by a strong cock to the center plate of the index.

The horizon glass F, is fixed in a frame that turns on the axes or pivots, which move in an exterior frame: the holes in which the pivots move may be tightened by four screws in the exterior frame; G is a screw by which the horizon glass may be set perpendicular to the plane of the instrument; should this screw become loose, or move too easy, it may be easily tightened by turning the capstan headed screw H, which is on one side the socket, through which the stem of the finger screw passes.

The sextant is furnished with a plain tube, *fig.* 7, without any glasses; and to render the objects still more distinct, it has also two achromatic telescopes, one, *fig.* 5, shewing the objects erect, or in their natural position; the longer one, *fig.* 6, shews them inverted. It has a large field of view, and other advantages; and a little use will soon accustom the observer to the inverted position, and the instrument will be as readily managed by it as by the plain tube only. By a telescope, the contact of the images is more perfectly distinguished; and by the place of the images in the field of the telescope, it is easy to perceive whether the sextant is held in the proper plane for observation. By sliding the tube that contains the eye glasses in the inside of the other tube, the image of the object is suited to different eyes, and made to appear perfectly distinct and well defined.

The telescopes are to be screwed into a circular ring at K; this ring rests on two points against an exterior ring, and is held thereto by two screws: by turning one, and tightening the other, the axis of the telescope may be set parallel to the plane of the sextant. The exterior ring is fixed on a triangular brass stem that slides in a socket, and by means of a screw at the back of the sextant, may be raised or lowered so as to move the center of the telescope to point to that part of the horizon glass which shall be judged the most fit for observation. *Fig.* 8, is a circular head, with tinged glasses to screw on the eye end of either of the telescopes, or the plain tube. The glasses are contained in a circular plate, which has four holes; three of these are fitted with tinged glasses, the fourth is open. By pressing the finger against the projecting edge of this circular plate, and turning it round, the open hole, or any of the tinged glasses, may be brought between the eye glass of the telescope and the eye.

Fig. 9, a small screw driver. *Fig.* 10, a magnifying glass, to read off the divisions by.

To find the index error of the sextant. To find the index error, is, in other words, to shew what number of degrees and minutes is indicated by the nonius, when the direct and reflected images of an object coincide with each other.

The most general and most certain method of ascertaining this error, is to measure the diameter of the sun, by bringing the limb of its image to coincide with the limb of the sun itself seen directly, *both* on the quadrantal arc, and on the arc of excess.

If the diameter taken by moving the index forward on the quadrantal arc be greater than that taken on the arc of excess, then half the difference is to be *subtracted*; but if the diameter taken on the arc of excess be greater than that by the quadrantal arc, half the difference is to be added. If the numbers shewn on the arc be the same in both cases, the glasses are truly parallel, and there is no index error; but if the numbers be different, then half the difference is the index error.*

It is however to be observed, that when the index is on the arc of excess, or to the right of o, the complement of the numbers shewn on the nonius to 20 ought to be set down.

Several observations of the sun's diameter should be made, and a mean taken as the result, which will give the index error to very great exactness.

Example. Let the numbers of minutes shewn by the index to the right of zero, when the limbs of the two images are in contact, be 20 minutes, and the odd number shewn by the nonius be 5, the complement of this to 20 is 15, which, added to 20, gives 35 minutes; and, secondly, that the number shewn by the index, when on the left of zero, and the opposite limbs are in contact, be 20 minutes, and by the nonius 9′ 30″, which makes together 29′ 30″; this subtracted from 35′ gives 5′ 30″, which divided by 2, gives 2′ 45″ for the index error; and because the greatest of the two numbers thus found, was, when the index was to the right of the o, this index error must be added to the number of degrees shewn on the arc at the time of an observation.

To set the horizon glass perpendicular to the plane of the sextant. Direct the telescope to the sun, a star, or any other well-defined object, and bring the direct object and reflected image to coincide nearly with each other, by moving the index; then set the two images parallel to the plane of the

* In other words, the difference between the degree and minute shewn by the index: first, when the lower reflected limb of the sun is exactly in contact with the upper limb of the sun; and secondly, when the upper edge of the image is in contact with the lower edge of the object, divided by 2, will be the index error.

sextant, by turning the screw, and the images will pass exactly over each other, and the mirror will then be adjusted in this direction.

To set the axis of the telescope parallel to the plane of the sextant. We have already observed, that in measuring angular distances, the line of sight, or plane of observation, should be parallel to the plane of the instrument, as a deviation in that respect will occasion great errors in the observation, and this is most sensible in large angles: to avoid these, a telescope is made use of, in whose field there are placed two wires parallel to each other, and equidistant from the center. These wires may be placed parallel to the plane of the sextant, by turning the eye glass tube, and, consequently, by bringing the object to the middle between them, the observer may be certain of having the axis of vision parallel to the plane of the quadrant.

To adjust the telescope. Screw the telescope in its place, and turn the eye tube round, that the wires in the focus of the eye glass may be parallel to the plane of the instrument; then seek two objects, as the sun and moon, or the moon and a star, whose distance should, for this purpose, exceed 90 degrees; the distance of the sun and moon is to be taken great, because the same deviation of the axis will cause a greater error, and will consequently be more easily discovered. Move the index, so as to bring the limbs of the sun and moon, if they are made use of, exactly in contact with that wire which is nearest to the plane of the sextant; fix the index there; then, by altering a little the position of your instrument, make the images appear on the wire furthest from the sextant. If the nearest limbs be now precisely in contact, as they were before, then the axis of the telescope is in its right situation. But if the limbs of the two objects appear to separate at the wire that is furthest from the plane of the instrument, it shews that the object end of the telescope inclines towards the plane of the instrument, and must be rectified by tightening the screw nearest the sextant, having previously unturned the screw furthest from it. If the images overlap each other at the wire furthest from the sextant, the object end of the telescope is inclined from the plane of the sextant, and the highest screw must be turned towards the right, and the lowest screw towards the left: by repeating this operation a few times, the contact will be precisely the same at both wires, and consequently the axis of the telescope will be parallel to the plane of the instrument.

To examine the glasses of a sextant, or quadrant. 1. To find whether the two surfaces of any one of the reflecting glasses be parallel, apply your eye at one end of it, and observe the image of some object reflected very obliquely from it; if that image appears singly, and well defined about the edges, it is a proof that the surfaces are parallel; on the contrary, if the edge of the reflected image appears misted, as if it threw a shadow from it, or separated like two edges, it is a proof that the two surfaces of the glass are inclined to each other: if the image in the speculum, particularly if that image be the sun, be viewed through a small telescope, the examination will be more perfect.

2. To find whether the surface of a reflecting glass be plane. Chuse two distant objects, nearly on a level with each other; hold the instrument in an horizontal position, view the left hand object directly through the transparent part of the horizon glass, and move the index till the reflected image of the other is seen below it in the silvered

part; make the two images unite just at the line of separation, then turn the instrument round slowly on its own plane, so as to make the united images move along the line of separation of the horizon glass. If the images continue united without receding from each other, or varying their respective position, the reflecting surface is a good plane. The observer must be careful that he does not give the instrument a motion about the axis of vision, as that will cause a separation if the planes be perfect.

3. To find if the two surfaces of a red or darkening glass are parallel and perfectly plane. It is difficult, nay, almost impossible, to procure the shades perfectly parallel and good; they will therefore, according to their different combinations, give different altitudes or measures of the sun and moon.

The best way to discover the error of the shades, is to take the sun's diameter with a piece of smoaked glass before your telescope, all the vanes being removed; then take away the smoaked glass, and view the sun through each shade and the several combinations thereof. If the two images still remain in contact, the glasses are good; but if they separate, the error is to be attributed to the dark glasses, which must either be changed, or the error found in each combination must be allowed for in the observations. If you use the *same* dark glasses in the observation as in the adjustment, there will be no error in the observed angle.*

* The dark glasses are generally left to turn in their cells; so that, after one observation has been made, turning either of them half round, after another observation, half the difference thus given is the error. EDIT.

OF HADLEY'S SEXTANT, AS USED IN SURVEYING.

No instrument can be so conveniently used for taking angles in maritime surveying as Hadley's sextant. It is used with equal facility at the mast head, as upon deck, by which its sphere of observation is much extended: for, supposing many islands to be visible from the mast head, and only one from deck, no useful observation can be made by any other instrument. But by this, angles may be taken at the mast head from the one visible object with great exactness; and further, taking angles from heights, as hills, or a ship's mast head, is almost the only way of exactly describing the figure and extent of the shoals.

It has been objected to the use of Hadley's sextant for surveying, that it does not measure the horizontal angles, by which alone a plan can be laid down. This observation, however true in theory, may be obviated in practice by a little caution.

If an angle be measured between an object on an elevation, and another near to it in a hollow, the difference between the base, which is the horizontal angle, and the hypothenuse, which is the angle observed, may be very great; but if these objects are measured, not from each other, but from some very distant object, the difference between the angles of each from the very distant object, will be very near the same as the horizontal angle. This may be still further corrected, by measuring the angle not between an object on a plane and an object on an elevation, but between the object on a plane and some object in the same direction as the elevated object, of which the eye is sufficiently able to judge.

How to observe the horizontal angle, or angular distance, between two objects. First adjust the sextant, and if the objects are not small, fix on a sharp top, or corner, or some small distinct part in each to observe; then, having set the index to o deg. hold the sextant horizontally, as above directed, and as nearly in a plane passing through the two objects as you can; direct the sight through the tube to the left hand object, till it is seen directly through the transparent part of the horizon glass; keeping that object still in sight, then move the index till the other object is seen by reflection in the silvered part of the horizon glass; then bring both objects together by the index, and by the inclination of the plane of the sextant when necessary, till they unite as one, or appear to join in one vertical line in the middle of the line which divides the transparent and reflecting parts of the horizon glass; the two objects thus coinciding, or one appearing directly below the other, the index then shews on the limb the angle which the two objects subtend at the naked eye. This angle is always double the inclination of the planes of the two reflecting glasses to one another; and, therefore, every degree and minute the index is actually moved from o, to bring the two objects together, the angle subtended by them at the eye will be twice that number of degrees and minutes, and is accordingly numbered so on the arc of the sextant; which is really an arc of 60 degrees only, but graduated into 120 degrees, as before observed.

The angle found in this manner between two objects that are near the observer, is not precise; and may be reckoned exact only when the objects are above *half a mile* off. For, to get the angle truly exact, the objects should be viewed from the center of the index glass, and not where the sight vane is placed; therefore, except the objects are so remote, that the distance between the index glass and sight vane vanishes, or is as nothing compared to it, the angle will not be quite exact. This inaccuracy in the angle between near objects is called the parallax of the instrument, and is the angle which the distance between the index glass and sight vane subtends at any near object. It is so small, that a surveyor will seldom have occasion to regard it; but if it shall happen that great accuracy is required, let him choose a distant object exactly in a line with each of the near ones, and take the angles between them, and that will be the true angle between the near objects. Or, observe the angle between near objects, when the sextant has been first properly adjusted by a distant object; then adjust it by the left hand object, which will bring the index on the arc of excess beyond o degrees; add that excess to the angle found between the objects, and the sum will be the true angle between them. If one of the objects is near, and the other distant, and no remote object to be found in a line with the near one, adjust the sextant to the near object, and then take the angle between them, and the parallax will be found.

Example. To measure the horizontal angle A B C, *fig.* 19, *plate* 9, with the sextant.

1. Set up such marks at A and C, as may be seen when you are standing at B. 2. Set o on the index to coincide with o on the quadrantal arc. 3. Hold the sextant in an horizontal position, look through the sight and horizon glass at the mark A, and observe whether the image is directly under the object; if not, move the index till they coincide. 4. if the index division of the

nonius is on the arc of excess, the indicated quantity is to be added to the observed angle; but if it be on the quadrantal arc, the quantity indicated is to be subtracted: let us suppose five to be added.
5. Now direct the sight through the transparent part of the horizon glass to A, keep that object in view, move the index till the object C is seen by reflection in the silvered part of the horizon glass.
6. The objects being now both in view, move the index till they unite as one, or appear in one vertical line, and the index will shew the angle subtended at the eye by the two objects: suppose 75.20, to which add 5 for the index error, and you obtain 75.25, the angle required; if the angle be greater than 120°, which seldom happens in practice, it may be subdivided by marks, and then measured.

No instrument can be more convenient or expeditious than the sextant, for setting of offsets. Adjust the instrument, and set the index to 90 degrees; walk along the station line with the octant in your hand, always directing the sight to the farther station staff; let the assistant walk along the boundary line; then, if you wish to make an offset from a given point in the station line, stop at that place, and wait till you see your assistant by reflection, he is then at the point in the boundary through which that offset passes; on the other hand, if you wish an offset from a given point or bend in the boundary, let the assistant stop at that place, and do you walk on in the station line till you see the assistant by reflection in the octant, and that will be the point where an offset from the proposed point or bend will fall.

The manner of using this instrument for the solution of those astronomical problems that are necessary in surveying, will be shewn in its proper place.

DESCRIPTION OF A NEW POCKET BOX SEXTANT, AND AN ARTIFICIAL HORIZON, BY THE EDITOR.

Fig. 11, *plate* 19, is a representation of a very convenient pocket sextant, and contains a material improvement on the reflecting cross staff before described, see *fig.* 4, *plate* 14. In military operations, as well as trigonometrical ones, it has been found of very essential service. AB a round brass box three inches in diameter, and one inch deep. AC is the index turning an index glass within the box. *a*, *a*, are the two outside ends of the screws that confine an horizon glass also within the box. An angle is observed by the sight being directed through an hole in the side of the box about D, upon and through the horizon glass and the second opening at E, and the angle is read off to one minute by the divided arc and nonius F, G, H. By sliding a pin projecting on the side of the box, a dark glass is brought before the sight hole, not shewn in the figure; by pushing the pin at *b*, a dark screen for the sun is interposed between the index and horizon glasses. I is an endless screw, sometimes applied to give a very accurate motion, like the tangent screw to the index of a sextant. Or a racked arc and pinion may be applied at about *c*, *d*, which I think in some respects better.

The following table is sometimes engraved upon the cover that goes over the box when shut up. By the sextant being set to any of the angles contained in this table, an height or distance of accessible or inaccessible objects is obtained in a very simple and expeditious manner.

Mul.	Angle.	Angle.	Div.
1	45° 00′	45° 00′	1
2	63 26	26 34	2
3	71 34	18 26	3
4	75 58	14 02	4
5	78 41	11 19	5
6	80 32	9 28	6
8	82 52	7 08	8
10	84 17	5 43	10

Make a mark upon the object, if accessible, equal to the height of your eye from the ground. Set the index to any of the angles from this table, and walk from the object, till the top is brought by the glasses to coincide with the mark; then, if the angle be greater than 45°, multiply the distance by the corresponding figure to the angle in the table; if it be less, divide, and the product, or quotient, will be the height of the object above the mark. If the object be inaccessible, set the index to the greatest angle in the table that the least distance from the object will admit of, when by moving backwards and forwards, till the top of the object is brought to a level with the eye, and at this place set up a mark equal to the height of the eye. Then set the index to any of the lesser angles, and go backwards in a line from the object, till the top is made to appear on the level with the eye, or mark before set; set here another mark, measure the distance between the two marks, and this divided by the difference of the figures in the last column, against the angle made use of, the quotient will give the height of the object above the height of the eye, or mark. *For the distance*, multiply the height of the object by the numbers against either of the angles made use of, and the product will be the distance of the object from the place where such angle was used.

If the index is set at 45° the distance is the height of the object, and *vice versa*. The index set to 90° becomes a reflecting cross staff, and is used according to the directions in page 282.

The sextants, as before described by the author, of the best kind, are made of brass, or other metal. The radii now most approved of are from six to ten inches, their arcs accurately divided by an engine, and the nonii shewing the angles to 30, 15, or even 10 seconds; but the fine divisions of the latter are liable to be obliterated by the frequent cleaning of the instrument.

THE ARTIFICIAL HORIZON.

In many cases it happens that altitudes are to be taken on land by the sextant; which, for want of a natural horizon, can only be obtained by an artificial one. There have been a variety of these sort of instruments made, but the kind now to be described is allowed to be the only one that can be depended upon. *Fig*. 12, *plate* 19, represent the horizon fixed up for use. A is a wood or metal framed roof containing two true parallel glasses of about 5 by 3½ inches, fixed not too tight in the frames of the roof. This serves to shelter from the air a wooden trough filled with quicksilver. In making an observation by it with the sextant, the reflected image of the sun, moon, or other object, is brought to coincide with the same object reflected from the glasses of the sextant; half the angle shewn upon the limb is the altitude above the horizon or level required. It is necessary in a set of observations that the roof be always placed the same way. When done with, the roof folds up flatways, and, with the quicksilver in a bottle, &c. is packed into a portable flat case.

TO SURVEY WITH THE CHAIN ONLY.

The difficulties that occur in measuring with accuracy a strait line, render this method of surveying altogether insufficient for measuring a piece of ground of any extent; it would be not only extremely tedious, but liable to many errors that could not be detected; indeed there are very few situations where it could be used without *King's surveying quadrant*, or some substitute for it. The method is indeed in itself so essentially defective, that those who have praised it most, have been forced to call in some instrument, as the surveying cross and optical square, to their aid. Little more need be said, as it is evident, as well from the nature of the subject, as from the practice of the most eminent surveyors, that the measuring of fields by the chain can only be proper for *level ground* and *small inclosures*; and that even then, it is better to go round the field and measure the angles thereof, taking offsets from the station lines to the fences. That this work may not be deemed imperfect, we shall introduce an example or two selected from some of the best writers on the subject; observing, however, *that fields that are plotted from measured lines, are always plotted nearest to the truth, when those lines form at their junction angles that approach nearly to a right angle.*

Example 1. To survey the triangular field A B C, *fig.* 22, *plate* 9, by the chain and cross. Set up marks at the corners, then begin at one of them, and measure from A to B, till you imagine that you are near the point D, where a perpendicular would fall from the angle C, then letting the chain lie in the line AB, fix the cross at D, so as to see through one pair of the sights the marks at A and B; then look through the other pair towards C, and if you see the marks there, the cross is at its right place; if not, you must move it backwards and forwards on the line AB, till you see the marks at C, and thus find the point D; place a mark at D, set down in your field book the distance AD, and complete the measure of AB, by measuring from D to B, 11.41. Set down this measure, then return to D, and measure the perpendicular DC, 6.43. Having obtained the base and perpendicular, the area is readily found: it is on this principle that irregular fields may be surveyed by the chain and cross; the *theodolite*, or *Hadley's sextant*, may even here be used to advantage for ascertaining perpendicular lines. Some authors have given the method of raising perpendiculars by the chain only; the principle is good, but the practice is too operose, tedious, and even inaccurate to be used in surveying; for the method, see *Geometry on the Ground*.

Example 2. To measure the four-sided figures, ABCD, *fig*. 34, *plate* 9.

AE	214	DE	210
AF	362	BF	306
AC	592		

Measure either of the diagonals, as AC, and the two perpendiculars DE, BF, as in the last problem, which gives you the above data for completing the figure.

Example 3. To survey the irregular field, *fig*. 22, *plate* 13. Having set up marks or station staves wherever it may be necessary, walk over the ground, and consider how it can be most conveniently divided into triangles and trapeziums, and then measure them by the last two problems.

It is best to subdivide the field into as few separate triangles as possible, but rather into trapeziums, by drawing diagonals from corner to corner, so that the perpendicular may fall within the figure; thus the figure is divided into two trapeziums A B C G, G D E F, and the triangle G C D. Measure the diagonal A C, and the two perpendiculars G M, B N, then the base G C, and the perpendicular D q; lastly, the diagonal D F, and the two perpendiculars, p E, O G, and you have obtained sufficient for your purpose.

OF SURVEYING BY THE PLAIN TABLE.

We have already given our opinion of this instrument, and shewn how far only it can be depended upon where accuracy is required; that there are many cases where it may be used to advantage, there is no doubt; that it is an expeditious mode of surveying, is allowed by all. I shall, therefore, here lay down the general modes of surveying with it, leaving it to the practitioner to select those best adapted to his peculiar circumstances, recommending him to use the modes laid down in example 3, in preference to others, where they may be readily applied. He will also be a better judge than I can be, of the advantages of Mr. *Break*'s method of using the plain table.

Example 1. To take by the plain table the plot of a piece of land A B C D E, *fig*. 36, *plate* 9, at one station near the middle, from whence all the corners may be seen.

Let RTSV, *fig*. 37, *plate* 9, represent the plain table covered with a sheet of paper, on which the plan of the field, *fig.* 36, is to be drawn; go round the field and set up objects at all the corners thereof, then put up and level your plain table, turning it about till the south point of the needle points to the N. point, or 360° in the compass box; screw the table fast in that position, and then draw a line P p parallel to one of the sides for a meridian line. Now choose some point on the paper for your station line, and make there a fine hole with a small circle of black lead round it; this is to represent the station point on the land, and to this the edge of the index is to be applied when directed to an object.

Thus, apply the edge of the index to the point ⊙, and direct the sight to the object at A, when this is cut by the hair, draw a blank line along the chamfered edge of the index from ⊙ towards A, after this move the index round the point ⊙ as a center, till you have successively observed through the sights, the several marks at A, B, C, D, E; and when these marks coincide with the sights, draw blank or obscure lines by the edge of the index to ⊙. Now measure the distance from the station point on the ground to each of the objects, and set off by your scale, which should be as large as your paper will admit of, these measures on their respective lines; join the points AB, BC, CD, DE, EA, by lines for the boundaries of the field, which, if the work be properly executed, will be truly represented on the paper.

N. B. It is necessary, before the lines are measured, to find by a plumb-line the place on the ground under the mark ⊙ on the paper, and to place an arrow at that point.

Example 2. Let *fig.* 33, *plate* 9, represent the piece of ground to be surveyed from one station point, whence all the angles may be seen, but not so near the middle as in the foregoing instance; go round the field, and set up your objects at all the corners, then plant the table where they may all be conveniently seen; and if in any place a near object and one more remote are in the same line, that situation is to be preferred. Thus, in the present case, as at ⊙, *g* coincides with *h*, and *c* with *b*; the table is planted thereon, making the lengthway of the table correspond to that of the field. Make your point-hole and circle to represent the place of the table on the land, and apply the edge of the index thereto, so as to see through the slit the mark at *a* cut by the hair; then with your pointrel draw a blank line from ⊙ towards *a*, do the same by viewing through the sights the several marks *c*, *d*, *e*, *f*, *g*, keeping the edge of the index always close to ⊙, and drawing blank lines from ⊙ towards each of these marks.

Find by a plumb-line the place on the ground under ⊙ on the paper, and from this point measure the distances first to *g*, and proceed on in the same line to *h*, writing down their lengths as you come to each; then go to *a*, and measure from it to ⊙, then set off from your scale the respective distance of each on its proper line; after this, measure to *c*, and continue on the line to *b*, and set off their distances; then measure from ⊙ to d, and from *e* to ⊙; and, lastly from ⊙ to f, and sett off their distances. Then draw lines in ink from each point thus found to the next for boundaries, and a line to cross the whole for a meridian line.

Fig. 33, may be supposed to be two fields, and the table to be planted in the N. E. angle of the lower field, where the other angles of both fields may be observed and measured to.

Example 3. To survey a field represented at *fig*. 32, *plate* 9, by going round the same either within or without, taking at the same time offsets to the boundaries; suppose inside.

Set up marks at *a*, *b*, *c*, *d*, at a small distance from the hedges, but at those places which you intend to make your station points.

Then, beginning at ⊙, plant your instrument there, and having adjusted it, make a fine point ⊙ 1 on that part of the paper, where it will be most probable to get the whole plan, if not too large, in one sheet; place the index to ⊙ 1, and direct the sight to the mark at ⊙ 6, draw a blank line from ⊙ 1 to ⊙ 6, then direct the index to the tree near the middle of the field, and afterwards to the mark at ⊙ 2, then dig a hole in the ground under ⊙ 1 in the plan, and taking up the table, set up an object in it exactly upright, and measure from it towards ⊙ 2, and find that perpendicular against 218, the offset to the angle at the boundary is 157 links, which set off in the plan; then measuring on at 375 the offset is but 6, and continues the same to 698, at both which set off 6 in the plan; then measure on to ⊙ 2, and find the whole 1041 links, which set off in the blank line drawn for it, and mark it ⊙ 2; then taking out the object, plant the table to have ⊙ 2 over the hole, when placed parallel to what it was on ⊙ 1; that is, the edge of the index-ruler touching both stations, the hair must cut the object at ⊙ 1, and then screw it fast.

Now setting up objects in the by-angles *a* and *b*, first turn the index to view that at *a*, and draw a blank line from ⊙ 2 towards it; then do the same towards *b*, ⊙ 3 and the tree, which last crossing that drawn towards it from ⊙ 1, the intersection determines the place of the tree, which being remarkable, as seen from all the stations, mark it in the plan; then measure to *a* 412 links, and from *b*

353, and set them off in the blank lines drawn towards them; then set off the distance to the boundaries in the two station lines produced, *viz.* 151 in the next produced backwards, and 15 in the first produced forwards; after this, draw the boundaries from the angle where the first offset was made to the next, and so on round by *a*, *b*, through the 151 to the next angle; then, taking up the table, fix again the object as before, and measure on the ⊙ 3, which set off 564 links, and the offset to the angle in the boundary 27, and then draw the boundary from it through the 15 to the angle at meeting that last drawn.

Now, taking out the object at ⊙ 3, plant the table so as to have ⊙ 3 over the hole, when placed parallel to what it was at the former stations, and screwed fast; then turn the index to make the edge touch the place of the tree and ⊙ 3 in the plan, and finding the hair cuts the tree, turn the index to view ⊙ 4, and draw a blank line towards it; then taking up the table, fix the object as before, and measure on to ⊙ 4, which set off 471 links, and the offset 23, and draw the boundary from the last angle through it to the next; then measure on in the station line produced to the next boundary 207 links, and the distance of ⊙ 4, from the nearest place in the same boundary 173, both which set off and draw the boundary from this last through the 207 to the angle.

Now taking out the mark, plant the table to have ⊙ 4 over the hole, when screwed fast in the same parallelism as at the other stations; then, after viewing again the tree, turn the index to view ⊙ 5, and draw a blank line towards it; then taking up the table, fix the object as before, and measuring on towards ⊙ 5, at 225 the nearest place of the boundary is distant 121, which set off bearing forwards, as the figure shews; at 388 the perpendicular offset is 9, and at 712 it is 12, both which set off in your plan; then measure on to ⊙ 5, and set it off at 912 links.

Take out the mark, plant the table to have ⊙ 5 over the hole, when screwed fast in the same parallelism as before; then set up objects in the by-angles *c* and *d*, and after viewing the tree, turn the index to view the objects at *c*, *d*, and ⊙ 6, and draw a blank line towards each; then measure to *c* 159, and from *d* 245, both which set off in your plan, and also the distance to the boundary in the next station line produced backward 95; and now make up the boundary round by the several offsets to the angles *c* and *d*; then taking up your table, fix the object as before, and measuring towards ⊙ 6, find at 162 the offset is 32, which set off; measure on to ⊙ 6, and set it off at 708, and the offset from it to the boundary is 36 links.

Finding the blank line drawn from ⊙ 1 to intersect the point-hole here made for ⊙ 6, do not plant the table at ⊙ 6, but begin measuring from it towards ⊙ 1, and finding at right angles to the line at ⊙ 6, the offset to the angle is 42, set that off in your plan; then measuring to ⊙ 1, 582, which, measuring the same by the scale in the plan, proves the truth of the work; the offset is here also 42, which set off, and draw the boundary from *d*, round by the several offsets, through this last to the angle; then measure on in the station line produced to the next boundary 88 links, and set that off also, and draw the boundary from the angle at the first offset, taken through it at the angle at meeting the last boundary; and then if a meridian line be drawn, as in the former, the rough plan is completed.

But if ⊙ 6 had not met in the intersection, or its distance from ⊙ 1 been too much, or too little, you would very likely have all your work, except the offsets, to measure and plot over again.

"The plain table *surveyors*, says Mr. *Gardiner*, when they find their work not to close right, do often close it wrong, not only to save time and labour, but the acknowledging an error to their assistants, which they are not sure they can amend, because in many cases it is not in their power, and may be more often the fault of the instrument than the *surveyor*; for in uneven land, where the table cannot at all stations be set horizontal, or in any other one plane, it is impossible the work should be true in all parts: but to prevent great errors, at every ⊙ after the second, view wherever it is possible, the object at some former ⊙, besides that which the table was last planted at; because if the edge of the index ruler do not quite touch, or but very little covers that ⊙ in the plan, whilst it touches the ⊙ you are at, the error may be amended before it is more increased, and if it varies much, it may be examined by planting again the table at the former station, or stations.

If a field is so hilly, that you cannot, without increasing the number of stations, see more than one object backward, and another forward, and there is nothing fit within the field, as the supposed tree in *fig.* 32, then set up an object on purpose to be viewed from all the stations, if possible, for such a rectifier.

The lengthening and shortening of the paper, as the weather is moister or drier, often causes no small error in plotting on the plain table; for between a dewy morning, and the sun shining hot at noon day, there is great difference, and care should be taken to allow for it; but that cannot be done in large surveys, and so ought not to be expected; indeed, those working by the degrees, without having their plan on it, are not liable to this error, though they are to the former; but both ways are liable to another error, which is, that the station lines drawn, or the degrees taken, are not in the line between the objects, nor parallel thereto; neither will this error be small in short distances, and may be great, if each ⊙ on the plan, or the center used with the degrees, is not exactly over the station holes; but to be most exact, it is the line of their sights that should be directly over the hole.

Mr. *Beighton* made such improvements to his plain tables, by a conical ferril fixed on the same staves as his theodolite, that the above errors, except that of the paper, are thereby remedied; for the line of the sights, in viewing, is always over the center of the table, which is as readily set perpendicular over the hole, as the center of the theodolite, and the station lines drawn parallel to those measured on the land; and the table is set horizontal with a spirit level by the same four screws that adjust the theodolite; therefore some choose to have both instruments, that they may use either, as they shall think most convenient.

Let *fig.* 32 now be a wood, to be measured and plotted on the *outside*; if on coming round to the first ⊙, the lines meet as they ought, the plan will be as truly made, as if done on the inside; but here having no rectifier of the work as you go on, you must trust to the closing of the last measured line; and if that does not truly close with the first, you must go over the work again; and, without a better instrument than the common plain table, you cannot be sure of not making an error in this case.

Suppose the table planted at ⊙ 1 on the outside, with paper fixed on it, and objects set up at all the other stations on the outside, and dry blank lines drawn from ⊙ 1 on the paper towards ⊙ 6 and ⊙ 2; these done, take up the table, and set up an object at ⊙ 1; then, measuring from it towards ⊙ 2, you find at 20 the offset to the first angle is 38, then at 280 the offset to the next angle is 26 links, both of which set off in the plan; then at 394 the perpendicular offset to the next angle is 206; then at 698 the distance of the same angle is 366 bearing backward, as may be seen in the figure, that by the intersection of these two offset lines the angle may be more truly plotted; then measure the distance from this angle to the next 323, and from that to 698 place in the station line 280, which is the perpendicular offset; then by the intersection of these, that angle will be well plotted; then at 776 the offset to angle *a* is 48, and at 1012 the offset to *b* is 22, both which set off in the plan, and at 1306 you make ⊙ 2; now draw the boundaries from the first offset to the next, *&c.* to the angle *b*. As there is no difficulty in taking the offsets from the other station lines, we shall not proceed farther in plotting it on the outside; for a sight of the figure is sufficient.

Some *surveyors* would plant their table at a place between 394 and 698 in the first station line, and take the two angles, which are here plotted by the intersection of lines, as the by-angles *a* and *b* were taken at ⊙ 2 within the field; but if the boundary should not be a strait line from one angle to the other, then their distance should be measured, and offsets taken to the several bends in it.

You plot an inaccessible distance in the same manner as the tree in *fig.* 32; for if you could come no nearer to it than the station line, yet you might with a scale measure its distance from ⊙ 1, or ⊙ 2, or any part of the line between them in the plan, the same as if you measured it with the chain on the land; observing to make the stations at such a distance from one another, that the lines drawn towards the tree may intersect each other as near as possible to right angles, drawing a line from each to ⊙ 1;* writing down the degree the north end of the needle points to, as it should point to the same degree at each station; remove the table, and set up a mark at ⊙ 1.

The imperfections in all the common methods of using the plain table, are so various, so tedious, and liable to such inaccuracies, that this instrument, so much esteemed at one time, is now disregarded by all those who aim at correctness in their work. Mr. *Break* has endeavoured to remedy the evils to which this instrument is liable, by adopting another method of using it; a method which I think does him considerable honour, and which I shall therefore extract from his complete "*System of Land Surveying*," for the information of the practitioner.

Example 1. *To take the plot of a field* ABCDEF, *fig.* 11, *plate* 13, *from one station therein.*

Choose a station from whence you can see every corner of the field, and place a mark at each, numbering these with the figures 1, 2, 3, 4, &c. at this station erect your plain table covered with

* In general the mark ⊙ always denotes a station or place where the instrument is planted. The dotted lines leading from one station to another, are the station lines; the black lines, the boundaries; the dotted lines from the boundary to the station line are offsets.

paper, and bring the south point of the needle to the flower-de-luce in the box; then draw a circle O P Q R upon the paper, and as large as the paper will hold.

Through the center of this circle draw the line NS parallel to that side of the table which is parallel to the meridian line in the box, and this will be the meridian of the plan.

Move the chamfered edge of the index on ⊙, till you observe through the sights the several marks A, B, C, and the edge thereof will cut the circle in the points 1, 2, 3, &c. Then having taken and protracted the several bearings, the distances must be measured as shewn before.

To draw the plan. Through the center ⊙ and the points 1, 2, 3, &c. draw lines ⊙ A, ⊙ B, ⊙ C, ⊙ D, ⊙ E, ⊙ F, and make each of them equal to its respective measure in the field; join the points A, B, C, &c. and the plan is finished.

Example 2. *To take the plot of a field* A B C D, *&c. from several stations, fig.* 14, *plate* 13.

Having chosen the necessary stations in the field, and drawn the circle *O P Q R*, which you must ever observe to do in every case, set up your instrument at the first station, and bring the needle to the meridian, which is called adjusting the instrument; move the index on the center ⊙, and take an observation at *A*, *B*, *C*, ⊙ 2, *H*, and the fiducial edge thereof will intersect the circle in 1, 2, 3, &c. Then remove your instrument to the second station in the field, and aplying the edge of the index to the center ⊙ and the mark ⊙ 2 in the circle, take a back sight to the first station, and fasten the table in this position; then move the index on the center ⊙, and direct the sights to the remaining angular marks, so will the fiducial edge thereof cut the circle in the points 4, 5, 6, &c. The several distances being measured with a chain, the work in the field is finished; and entered in the book thus:

THE FIELD BOOK EXPLAINED.

No.	Dist.	No.	Dist.
	⊙1		⊙2
1	520	3	
2	344	4	370
3	360	5	470
⊙2	730	6	550
8	386	7	550
		8	

To draw the Plan.

Having chosen ⊙ 1 upon paper to represent the first station in the field, lay the edge of a parallel ruler to ⊙ and the mark 1, and extend the other edge till it touch or lay upon ⊙ 1, and close by its edge draw a line 1,1=520. Then lay the ruler as before to ⊙ and the mark 2, and, extending the other edge to ⊙ 1, draw thereby the line 1,2=344, which gives the corner *B*, as the line 1,1 does the corner *A*. After the same manner project ⊙ 2, together with the corners *C*, *H*. Again, apply the edge of the ruler to ⊙ and the point 4, and extend the other edge till it touch ⊙ 2, and draw the line 2,4=370, which will give the point or corner *D*. Thus project the remaining corners *E*, *F*, *G*, and the plan is ready for closing.

Example 3. *To take the plot of several fields* ABCD, BECF, DIHK, *and* IFGH, *from* *stations chosen at or near the middle of each, fig.* 15, *plate* 13.

Adjust your plain table at the first station in *ABCD*, and draw the protracting circles and meridians *NS*; then by proposition 1, project the angles or corners of *ABCD*, *BEFC*, and also the second station, into the points 1, 2, ⊙ 2, 3, 4, 5, 6. Again, erect your instrument at the first station, and lay the index on the center ⊙ and the mark ⊙ 2, and direct the sights by turning the table to the second station, then move the index till you observe the third station, and the edge thereof will cut the circle in ⊙ 3. Then remove the instrument to the third station, lay the index on the center ⊙ and the mark ⊙ 3, and take a back observation to the first station; after which by the last proposition find the points 7, 8, ⊙ 4, 9, in the circle *OPQR*. As to the measuring of distances, both in this and the two succeeding propositions that shall be passed over in silence, having sufficiently displayed the same heretofore; what I intend to treat of hereafter, is the method of taking and protracting the bearings in the field, with the manner of deducing a plan therefrom.

To draw the Plan.

Choose any point ⊙ 1 for your first station; apply the edge of a parallel ruler to the center ⊙ and the point 1, and having extended the other edge to ⊙ 1, draw the line 1,1=370, which will give the corner *A*. In like manner find the other corners *B*, *C*, *D*, together with ⊙ 2; which being joined, finishes the field *ABCD*. After the same method construct the other fields *BEFC*, *DIHK*, *IFGH*, and you have done.

THE FIELD BOOK.

REMARKS.	N°.	DIST.	REMARKS.
		$\odot$ 1	In *ABCD.*
The open	1	370	Field.
	2	380	*BEFI.*
	$\odot$ 2	580	
	3	403	*DIHK.*
Sir Will. Jones's	4	440	Ground.
		$\odot$ 2	In *BEFI.*
	2		
John Simpson's	5	400	Ground.
	6	460	*IFGH.*
	7		
BEFI closes at	3		
		$\odot$ 1	In *ABCD.*
Return to	$\odot$ 3	600	
		$\odot$ 3	In *DIHK.*
	7		*IFGH.*
	$\odot$ 4	630	
South	8	432	Field.
John Spencer's	9	400	Ground.
DIHK closes at	4		
		$\odot$ 4	In *IFGH.*
Ed. Johnstone's	6	—	Ground.
South	10	380	Field.
IFGH closes at	8		

Example 4. *To take the plot of a field* ABCD EF, *by going round the same, fig.* 12, *plate* 13.

Set up your plain table at the first station in the field; move the fiducial edge of the index on the center $\odot$, and take an observation at the mark placed at the second station, then will the same fiducial edge cut the circle *OPQR* in the point 1. Then remove your instrument to the second station, and placing the edge of the index on $\odot$ and the point 1, take a back sight to the first, or last station; then directing the index on the center ⊙ to the third, or next station, the edge thereof will cross the circle in the point 2. In like manner the instrument being planted at every station, a back sight taken to the last preceding one, and the index directed forward to the next succeeding station, will give the protracted points 3, 4, 5, 6.

THE FIELD BOOK.

REMARKS.	*O.*	⊙ *L.*	*O.*	REMARKS.
		⊙ 1		In *ABCDEF.*
	70	0		
	50	250		
	85	550		
		⊙ 2		In Ditto.
	84	0		
Corner	65	440		
		⊙ 3		In Ditto.
		0		
	60	465		
		⊙ 4		In Ditto.
	72	0		
	58	365		
	80	750		
		⊙ 5		In Ditto.
	40	0		
	68	302		
	60	680		
		⊙ 6		In Ditto.
	58	0		
	50	355		
	67	663	to ⊙ 1	Close Ditto.

To draw the Plan.

Choose any point ⊙ 1, to denote the first station. Lay the edge of a parallel ruler on the center ⊙ and the point 1, and extend the other edge till it touch ⊙ 1, and draw by the side thereof the line 1,2=550; then apply the ruler to ⊙ and the mark 2, and extend the other edge to ⊙ 2, and draw thereby the line 2,3=440; again, lay the edge of the ruler to ⊙ and the point 3, and the other edge being extended to ⊙ 3, draw the line 3,4=465; after the same method lay down the remaining stations, and the traverse is delineated. As for drawing the edges, that shall be left for the learner's exercise.

Example 5. *To take the plot of several fields* A, B, C, D, *by circulation, fig.* 16, *plate* 13.

From the projecting point ⊙ by last example, project the stations in *A*, into the points 1, 2, 3, 4; then the instrument being planted at the second station, from the same projecting point ⊙ project that station the second in *A* into the point 2^2, 2^2 denoting the instrument being planted a second time at that station, which is done thus: lay the index to ⊙ and the point 2, and take a back sight to the first station, that being the station immediately preceding that you are at in the field book; then on the center ⊙ take a fore observation at the next succeeding station, and the index will cut the circle in the point 2^2. Thus project every other remaining station.

THE FIELD BOOK.

REMARKS.	O.	⊙ L.	O.	REMARKS.
		⊙ 1		In Field *A*.
	53	0		
	80	290		
Corner	85	610		
	60	695		
		⊙ 2		In Ditto.
	70	0		
	65	560		
		⊙ 3		In Ditto.
	60	0		
	75	368		
	55	680		
		⊙ 4		In Ditto.
	50	0		
	48	440	to ⊙ 1	Close here *A*.
Return to		⊙ 2		In *A*.
		0		
Corner	60	80	into	*B*.
	50	402		
		⊙ 5		In *B*.
	50	0		
	60	340		
	50	650		
		⊙ 6		In Ditto.
	50	0		
Ag. Hedge	51	500		
		560	by ⊙	for closing *C*,
Closes at Cor. of *A*	50	620	+	the Hedge,
		682	to ⊙ 3	in *A*.
Return to		⊙ 4		In *A*.
		0		
		60	70	
		into *C*.		
		380	60	
		682	68	

THE FIELD BOOK.

REMARKS.	*O.*	⊙ *L.*	*O.*	REMARKS.
		⊙ 7		In *C.*
		0	40	
		300	60	
		720	50	
		⊙ 8		In Ditto.
		0	70	
		400	24	
		780	63	Close *C.*
		Cross	hedge	
		820	bye	⊙ at 560l. 6, 4.
Return to		⊙ 8		In Ditto.
		0		
		60	50	Corner.
		into	*D.*	
		220	50	
		554	40	
		⊙ 9		In *D.*
		0	40	
		250	38	
		⊙ 10		In Ditto.
		0		
		64	50	
		530	64	Close *D* at Cor. *B.*
		into	*B.*	
		570	to ⊙ 6	in *B.*

The bearing being protracted, the plan may be readily drawn from what has been already described.

GENERAL METHODS OF SURVEYING DETACHED PIECES OF GROUND.

To survey the triangular field, fig. 22, plate 9, with any instrument used for measuring angles.

1. Set up marks at the three corners A, B, C.

2. Measure the angle AC B. 3. Measure the two sides AC, C B.

This method of measuring two sides and the included angle, is far more accurate than the old method of going round the field, and measuring all the angles. It was first introduced into practice by Mr. *Talbot*.

If the field contain four sides, *fig.* 23, *plate* 9, begin at one of the corners A. 1. Measure the angle BAD. 2. Measure the side AB. 3. Take the angle ABC. 4. Measure the side B C, and angle B C D. 5. Measure the side C D, and the angle C DA, and side DA, the dimensions are finished; add the four angles together, and if the sum makes 360°, you may conclude that your operations are correct; the above figure may be measured by any other method as taught before, by measuring the diagonal, &c.

If the field contain more than four sides, *fig.* 24, 25, 26, *plate* 9, having set up your marks, endeavour to get an idea of the largest four-sided figure, that can be formed in the field you are going to measure; this figure is represented in the figures by the dotted lines.

Then beginning at A, take the angle BAD, measure in a right line towards B, till you come against the angle *f*, there with your sextant, or cross, let fall the perpendicular *f e*, as taught in the method of the triangle, observing at how many chains and links this offset or perpendicular falls from the beginning or point A, which note in your field book, and measure *e f*, noting it also in your field book; then continue the measure of the line AB to B; take the angle AB C, and measure the line B C; take the angle B C D, and measure the line C D; take the angle C DA, and measure the line DA; observing as you go round, to let fall perpendiculars where necessary, and measure them as specified in the line AB.

Example to *fig*. 26, containing seven sides.

FIELD BOOK.

∠A=	86°,,35′	
A e=	2,08 5,32	80 = ef. = 1st side AB.
∠B=	88°,,25′	
	4,80	= 2d side BC.
∠C=	93°,,00′	
C g=	1,20 4,92	88 = gh. = 3d side CD.
∠D=	92°,,00′	
D i=	2,00 4,84	1,08 = ik. = 4th side DA.

N. B. Set the offsets to the right or left of your column of angles and chains, according as they fall to the right and left of your chain line in the field.

TO USE THE COMMON CIRCUMFERENTOR.

We have already observed that this instrument should never be used where much accuracy is required, for it is scarcely possible to obtain with any certainty the measure of an angle nearer than to two degrees, and often not so near; it has therefore long been rejected by accurate surveyors.*

* See Gardiner's Practical Surveying, p. 54.

This instrument takes the bearing of objects from station to station, by moving the index till the line of the sights coincides with the next station mark; then counting the degrees between the point of the compass box marked N, and the point of the needle in the circle of quadrants.

Thus let it be required to survey a large wood, *fig.* 31, *plate* 9, by going round it, and observing the bearing of the several station lines which encompass that wood.*

The station marks being set up, plant the circumferentor at some convenient station as at *a*, the flower-de-luce in the compass box being from you; direct the sights to the next station rod *b*, and set down the division indicated by the north end of the needle, namely 260° 30′, for the bearing of the needle.

Remove the station rod *b* to *c*, and place the circumferentor exactly over the hole where the rod *b* was placed, measuring the station lines, and the offsets from them to the boundaries; now move the instrument, and place the center thereof exactly over the hole from whence the rod *b* was taken. The flower-de-luce being from you, turn the instrument till the hair in the sights coincides with the object at the station *c*, then will the north end of the needle point to 292° 12′, the bearing of *b c*; the instrument being planted at *c*, and the sights directed to *d*, the bearing of *c d*, will be 331° 45′. In the same manner proceed to take the bearing of other lines round the wood, observing carefully the following general rule:

Keep the flower-de-luce from you, and take the bearing of each line from the north end of the needle.

* Wyld's Practical Surveyor, p. 77.

lines.	bearings.	links.
a b	260. 30	1242
b c	292. 12	1015
c d	331. 45	1050
d e	59. 00	1428
e f	112. 15	645
f a	151. 30	1806

Instead of planting the circumferentor at every station in the field, the bearings of the several lines may be taken if it be planted only at every other station.

So if the instrument had been planted at *b*, and the flower-de-luce in the box kept towards you when you look back to the station *a*, and from you when you look forwards to the station *c*, the bearings of the lines *a b*, and *b c*, would be the same as before observed; also the bearings of the lines *c d*, and *d e*, might be observed at *d*, and *e f*, and *f a*, at *f*; so that instead of planting the instrument six times, you need in this case plant it but three times, which saves some labour.

But, since you must go along every station line, to measure it, or see it measured, the trouble of setting down the instrument is not very great, and then also you may examine the bearing of each line as you go along; and, if you suspect an error in the work, by the needles being acted upon by some hidden magnetic power, or from your own mistake, in observing the degrees that the needle points to, you may correct such error at the next station before you proceed.

As when the instrument was planted at *a*, and the sights directed at *b*, the flower-de-luce from you, the north end of the needle pointed to 260° 30′; now being come to *b*, direct the sights back to a mark at *a*, keeping the flower-de-luce towards you: so shall the north end of the needle point to 260° 30′, as before at *a*, and then you may be sure the bearing of the line *a b* is truly observed.

But if the needle doth not point to the same number of degrees, &c. there hath been some error in that observation, which must be corrected.

OF THE IMPROVED CIRCUMFERENTOR, WITH MR. GALE'S METHOD OF USING IT; A METHOD THAT IS APPLICABLE WITH EQUAL ADVANTAGE TO THE THEODOLITE, &C.

For the sake of perspicuity, it will be necessary to give again the example before used in page 222, and that not only because it will exhibit more clearly the advantages of Mr. *Gale*'s method,* but because we shall have occasion to refer to it when we come to his improved method of plotting; and further, because I have thought this mode so advantageous, and the tables so conducive to accuracy and expedition, that I have caused occasionally the traversing quadrant to be engraved in smaller figures under the usual one of the limb of the theodolite.

Set the circumferentor up at B, *fig.* 1, *plate* 18, take the course and bearing of B C, and measure the length thereof, and so proceed with the sides C D, D E, E F, F G, G A, all the way round to the place of beginning, noting the several courses or bearings, and the lengths of the several sides in a field book, which let us suppose to be as follow:

			°	′		chains.	links.
1.	AB	North	7		West	21.	00.
2.	BC	North	55	15	East	18.	20.

* See page 219 of this work for a fuller account of this method.

							chains.	links.
			°	′				
3.	CD	South	62	30	East		14.	40.
4.	DE	South	40		West		11.	
5.	EF	South	4	15	East		14.	
6.	FG	North	73	45	West		12.	40.
7.	GA	South	52		West		9.	17.

N. B. By north 7° west, is meant seven degrees to the westward, or left hand of the north, as shewn by the needle; by north 55° 15′ east, fifty-five degrees fifteen minutes to the eastward, or right hand of the north, as shewn by the needle.

In like manner, by south 62° 30′ east, is meant sixty-two degrees and thirty minutes to the eastward, or left-hand of the south; and by south 40° west, forty degrees to the westward, or right hand of the south.

The 21 chains, 18 chains 20 links, &c. are the lengths or distances of the respective sides, as measured by the chain.

To survey a field, or tract of land, having irregular boundaries.

When the boundaries of a survey have crooks and bends in them, it is by no means necessary to take a new course for every small bend; the best and most usual way, is to proceed in a strait line from one principal corner to another, and when you are opposite to any bend in the boundary, to measure the rectangular distance; termed the off-set, from the strait line to the bend, noting the same in the field book, together with the distance on the strait line from whence such offsets were made. The offsets, as already observed, are generally measured with an offset staff.

For the purpose of noting these offsets, it is necessary that the field book should be ruled into

FIVE COLUMNS. The middle column to contain the courses and distances; the adjoining columns on the right and left hand to contain the measure of the offsets, made to the right or left hand respectively; and the outside columns on the right and left hand, to contain remarks made on the right and left hand, respectively, such as the names of the adjoining fields, or the bearing of any remarkable object, &c.

Example. Let *fig.* 3, *plate* 18, represent a field to be surveyed, whose boundaries are crooked.

Set up the instrument at, or near any convenient corner, as at 1, and take the course, and bearing, as before directed, *north* 7° *west*, note this down in the middle column of the field book, and measure with the chain as before directed, till you come opposite to the first bend, so that the bend be at right angles to the station line; note the distance thus measured, 3 chains 60 links, in the middle column, and measure the offset from thence to the bend 40 links, noting the same in the adjoining left hand column, because the boundary is on the left hand of the station line, and note in the outside column, the name or owner of the adjoining field, proceeding in the same manner all round the field, noting the courses, distances, offsets, and remarks, as in the following:

FIELD BOOK.

REMARKS.	Off-sets.	Station lines.	Off-sets.	REMARKS.
	1st. At the corner, against Wm. Humphrey's, and H. Derman's land.			
		N. 7° W.		
	0 0	0 0		
H. Derman's land	0 40	3 60		
	0 10	8 45		
	0 65	15 60		
A corner	0 0	21 00		
		2d.		
W. Higgin's land		N. 55° 15. E.		
	0 0	0 0	0 0	
		6 10	0 60	To the boundary of the field.
A corner	0 0	18 20	0 0	
		3d.		
		S. 62° 30. E.		
D. Horne's land	0 0	0 0	0 0	
		7 40	0 90	To the boundary of the field.
A corner	0 0	14 40	0 0	
		4th.		
C. Ward's land		S. 40° W.		
	0 0	0 0		
	0 40	8 00		
To a corner	0 95	11 00		
		5th.		
		S. 4° 15. E.		
To the above corner	0 68	0 66		
C. Ward's land	1 25	7 50		
A corner	0 0	14 00		
		6th.		
W. Humphrey's land		N. 73° 45. N.		
	0 0	0 0	0 0	
		3 20	0 30	To the boundary of the field.
		7 50	1 25	
		12 40	0 0	A corner.

FIELD BOOK.

REMARKS.	Off-sets.	Station lines.	Off-sets.	REMARKS.
		7th.		
W. Humphrey's land		S. 52° W.		
	0 0	0 0	0 0	
		4 10	0 60	To the boundary of the field.
		9 17	0 0	

To the place of beginning.

To survey a tract of land, consisting of any number of fields lying together.

1. Take the outside boundaries of the whole tract as before directed, noting in your field book where the particular fields intersect the outside boundaries; and then take the internal boundaries of the several fields from the place where they so butt on the outside bounds.

Let *fig.* 4, *plate* 18, represent a tract of land to be surveyed, consisting of three fields.

First, begin at any convenient corner, as at A, and proceed taking the courses, distances, offsets, and remarks, as in the following field book.

FIELD BOOK.

REMARKS.	Off-sets.	Station lines.	Off-sets.	REMARKS.
	1st. At a corner against Winterton farm, and H. Smith's land.			
		S. 87° W.		
Lands of Henry Smith.	0 00	0 00	0 00	East field.
		5 50	1 10	To the corner of east and west fields.
		11 30	0 00	
		15 00	0 60	Corner of west field.

FIELD BOOK.

REMARKS.	Off-sets.	Station lines.	Off-sets.	REMARKS.
		2d.		
		N. 18°30. W		
		0 58	0 16	The above corner.
Hunterdon farm.		9 60	0 00	
		3d.		
		N. 12° 15. E.		
		0 00	0 00	West field.
		4 10	0 60	
		8 90	0 00	
		4th.		
		N. 37° E.		
		5 40	0 40	To the corner of west
	0 00	17 00	0 00	field and north field.
		5th.		
		S. 76° 45. E.		
Lands of	0 70	6 50		North field.
Jacob Williams.	0 00	16 00	0 00	
		6th.		
		S. 5° 15. W.		To the corner of north
		9 60	0 56	and east fields. N. B.
Winterton farm.		14 30	0 10	A gate into each field
		19 00	0 70	20 links from the
		25 00	0 00	corner.
		7th.		
		S. 74°30. W.		
		2 30	0 86	East field.
		7 65	0 00	
	To the place of beginning.			
	8th. At the corner of east and west field. viz. The first offset on the first station line.			
		North.		
West field.		8 30	0 75	East field.
		16 61	0 00	

FIELD BOOK.

REMARKS.	Off-sets.	Station lines.	Off-sets.	REMARKS.
		9th. East.		East field.
	0 00	0	0 00	To the corner of north
North field.		5 40	0 60	and east fields against
		13 77	0 00	Winterton farm.
		10th.		
West field.		Back to the last station.		
Corner of west		N.62°03.W		
and north fields	0 00	0	0 00	North field.
against Hunten-	0 70	3 40		
don farm.	0 00	7 92	0 00	

To take a survey of an estate, manor, and lordship. An estate, manor, or lordship, is in reality a tract of land, consisting of a number of fields; it differs in no respect from the last article, excepting in the number of fields it may contain, and the roads, lanes, or waters that may run through it, and is of course surveyed in the same manner.

It is best in the first place to take the whole of the outside boundaries, noting as above directed the several offsets, the several places where the boundaries are intersected by roads, lanes, or waters, the places where the boundaries of the respective fields butt on the outside bounds, and where the gates lead into the respective fields, and whatever other objects, as windmills, houses, &c. that may happen to be worthy of being taken notice of. If, however, there should be a large stream of water running through the estate, thereby dividing it into two parts, and no bridge near the boundary, then it will be best to survey that part which lies on one side of the stream first, and afterwards that part which lies on the other side thereof: if the stream be of an irregular breadth, both its banks forming boundaries to several fields should be surveyed, and its breadth, where it enters, and where it leaves the estate, be determined by the rules of trigonometry.

In the next place, take the lanes or roads, (if any such there be) that go through the estate, noting in the same manner as before, where the divisions between the several fields butt on those lanes or roads, and where the gates enter into those fields, and what other objects there may be worth noticing. Where a lane runs through an estate, it is best to survey in the lane, because in so doing, you can take the offsets and remarks both on the right hand and the left, and thereby carry on the boundaries on each side at once. If a large stream run through and separate the estate, it should be surveyed as above-mentioned; but small brooks running through a meadow, require only a few offsets to be taken from the nearest station line, to the principal bends or turning in the brook.

In the last place take the internal divisions or boundaries between the several fields, beginning at any convenient place, before noted in the field book, where the internal divisions butt on the outside of the grounds, or on the lanes, &c. noting always every remarkable object in the field book.

Example. Let *fig.* 5, *plate* 18, represent an estate to be surveyed; begin at any convenient place as at A, where the two lanes meet, proceed noting the courses, distances, &c. as before directed from A to B, from B to C, and so on to D, E, F, G, H, I, K, L, M and A, quite round the estate.

Then proceed along the lane from A to N, O and I, setting the courses, distances, offsets, and remarks, as before.

This done, proceed to the internal divisions, beginning at any convenient place, as at O, and proceed, always taking your notes as before directed, from O to P, Q and R, so will you have with the notes previously taken, the dimensions of the north field; go back to Q, and proceed from Q to H, and you obtain the dimensions of the copse. Take E S and S P, and you will have the dimensions of the home field; go back to S, and take S T, and you will have the dimensions of the land, applied to domestic purposes of buildings, yards, gardens, and orchards, the particulars and separate divisions of which being small, had better be taken last of all; go down to N, and take N D, noting the offsets as well to the brook, as to the fences, which divide the meadow from the south and west fields, so will you have the dimensions of the long meadow, together with the minutes for laying down the brook therein; go back to U, and take U B, and you obtain the dimensions of the west field, and also of the south field; go back again to N, and take N W, noting the offsets as well to the brook as to the fence which divides the meadow from the east field, thus will you have the dimensions of the east field, and the minutes for laying down the brooks in the meadow; then go to L, and take L X, which gives you the dimensions of the east meadow, and of the great field; and lastly, take the internal divisions of the land, appropriated to the domestic purposes of buildings, yards, orchards, gardens, &c.

The method of taking the field notes is so intirely similar to the examples already given, that they would be altogether unnecessary to repeat here.

To the surveyor there can need no apology for introducing, in this place, the method used by Mr. *Milne*, one of the most able and expert surveyors of the present day; and I think he will consider himself obliged to Mr. *Milne*, for communicating, with so much liberality, his deviations from the common practice, as those who have hitherto made any improvements in the practical part of surveying, have kept them as profound secrets, to the detriment of science and the young practitioner. As every man can describe his own methods, in the clearest and most intelligible manner, I have left Mr. *Milne*'s in his own language.

Mr. Milne's method of surveying.

"The method I take to keep my field notes in surveying land, differing materially from those yet published; if, upon examination, you shall think it may be useful or worthy of adoption, you may, if you please, give it a place in your treatise on surveying.

What I, as well as all surveyors aim at, in going about a survey, is accuracy and dispatch; the first is only to be acquired by care and good instruments, the latter by diligence and long practice.

From twenty years experience, in the course of which I have tried various methods, the following is what I at last adopted, as the most eligible for carrying on an extensive survey, either in England, Scotland, or any other cleared country.

Having taken a cursory view of the ground that I am first to proceed upon, and observed the plainest and clearest tract, upon which I can measure a circuit of three or four miles, I begin at a point convenient for placing the theodolite upon, and making a small hole in the ground, as at A, *plate* 20, one assistant leading the chain, the other having a spade for making marks in the ground, I measure in a direct line from the hole at A to B, noting down, as I go along, on the field sketch the several distances from place to place, and sketching in the figure of the road, as 75 links to the east side of the avenue, 400 links, touching the south side of the road, with an offset of 40 links to the other side, 500 links to the corner of a wood, and 817 to the corner of another wood; which last not allowing me to carry the line farther, I make a mark at B, where I mean to plant the instrument, and beginning a new line, measure along the side of the road 885 links to C, where I make a mark in the ground, writing the same in my field sketch, taking care always to stop at a place from which the last station can be seen, when a pole is placed at it. Beginning a new line, and measuring in a direct line towards D, I have 345 links to a line of trees, 390 links opposite a corner of paled inclosures, where I make a mark to have recourse to, and 893 links to the end of the line, where I make a mark in the ground at D; from thence measuring in a direct line to E, I here have 600 links of steep ground, therefore make a mark in the ground at the bottom of the steepness, so that when I come to take the angles with the theodolite, I may take the depression thereof; continuing out the line, I have 1050 links to a bridge, with an offset of 20 links to the bridge, and 80 links to the paling; at 1247 there is a line of trees on the left, and here also the line comes to the north side of the road, and 1388 links to the end of the line at E, and so proceed in like manner round the circuit to F, G, H, I, K, L, M, N; and from thence to A where I began. The holes, or marks made in the ground, are represented by round dots on the field sketch; steepness of ground is expressed on the sketch by faint strokes of the pen, as at D, F, &c. Figures ending a station line are written larger than the intermediate ones; offsets are written down opposite the places they were taken at, and are marked either to the right, or left of the line on the field sketch, just as they happen to be on the ground. If the field sketch here given was enlarged, so as to fill a sheet of paper, there would then be room for inserting the figures of all the offsets, which the smallness of this does not admit of.

In measuring these circuits, or station lines, too much care cannot be taken by the surveyor to measure exact; I therefore, in doing them, always choose to hold the hindermost end of the chain myself.

The next thing to be done is to take the angles or bearings of the above described circuit, and also the altitude or depression of the different declivities that have been measured up or down.

Having previously prepared a sheet of Dutch paper with meridian lines drawn upon it, as in *plate* 21, also a horn protractor with a scale of chains upon the edge thereof, and a small ruler about a foot in length; and having two assistants provided with a pole each, to which are attached plumb lines for keeping them perpendicular, and a third assistant for carrying the theodolite; I proceed to plant the instrument where I began to measure, or at any other angular point in the circuit; if the wind blows high, I choose a point to begin at, that is sheltered from it, so that the needle may settle steady at the magnetic north, which is indispensably necessary at first setting off, at the same time taking care that no iron is so near the place as to attract the needle.

The best theodolite for this purpose is the large one, see *fig.* 2, *plate* 16, the manner of using of which I shall here describe.

The spirit level C, having been previously adjusted to the telescope A, and the two telescopes pointing to the same object, I begin by levelling the instrument, by means of the four screws M acting between the two parallel plates N, first in a line with the magnetic north, and then at right angles thereto. This accomplished, I turn the moveable index, by means of the screw G, till it coincides with 180° and 360°, and with the magnetic north nearly, screwing it fast by means of the nut H, and also the head of the instrument by means of the pin L; I make the north point in the compass box coincide with the needle very exactly, by turning the screw K; both telescopes being then in the magnetic meridian, I look through the lower one I, and notice what distant object it points to.

Then unscrewing the moveable index by the aforesaid nut H, the first assistant having been previously sent to the station point marked N, I turn about the telescope A, or moveable index, by means of the nut G, till it takes up the pole now placed at N, raising or depressing the telescope by means of the nut E, till the cross hairs or wires cut the pole near the ground. This done, I look through the lower telescope to see that it points to the same object it did at first; if so, the bearing or angle is truly taken, and reading it upon the limb of the instrument F, find it to be 40° 55′ S. W. I then take my sheet of paper, and placing the horn protractor upon the point A, *plate* 21, along the meridian line, passing through it, I prick off the same angle, and with the first ruler draw a faint line with the pen, and by the scale set off the length of the line, which I find to be, by my first sketch, *plate* 20, 7690 links; and writing both bearing and distance down, as in *plate* 21, and again reading off the angle to compare it with what I have wrote down, I then make a signal to the first assistant to come forward with his pole; in the mean time I turn the moveable index about, till the hair or wire cut the pole which the second assistant holds up at B, and looking through the lower telescope, to see that it points on the same object as at first, I read the angle upon the limb of the theodolite, 48° 55′ S. E. and plotting it off upon the sketch with the horn protractor, draw a strait line, and prick off by the scale the length to B, 817 links; writing the same down on the field sketch, and again reading the angle, to see that I have wrote it down right, I screw fast the moveable index to the limb, by means of the screw H, and making a signal to the second assistant to proceed with his pole, and plant it at the third station C, while I with the theodolite proceed to B, leaving the first assistant with his pole where the instrument stood at A.

Planting the instrument by means of a plumb-line over the hole, which the pole made in the ground at the second station, and holding the moveable index at 48° 55′ as before, and the limb levelled, I turn the limb till the vertical wire in the telescope cuts the pole at A nearly, and there screw it fast. I then make the vertical wire cut the pole very nicely by means of the screw K; then looking through the lower telescope to see and remark what distant object it points on, I loosen the index screw, and turn the moveable index till the vertical hair in the telescope cuts the pole at C nearly, and, making the limb fast, I make it do so very nicely by means of the screw K, and then fix the index. Reading the degrees and minutes which the index now points to on the limb, I plot it in my field sketch as before, the bearing or angle being 48° 30′, and the length 885 links. Sending the theodolite to the next station C, the same operation is repeated; coming to D, besides taking the bearing of the line D E, I take the depression from D, to the foot of the declivity 600 links below it, which I perform thus: Having the instrument and the double quadrant level, I try what part of the body of the assistant, who accompanies me for carrying the instrument, the telescope is against, and then send him to stand at the mark at the bottom of the declivity, and making the cross wires of the telescope cut the same part of his body equal the height of the instrument, I find the depression pointed out upon the quadrant to be ⅔ of a link upon each chain, which upon six chains is four links to be subtracted from 1388 links, the measured length of the line, leaving 1384 links for the horizontal length, which I mark down in my field sketch, *plate* 2, in the manner there written. Also from the high ground at D, seeing the temple O, I take a bearing to it and plotting off the same in my field sketch, draw a strait line; the same temple being seen from station L, and station M, I take bearings to it; from each; and from the extension of these three bearings intersecting each other in the point O, is a proof that the lines have been truly measured, and the angles right taken.

Again, from station E to station F, the ground rises considerably, therefore, besides the bearing, I take the altitude in the same manner I did the depression; and do the same with every considerable rise and fall round the circuit.

Coming to station N, and having the moveable index at 61° 15′ N. W. taking up the pole at M, and making fast the limb, I loosen the index screw, and turn it till I take up the pole at the first station A. This bearing ought to be 40° 55′ N. E. being the same number of degrees and minutes it bore from A south-west, and if it turn out so, or within a minute or two, we call it a good closure of the circuit, and is a proof that the angles have been accurately taken.

But if a greater error appears, I rectify the moveable index to 40° 55′, and taking up the pole at A, make fast the limb, and take the bearing to M, and so return back upon the the circuit again, till I find out the error. But an error will seldom or never happen with such an instrument as here described, if attention is paid to the lower telescope; and besides, the needle will settle at the same degree (minutes by it cannot be counted) in the box, as the moveable index will point out on the limb, provided it is a calm day, and no extraneous matter to attract it. If it does not, I then suspect some error has been committed, and return to the last station to prove it, before I go farther.

With regard to plotting off the angles with the horn protractor in the field, much accuracy is not necessary; the use of it being only to keep the field sketch regular, and to preserve the figure of the ground nearly in its just proportion.

Coming home, I transfer from the first field sketch, *plate* 20, all the intermediate distances, offsets, and objects, into the second, *plate* 21, nearly to their just proportions; and then I am ready to proceed upon surveying the interior parts of the circuit. For doing this, the little light theodolite represented at *fig*. 4, *plate* 15, or the more complete one, *fig*. 7, *plate* 14, will be sufficiently accurate for common surveying. It is unnecessary for me to describe farther the taking the angles and mensurations of the interior subdivisions of the circuit, but in doing of which, the young surveyor had best make use of the horn protractor and scale; because, if he mistakes a chain, or takes an angle wrong, he will be soon sensible thereof by the lines not closing as he goes along; the more experienced will be able to fill in the work sufficiently clear and distinct without them. The field sketch will then be such as is given in *plate* 22, which indeed appears rather confused, owing to the smallness of the scale made use of to bring it within compass; but if this figure was enlarged four times, so as to fill a sheet of paper, there would then be room for entering all the figures and lines very distinctly.

Having finished this first circuit; before I begin to measure another, I examine the chain I have been using, by another that has not been in use, and find that it has lengthened more or less, as the ground it has gone through was rugged or smooth, or as the wire of which it is made is thick or small; the thick or great wire drawing or lengthening more than the small.

The careful surveyor, if he has more than two or three days chain work to do, will take care to have a spare chain, so that he may every now and then correct the one by the other. The offset staff is inadequate for this purpose, being too short. For field surveying, when no uncommon accuracy is required, cutting a bit off any link on one side, 50 links, and as much on the other side, answers the purpose better than taking away the rings.

The above method is what I reckon the best for taking surveys to the extent of 100,000 acres; beyond this, an error from small beginnings in the mensuration becomes very sensible, notwithstanding the utmost care; therefore, surveying larger tracts of country, as counties or kingdoms, requires a different process. Surveys of this kind are made from a judicious series of triangles, proceeding from a base line, in length not less than three miles, measured upon an horizontal plane with the greatest possible accuracy. *Thomas Milne.*"

The following Tables are inserted for the occasional use of the young Surveyor.

1. *A Table of Long Measures.*

7.92 inch	1 link						
12	$1\frac{17}{33}$	1 foot					
36	$4\frac{6}{11}$	3	1 yard				
198	25	$16\frac{1}{2}$	$5\frac{1}{2}$	1 pole			
792	100	66	22	4	1 chain		
7920	1000	660	220	40	10	1.furl.	
63360	8000	5280	1760	320	80	8	1 mile

2. *A Table of Square Measures.*

9 feet	1 yard					
$272\frac{1}{4}$	$30\frac{1}{4}$	1 perch				
4356	484	16	1 chain			
10890	1210	40	$2\frac{1}{2}$	1 rood		
43560	4840	160	10	4	1 acre	
27878400	3097600	102400	6400	2560	640	1 mile

In these Tables, each perpendicular column is of one denomination throughout; and all the lateral ones are equal, but of different denominations.

N. B. Perch the same as pole, and a square chain is the tenth part of an acre.

OF

PLOTTING,

OR MAKING A DRAUGHT OF THE LAND FROM THE FIELD NOTES.

OF PLOTTING, AND OF THE INSTRUMENTS USED IN PLOTTING.

By plotting, we mean the making a draught of the land from the field notes. As the instruments necessary to be used by the surveyor in taking the dimensions of land, are such wherewith he may measure the length of a side, and the quantity of an angle in the field; so the instruments commonly used in making a plot or draught thereof, are such wherewith he may lay down the length of a side, and the quantity of an angle on paper. They therefore consist in *scales of equal parts* for laying down the lengths or distances, and *protractors* for laying down the angles.

Scales of equal parts are of different lengths and differently divided; the scales commonly used by surveyors, are called feather-edge scales; these are made of brass, ivory, or box; in length about 10 or 12 inches, but may be made longer or shorter at pleasure. Each scale is decimally divided, the whole length, close by the edges, which are made sloping in order to lay close to the paper, and numbered 0, 1, 2, 3, 4, &c. which are called chains, and every one of the intermediate divisions is ten links, the numbers are so placed as to reckon backwards and forwards; the commencement of the scale is about two or three of the larger divisions from the fore end of the scale; these are numbered backwards from o towards the left hand with the numbers or figures 1, 2, 3, &c. these scales are often sold in sets. See *fig.* 2, *plate* 22.

The application and use of this scale is easy and expeditious, for to lay down any number of chains from a given point in a given line; place the edge of the scale in such a manner that the o of the scale may coincide with the given point, and the edge of the scale with the given line; then with the protracting pin, point off from the scale the given distance in chains, or chains and links.

Fig. F G H, *plate* 2, represents a new scale or rather scales of equal parts, as several may be laid down on the same instrument; each scale is divided to every ten links, or tenths of a chain, which are again subdivided by their respective nonius divisions into single links; the protracting pin is moved with the nonius by means of the screw *d*, so that the distances may be set off with such great accuracy, as not to err a single link in setting off any extent, which in the scale of four chains in an inch, does not amount to more than the 400th part of an inch.

Fig. I K L, *plate* 2, answers the same purpose as the foregoing instrument, and may be used as a protractor also.

For Mr. *Gale*'s method of plotting, a method which will recommend itself to every attentive surveyor, two scales should be used, one of about 15 inches long, the other about 10 inches, each divided on the edge from one end to the other; a clasp should be fixed on the shorter scale, whereby it may at pleasure be so fixed to the other scale, as to move along the edge thereof at right angles, for the purpose of laying off perpendicular lines.

Fig. F G H, *plate* 2, represents an instrument for this purpose, with different scales on the part F G; both this, and that represented at *fig.* I K L, form excellent parallel rules. They are also made only with the clasp, if desired.

The protractor is a circle or semicircle of thin brass, divided into degrees, and parts of a degree, on the outer edge.

The common semicircular protractor, *fig.* 2, *plate* 3, is of six or eight inches diameter, the limb divided into 180 degrees, and numbered both ways, 10, 20, 30, &c, to 180; each degree is subdivided into two parts. In the middle of the diameter is a small mark, to indicate the center of the protractor; this mark must be always placed on the given angular point.

The common circular protractor is more useful then the semicircular one; the outside edge is divided into 360 degrees, and numbered 10, 20, 30, to 360; each degree is subdivided into halves. In the middle of the diameter is a small mark, which is to be placed on the angular point, when an angle is to be protracted; the diameter, representing a meridian, must be placed on the meridian of any plan, where the bearing of any object is to be laid down. The application of this instrument is so easy and simple, that examples of its use are unnecessary here.

Fig. 7, *plate* 11, and 4 and 5, *plate* 17, represent the three best circular protractors; the principle is the same in all; the difference consists in superiority of execution, and the conveniences that arise from the construction. *Fig.* 4, *plate* 17, is a round protractor, the limb accurately divided to 360°, each degree is divided into two parts, which are again subdivided to every minute by the nonius, which moves round the limb of the protractor, on a conical center; that part of the index beyond the limb has a steel point fixed at the end, whose use is to prick off the angles; it is in a direct line with the center of the protractor, and the index division of the nonius, all which is evident from an inspection of the figure.

OF PLOTTING, OR MAKING A DRAUGHT OF THE LAND FROM THE FIELD NOTES.

The common method of plotting is this: take a sheet of paper of convenient size, draw a line thereon, to represent the magnetic meridian, and assign any convenient point therein to represent the place where the survey commenced; lay the edge of the protractor on this meridian line, and bringing the center thereof to the point so assigned to represent the place of beginning, mark off the degrees and minutes of the first course or bearing by the limb of the protractor, and draw a line from the place of beginning through the point so marked, laying off its proper length or distance by the scale of equal parts; this line will represent the first line of the survey. Through the point or termination of the said first line of the survey, draw another line, representing the magnetic meridian, parallel to the former; and lay off the course or bearing of the second line of the survey by the protractor, and its length by the scale of equal parts, as before; and so proceed, till the whole be laid down; and you will have a plot or draught of the survey.

For example. Let it be required to make a plot or draught of the field notes.

1st. Draw any line, as NAS, *fig.* 1, *plate* 18, to represent the magnetic meridian, and assign any convenient point therein, as A, to represent the place of beginning the survey; lay the edge of the protractor on the line N A S, with the center thereof at the point A, and mark off *seven degrees on the limb* to the westward, or left hand of the north, and draw the line A B through the point so marked off, making the length thereof 21 chains by the scale of equal parts. 2nly. Draw another meridian line N B S, through the point B, parallel to the former.* Lay the edge of the protractor on this second meridian line N B S, with the center thereof at the point B, and mark off $55^\circ\ 15'$ on the limb to the right hand, or eastward of the north; and draw the line B C through the point so marked off, making the length thereof 18 chains 20 links by the scale of equal parts. 3dly. Draw another meridian line N C S, through the point C, parallel to the former; lay the edge of the protractor to this third meridian line N C S, with the center thereof at the point C, and lay down the third course and distance C D, in the same manner as before; and so proceed with all the other lines, D E, E F, F G, G A; and if the last line shall terminate in the place of beginning, the work *closes*, as it is called, and all is right. But if the last line do not terminate in the place of beginning, there must have been a mistake either in taking the notes, or in the protraction of them: in such case, therefore, it will be necessary to go over the protraction again, and if it be not found then, it must of course be in the field notes, to correct which, if the error is material, they must be taken again.

* These parallel lines may be drawn with the least trouble by a parallel rule.

Mr. Gale's method of plotting.

The foregoing method of plotting is liable to some inaccuracies of practice, on account of having a new meridian for every particular line of the survey, and on account of laying off every new line *from the point of termination of the preceeding one*, whereby any little inaccuracy that may happen in laying down of one line is communicated to the rest. But there is a second method of plotting, by which these inconveniences are avoided, and by which also the accuracy or inaccuracy of the field work is decided with precision and certainty; I would, therefore, recommend this second method to the practitioner, as far preferable to any other I have seen.

2d Method. Take out from the first table in the *Appendix to this Work* the *northings*, *southings*, *eastings*, and *westings*, made on each of the several lines of the survey, placing them in a kind of table in their respective columns; and, if the sum of the northings be equal to the sum of the southings, and the sum of the eastings equal to the sum of the westings, the work is right, otherwise not.*

Then in an additional column put the whole quantity of northing or of southing made at the termination of each of the several lines of the sur-

* The truth of this observation cannot but appear self-evident to the reader. For the meridians within the limits of an ordinary survey having no sensible difference from parallelism, it must necessarily follow, that if a person travel any way soever with such small limits, and at length come round to the place where he sat out, he must have travelled as far to the northward as to the southward, and to the eastward as to the westward, though the practical surveyor will always find it difficult to make his work close with this perfect degree of exactness.

vey; which will be determined by adding or subtracting the northing or southing made on each particular line, *to* or *from* the northing or southing made on the preceding line or lines. And in another additional column, put the whole quantity of easting or westing made at the termination of each of the several lines of the survey, which will be determined in like manner by adding or subtracting the easting or westing made on each particular line, *to* or *from* the easting or westing made on the preceding line or lines.

The whole quantity of the northings or southings, and of the eastings or westings, made at the terminations of each of the several lines, being thus contained in these two additional columns, the plot may be easily laid down from thence, by a scale of equal parts, without the help of a protractor.

It is best, however, to use a pair of scales with a clasp,* whereby the one may at pleasure be so fastened to the other, as to move along the edge thereof at right angles, so that the one scale may represent the meridian, or north and south line, the other, an east and west line. It may also be observed, that in order to avoid taking the scales apart during the work, it will be necessary that the whole of the plot lay on one side of that scale which represents the meridian; or, in other words, the stationary scale should represent *an assumed meridian*, laying wholly on one side of the survey. On this account it will be necessary to note in a third additional column of the preparatory table, the distances of each of the corners or terminations of the lines of the survey, east or west from such assumed meridian. In practice, it is rather

* Or one of those at *fig.* F G H, or I K L, *plate* 2.

more convenient that this assumed meridian should lay on the west side, than on the east side of the survey.

The above-mentioned additional column, as has been already observed, contains the distance of each of the corners or terminations of the lines of the survey, eastward or westward, from that magnetic meridian which passes through the place of beginning the survey. In order, therefore, that the assumed meridian should lay entirely without the survey, and on the west side thereof, it will only be necessary that its distance from that meridian, which passes through the place of beginning, should be somewhat greater than the greatest quantity of westing contained in the said second additional column. Let then an assumed number somewhat greater than the greatest quantity of westing contained in the second additional column, be placed at the top of the third additional column, to represent the distance of the place of beginning of the survey from the assumed meridian. Let the several eastings, contained in the second additional column, be added to this assumed number, and the several westings subtracted from it; and these sums and remainders being respectively placed in the third column, will shew the distance of the several corners or terminations of the lines of the survey from the assumed meridian.

The preparatory table being thus made, take a sheet of paper of a convenient size, or two or more sheets pasted together with a little paste or mouth glue, in case a single sheet should not be large enough; and on the left hand of the intended plot draw a pencil line to represent the assumed meridian, on which lay the stationary scale. Place the moveable scale to any convenient point on the edge of the former, and point off by the edge of the latter, according to any desired number of chains to an inch, the assumed distance or number of chains contained at the top of the above-mentioned third column of the preparatory table. Move the moveable scale along the edge of the stationary one, to the several north or south distances contained in the first of the above-mentioned additional columns; so will the points thus marked off represent the several corners or terminations of the lines of the survey; and lines being drawn from one point to the other, will of course represent the several lines of the survey.

Example. Let it be required to make a plot or draught of the field notes, p. 222, according to Mr. *Gale*'s method.

The preparatory table will be as follows:

Courses and distances as taken in the field.	Northings, southings, eastings, and westings, made on those respective courses and distances.				N. and S. distances from the place of beginning to the end of each line.	E. and W. distances from the place of beginning to the end of each line.	Distances of the ends of each line from the assumed meridian.
	N.	S.	E.	W.			
° ch.							E. 3 00
1. N. 7 W. 21 00	20 84			2 56	N. 20 86	W. 2 56	E. 0 44
2. N. 55 15 E. 18 20	10 37		14 95		N. 31 21	E. 12 39	15 39
3. S. 62 30 E. 14 40		6 65	12 77		N. 24 56	E. 25 16	28 16
4. S. 40 W. 11		8 43		7 07	N. 16 13	E. 18 09	21 09
5. S. 4 15 E. 14		13 96	1 04		N. 2 17	E. 19 13	22 13
6. N. 73 45 W. 12 40	3 47			11 90	N. 5 64	E. 7 23	10 23
7. S. 52 W. 9 17		5 64		7 23	0 00	0 00	3 00
	34 68	34 68	28 76	28 76			

The northings and southings, eastings and westings in the above table, are taken from the first table in the Appendix; thus, first find the course 7 degrees in the table, and over against 21 chains, in the column marked dist. you have 20.843 in the column marked N. S. which, rejecting the right hand figure 3 for its insignificancy, is 20 chains

84 links for the quantity of *northing*, and in the column marked E. W. you have 2.559, very near 2 chains and 56 links for the westing made on that course and distance. 2. Find the next course 55.15 in the table, and over against 18 chains in the column dist. you have 10 chains 26 links in the column marked N. S. and 14 chains 79 links in the column marked E. W. and over against 20 links in the column dist. you have 11 links in the column N. S. and 16 links in the column marked E. W. which, put together, make 10 chains 37 links for the northing, and 14 chains 95 links for the easting made on the second course; and so of the rest.

The north and south distances made from the place of beginning to the end of each line, contained in the next column, are determined thus. On the first course, 20 chains and 84 links of northing was made; on the second course, 10 chains 37 links of northing, which, added to the preceding, makes 31 chains 21 links of northing; on the third course was made 6 chains 65 links of southing, which subtracted from the preceding 31 chains 21 links of northing, makes 24 chains 56 links of northing; and so of the rest.

The east and west distances made from the place of beginning to the end of each line, contained in the next right hand column, are determined in the same manner; thus, on the first course was made 2 chains 56 links of westing; on the second course 14 chains 25 links of easting, from which subtracting the preceding 2 chains 56 links of westing, there remains 12 chains 39 links of easting; on the third course was made 12 chains 77 links of easting, which added to the preceding 12 chains 39 links, makes 25 chains 16 links of easting; and so of the rest.

The distances of the end of each line from the assumed meridian, contained in the next right hand column, are thus determined. The first assumed number may be taken at pleasure, provided only that it exceeds the greatest quantity of westing contained in the preceding column, whereby the assumed meridian shall be entirely out of the survey. In the foregoing example, the greatest quantity of westing contained in the preceding column is 2 chains 56 links; the nearest whole number greater than this is 3 chains, which is accordingly taken and placed at the top, to represent the distance between the assumed meridian and the place of beginning of the survey; from this 3 chains subtract 2 chains 56 links of westing, there remains 44 links for the distance between the terminations of the first line and the assumed meridian. The 12 chains 39 links of easting in the next step, is added to the assumed 3 chains, which make 15 chains 39 links for the distance of the termination, by the second line from the assumed meridian. The 25 chains 16 links of easting in the next step, being added in like manner to the assumed 3 chains make 28 chains 16 links for the distance of the termination of the third line from the assumed meridian; and so on, always adding the eastings and subtracting the westings from the first assumed number.

The preparatory table being completed, take a sheet of paper or more, joined together if necessary, and near the left hand edge thereof rule a line as N. S. *fig.* 2, *plate* 18, to represent the assumed meridian; on this line lay the stationary scale, and assuming *a* as a convenient point therein to represent the point directly west from the place of beginning, bring the moveable scale to the point *a*, and lay off the first number contained in the last-mentioned column of the preparatory table, viz. 3 chains, from a to A, and A will represent the place of beginning of the survey. Move the moveable scale along the stationary one to the first north and south distance from the place of beginning, viz. 20.84 from a to b, and lay off the corresponding distance from the assumed meridian, 0.44, from b to B, and draw AB, so will AB represent the first line of the survey; again, move the moveable scale along the stationary one, to the third north and south distance from the place of beginning, viz. N. 31.21 from a to c; and lay off the corresponding distance from the assumed meridian, viz. 15.39, from c to C, and draw BC, so will BC represent the second line of the survey. Again move the moveable scale along the stationary one to the third north and south distance from the place of beginning, viz. 24.56, from a to d, and lay off the corresponding distance from the assumed meridian, viz. 28.16, and draw CD, which will be the third line of the survey. Proceed in the same manner till the whole be laid down, and ABCDEFG will be the required plot.

This method of plotting is by far the most perfect, and the least liable to error of any that has been contrived. It may appear to some to require more labour than the common method, on account of the computations required to be made for the preparatory table. These computations are however made with so much ease and expedition, by the help of the table in the appendix, that this objection would vanish, even *if the computation were of no other use but merely for plotting; but it must be observed that these computations are of much further use* in determining the area or quantity of land contained in the survey, which cannot be ascertained with equal accuracy in any other way. When this is considered, it will be found that this method is not only preferable on account of its superior accuracy, but is attended with less labour on the whole than the common method.

If a pair of scales, such as are above recommended, be not at hand, the work may be laid down from a single scale, by first marking off the N. and S. distance on the line N. S. and afterwards laying off the corresponding east distances at right angles thereto.

To plot the field notes, p. 313. 1. Lay down all the station lines, viz. 7° W. 21 chains, N. 55. 15, E. 18. 20, &c. contained in the middle column of the field book, as before directed, without paying any regard to the offsets, until all the station lines, represented in *fig*. 3, *pl*. 18, by dotted lines, be laid down; then lay off the respective offsets at right angles from the station line, at the respective distances at which they were taken, as is done at *fig*. 3, a bare inspection of which will make the work perfectly plain.

To plot the field notes, p. 314. 1. Lay down the whole of the outside boundaries, the station lines first, the offsets afterwards, as directed in the preceding articles; and then lay down the internal divisions or boundaries of the respective fields, in the same manner as is done in *fig*. 4.

OBSERVATIONS ON PLOTTING, BY MR. MILNE.

The protractor, whether a whole or a semicircle, ought not to be less in diameter than seven or eight inches, to insure the necessary degree of accuracy in plotting the angles of a survey. The degrees on the limb are numbered various ways, but most commonly from 10° 20°, &c. to 360°.

It would be right to repeat the numbers the contrary way, when the breadth of the limb will admit of it. Others are numbered 10° 20°, &c. to 180°, and the contrary. Surveyors will suit themselves with that kind best adapted to their mode of taking angles in the field; so that in which ever way the limb of the instrument they survey with is graduated, the protractor had best be the same.

It is true, a surveyor of much practice will read off an angle mentally very readily, in which ever way the instrument happens to be graduated.

For instance, I have in the field taken the angles made with the magnetic meridian, in which case I count no angle above 89° 59′; if the angle comes to 90° 0′, I call it due E. or due W. if it is 89° 59′, I write to it N. E. or S. E. or else N. W. or S. W. as it happens to turn out.

Now 89° 59′ N. E. upon a protractor numbered to 360° 0′, and placing 360 to the north, reads the same; but 89° 59′ S. E. reads 90° 1′, and 89° 59′ S. W. reads 269° 59′; and lastly, 89° 59′ N. W. reads 270° 1′.

Again, if the angle or bearing is due south, it reads on this protractor 180°; if 0° 1′ S. E. it reads 179° 59′; and if 0° 1′ S. W. it is 180° 1′; and so of any other intermediate angle or bearing.

The great inducement to surveyors for taking the angles by the bearings the lines make with the magnetic meridian, is the having the needle as a check in the course of the survey; and when the circuit comes to be plotted, having it in his power to prick off all the angles, or bearings of the circuit, at once planting of the protractor.

This mode of surveying, or taking angles, is also more expeditious, and more accurate than any other, provided the index of the instrument is furnished with a clamp for making it fast to the limb, while it is carrying from one station to another.

The protractor for plotting this way of surveying had best be graduated 10, 20, &c. to 180, on the right hand, and the same repeated on the left, and again repeated contrary to the former; but as protractors for general use are graduated 10, 20, &c. to 360, I shall here describe a very useful and convenient one of this latter kind, and then proceed to give an example of plotting all the angles of a circuit by it from one station.

This protractor is represented at *fig.* 5, *plate* 17. Its diameter, from pointer to pointer, is 9½ inches; the center point is formed by two lines crossing each other at right angles, which are cut on a piece of glass. The limb is divided into degrees and half degrees, having an index with a nonius graduated to count to a single minute, and is furnished with teeth and pinion, by means of which the index is moved round, by turning a small nut. It has two pointers, one at each end of the index, furnished with springs for keeping them suspended while they are bringing to any angle; and being brought, applying a finger to the top of the pointer, and pressing it down, pricks off the angle. There is this advantage in having two pointers, that all the bearings round a circuit may be laid, or pricked off, although the index traverses but one half of the protractor.*

* The protractor, represented *fig.* 7, *plate* 11, is circular, and of the same diameter as the foregoing; the center is also formed by the intersection of two lines at right angles to each other, which are cut on glass, that all parallax may thereby be avoided; the index is moved round by teeth and pinion. The limb is divided into degrees and half degrees, and subdivided to every minute by the nonius; the pointer may be set at any convenient distance from the center, as the socket which carries it moves

Let *fig*. 5, *plate* 13, be the circuit to be plotted. I draw the magnetic meridian N. S. *fig*. 6, *plate* 13, and assigning a point therein for station 1, place the center of the above described protractor upon it, with 360° exactly to the north, and 180° to the south of the magnetic meridian line N. S. For the conveniency of more easily reading off degrees and minutes, I bring the nonius to the south side of the protractor, or side next to me; and seeing by the field sketch, or eye draught, *fig*. 5, that the bearing from station 1 to station 2 is 87° 30′ S. E. I readily conceive it wanting 2° 30′ of being due east, or 90° 0′; therefore, adding mentally 2° 30′ to 90° 0′, I make 92° 30′ on this protractor, to which I bring the nonius, and then make a prick by pressing down the pointer of the protractor, and with my lead pencil mark it 1. The bearing from station 2 to station 3 being 10° 15′ N. E. see *fig*. 5, I bring the nonius, for the sake of expedition and ease of reading off, to 10° 15′ S. W. which is upon this protractor 190° 15′, and pressing down the north pointer, it will prick off 10° 15′ N. E. which I mark with my pencil, 2.

The bearing from station 3 to station 4 being 85° 13′ N. E. I readily see this is wanting 4° 47′ of being due east; therefore I bring the nonius to within that of due west, which upon this protractor is 265° 13′, and pressing down the N. E. pointer, it will prick off 85° 13′ N. E. which I mark 3.

upon the bar B C, and is fixed thereto by the nut D. At right angles to the bar B C, and moveable with it, there is another bar E F; on this bar different scales of equal parts are placed, so that by moving a square against the inner edge thereof, angles may be transferred to any distance from the center, containing the same number of degrees marked out by the index; the use of which will be evident from Mr. Milne's observations.

The bearing from station 4 to station 5 being 11° 5′ N. W. I bring the nonius to 168° 55′, and with the opposite, or north-west pointer, prick off 11° 5′ N. W. which I mark 4.

In like manner I prick off the bearings from station 5 to 6, from 6 to 7, and from 7 to 1, which closes the circuit; marking them severally with my pencil 5, 6, 7.

Then laying aside the protractor, and casting my eye about the tract traced by the pointer of the protractor for the bearing from station 1, marked 1, I apply the ruler to it and station 1, and drawing a line, mark off its length by compasses and scale, eight chains, which fixes station 2.

Again, casting my eye about the tract traced by the pointer of the protractor, for the 2 bearing, marked 2, I apply the parallel ruler to it and station 1, and moving the ruler parallel eastward, till its edge touches station 2, I from thence draw a line northward, and by scale and compasses mark its length, five chains, to station 3. Thus the bearing 10° 15′ N. E. at station 1, is transferred to station 2, as will easily be conceived by supposing the points a, b, moved eastward, preserving their parallelism till they fall into the points of station 2 and 3, forming in *fig.* 6, the parallelogram a, b, 3, 2.

It is evident this bearing of 10° 15′ N. E. is by this means as truly plotted as if a second meridian line had been drawn parallel to the first from station 2, and the bearing 10° 15′ N. E. laid off from it, as was the common method of plotting previous to the ingenious Mr. *William Gardiner* communicating this more expeditious and more accurate mode to the public. However, its accuracy depends entirely upon using a parallel ruler that moves truly parallel, which the artist will do well to look to, before he proceeds to this way of plotting.

Station 3 being thus fixed, to find station 4, I cast my eye about the tract traced by the pointer of the protractor for the bearing marked 3, and applying the parallel ruler to it and station 1, move the ruler parallel thereto northward, till it touch station 3; and from thence draw a line north-eastward, and by scale and compasses marking its length, three chains, station 4 is found.

Now conceive the points c, d, moved northward, preserving their parallelism till they fall into the points of stations 3 and 4; and thus station 4 has the same bearing from station 3, as c or d has from station 1.

In the same manner I proceed to the next bearing 4; and placing the ruler to the point made by the pointer of the protractor at 4 and station 1, and moving the ruler parallel till its edge touch station 4; from thence draw a line, and mark off its length, 5 chains, to station 5. Here e, f, is transferred to 4th and 5th stations, and the 5th station makes the same angle with the meridian from the 4th station, as e, or f, does with the meridian from the first station.

In like manner, applying the parallel ruler to station 1, and the several other bearings of the circuit 5, 6, and 7; the points g, h, will be transferred to 5th and 6th, and i, k, to 6th and 7th; and lastly, the bearing from the 7th station will fall exactly into the first, which closes the plot of the circuit.

It is almost unnecessary to observe, that the dotted lines marked a, b, c, &c. *fig*. 6, are drawn there only for illustrating the operation: all that is necessary to mark in practice are the figures 1, 2, 3, &c. round the tract traced by the pointer of the protractor. T. MILNE.

OF DETERMINING

THE AREA OR CONTENT *OF LAND.*

1. The *area* or content of land is denominated by acres, roods, and perches.

Forty square perches make a *rood.*

Four roods, or 160 square perches, make an *English acre.*

And hence 10 square chains, of four perches each, make an English acre.

2. The most simple surface for admensuration is the rectangular parallelogram, which is a plane four-sided figure, having its opposite sides equal and parallel, and the adjacent sides rectangular to one another; if the length and breadth be equal, it is called a *square*; but, if it be longer one way than the other, it is commonly called an *oblong*.

TO DETERMINE THE AREA OF A RECTANGULAR PARALLELOGRAM.

Rule. Multiply the length by the breadth, and that product will be the area.*

It has been already observed, that the best method of taking the lengths is in *chains* and *links*; hence, the above-mentioned product will give the area in chains and decimal parts of a chain. And as 10 square chains make an acre, the area in acres will be shewn by pointing off one place of the decimals more in the *product* than there are both in the *multiplicand* and *multiplier*; which answers the same purpose as dividing the area into chains by 10;

* Euclid, Book i. Prop. 35 Scholium.

and the roods and perches will be determined by multiplying the decimal parts by 4 and by 40.

Example. Suppose the side of the foregoing square be 6 chains and 75 links; what is the area?

```
Multiply 6.75
      by 6.75
         ----
         3375
        4725
       4050
       -------
Acres  4.55625
             4 Roods in an acre.
       -------
Roods  2.22500
            40 Perches in a rood.
       -------
Perches 9.00000
       -------
      Acres. Roods. Perches.
Ans.    4      2      9.
```

Suppose the length of the foregoing oblong be 9 chains and 45 links, and the breadth 5 chains and 15 links; how many acres doth it contain?

```
Multiply 9.45
      by 5.15
         ----
         4725
         945
       4725
       -------
Acres  4.86675
             4
       -------
Roods  3.46700
            40
       -------
Perches 18.68000
       --------
      A.  R.  P.
Ans.  4   3   18.
```

(The decimal parts .68 are not worth regarding.)

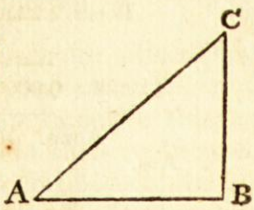
If the parallelogram be not rectangular, but oblique angled, the length must be multiplied by the perpendicular breadth, in order to give the area.*

3. After the parallelogram, the next most simple surface for admensuration is the triangle.

TO DETERMINE THE AREA OF A TRIANGLE.

Rule. Multiply the base, or length, by half the perpendicular, let fall thereon from the opposite angle, and that product will be the area of the triangle.†

Examples. In the triangle ABC, right angled at B, let the base AB be 8 chains and 25 links, and the perpendicular BC 6 chains and 40 links; how many acres doth it contain?

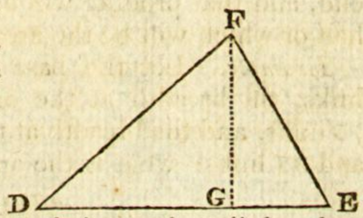


Multiply	8.25
by (the half of 6.40, viz.)	3.20
	16500
	2475
Acres	2.64000
	4
Roods	2.56000
	40
Perches	22.40000

	A.	R.	P.
Ans.	2	2	22.

* See Euclid, Book i. Prop. 35 and 36.

† Euclid, Book i. Prop. 41, Scholium. Every triangle is equal to the one half of a parallelogram, having the same base and altitude.



In the oblique angled triangle D E F, let the base D E be 12 chains and 25 links, and the perpendicular F G, let fall on the base D E from the opposite angle F, be 7 chains and 30 links; how many acres doth it contain?

Multiply	12.25
by (the half of 7.30, viz.)	3.65
	6125
	7350
	3675
Acres	4.47125
	4
Roods	1.88500
	40
Perches	35.40000

	A.	R.	P.
Ans.	4	1	35.

It would be the same thing, if the perpendicular were to be multiplied by half the base; or, if the base and perpendicular were to be multiplied together, and half the product taken for the area.

4. The next most simple surface for admensuration is a four-sided figure, broader at one end than at the other, but having its ends parallel to one another, and rectangular to the base. Its area is determined by the following

Rule. Add the breadths at each end together, and multiply the base by half their sum, and that product will be the area. Or otherwise, multiply the base by the whole sum of the breadths at each end, and that product will be double the area, the half of which will be the area required.

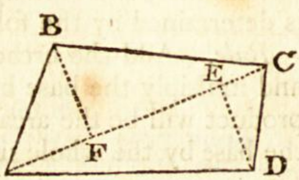
Example. Let the base be 10 chains and 90 links, the breadth at the one end 4 chains and 75 links, and the breadth at the other end 7 chains and 35 links; what is the area?

$$\begin{array}{r}
\left.\begin{array}{r}4.75\\7.35\end{array}\right\}\\
\hline
2)12.10 \text{ sum}\\
\hline
6.05 \text{ half sum}\\
10.90\\
\hline
54450\\
6050\\
\hline
\text{Acres } 6.59450\\
4\\
\hline
\text{Roods } 2.37800\\
40\\
\hline
\text{Perches } 15.12000\\
\hline
\end{array}$$

	A.	R.	P.
Ans.	6	2	15.

5. Irregular four-sided figures are called *trapezia.* The area of a trapezium may be determined by the following

Rule. Take the diagonal length from one extreme corner to the other as a base, and multiply it by half the sum of the perpendiculars, falling thereon from the other two corners, and that product will be the area.



Example. In the trapezium A B C D, let the diagonal A C be 11 chains and 90 links; the perpendicular B F 4 chains and 40 links, and the perpendicular D E 3 chains and 90 links; what is the area?

BF	4.40	
DE	3.90	
	2)8.30	sum
	4.15	half sum
AC	11.90	
	37350	
	415	
	415	
Acres	4.93850	
	4	
Roods	3.75400	
	40	
Perches	30.16000	

	A.	R.	P.
Ans.	4	3	30.

If the diagonal AC were multiplied by the whole sum of the perpendiculars, that product would be *double* the area; the half of which would be the area required.

6. The area of all the other figures, whether regular* or irregular, of how many sides soever the figure may consist, may be determined either by dividing the given figure into triangles, or trapezia, and measuring those triangles, or trapezia, separately, the sum total of which will be the area required; *or otherwise*, the area of any figure may be determined by a computation made from the

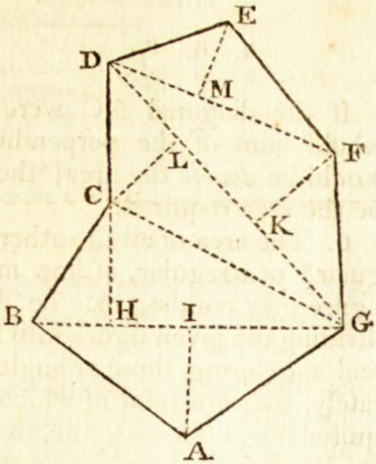
* *The regular figures*, polygons and circles, do not occur in practical surveying. On this head, therefore, I shall only observe, that the method of determining the area of a regular polygon, which is a figure containing any number of equal sides and equal angles, is, to multiply the length of one of the sides by the number of sides which the polygon contains, and then to multiply the product by half the perpendicular, let fall from the center to any one of the sides, and this last product will be the area.

The length of the perpendicular, if not given, may be determined by trigonometry, thus:

courses and distances of the boundary lines, according to an universal theorem, which will be mentioned presently.

7. The method of determining the area by dividing the given figure into trapezia and triangles, and measuring those trapezia and triangles separately, as in article 5 and article 3 of this part, is generally practised by those land measurers, who are employed to ascertain the number of acres in any piece of land, when a regular land surveyor is not at hand; and for this purpose, they measure with the chain the bases and perpendiculars of the several trapezia and triangles in the field.

Thus, for example, suppose the annexed figure be a field to be measured according to this method, the land measurer would measure with the chain the base BG, and the perpendiculars HC, and IA, of the trapezium ABCG; the base GD, and the perpendiculars KF, and LC, of the tra-



Divide 360 degrees by double the number of sides contained in the polygon, and that quotient will be half the angle at the center; then say, as the *tangent*, or half the angle at the center, is to half the length of one of the sides; so is *radius*, to the perpendicular sought.

The area of a CIRCLE is thus determined; if the diameter be given, say, as 1 : is to 3.141592 : : so is the diameter : to the circumference; or, if the circumference be given, say, as 3.141592 is to 1 : : so is the circumference : to the diameter. Then multiply the circumference by $\frac{1}{4}$ of the diameter, and the product will be the area.

pezium G C D F; and the base D F, and perpendicular M E, of the triangle D E F.

In order to determine where on the base lines the perpendiculars shall fall, the land measurer commonly makes use of a *square*, consisting of a piece of wood about three inches square, with apertures therein, made with a fine saw, from corner to corner, crossing each other at right angles in the center; which, when in use, is fixed on a staff, or the crosss; see *figs.* 2, 3, and 6, *plate* 14.

The bases and perpendiculars being thus measured, the area of each trapezium is determined as in article 5, and the area of the triangle as in article 3 of this part; which, being added together, gives the area of the whole figure required.

This method of determining the area, where the whole of each trapezium and triangle can be seen at one view, is equal in point of accuracy to any method whatever. It may likewise be observed, that a plot or map may be very accurately laid down from the bases and perpendiculars thus measured; this method, however, is practicable only in open or cleared countries; in countries covered with wood, it would be altogether impracticable, because it would be impossible to see from one part of the tract to another, so as to divide it into the necessary trapezia and triangles. But in open countries, the only objection to this method, is the great labour and tediousness of the field work; in a large survey, the number of the trapezia and triangles would be so very great, that the measuring of all the several bases and perpendiculars would be a very tedious business. The same ends are as effectually answered, with infinitely less labour, by taking the courses and distances of the boundary lines, in the manner mentioned p. 310.

8. Many surveyors who take their field notes in courses and distances, make a plot or map thereof, and then determine the *area* from the plot or map so made, by dividing such plot or map into trapezia and triangles, as in the above figure, and measuring the several bases and perpendiculars by the scale of equal parts, from which the plot or map was laid down.

By this method, the area will be determined something nearer the truth, but it falls short of that accuracy which might be wished, because there is no determining the lengths of the bases and perpendiculars by the scale, within many links; and the smaller the scale shall be by which the plot is laid down, the greater, of course, must be the inaccuracy of this method of determining the area.

This method, however, notwithstanding its inaccuracy, will always be made use of where tables of the northing, southing, easting and westing, are not at hand.

9. Proper tables of the northing, southing, easting and westing, fitted purposely for the surveyor's use, such as are contained in the *Appendix to this Tract*, ought to be in the hands of every surveyor. By these tables the area of any survey is determined with great accuracy and expedition, by an easy computation from the following

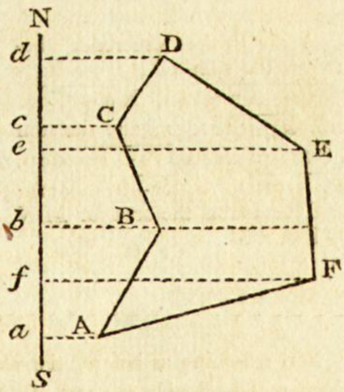
UNIVERSAL THEOREM.

If the sum of the DISTANCES, *on an east and west line, of the two ends of each line of the survey, from any assumed meridian laying entirely out of the survey, be multiplied by the respective* NORTHING *or* SOUTHING *made on each respective line; the* DIFFERENCE *between the sum of the* NORTH PRO-

DUCTS, *and the sum of the* SOUTH PRODUCTS, *will be double the area of the survey.**

For example. Let it be required to determine the area of the survey mentioned in p. 310, from the field notes there given.

The northings, southings, eastings and westings, as also the *north and south distances*, and the *east and west distances* made from the place of beginning to the end of each line; and likewise the distances of the ends of each line from the assumed



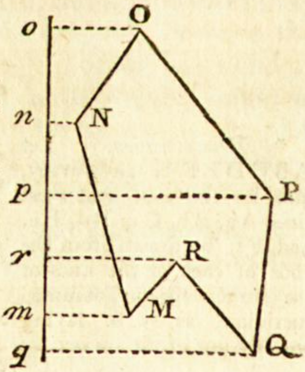
* *Demonstration.* Let A B C D E F be any survey, and let the east and west lines A a, B b, C c, D d, E e, and F f, be drawn from the ends of each of the lines of the survey to any assumed meridian, as N. S. laying entirely out of the survey.

Then, I say, the difference between the sum of the products $\overline{Aa+Bb}\times ab$, $\overline{Bb+Cc}\times bc$, $\overline{Cc+Dd}\times cd$; and the sum of the products $\overline{Dd+Ee}\times de$, $\overline{Ee+Ff}\times ef$, $\overline{Ff+Aa}\times fa$; will be double the area of A B C D E F A.

For, by article 4, the former products will be, respectively, double the areas of the spaces ABba, BCcb, and CDdc, and their sum will of course be double the area of the space Aad DCBA. In like manner, the latter products will be, respectively, double the areas of the spaces DEed, EFfe, and FAaf; and their sum will of course be double the area of the space AadDEFA. But it is evident, that the difference between the areas of the spaces AadDEFA, and AadDCBA, must be the area of the space ABCDEFA; consequently, the difference between their doubles must be double the area of ABC DEFA.

meridian, have been already placed in their respective columns, and explained in the preparatory to the explanation of the method of plotting there recommended; but it will not be improper to give them a place here again, together with the further columns for determining the area; in order that the whole, together with the respective uses of the several parts, may appear under one view, as follows.

Again, the difference between the sum of the products $\overline{Mm+Nn}\times mn$, $\overline{Nn+Oo}\times no$, $\overline{Qq+Rr}\times qr$; and the sum of the products $\overline{Oo+Pp}\times op$, $\overline{Pp+Qq}\times pq$, $\overline{Rr+Mm}\times rm$; will be double the area of MN OPQRM.



For, reasoning as before, the sum of the first mentioned products will be double the area of $\overline{QqoONMRQ+MRrmM}$; and the sum of the last mentioned products will be double the area of $\overline{QqoOPQ+MRrmM}$. But it is evident, that $QqoOPQ-QqoONMRQ=MNOPQRM$: consequently, $\overline{QqoOPQ+MRrmM}-\overline{QqoONMRQ+MRrmM}=M$ $NOPQRM$; and, consequently, the difference of their *doubles* must be equal to *double* MNOPQRM. And the like in every other possible figure.

Field Notes.	For a Proof of the Field Work.				For the Plotting.			For the Area.		
Courses and distances as taken in the field.	Northings, southings, eastings, and westings, made on those respective courses and distances.				North and south distances made on the whole from the place of beginning to the end of each line.	East and west distances made on the whole from the place of beginning to the end of each line.	Distances of the ends of each line from an assumed meridian laying out of the survey.	Sums of the distances of the two ends of each line from the assumed meridian, for multipliers.	North products, in acres.	South products, in acres.
	N.	S.	E.	W.						
° ′ ch. l.							E. 3 00			
1. N. 7 W. 21 00	20 84			2 56	N. 20 84	W. 2 56	E. 0 44	3 44	7 16896	
2. N. 55 15 E. 18 20	10 37		14 95		N. 31 21	E. 12 39	15 39	15 83	16 41571	
3. S. 62 30 E. 14 40		6 65	12 77		N. 24 56	E. 25 16	28 16	43 55		28 96075
4. S. 40 W. 11 00		8 43		7 07	N. 16 13	E. 18 09	21 09	49 25		41 51775
5. S. 4 15 E. 14 00		13 96	1 04		N. 2 17	E. 19 13	22 13	43 22		60 33512
6. N. 73 45 W. 12 40	3 47			11 90	N. 5 64	E. 7 23	10 23	32 36	11 22892	
7. S. 52 W. 9 17		5 64		7 23	0 00	0 00	3 00	13 23		7 46172
Totals	34 68	34 68	28 76	28 76					34 81359	138 27534
								North prod.	Subtract	34 81359
										2)103 46175
								Area.	Acres	51 73087
										4
									Roods	2 92348
										40
									Perches	36 93920

The agreement between the sum of the northings, and the sum of the southings, and between the sum of the eastings, and the sum of the westings shews that the survey has been truly taken.

The *first* and *third* of these columns only are used in plotting; the middle column is only a preparatory step for the distances contained in the third column.

The three last columns relating to the *area*, are the only ones that remain to be explained.

The first of these three columns contains the sums of the distances of the two ends of each line of the survey from the assumed meridian; which, according to the foregoing universal theorem, are *multipliers* for determining the area. These are formed by adding each two of the successive numbers in the preceding left hand column together. Thus 3.00 is the distance of the place of beginning from the assumed meridian, and 0.44 is the distance of the end of the first line from the assumed meridian; and their sum is 3.44 for the first multiplier. Again 0.44 added to 15.39 makes 15.83 for the second multiplier; and 15.39 added to 28.16 makes 43.55 for the third multiplier; and so of the rest.

The north products contained in the next column, are the products of the multiplication of the several *northings* contained in the columns marked N, by their corresponding *multipliers* in the last mentioned column. Thus 7.16896 is the product of the multiplication of 20.84 by 3.44; an additional place of decimals being cut off, to give the product in acres.

The south products contained in the next right hand column are, in like manner, the products of the multiplication of the several *southings* in the column marked S, by their corresponding multipliers. Thus 28.96075 is the product of the multiplication of 6.65 by 43.55; an additional place of decimals being cut off, as before, to give the product in acres; and so of the rest.

The *north products* being then added together into one sum, and the south products into another sum, and the lesser of these sums being subtracted from the *greater*, the *remainder*, by the above universal theorem, is *double the area of the survey:* which being divided by 2, gives the *area* required: viz. 51.73087, equal, when reduced, to 51 acres, 2 roods, and 36 perches 9-10.*

Nothing can be more simple or easy in the operation, or more accurate in answering the desired ends, both with respect to the proof of the work, the plotting, and the computation of the area, than this process. The northings, southings, eastings and westings, which give the most decisive proof of the accuracy or inaccuracy of the field work, are shewn by inspection in table 1. From these the numbers or distances for the plotting are formed by a simple addition or subtraction. And the multipliers, for determining the area, are formed from thence, by a simple addition, with much less labour, than by the common method of dividing the plot into trapezia, and taking the multipliers by the scale of equal parts.

The superior accuracy and ease with which every part of the process is performed, cannot, it is imagined, fail to recommend it to every practitioner, into whose hands this tract shall fall.

10. If the boundary lines of the survey shall have crooks and bends in them, which is generally the case in old settled countries, those crooks and bends are taken by making offsets from the station lines; as explained in page 311. In these cases, the area comprehended within the station lines is determined as in the last example; and the areas

* It must be observed, that, in order to determine the area by this method of computation, it is indispensibly necessary that the several lines of the survey be arranged regularly, one after another, all the way round the survey, in case they should not have been so taken in the field. because, without such arrangement, the northings, or southings made on each line, and the distances from the assumed meridian, would not correspond with one another.

comprehended between the station lines and the boundaries are determined separately, as in the *third* and *fourth* articles of this part, from the *off-sets* and *base lines*, noted in the field book; and are *added* or *subtracted* respectively, according as the station lines shall happen to be *within* or *without* the field surveyed; which, of course, gives the area required.

For example. Let it be required to determine the area of the survey mentioned in the example, page 312, from the field book there given.*

The *bearings* and *lengths* of the station lines are the same as in the last example; and, consequently, the area comprehended within those station lines must be the same; viz. 51.73087.

Then, for the area comprehended between the first station line and the boundary of the field, we have (see the field book, page 313) first, a base line 3.60, and a perpendicular offset 0.40; which, according to article 3 of this part, contains an area of 0.07200. *Secondly*, a base line 4.85, (being the difference between 3.60 and 8.45 on the station line, at which points the offsets were taken,) and an offset or breadth at the one end 0.40, and at the other end 0.10; which, according to article 4 of this part, contains an area of 0.12125. *Thirdly*, a base line 7.15, (being the difference between 8.45 and 15.60 on the station line,) and an offset or breadth at the one end 0.10, and at the other end 0.65; which according to the 4th article of this part, contains an area of 0.26812. *Fourthly*, a base line 5.40, (being the difference between 15.60 and 21.00 on the station line,) and a perpendicular offset 0.65; which, according to the 3d article of this part, contains an area of 0.17550. These added together make 0.63687 for the area

* A plot of this survey is given in *fig.* 3, *plate* 18.

of the offsets on the first station line; and, as this station line is *within* the survey, this area must, of course, be *added* to the above-mentioned area comprehended within the station lines.

Next, for the area comprehended between the second station line and the boundary of the field, we have a base line 18.20, and a perpendicular or offset 0.60; which, according to article 3 of this part, contains an area of 0.54600. And, as this station line is without the survey, this area must of course be *subtracted* from the above-mentioned area.

The areas of all the other offsets are determined in the same manner, and are respectively as follows; viz.

	Areas of the Offsets.	
	To be added.	To be subtracted.
On the *first* station line	A. 0.63687	
On the *second* - -		0.54600
On the *third* - -		0.64800
On the *fourth* - -	0.36250	
On the *fifth* - -	1.08875	
On the *sixth* - -		0.68750
On the *seventh* - -		0.27510
Totals - -	2.08812	2.15660

The area comprehended within the station lines, as above	A 51.73087
Offsets to be added	2.08812
Sum	53.81899
Offsets to be subtracted	2.15660
Remains the area of the survey required	A 51.66239
	4
Roods	2.64956
	40
Perches	25.98240

11. When the survey consists of a number of fields lying together, it is best to determine the area of the *whole* first; and afterwards, the area of each of the fields separately; the sum of which, if the work be true, will of course agree with the area of the whole; which forms a check on the truth of the computations.

But here it must be observed, that the boundaries of each field must be arranged for the computation, so as to proceed regularly one after another all the way round the field; because, as was observed in the note to the 9th article of this part, without such arrangement, the *northing* or *southing* made on each line, and the distances from the *assumed meridian*, would not correspond with one another.

For example. Suppose it were required to ascertain the area of the survey mentioned in page 314, of the several fields therein contained, from the field book there given.*

The bearings and lengths of the station lines on the outside boundaries are as follow; viz.

		°	′		ch. l.
1st.	S.	87		W.	15.00
2d.	N.	18	30	W.	9.60
3d.	N.	12	15	E.	8.90
4th.	N.	37		E.	17.00
5th.	S.	76	45	E.	16.00
6th.	S.	5	15	W.	25.00
7th.	S.	74	30	W.	7.65

The area comprehended within these station lines, computed according to the above-mentioned universal theorem, is .... A. 66.90493

The area of the offsets to be added 0.56715
(computed as in the last example)

Sum 67.47208

* See the plot of this survey in *fig.* 4, *plate* 18.

(brought forward) Sum	67.47208
The area of the offsets to be subtracted	2.56715
The area of the whole survey Acres	64.90493
	4
Roods	3.61972
	40
Perches	24.78880

Then, for determining the area of the east field, we have the following lines, viz.

			ch. l.
1st. Part of the first station line in the field book, ...	S. 87°	W.	5.50
2d. The offset to the corner of the field, ...	N. 30°	W.	1.10
3d. The eighth station line in the field book, ...	North	—	16.61
4th. The ninth station line in the field book, ...	East	—	13.77
5th. The offset from the corner of the field to the sixth station line in the field book, ...	S. 84° 45′ E.		0.56
6th. Part of the sixth station line in the field book, ...	S. 5° 15′ W.		15.40
7th. The seventh station line in the field book, ...	S. 74° 30′ W.		7.65

The area comprehended within these lines, computed according to the foregoing universal theorem, is	A. 22.91936
There are no offsets to be added in this field, the areas of the offsets to be subtracted, computed as in the last example, are	2.22052
Remains the area of the east field, Acres	20.69884
	4
Roods	2.79536
	40
Perches	31.81440

For ascertaining the area of the west field, we have the following lines, viz.

			ch. l.
1st. The offset from the south-easterly corner to the first station line in the field book, ...	S. 3°	E.	1.10
2d. Part of the first station line in the field book, ...	S. 87°	W.	9.50
3d. The second station line in the field book, ...	N. 18° 30′	W.	9.60
4th. The third station line in the field book, ...	N. 12° 15′	E.	8.90
5th. Part of the fourth station line in the field book, ...	N. 37°	E.	5.40
6th. The offset to the corner, ...	S. 53°	E.	0.40
7th. The tenth station line in the field book, reversed, ...	S. 62° 30′	E.	7.92
8th. The eighth station line in the field book, reversed, ...	South	—	16.61

The area comprehended within these lines, computed according to the foregoing universal theorem, is . . . . A.	22.18048
The areas of the offsets to be added, computed as in the last example, ...	0.62287
Sum	22.80335
The areas of the offsets to be subtracted	1.15900
Remains the area of the west field, Acres	21.64435
	4
Roods	2.57740
	40
Perches	23.09600

For ascertaining the area of the north field, we have the following lines, viz.

			ch. l.
1st. The offset from the south-westerly corner to the fourth station line in the field book, ...	N. 53°	W.	0.40
2d. Part of the fourth station line in the field book, ...	N. 37°	E.	11.60
3d. The fifth station line in the field book, ...	S. 76° 45′	E.	16.00

		ch. l.
4th. Part of the sixth station line in field book, ...	S. 5° 15′ W.	9.60
5th. The offset to the corner, ...	N. 84° 45′ W.	0.56
6th. The ninth station line in the field book, reversed, ...	West —	13.77
7th. The tenth station line in the field book, ...	N. 62° 30′ W.	7.92

The area comprehended within these lines, computed according to the foregoing universal theorem, is	A. 21.81422
The areas of the offsets to be added, computed as in the last example, ...	1.25030
Sum	23.06452
The areas of the offsets to be subtracted	0.50080
Remains the area of the north field, Acres	22.56372
	4
Roods	2.25488
	40
Perches	10.19520

	A.	R.	P.
The area of the east field, ...	20	2	31
West field, ...	21	2	23
North field, ...	22	2	10
Total .... A.	64	3	24

The agreement of this *total* with the area of *the whole*, as first ascertained, shews the work to be right. We must not, however, in actual practice expect to find so perfect an agreement as the above. A difference of a few perches, in actual practice, is no impeachment either of the field work, or of the computations.

For other methods of surveying, and a new plan of a field book, see the *Addenda* to this work.

OF

MARITIME SURVEYING.

It was not my intention at first to say any thing concerning maritime surveying, as that subject had been already very well digested by Mr. *Murdoch Mackenzie*, whose treatise on *maritime surveying* ought to be in every person's hands who is engaged in this branch of surveying, a branch which has been hitherto too much neglected. A few general principles only can be laid down in this work; these, however, it is presumed, will be found sufficient for most purposes; when the practice is seen to be easy, and the knowledge thereof readily attained, it is to be hoped, that it will constitute a part of every seaman's education, and the more so, as it is a subject in which the safety of shipping and sailors is very much concerned.

1. Make a rough sketch of the coast or harbour, and mark every point of land, or particular variation of the coast, with some letter of the alphabet; either walk or sail round the coast, and fix a staff with a white rag at the top at each of the places marked with the letters of the alphabet. If there be a tree, house, white cliff, or other remarkable object at any of these places, it may serve instead of a station staff.*

2. Choose some level spot of ground upon which a right line, called *a fundamental base*, may be measured either by a chain, a measuring pole, or a piece of log-line marked into feet; generally speaking, the longer this line is, the better; its

* See Nicholson's Navigator's Assistant.

situation must be such, that the whole, or most part of the station-staves may be seen from both ends thereof; and its length and direction must, if possible, be such, that the bearing of any station-staff, taken from one of its ends, may differ at least ten degrees from the bearing of the same staff taken from the other end; station-staves must be set at each end of the fundamental base.

If a convenient right line cannot be had, two lines and the interjacent angles may be measured, and the distance of their extremes found by construction, may be taken as the fundamental base.

If the sand measured has a sensible and gradual declivity, as from high-water mark to low-water, then the length measured may be reduced to the horizontal distance, which is the proper distance, by making the perpendicular rise of the tide one side of a right-angled triangle; the distance measured along the sand, the hypothenuse; and from thence finding the other side trigonometrically, or by protraction, on paper; which will be the true length of the base line. If the plane measured be on the dry land, and there is a sensible declivity there, the height of the descent must be taken by a spirit-level, or by a quadrant, and that made the perpendicular side of the triangle.

If in a bay one strait line of a sufficient length cannot be measured, let two or three lines, forming angles with each other, like the sides of a polygon, be measured on the sand along the circuit of the bay; these angles carefully taken with a theodolite, and exactly protracted and calculated, will give the strait distance betweeen the two farthest extremities of the first and last line.

3. Find the bearing of the fundamental base by the compasses, as accurately as possible, with Hadley's quadrant, or any other instrument equally

exact; take the angles formed at one end of the base, between the base line and lines drawn to each of the station-staves; take likewise the angles formed between the base line and lines drawn to every remarkable object near the shore, as houses, trees, windmills, churches, &c. which may be supposed useful as pilot marks; from the other end of the base, take the angles formed between the base and lines drawn to every one of the station staves and objects; if any angle be greater than the arc of the quadrant, measure it at twice, by taking the angular distance of some intermediate object from each extreme object; enter all these angles in a book as they are taken.

4. Draw the fundamental base upon paper from a scale of equal parts, and from its ends respectively draw unlimitted lines, forming with it the angles taken in the survey, and mark the extreme of each line with the letter of the station to which its angle corresponds. The intersection of every two lines, whose extremes are marked with the same letter, will denote the situation of the station or object to which, in the rough draught, that letter belongs; through, or near all the points of intersection which represent station staves draw a waving line with a pencil to represent the coast.

5. At low water sail about the harbour, and take the soundings, observing whether the ground be rocky, sandy, shelly, &c. These soundings may be entered by small numeral figures in the chart, by taking at the same time the bearings of two remarkable objects; in this excursion, be particular in examining the ground off points of land which project out into the sea, or where the water is remarkably smooth, without a visible cause, or in the vicinity of small islands, &c. observe the set and velocity of the tide of flood, by heaving the

log while at anchor, and denote the same in the chart by small darts. The time of high water is denoted by Roman numeral letters; rocks are denoted by small crosses; sands, by dotted shading, the figures upon which usually shew the depth at low water in feet; good anchoring places are marked by a small anchor. Upon coming near the shore, care must be taken to examine and correct the outline of the chart, by observing the inflections, creeks, &c. more minutely.

6. In a small sailing vessel go out to sea, and take drawings of the appearance of the land, with its bearings; sail into the harbour, observe the appearance of its entrance, and particularly whether there be any false resemblance of an entrance by which ships may be deceived into danger; remark the signs or objects, by attending to which, the harbour may be entered in safety; more especially where it can be done, let the ship steer to the anchoring place, keeping two remarkable objects in one, or on a line.

7. Coasts are shaded off on the land side; houses, churches, trees, &c. on shore, are drawn small in their proper figures; in a proper place of the chart, draw a mariner's compass, to denote the situation of the rhumbs, &c.; and on one side of the flower-de-luce, draw a faint half flower-de-luce at the point of north by compass; draw also a divided scale of miles, or leagues, which must be taken from the same original scale as the fundamental base.

Charts or plans may be neatly drawn with Indian ink, or the pen and common ink, and are as useful as any others, but they are frequently done in water colours; for which purpose the beginner will derive more advantage from viewing a proper drawing, or from overlooking a proficient at work, than from a multitude of written instructions.

An example to illustrate the foregoing precepts. Let A B C D E F G H, *fig.* 11, *plate* 23, represent a coast to be surveyed, and I K L, an adjacent island; it is required to make an accurate chart of the same.

To make the actual observations on shore. By sailing or walking along the shore, a rough sketch is made, and station staves set up at the points of land, A, B, C, D, E, F, G, H, and also at I, K, L, on the island, precept 1. During this operation, it is observed, that no proper place offers itself at which a fundamental base can be drawn, so as to command at once a view of all the stations; it will, therefore, be necessary to survey the coast in two separate parts, by making use of two base lines; if the coast had been more extended, or irregular, a greater number of base lines might have been necessary. Between the points B and C, the ground is level, and a considerable number of the southernmost points may be seen from both points, conformable to precept 2; make B C, therefore, the first fundamental base, its length by admeasurement is found to be 812 fathoms, and its bearing by compass from B to C, is N. 11° 14′ E. from each end of this base, measure with *Hadley's* quadrant the angles formed between it, and lines drawn to each station in sight, precept 3, enter, or tabulate them as follows:

First fundamental base B C=812 fathoms, bearing of C from B, N. 11° 14′ E.

	° ′		° ′
Angle A B C =	135 07	Angle A C B =	22 54
L B C =	85 00	L C B =	71 02
K B C =	56 28	K C B =	91 40
E B C =	43 32	E C B =	113 50
D B C =	13 30	D C B =	145 20

After having made these observations, it will be requisite to proceed to the northern part of the coast; in all cases where a coast is surveyed in separate parts, it is best to measure a new fundamental base for each part, when it can be conveniently done; a line from the station E drawn towards F, is well adapted to our purpose; let E P, therefore, be the second base line, its length by admeasurement is found to be 778 fathoms, and its bearing by compass from E to P, N. 38° 20′ E. Measure the angles formed by lines drawn from each end of this base, as before directed, and tabulate them as follows:

Second fundamental base E P=778 fathoms, bearing of P, from E. N. 38° 20′ E.

	° ′		° ′
Angle H E P =	88 30	Angle H P E =	73 00
I E P =	71 32	I P E =	28 54
G E P =	43 59	G P E =	111 00
F E P =	27 51	F P E =	127 18

It is sufficiently apparent, that the connection between the two parts of this survey is preserved by the second fundamental base being drawn from the point E, whose situation was before determined by observations from the first base line; if this particular position of the second base had not been convenient, and it had been taken at a distance from every point determined in situation from the first base, the connection would have required an observation of the bearing of one of the said points from each end of the second base: thus, suppose the line I P to be the second base line, instead of E P; the position of I P with respect to the given point E, may be known by taking the bearings of E, from I and P.

All the observations which are required to be made on shore, being now completed, it will be adviseable to construct the chart before we proceed to the other observations on the water, for, by this method, an opportunity is offered of drawing the waving line of the coast with more correctness than could otherwise be obtained.

GEOMETRICAL CONSTRUCTION, *fig.* 12, *pl.* 23.

On the point B as a center, describe the circle n e s w, the diameters n s and w e being at right angles to each other, and representing the meridian and parallel; from n set off the arc n N, equal to the quantity of the variation of the compass; draw the diameter N S, and the diameter W E at right angles to the same; these lines will represent the magnetic meridian, and its parallel of latitude.

From the center B, draw the first base line B C=812 fathoms, N. 11° 14′ E. by compass; from one extremity B, draw the unlimited lines B a, B l, B k, B e, B d, forming, respectively with the base, the angles observed from thence; from the other extremity C, draw the unlimited lines C a, C l, C k, C e, C d, forming, respectively with the base, the angles observed from thence; the intersections of lines terminated by the same letter, will give the stations A, L, K, E, D, precept 4.

Draw the line BQ, N. 38° 20′ E. by compass, and from E, draw EP=778 fathoms in the same direction or parallel to it, EP will be the second base line; from one extremity E, draw the unlimited lines Eh, Ei, Eg, Ef, forming, respectively with the base, the angles observed from thence; from the other extremity P, draw the unlimited lines Ph, Pi, Pg, Pf; the intersections of lines terminated by the same letter will give the stations H, I, G, F, precept 4.

Near the points A, B, C, D, E, F, G, H, draw a waving or irregular line to represent the coast; draw also by the help of the points I, K, L, a line to represent the coast of the island.

The chart being thus far delineated, it will be proper to make the observations which are required on the water.

TO MAKE THE ACTUAL OBSERVATIONS ON THE WATER.

Precept 5, sufficiently explains the business to which it is necessary to attend on the water; nothing more, therefore, need be added here, except an elucidation of the method, by the place at which any observation is made, may be found on the chart; the place of an observer may be determined by the bearings of two known objects; the intelligent practitioner will find reasons for occasionally preferring each of these methods; but, as the distance in the second method is known only by estimation, we have given the preference to the first, in the present example; all the soundings, &c. are supposed to be laid down by these methods, but, to prevent confusion and prolixity, the lines of bearing are drawn only from the points M, N, and O.

From the southern extremity M, of the reef of rocks, which runs off the point L, the bearing of the stations C and B are taken by the compass, as follows, viz. C, N. 62° E. and B, N. 85° E.

On the chart, therefore, from C, parallel to the rhumb B R, draw the opposite rhumb C m, S. 62° W. and from B, draw the opposite rhumb B m, S. 85° W. their intersection M, will be the extremity of the reef of rocks.

Again, from the sand head N, the bearings of the stations L and K are taken by the compass, as follows, viz. L, N. 17° W. and K, N. 5° E.

On the chart draw the rhumb B n, N. 17° W. and from L, parallel to the same, draw the opposite rhumb L n; draw likewise the rhumb B T, N. 5° E. and from K, parallel to the same, draw the opposite rhumb K n; the intersection of these two last lines will be the sand head, and the situation of the point of land O, which lies so that it cannot be seen from either of these base lines, is determined in like manner from the bearings of L and K, though in practice it may have been more convenient to settle its place from a single bearing and distance of the station A.

The position of the compass is given from that of the circle N E S W, made use of in the construction, and the scale of geographical miles is found from the consideration, that 1021 fathoms make one geographical mile; take therefore 1021 from the same scale of equal parts, as was used in laying down the fundamental bases, and it will give one primary division for the intended scale of miles.

To measure a strait line on the surface of the sea. First, prepare a measuring line of strong chord, two or three hundred yards in length, with small pieces of cork of equal thickness made fast to it at small distances, all along, like a fishingnet, so that it may float strait on the surface of the water: if the line has been well stretched, or much used before, it is the better; also prepare two ropes somewhat longer than the greatest depth of the water to be measured, with a pig of lead or iron ballast, which we call an anchor, 50 or 60 pounds weight, tied by the middle to one end of each rope, that when it is at the bottom it may be able to anchor a boat, and bear to be stretched strait without shifting the place of the anchor. Let the measuring line be thoroughly wet immediately before you begin to use it, and then stretched on the water close by the shore, and its length measured there with a pole. Then, in the direction intended to be measured, take two remarkable sharp objects on the land in a line, one near the shore, the other as far up in the country as you can: if such are not to be had, place buoys on the water at proper distances in that direction.

2. Take the objects, or buoys, in a line, and holding one end of the measuring line fast on the shore, carry out the other in a boat, in that direction, till it is stretched strait at its full length by one man in the boat, and exactly at the end of the line let another man drop the anchor, which will mark one length of it. There keep the boat, and the end of the measuring line, close to the anchor rope, drawn tight up and down, till another boat takes in the other end which was on the shore, and rows farther on, and lays it strait in the direction of the land marks, or buoys, and there drops another anchor, which will mark the second length of the measuring line. Go on thus till the whole proposed distance is measured; and immediately after let the measuring line be again measured with a pole on the water near the shore, as at first, and if the lengths differ, take the mean between them

for the true length. It is obvious, that to measure with any exactness this way, the sea must not only be smooth, but void of a swell, and of all stream of tide; either of which will hinder the line from lying strait. This method of measuring a strait line may be convenient on some occasions; and if care is taken to keep the anchor rope right up and down when the measuring line is applied to it, will be found sufficiently exact for many purposes, but not for a fundamental base line from which other distances are to be deduced.

There is another way of measuring a strait line, mechanically, on the sea, which is so well known to seamen, that it is needless to describe it particularly here: and that is, by heaving the log over a ship's stern while she is under sail, and observing how many knots of the log line run out in half a minute; for the line is so divided that the ship will run, or is supposed to run, so many miles in an hour, in a strait course; and twice as much in two hours, and so on. But this conclusion is founded on three suppositions, neither of which is certain, viz. that the log remains in the same place during the whole half minute that the line is running out from the ship's stern; that the ship continues to sail with the same velocity, and also in the same direction, during an hour, or two, that she did during the half minute; the contrary of which is more likely in most cases. For the log line may shrink, or stretch, while it is running out; or may drag after the vessel by the weight of the line, or by not running easily and readily off the reel; the swell of the sea may alter the place of the log; and currents, or streams of tide, stronger or weaker below the surface than on it, an unsteady helm, lee-way, and varying winds, may change the direction, or celerity of the ship's motion; for neither of which can any certain allowance be made. This way, therefore, of measuring a strait line, or distance, is not to be depended on as exact; but is mentioned here, because rocks, shoals, or islands, sometimes lie so far from the coast, that there is no other way of forming any notion of their distance. If any such distance is to be measured after this manner, let the log-line be thoroughly wet when it is measured; let the length between each knot be 51 feet, which is the 120th part of a geometrical mile, as half a minute is $\frac{1}{120}$th part of an hour. Choose neap-tide, as much slack water as can be got, and a moderate breeze of following wind; let the line be run off the reel so as never to be stretched quite strait; and if the half minute is measured by a watch that shews seconds, rather than by a glass, it will generally be more exact. Perhaps one second should be allowed for the loss of time in calling out at the beginning, and stopping it at the end of the time; except the person who holds the watch can contrive to observe the going out of the red rag at the beginning, and also to stop the line himself at the end of the time; which does not seem a difficult matter.

To find the distance of two places by the flash and report of a gun. Sound moves 1142 English feet in one second of time, or 6120 feet, which makes a geographical mile, in $5\frac{1}{3}''$ nearly; therefore, let a gun be fired at one place at an appointed time, and observe the time that elapses between the flash and report, and so many seconds as you observe, so many times 1142 feet are you distant from the place; the operation should be repeated two or three times for greater certainty. The distance to be measured in this way should never be less than two miles, on account of errors that may arise in taking the time.

TO TRANSFER A PLAN FROM ONE PAPER TO ANOTHER.

Method 1. *By points.* Lay the rough plan upon the clean paper, on which you intend to draw the fair copy, press them close together by weights, and keep them as flat as possible; then with a pointrel, or needle point, prick through all the corners of the plan to be copied; separate the papers, and join by lines the points on the clean paper. This method can only be used in plans, whose figures are small, regular, and bounded by strait lines.

Method 2. *By tracing paper.* Rub the back of the rough plan with black-lead powder, and having wiped off the superfluous lead, lay the blacked part upon the clean paper, or place a sheet of black tracing paper between the rough plan and the clean paper; weights are to be placed as in the former method, to maintain the papers in the same position.

Then, with a blunt point of brass, steel, or ivory, trace exactly the lines of the plan, pressing the paper so much, that the black lead under the lines may be transferred to the clean paper; when the whole of the plan has been thus delineated, go over the black-lead marks with common, or Indian ink.

Method 3. *By squares.* See prob. 65, page 90.

Method 4. *By a copying glass.* This is a large square, or rectangular piece of looking glass, fixed in a frame of wood, which can be raised to any angle, like a desk, the lower end resting upon the table; a screen of blue paper fits to the upper edge, and stands at right angles to it.

Place this frame at a convenient angle against a strong light: fix the old plan and clean paper firmly together by pins, the clean paper uppermost, and on the face of the plan to be copied; lay them with the back of the old plan next the glass, namely, that part which you intend to copy first. The light through the glass will enable you to perceive distinctly every line of the plan upon the clean paper, and you can easily trace over them with a pencil; and having finished that part which covers the glass, slide another part over it, and copy this, and thus continue till the whole be copied.

Method 5. *By the assistance of proportionable and triangular compasses*, *fig*. A and N, *plate* 1, and *fig*. 12, *plate* 3. These will, in many instanstances, assist the draughtsman very much, and lessen the labour of copying.

Method 6. *By the pantographer*. There is no method so easy, so expeditious, nor even so accurate, as the pantographer. It is an instrument as useful to the experienced draughtsman, as to those who have made but little progress in the art. It saves a great deal of time either in reducing, enlarging, or copying of the same size, giving the outlines of any drawing, however crooked or complex, with the utmost exactness; nor is it confined to any particular kind, but may with equal facility be used for copying figures, plans, sea charts, maps, profiles, landscapes, &c.

DESCRIPTION AND USE OF THE PANTOGRAPHER, OR PANTAGRAPH.

I have not been able to ascertain who was the inventor of this useful instrument. The earliest account I find, is that of the Jesuit *Scheiner*, about the year 1631, in a small tract entitled, *Pantagraphice, sive Ars nova Delineandi.* The principles are self-evident to every geometrician; the mechanical construction was first improved by my father, about the year 1750. It is one, among other scientific improvements completed by him, that others have many years after, assumed to themselves.

The pantographer is usually made of wood, or brass, from 12 inches to two feet in length, and consists of four flat rules, *fig.* 19, *plate* 28, two of them long and two short. The two longer are joined at the end A by a double pivot, which is fixed to one of the rules, and works in two small holes placed at the end of the other. Under the joint is an ivory castor to support this end of the instrument. The two smaller rules are fixed by pivots at E and H, near the middle of the larger rules, and are also joined together at their other end, G.

By the construction of this instrument, the four rules always form a parallelogram. There is a sliding box on the longer arm, and another on the shorter arm. These boxes may be fixed at any part of the rules by means of their milled screws; each of these boxes are furnished with a cylindric tube, to carry either the tracing point, crayon, or fulcrum.

The fulcrum, or support K, is a leaden weight; on this the whole instrument moves when in use. To the longer instruments are sometimes placed two moveable rollers, to support and facilitate the motions of the pantographer; their situation may be varied as occasion requires.

The graduations are placed on two of the rules, B and D, with the proportions of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. to $\frac{1}{12}$, marked on them.

The pencil holder, tracer, and fulcrum, must in all cases be in a right line, so that when they are set to any number, if a string be stretched over them, and they do not coincide with it, there is an error either in the setting or graduations.

The long tube which carries the pencil, or crayon, moves easily up or down in another tube; there is a string affixed to the long, or inner tube, passing afterwards through the holes in the three small knobs to the tracing point, where it may, if necessary, be fastened. By pulling this string, the pencil is lifted up occasionally, and thus prevented from making false or improper marks upon the copy.

To reduce in any of the proportions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *&c. as marked on the two bars* B, *and* D. *Suppose, for example,* $\frac{1}{2}$ *is required.* Place the two sockets at $\frac{1}{2}$ on the bars B and D. Place the fulcrum, or lead weight at B, the pencil socket, with pencil at D, and the tracing point at C. Fasten down upon a smooth board, or table, a sheet of white paper under the pencil D, and the original map, &c. under the tracing point C, allowing yourself room enough for the various openings of the instrument. Then with a steady hand carefully move the tracing point C over the outlines of the map, and the pencil at D will describe exactly the same figure as the original, but $\frac{1}{2}$ the size. In the same manner for any other proportion, by only setting the two sockets to the number of the required proportion.

The pencil-holder moves easily in the socket, to give way to any irregularity in the paper. There is a cup at the top for receiving an additional weight, either to keep down the pencil to the paper, or to increase the strength of its mark.

There is a silken string fastened to the pencil-holder, in order that the pencil may be drawn up off the paper, to prevent false marks when crossing the original in the operation.

If the original should be so large, that the instrument will not extend over it at any one operation, two or three points must be marked on the original, and the same to correspond upon the copy. The fulcrum and copy may then be removed into such situations, as to admit the copying of the remaining part of the original; first observing, that when the tracing point is applied to the three points marked on the original, the pencil falls on the three corresponding points upon the copy. In this manner, by repeated shiftings, a pentagraph may be made to copy an original of ever so large dimensions.

To enlarge in any of the proportions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, *&c. Suppose* $\frac{1}{2}$. You set the two sockets at $\frac{1}{2}$, as before, and have only to change places between the pencil and tracing point, viz. to place the tracing point at D, and the pencil at C.

To copy of the same size, but reversed. Place the two sockets at $\frac{1}{2}$, the fulcrum at D, and the pencil at B.

There are sometimes divisions of 100 unequal parts laid down on the bars B and D, to give any intermediate proportion, not shewn by the fractional numbers commonly placed.

Pentagraphs of a greater length than two feet are best made of hard wood, mounted in brass, with steel centers, upon the truth of which depends entirely the equable action of this useful instrument.

OF

LEVELLING.

THE necessity of finding a proper channel for conveying water occurs so often to the surveyor, that any work on that subject, which neglected to treat on the art of levelling, would be manifestly imperfect; I shall therefore endeavour to give the reader a satisfactory account of the instruments used, and the mode of using them.

A DESCRIPTION OF THE BEST SPIRIT LEVEL.

Fig. 3, *plate* 17, represents one of the best constructed spirit levels, mounted on the most complete staves, similar to those affixed to a best theodolite.

The achromatic telescope, A, B, C, is moveable, either in the plane of the horizon, or with a small inclination thereto, so as to cut any object whose elevation, or depression, from that plane, does not exceed 12 degrees; the telescope is about two feet long, is furnished with fine cross wires, the screws to adjust which are shewn at *a*, for determining the axis of the tube, and forming a just line of sight. By turning the milled screw B, on the side of the telescope, the object glass is moved outwards, and thus the telescope suited to different eyes.

The tube *c c* with the spirit bubble is fixed to the telescope by a joint at one end, and a capstan headed screw at the other, to raise or depress it for the adjustment. The two supporters D, E, on which the telescope is placed, are nearly in the shape of the letter Y, the inner sides of the Y's are tangents to the cylindric ring of the telescope. The lower ends of these supporters are let perpendicularly into a strong bar; to the lower end of the support E, is a milled nut F, to bring the instrument accurately to a level; and at the other end of the bar at H, is a screw for tightening the support D at any height. Between the two supports is a compass box G, divided into four quarters of 90° each, and also into 360°, with a magnetical needle, and a contrivance to throw the needle off the center when it is not used; and thus constituting a perfect circumferentor.

The compass is in one piece with the bar, or is sometimes made to take on and off by two screws. To the under part of the compass is fixed a conical brass ferril K, which is fitted to the bell-metal frustum of a cone at the top of the brass head of the staves, having at its bottom a ball, moving in a socket, in the plate fixed at the top of the three metal joints for the legs. L, L, are two strong brass parallel plates, with four adjusting screws, *b*, *b*, *b*, *b*, which are used for adjusting the horizontal motion. The screw at M is for regulating this motion, and the screw N for making fast the ferril, or whole instrument, when necessary. By these two screws the instrument is either moved through a small space, or fixed in any position with the utmost accuracy. The staves being exactly the same as those applied to the best theodolites, render any further description of them here unnecessary.

It is evident from the nature of this instrument that three adjustments are necessary. 1. To place the intersection of the wires in the telescope, so that it shall coincide with the axis of the cylindrical rings on which the telescope turns. 2. To render the level parallel to this axis. 3. To adjust for the horizontal motion quite round upon the staves.

To adjust the cross wires. If, while looking through the telescope at any object, the intersection of the wires does not cut precisely the same part during a revolution of the telescope on its axis, their adjustment is necessary, and is easily obtained by turning the little screws *a, a, a.* The two horizontal screws are to move the vertical wire, and act in opposite directions to each other; one of which is to be tightened as the opposite is slackened, (this not being attended to, will endanger the breaking of the wire) till the wire has been moved sufficiently. The upper screw to the horizontal wire is generally made with a capstan head, so that by simply turning it to the right or left hand, the requisite motion of the wire is produced, and thus the intersection brought exactly in the axis of the telescope.

To adjust the spirit level at only one station. When the spirit level is adjusted to the telescope, the bubble of air will settle in the middle, or nearly so, whether the telescope be reversed or not on its supports D, E, which in this case are not to be moved. The whole level being placed firmly on its staves, the bubble of air brought to the middle by turning the screw F, the rims *f, f,* of the Y's open, and when the telescope is taken off and laid the contrary way on its supports, should the bubble of air not come to rest in the middle, it then proves that the spirit level is not true to the axis of the tube, and requires adjustment. The end to which the bubble of air goes must be noticed, and the distance of the bubble case, and height of one support, so altered by turning the screws at *c* and F, till by trial the bubble comes to the middle in both positions of the telescope. This very facile mode of adjustment of the level is one great improvement in the instrument; for, in all the old and common constructed levels, which did not admit of a reversion of the telescope, the spirit level could only be well adjusted by carrying the instrument into the field, and at two distant stations observing both forwards and backwards the deviation upon the station staves, and correcting accordingly; which occasioned no small degree of time and trouble not necessary by the improved level.

To adjust for the horizontal motion. The level is said to be completely adjusted, when, after the two previous ones, it may be moved entirely round upon its staff-head, without the bubble changing materially its place. To perform which, bring the telescope over two of the parallel plate screws *b*, *b*, and make it level by unscrewing one of these screws while you are screwing up the opposite one, till the air bubble is in the middle, and the screws up firm. Then turn the instrument a quarter round on its staff, till the telescope is directly over the other two screws *b*, *b*, unscrewing one screw, and screwing up at the same time the opposite one, as before, till the bubble settles again in the middle. The adjustment of the staff plates are thus made for the horizontal motion of the instrument, and the telescope may be moved round on its staff without any material change of the place of the bubble, and the observer enabled to take a range of level points. The level tube is also made to adjust in the horizontal direction by two opposite screws at its joint *e*, so that its axis may be brought into perfect parallelism to that of the telescope.

The telescope is generally glassed to shew objects erect. In the common sort of levels with a shorter telescope from 12 to 20 inches in length, and without a circumferentor, they are glassed to shew the objects inverted. To an expert observer this will make no difference; and, there being but two eye-glasses instead of four, as in the other, about three or four inches are saved in the length of the telescope.

A short brass tube, to screen the sun's rays from the object glass, is sometimes made to go on the object end c of the telescope, and a screw-driver, and steel pin for the capstan-headed screws are packed in the same case with the instrument.*

Two mahogany station staves often accompany this instrument; they consist of two parts, each part is about five feet ten inches long, so that when they are pulled out to their greatest extent, they form a ten feet rod; and every foot is divided into 100 equal parts. To each of these staves

* *Fig.* 10, is a representation of a brass mounted pocket spirit level, about six inches in length, with sights, having a ground bottom, intended for ordinary purposes either in levelling a plane or determining level points. When about 12 inches in length, with double sights and adapted to a staff, it may serve for conducting small parcels of water, or draining a field, &c.

Fig. 11, is one that I have constructed with some improvement. The brass perpendicular piece A, is made to slide on occasionally by a dovetail. By this piece, and the bottom of the level together, any standing square pillar, or other object, may be set to the level, and perpendicular at the same time. The case of the spirit level B, swings on two pivots; the horizontal position, therefore, of a ceiling or under surface of a plane may be as readily obtained as any inferior plane in the common way; the spirit level thereby always remaining the same, and the position of the base only changing to admit of a contact with the plane to be levelled. In which case also the perpendicular side A, becomes equally useful.

The adjustment of these pocket levels is very easily proved, or made, by bringing the bubble in the middle upon any table or there is a sliding vane A, *fig.* 9, *plate* 17, with a brass wire across a square hole made in the vane; when this wire coincides with the horizontal wire of the telescope, it shews the height of the apparent level above the ground at that place.

OF LEVELLING.

Levelling is the art which instructs us in finding how much higher or lower any given point on the surface of the earth is, than another given point on the same surface; or in other words, the difference in their distance from the center of the earth.

Those points are said to be *level*, which are equi-distant from the center of the earth. The art of levelling consists, therefore, 1st. In finding and marking two, or more level points that shall be in the circumference of a circle, whose center is that of the earth. 2dly. In comparing the points thus found, with other points, in order to ascertain the difference in their distances from the earth's center.

Let *fig.* 1, *plate* 23, represent the earth; A, its center; the points B, C, D, E, F, upon the circumference thereof are level, because they are equally distant from the center; such are the waters of the sea, lakes, &c.

base; if, upon reversion afterwards on the same place precisely, the bubble keeps to the middle, it is adjusted; if not, by means of a screw-driver turn one of the screws at the proper end, till it be so raised or depressed as to cause the bubble to stand the reversing, at the same time altering the inclination of the plane on which the level is tried. If a bubble stand the reversion of the level upon the plane, both level and plane are right, and it is most expeditious to adjust small levels on a true horizontal plane; if the bubble does not stand the reversion, both bubble and plane are inclined, and both require to be corrected by half the whole angle of the deviation shewn by reversion. EDIT.

To know how much higher the point B, *fig.* 2, *plate* 23, *is than* C, *and* C *lower than* D. We must find, and mark the level points, E, F, G, upon the radii AB, AC, AD, thereby comparing B with E, C with F, and D with G; we shall discover how much B is nearer the circumference of the circle than C, and, consequently, how much further from the center of the earth, and so of the other points.

OF THE DIFFERENT METHODS OF MARKING OUT THE LEVEL POINTS.

The first, which is the most simple and independent, is by the tangent of a circle, for the two extremities of the tangent give the true level points, when the point of contact is precisely in the middle of the line. But if the point of contact with the circumference be at one of the extremities of the line, or in any other part except the middle, it will then only shew the apparent level, as one of its extremities is further from the circumference than the other. Thus the tangent B C, *fig.* 3, *plate* 23, marks out two true level points at B and C, because the point of contact D is exactly in the middle of the line B C, and its two extremities are equally distant from the circumference and the center A.

But the tangent B D, *fig.* 7, *plate* 23, marks two points of apparent level, because the point B, where it touches the circumference, is not the middle of the line; and, therefore, one of its extremities B, is nearer to the center than the other D; D is further from the center, in proportion as it is more distant from the point of contact B; which constitutes the difference between the true and apparent level, of which we shall speak presently. Every point of the apparent level, except the point of contact, is higher than the true level.

As the tangent to a circle is perpendicular to the radius, we may make use of the radius to determine the tangent, and thus mark the level points. Thus let A, *fig.* 5, *plate* 23, represent the center of the earth, A B the radius, and C, B, D, the tangent; the two extremities C, D, are equally distant from the point of contact B, consequently the angles B C A, B D A, will be equal; the angles at the tangent point are right angles, and the radius common to both triangles, and the sides C A, D A, are equal, and the points C D, are two level points, because equally distant from the center.

It is evident from this, that if from any point of the radius two lines be drawn, one on each side, making equal angles with it, and being of an equal length, the extreme points of these lines will be level points. Thus, if from B, of the radius B A, two equal lines B C, B C, *fig.* 6, *plate* 23, be drawn, making equal angles, C B A, D B A, then will C and D be equally distant from the center; though the level may be obtained by these oblique lines, yet it is far easier to obtain it by a line perpendicular to the radius.

When the level line is perpendicular to the radius, and touches it at one of its extremes, the other extremity will mark the apparent level, and the true level is found by knowing how much the apparent one exceeds it in height.

To find the height of the apparent above the true level for a certain distance, square that distance, and then divide the product by the diameter of the earth, and the quotient will be the required difference; it follows clearly, that the heights of the apparent level at different distances are as the squares of those distances; and, consequently, that the difference is greater, or smaller, in proportion to the extent of the line; for the extremity of this line separates more from the circumference of the circle, in proportion as it recedes from the point of contact. Thus, let A, *fig*, 7, *plate* 23, be the center of the earth, BC the arc which marks the true level, and B, E, D, the tangent that marks the apparent level; it is evident, that the secant AD exceeds the radius AB, by CD, which is the difference between the apparent and true level; and it is equally evident, that if the line extended no farther than E, this difference would not be so great as when it is extended to D, and that increases as the line is lengthened.

When the distance does not exceed 25 yards, the difference between the two levels may be neglected; but if it be 50, 100, &c. yards, then the error resulting from the difference will become sensible and require to be noticed.

A TABLE,

Which shews the quantity of curvature below the apparent level, in inches, for every chain up to 100.

Chain	Inches	Chain	Inches	Chain	Inches	Chain	Inches
1	0,00125	14	0,24	27	0,91	40	2,00
2	0,005	15	0,28	28	0,98	45	2,28
3	0,01125	16	0,32	29	1,05	50	3,12
4	0,02	17	0,36	30	1,12	55	3,78
5	0,03	18	0,40	31	1,19	60	4,50
6	0,04	19	0,45	32	1,27	65	5,31
7	0,06	20	0,50	33	1,35	70	6,12
8	0,08	21	0,55	34	1,44	75	7,03
9	0,10	22	0,60	35	1,53	80	8,00
10	0,12	23	0,67	36	1,62	85	9,03
11	0,15	24	0,72	37	1,71	90	10,12
12	0,18	25	0,78	38	1,80	95	11,28
13	0,21	26	0,84	39	1,91	100	12,50

The *first method* of finding and marking two level points, is by a tangent whose point of contact is exactly in the middle of the level line; this method may be practised without regarding the difference between the apparent and true level; but, to be used with success, it would be well to place your instrument as often as possible at an equal distance from the stations; for it is clear, that if from one and the same station, the instrument remaining at the same height, and used in the same manner, two or more points of sight be observed, equally distant from the eye of the observer, they will also be equi-distant from the center of the earth.

Thus, let the instrument, *fig.* 8, *plate* 23, be placed at equal distances from C, D; E, F, the two points of sight marked upon the station-staves C G, D H, will be the level points, and the difference in the height af the points E, F, will shew how much one place is higher than the other.

Second method. This consists in levelling from one point immediately to another, placing the instrument at the stations where the staves were fixed. This may also be performed without noticing the difference between the true and apparent level; but then it requires a double levelling, made from the first to the second station, and reciprocally from the second to the first.

Thus, let the two stations be B E, *fig.* 9, *plate* 23, the station staves C B, D E, which in the practice of levelling may be considered as parts of the radii A B, A E, though they be really two perpendiculars parallel to each other, without the risk of any error. In order to level by this method, plant the instrument at B, let the height of the eye, at the first observation, be at F, and the point of sight found be G; then remove the instrument to E, and fix it so that the eye may be at G; then, if the line of sight cuts F, the points F and G are level, being equally distant from the center of the earth, as is evident from the figure.

But if the situation of the two stations be such, that the height of the eye at the second station could not be made to coincide with G, but only with H, *fig.* 10, *plate* 23, yet if the line of sight gives I as far distant from F as G is from H, the two lines F G, H I, will be parallel, and their extremities level points; but if that is not the case, the lines of sight are not parallel, and do not give level points, which however might still be obtained by further observations; but, as this mode is not practised, we need not dwell further upon it.

The velocity of running water is proportional to the fall; where the fall is only three or four inches in a mile, the velocity is very small; some cuts have been made with a fall from four to six inches; a four inch fall in a strait line is said to answer as well as one of six inches with many windings.

The distance from the telescope to the staff should not, if possible, exceed 100 yards, or five chains; but 50 or 60 yards are to be preferred.

OF THE PRACTICE OF LEVELLING.

1st. *Of simple Levelling.*

We term that simple levelling, when level points are determined from one station, whether the level be fixed at one of the points, or between them.

Thus, let AB, *fig.* 1, *plate* 24, be the station points of the level, C, D, the two points ascertained, and let the height

	Feet	Inches.	
from A to C be	6	0	0
from B to D be	9	0	0
their difference is	3	0	0

shewing that B is three feet lower than A.

In this example, the station points of the level are below the line of sight and level points, as is generally the case; but if they were above, as in *fig.* 2, *plate* 24, and the distance of A to C be six feet, and from B to D nine feet, the difference will be still three feet, that B is higher than A.

2d. *Of compound Levelling.*

Compound levelling is nothing more than a collection of many single or simple operations compared together. To render this subject clearer, we shall suppose, that for a particular purpose it were necessary to know the difference in the level of the two points A and N, *fig.* 3, *plate* 24; A on the river *Zome*, N on the river *Belann.*

As I could find no satisfactory examples in any English writer that I was acquainted with, and not being myself in the habits of making actual surveys, or conducting water from one place to another, I was under the necessity of using those given by Mr. *Le Febvre.*

Stakes should be driven down at A and N, exactly level with the surface of the water, and these should be so fixed, that they may not be changed until the whole operation is finished; the ground between the two rivers should then be surveyed, and a plan thereof made, which will greatly assist the operator in the conducting of his business; by this he will discover that the shortest way from A to N, is by the dotted line AC, GH, HN, and he will from hence also determine the necessary number of stations, and distribute them properly, some longer than the others, according to the nature of the case, and the situation of the ground. In this instance 12 stations are used; stakes should then be driven at the limits of each station, as at A, B, C, D, E, F, &c. they should be driven about 18 inches into the ground, and be about two or three inches above its surface; stakes should also be driven at each station of the instrument, as at 1, 2, 3, 4, &c.

Things being thus prepared, he may begin his work; the first station will be at 1, equi-distant from the two limits A, B; the distance from A to B, 166 yards; and, consequently, the distance on each side of the instrument, or from the station stake, will be 83 yards.

Write down in the first column, the first limit A; in the second, the number of feet, inches, and tenths, the points of sight, indicated on the station staff at A, namely, 7.6.0. In the third column, the second limit B; in the fourth, the height indicated at the station staff B, namely, 6. 0. 0. Lastly, in the fifth column, the distance from one station staff to the other, in this instance 166 yards.

Now remove the level to the point marked 2, which is in the middle between B and C, the two places where the station staves are to be held, observing that B, which was the first limit in the former operation, is the first in this; then write down the observed heights as before, in the first column B; in the second 4. 6. 0; in the third, C; in the fourth, 5. 6. 2; in the fifth, 560, the distance between B and C.

As, from the inequality of the ground at the third station, it is not possible to place the instrument in the middle between the two station staves, find the most convenient point for your station, as at 3; then measure exactly how far this is from each station staff, and you will find from 3 to C, 160 yards; from 3 to D, 80 yards: the remainder of the operations will be as in the preceding station.

In the fourth operation, it will be necessary to fix the station, so as to compensate for any error that might arise from the inequality of the last; therefore, mark out 80 yards from the station staff D, to the point 4; and 160 yards from 4 to E; and this must be carefully attended to, as by such compensations the work may be much facilitated.

Proceed in the same manner with the eight remaining stations, as in the four former, observing to enter every thing in its respective column; when the whole is finished, add the sums of each column together, and then subtract the less from the greater, thus from 82 : 2 : 5, take 76 : 9 : 7, and the remainder 5 : 4 : 8, is what the ground at N is lower than the ground at A.

	f.	in.			f.	in.		yards.
A	7	6		B	6	0	0	160
B	4	6		C	5	6	2	250
C	12	8	6	D	8	4	0	240
D	0	0	0	E	4	1	0	250
E	6	10	0	F	2	11	0	250
F	7	0	4	G	4	8	6	300
G	7	7	6	H	10	0	0	250
H	4	6	4	I	8	10	0	110
I	6	3	0	K	10	0	0	130
K	6	4	3	L	5	8	6	250
L	7	0	0	M	8	4	3	250
M	6	5	0	N	7	10	0	250
	76	9	7		82	2	5	2680

82	2	5
76	9	7
5	4	6

OF THE PROFILE OF THE LEVEL.

The next thing to be obtained is a section of this level; for this purpose draw a dotted line, as o o, *fig.* 4, *plate* 24, either above or below the plan, which line may be taken for the level or horizontal line; then let fall perpendiculars upon this line from all the station points and places, where the station staves were fixed.

Beginning at A, set off seven feet six inches upon this line from A to a; for the height of the level point determined on the staff at this place, draw a line through *a*, parallel to the dotted line o o, which will cut the third perpendicular at b, the second station staff; set off from this point downwards six feet to B, which shews the second limit of the first operation, and that the ground at B, is one foot six inches higher than at A; place your instrument between these two lines at the height of the level line, and trace the ground according to its different heights.

Now set off on the second station staff B, four feet six inches to C, the height determined by the level at the second station; and from *c*, draw a line parallel to o o, which will cut the fifth perpendicular at *d*, the third station staff from this point set off 5 feet 6 inches $\frac{3}{10}$ downwards to C, which will be our second limit with respect to the preceding one, and the third with respect to the first; then draw your instrument in the middle between B and C, and delineate the ground with its different inequalities. Proceed in the same manner, from station to station, to the last N, and you will have the profile of the ground over which the level was taken.

To trace more particularly the profile of each station. It is necessary to observe in this place, that if the object of the operation be only to know the reciprocal height of the two extreme terms, as in the preceding example, then the method there laid down for the profile or section will be sufficient; but if it be necessary to have an exact detail of the ground between the said limits, the foregoing method is too general; we shall, therefore, institute another example, in which we shall suppose the level to have been taken from A to N by another route, but on more uniform ground, and less elevated above the level of the two rivers, in order to form a canal, marked O, P, Q, R, S, T, U, X, Y, for a communication from one river to the other.

Draw at pleasure a line Z Y, *fig.* 5, *plate* 24, to represent the level line and regulate the rest; then let fall on this line, perpendiculars to represent the staves at the limits of each station, taking care that they be fixed accurately at their respective distances from each other.

As the difference between the extreme limits ought to be the same as in the former example, is 5 feet 4 inches $\frac{6}{10}$, set off this measure upon the perpendicular at o, the first limit, and from o, prolonging the perpendicular, mark off at *a*, the height determined at the first station staff, as in the preceding example; then do the same with the second and third, and so on with the following, till this part of the work is finished; there then remains only to delineate in detail the ground between the station staves; the distances are assumed larger in this instance on account of the detail.

To get the section of the ground between O and P, place your instrument at one of the limits as P, fixing it so that the cross hairs may answer to the point b; then look towards the first limit o, raising or depressing the vane, till it coincides with the intersection of the cross hairs, and the line of sight, from one point to the other, will mark the level or horizontal line.

Now to set off the height of the brink of the river above the first limit, drive a stake down close to the ground at *a*, upon which place your station staff, and observe at what height the hairs intersect the vane, it will be at 4. 10; then laying off upon the line o z, the distance from the first term to the first stake, from whence let fall a perpendicular, and set off thereon 4. 10 to *a*, which gives the height at the first stake, or, which is the same, the height from the edge of the river above the surface of the water, as is plain from the section.

Drive a second stake at b, in a line between the limits; place the station staff upon this stake, and observe the height 4. 6. intersected by the cross hairs, the instrument still remaining in the same situation; set off on the level line the distance from the first stake *a* to the second *b*, and then let fall a perpendicular, and mark upon it 4. 6. to *b*, which gives us the height of the ground at this place.

To mark out the small hollow *c*, drive down a third stake even with the ground, in the middle of it at *c*; but as before in the station line, mark upon the level line the exact distance from the second stake *b*, to the third *c*; then let fall a perpendicular from *c*, and set off thereon 6 : 8 : 0 pointed out by the cross hairs on the staff, which determines the depth of the hollow, as may be seen by the section.

With respect to the ground between each stake, as the distances are now very short, it will be easily expressed by the operator, whose judgment will settle the small inequalities by a comparison with those already ascertained.

Proceed thus with the other stations, until you arrive at the last, and you will obtain an accurate section in detail of your work; by such a section, it is easy to form a just estimation of the land to be dug away, in order to form the canal, by adding thereto the depth to be given to the canal.

Another example of compound levelling. In the levelling exhibited in *plate* 25, we have an example taken where the situation was so steep and mountainous, that it was impossible to place the staves at equal distances from the instrument, or even to make a reciprocal levelling from one station to another.

Such is the case between the first point A, taken from the surface of a piece of water, which falls from the mountains, and the last point K, at the bottom of a bason, where it is proposed to make a fountain, and the height is required, at which a jet d'eau will play by conducting the water from the reservoir A, to the point K of the bason, by tubes or pipes properly made, and disposed with all the usual precaution.

From the manner in which the operator must proceed in this instance, it is evident, that the instrument should be adjusted with more than ordinary care, as the true distance from one mountain to the other cannot be attained without much trouble.

The height is here so great, that it will be necessary to go by small ascents from A to D, and it will probably be commodious in some part of the work to use a smaller instrument; underneath is a table of the different level points as ascertained, which, with the profile and plan, render this part of the work sufficiently clear, without further explication.

	f.	in.				f.	in.		yards.
A	21	6	0	C		0	9	0	90
C	4	3	0	D		0	3	0	40
D	3	9	0	E		16	3	0	350
E	5	0	0	F		17	9	0	250
F	10	6	0	G		5	0	0	375
G	5	0	0	H		19	0	0	300
H	5	0	0	K		47	3	0	1000
	55	0	0			106	9	0	2405

	106	9	0
	55	0	0
Difference	51	9	0

Only two levellings are made here between A and D, though it is evident from the plan, that more would have been necessary; but as our design is only to shew the manner of proceeding in this particular case, and as more would have confused the plan and section, they are here neglected.

In the fourth, the height found was 16. 8; but as the distance E, 350 yards, was considerable, it was necessary to reduce the apparent level to the true one; the same was also done in all other cases where it was necessary; at the last limit we first get the height from N to *o*, then from *o* to I, then from I to K, which added together, and then corrected for the curvature, gives 47. 3. 0. Now, by adding each column together, and then subtracting one from the other, we obtain 51 feet nine inches for the height, which the point A is above the bottom of the bason, and which will cause the jet d'eau to rise about 45 feet.

The general section of this operation is delineated under the plan at *fig.* 2, and is sufficiently plain from what has been already said in the preceding instances. But an exact profile of the mountain is not so easy, as it would require many operations, as may be seen in the section; some of these might however be obtained, by measuring from the level line already mentioned, without moving the instrument, as at *fig.* 3.

Example 3, *fig.* 1, *plate* 26, Mr. *Le Febvre* gives us in that of a river, being one part of the river *Haynox*, from *Lignebruk* to *Villeburg*, and the mode he observed in taking this level.

The first operation was that of having stakes driven at several parts of the river, even with the water's edge; the first stake A, a little above the mills at *Lignebruk*, shews the upper water mark when the water is highest, and is our first limit; the stake *b* shews the low water mark at the same mills, the stake B is the second limit.

The stakes C and D, above and below the mills of *Mazurance*, shew the height of the waters when at the highest and lowest, and their difference; these stakes form our third and fourth limits. Lastly, the stakes at E and F, above and below the mills at *Villebourg*, mark as before the difference between the highest and lowest stations of the water, and are also the last limits of the operation.

Particular care was taken, that the marks should all be made exactly even with the edge of the water, and they were all made at the different parts of the river, as nearly as possible at the same instant of time.

The principal limits of the levelling being now determined and fixed, it only remains to bind the level between the limits, according to the methods already pointed out, using every advantage that may contribute to the success of the work, and at the same time avoiding all obstacles and difficulties that might retard, or injure the operations.

The first rule is always to take the shortest possible way from one limit to the other.

However, this rule must not be followed if there are considerable obstacles in the way, as hills, woods, marshy ground, &c. or, if by going aside, any advantage can be obtained; thus, in the present instance, it was found most convenient to go from A 2 to B, by the dotted line A c d e f g h i k B, which, although it appears the longest, was in effect the shortest, as you have only to level from one pond to the other, at A c, d e, f g, h i, k B (at the top of the plate,) the distances c d, e f, g h, i k, the surfaces of the several ponds being assumed as level lines, thereby abridging the work without rendering it less exact; more so, as it was not the length of the river that was required, but only the declivity.

Having levelled from A to B, proceed from B to C, following the dotted line B l m n o C, whence we obtain the difference in height between the surface of the water at A, and that at C.

The next step was to level between C and D, above and below the mills, to find the difference between the water when at its highest and lowest situations. From D, levels were taken across the country to *p*; leaving *p* on account of the pond or lake which was assumed as level, we began at *q*, from thence to *r*, where we left off; beginning again at *s*, then levelling from thence to *t*, and so on to L, above the mills at *Villebourg*, and finished at F, below them.

By these operations we obtained the knowledge of how much the waters above and below the mills of *Linebruk* are higher than those of *Mazurance*, and these, than those of *Villebourg*, with all the necessary consequences. From this example, the importance of a thorough knowledge of the ground in order to carry on such a work is very evident; this piece of levelling was near five German miles in length, in a strait line, and nine or ten with the bendings of the river. For the profile or section of the foregoing operation, *fig.* 2, *plate* 26, first draw the dotted line AG, on which let fall perpendiculars from the principal limits ABCD produced; then beginning at the highest watermark at *Lignebruk*, set off three feet to six, for the difference between high and low water; from b draw the dotted line bc, parallel to AG. From the point set off on the perpendicular four feet to B, the difference found between b and B; from B draw Bd, parallel also to AG; then set off three feet from d to E downwards, for the difference found between B and C, and 4½ feet from C to D, for the difference in height of the mills at *Mazurance*. From D draw the line De parallel to AG, and from the point e to L, set off three feet for the difference of the level between D and F. And lastly, from E to F, set off one foot six inches for the difference between the higher and lower waters at the mills of *Villebourg*, shewing that there are 19 feet difference which the upper waters at *Lingebruk* are higher than the lower waters of *Villebourg*.

The pocket measuring tape of 100 feet in length, with the centesimals of a yard, is found to be a useful article in the practice of levelling.

ASTRONOMICAL OBSERVATIONS AT LAND, WITH HADLEY'S OCTANT AND SEXTANT.

The portability of the sextant, its cheapness compared with other instruments, the ease with which it is used, the accuracy of the observations made with it, strongly recommend it to the attention of surveyors, &c. the only addition necessary to employ it advantageously in astronomical observations at land, is an artificial or reflecting horizon.

The best artificial horizon is quicksilver or water; but as these are always more or less affected by the air, the trough which contains them should be covered with a roof, consisting of two parallel planes of glass. See page 285.

If these are parallel, it matters little at what an gle they are set, and if any error be suspected, the frame may be placed sometimes with one side, sometimes with the other, foremost, taking a mean of the observations.

Reflecting surfaces, whether of glass or metal, circular levels, floating planes of glass, &c. are not to be depended upon, as they always give a different altitude or diameter from that observed from the surface of the mercury, or water, proved by taking the sun's meridian altitude, or its diameter successively from these different surfaces; this arises from the imperfection of the surface of metal or glass, which has never been ground perfectly *flat.*

The parallelism of a glass may be readily examined, and its defects easily discovered; whereas the want of a flat surface has scarce been suspected. The parallelism of a glass may be discovered by looking at the moon with it, and receiving her rays in a very oblique manner, so as to make the angle between the direct and reflected rays as obtuse as possible; if it appears single and well defined, the glass is parallel. Or a glass may be accurately examined by laying it on a piece of paper, and viewing the top of a wall or chimney, &c. about 15 or 20 yards distant; for, if the two surfaces be not parallel, the object will appear double and surrounded with a light fringe, and thus may every part of the glass be examined, and its defects discovered; the examination will be more perfect if a small telescope be used.

To examine whether a surface be a perfect plane, take the sun's diameter very accurately with your sextant, when its altitude is considerable; then examine in the surface you wish to try the two images, without altering the index; if it be concave, the two images will lap over, if convex, they will separate, and the quantity of this error may be found by the sextant. If, therefore, you use glass, &c. as a reflecting surface for an artificial horizon, you must either allow for the error, which makes the given altitude too great, if the glass be convex, and too small, if concave, or you must make both your adjustments and observations from the same reflecting surface. But this will not entirely obviate the difficulty, as the surface is apt to vary from the sun's heat during a long course of observations.

The angle, observed by means of the artificial horizon, is always double the altitude of the star, &c. above the horizon; consequently, you cannot take an altitude of more than 45° with an octant, or of 60° by the sextant.

Every thing being ready, and the instrument properly adjusted, move backward, till you see the reflected image of the sun in the water. If this image be bright, turn one or more of the dark glasses behind the horizon glass.

Hold now the sextant in a vertical plane, and direct the sight to the sun's image in the artificial horizon. Then move the index tell you see the other image reflected from the mirrors come down to the sun's image seen in the horizon, so as to touch, but not pass it; then bring the edges in contact in the middle, between the wires of the telescope, as before directed, and the divisions on the arc will shew the double altitude.

Correct the double altitude for the index error, before you halve it. Then to this half altitude add the sun's semi-diameter, and subtract the correction for refraction, and you will have the true altitude of the sun's center above the real horizon.

The altitude of a star must be taken in the same manner as that of the sun; the double altitude must be corrected for the index error, if any, then halved, and this half corrected for refraction gives the true altitude above the real horizon.

In taking the sun's altitude, whether for the purpose of calculating time, or for double altitudes, it is best to fix the index to some particular division of the instrument with great nicety, and then wait till the sun is risen or fallen to that altitude.

This is much better than observing its altitude and moving the screw to it, as the screw when thus suddenly moved is very apt to alter a small trifle by the inequality and pressure of the threads, after the hand is removed from it; whereas, when it is fixed to some division previous to the observation, it may be repeatedly tried and examined before the observation is taken.

An accurate observer will find that the error of his sextant will vary according as he takes it, by moving the index backwards or forwards in taking the ☉'s (sun's) diameter on the quadrantal arc of excess; this is owing to some spring in the index, or inequality in the adjusting screw, which it is very difficult totally to obviate.

The best way to correct this, is always to move the index the same way in making your observations, as you did in taking the error of adjustment; though, where a great number of observations are taken, it were best both to settle the adjustment, and take the observations alternately by moving the index backwards and forwards, or by setting the objects open, and making them lap over alternately. A mean of all these will certainly be the most free from error, as the errors will counteract each other.

This may also correct a faulty habit which an observer may have contracted, in forming the contact between the two objects; and though there may seem to be some impropriety in the mode, yet a mean of them will be much nearer the truth than any single observation, where a person prefers seeming to real accuracy.

The lower limb of the ☉ or ☾ always comes first into contact, when you move the index forward, and the index shews the double altitude of the ☉'s upper limb, if the moveable sun is uppermost, but of the lower limb when the moveable sun is lowermost.

At sea, they generally take the altitude of the ☉'s lower limb, because it is most natural to bring that to sweep the horizon. But, by land, it is most correct to take the altitude of the ☉'s upper limb; 1. Because it is highest, and less liable to be affected by refraction. 2. Because the semidiameter and refraction are both subtractive, and the operation is more direct than when one is plus, the other minus.

TO TAKE THE ALTITUDE OF THE SUN BY THE FORE OBSERVATION.

Observations taken by means of the fore horizon glass are called fore observations, because in them both objects are before the observer.

Previous to every observation, the instrument should be examined, in order to see whether the index or horizon glasses be firm, or whether any of the screws be loose; the horizon glass must also be adjusted.

One or two of the dark glasses should be placed before the horizon glass, always proportioning the strength of the shades to the brightness of the sun's rays, that the image may be looked at without injuring the eye.

Hold the quadrant in a vertical position, the arc downwards, either by the braces or the radii, as may be most convenient, or still better according to the foregoing directions. Let the eye be at the upper hole in the sight vane, and the lower part of the limb against the breast.

Turn yourself towards the sun, and direct the sight to that part of the horizon that lies directly under it, keeping the quadrant, as near as you can judge, in a plane passing through the sun's center and the nearest part of the horizon, moving at the same time the index with the left hand, so as to bring the image of the sun down towards the horizon; then swing the quadrant round in a line parallel to the line of sight; by this means the image of the sun may be made to describe the arc of a circle, with the convex side downwards. Now if that edge of the sun which is observed, just grazes upon the horizon, or if the horizon just touches it like a tangent, without cutting it, the observation is rightly made, and the degrees and minutes pointed out by the nonius on the arc shew the apparent altitude of the sun. But if the sun's edge dip below, or cut the horizon, the index must be moved backward; if, on the contrary, it falls short of it, the index must be moved forward, until it just grazes the horizon.

Dr. *Maskelyne* gives the following advice: that in taking the sun's altitude, the observer should turn his quadrant round upon the axis of vision, and at the same time turn himself upon his heel, so as to keep the sun always in that part of the horizon glass which is at the same distance as the eye from the plane of the quadrant; and that unless care be taken to observe the objects in the proper part of the horizon glass, the measured angles cannot be true. In this method the reflected sun will describe an arc of a parallel circle round the true sun, whose convex side will be downwards, and consequently when by moving the index, the lowest point of the arc is made to touch the horizon, the quadrant will stand in a vertical plane, and the altitude above the visible horizon will be properly observed.

Great care should be taken that the situation of the index be not altered, before the quantity it makes is read off.

The observed or apparent altitude of the sun requires three corrections, in order to obtain the true altitude of the sun's center above the horizon.

1. The first correction is to obtain the observed altitude of the sun's center.

All astronomical calculations respecting the heavenly bodies, are adapted to their centers; but in taking altitudes of the sun, it is usual to bring his lower limb in apparent contact with the horizon. In this case it is evident, that a quantity equal to the semidiameter of the sun must be added to the observed altitude, to give the altitude of his center. But if on any occasion, as from clouds, the altitude of the upper limb be taken, the semidiameter of the sun must be subtracted.

The mean semidiameter of the sun is 16 minutes, which may be taken as a constant quantity in common observations, as the greatest variation from this quantity scarcely exceeds one quarter of a minute.

2. The second correction is, to rectify the errors arising from refraction.

One of the principal objects of astronomy is to fix the situation of the several heavenly bodies. It is necessary, as a first step, to understand the causes which occasion a variation in the appearance of the place of those objects, and make us suppose them to be in a different situation from what they really are: among these causes is to be reckoned the following.*

The rays of light, in their passage from the celestial luminaries to our eyes, are bent from their true direction by the atmosphere; this bending is called refraction, and they are more or less refracted, according to the degree of obliquity with which they enter the atmosphere, that is according to the altitude of the object; from this cause, their apparent altitude is always too great; the quantity to be subtracted from the observed altitude may be found in any treatise on navigation.

The following pleasing and easy experiment will give the reader an idea of what is meant by the refraction of the rays of light; a wonderful

* Adams's Astronomical and Geographical Essays.

property, to which we are indebted for all the advantages of vision, and the assistance we receive from telescopes, &c.

Experiment. Into any shallow vessel, a bason, put a shilling, and retire to such a distance, as that you can just see the farther edge of the shilling, but no more; let the vessel, the shilling, and your eye, remain in the same situation, while an assistant fills up the vessel with water, and the whole shilling will become visible, the rays coming from the shilling being lifted or bent upwards in their passage through the water. For the same reason, a strait stick put partly into water appears bent.

3. The third correction is for the dip, or depression of the horizon.

The dip of the horizon is the quantity that the apparent horizon appears below the true horizon, and is principally occasioned by the height of the observer's eye above the water; for, as he is elevated above the level of the sea, the horizon he views is below the true one, and the observed altitude is too great, by a quantity proportioned to the height of the eye above the sea: the quantity to be subtracted from the altitude will be found in any Treatise upon Navigation.*

Mr. *Nicholson* says, that observers at sea generally choose to stand in the ship's waist when they take altitudes, because the height of the eye above the water is not so much altered by the motion of the ship; but this is of no consequence, for in rough weather the edge of the sea beheld from a small elevation is made uneven by waves, whose altitudes amount to two or three minutes, or more;

* See W. Jones's Description of the Hadley's Quadrant.

which circumstance produces as great an uncertainty as the rise and fall of the object seen from the poop, when the ship pitches. These are minute causes of error, but not to be disregarded by those who wish to obtain habits of accuracy and exactness.*

MERIDIONAL ALTITUDES.

The meridian altitude of the sun† is found by attending a few minutes before noon, and taking his altitude from time to time; when the sun's altitude remains for some time without any considerable increase, the observer must be attentive to mark the coincidence of the limb of the sun with the horizon, till it perceptibly dips below the edge of the sea. The quantity thus observed is the meridional altitude.

TO TAKE THE ALTITUDE OF A STAR.

Before an observer attempts to take the altitude of a star, it will be proper for him to exercise himself by viewing a star with the quadrant, and learning to follow the motion of the reflected image without losing it, lest he should take the image of some other star, instead of that whose altitude he is desirous to obtain. His quadrant being properly adjusted, let him turn the dark glasses out of the way, and then,

1. Set the index of the nonius to the o line of the limb.
2. Hold the quadrant in a vertical position, agreeable to the foregoing directions.
3. Look, through the sight vane and the transparent part of the horizon glass, strait up to the star, which will coincide with the image seen in

* Nicholson's Navigator's Assistant.
† At places where it rises and sets.

the silvered part, and form one star; but, as soon as you move the index forward, the reflected image will descend below the real star; you must follow this image, by moving the whole body of the quadrant downwards, so as to keep it in the silvered part of the horizon glass, as the motion of the index depresses it, until it comes down exactly to the edge of the horizon.

It is reckoned better to observe close than open; that is, to be well assured that the objects touch each other; and this opinion is well founded, as many persons are near-sighted without knowing it, and see distant objects a little enlarged, by the addition of a kind of penumbra, or indistinct shading off into the adjacent air.

There are but two corrections to be made to the observed altitude of a star, the one for the dip of the horizon, the other for refraction.

RULES FOR FINDING THE LATITUDE; THE SUN'S ZENITH, DISTANCE, AND DECLINATION AT NOON BEING GIVEN.

The first subject for consideration is, whether the sun's declination be north or south; and, secondly, whether the required latitude be north or south. If the latitude, or declination, be both north, or both south, they are said to be of the same denomination; but if one be north, and the other south, they are said to be of different denominations. Thirdly, take the given altitude from 90° to obtain the zenith distance.

Rule 1. If the zenith distance and declination be of the same name, then their difference will give the latitude, whose demomination is the same with the declination, if it be greater than the zenith distance; but the latitude is of a contrary denomination, if the zenith distance be less than the declination.

Rule 2. If the zenith distance and declination have contrary names, their sum shews the required latitude, whose name will be the same as the declination.

Or, by the two following rules you may find the latitude of the place from the altitude and declination given.

Rule 1. If the altitude and declination are of different names, that is, the one north, the other south, add 90 degrees to the declination; from that sum subtract the meridian altitude, the remainder is the latitude, and is of the same name as the declination.

Rule 2. If the meridian altitude and declination are of the same denomination, that is, both north or both south, then add the declination and altitude together, and subtract that sum from 90 degrees, if it be less, and the remainder will be the latitude, but of a contrary name. But if the sum exceeds 90 degrees, the excess will be the latitude, of the same name with the declination and altitude.

EXAMPLES FOR FINDING THE LATITUDE BY MERIDIAN OBSERVATION.*

Example 1. Being at sea, July 29, 1779, the meridian altitude of the sun's lower limb was observed to be 34° 10′ N. the eye of the observer being 25 feet above the sea. The latitude of the place is required.

* Nicholson's Navigator's Assistant.

The sun's declination for the third year * after leap year on July 29, is found in Table III. to be 18° 46′ N. the dip for 25 feet elevation is five minutes, the refraction for 34° is one minute; therefore,

Altitude of sun's lower limb	34° 10′ N.
Add semidiameter	16
	34 26
Subtract dip and refraction	6
Correct altitude	34 20
Zenith distance or co-alt.	55 40
Subtract declination	18 46 N.
Remains latitude	36 54 S. or of the contrary name to the declin.

Example 2. October 26, 1780, sun's meridional altitude, lower limb 62° 09′ S. required the latitude. Height of the eye 30 feet.

1780 is leap year, and the sun's declination in October 26, is 12° 45′ S. The dip for 30 feet elevation is six minutes, the refraction for 62° is ½ minute. Therefore,

* The annual course of the seasons, or the natural year, consisting of nearly 365 days six hours, and the current year being reckoned 365 days, it is evident that one whole day would be lost in four years if the six hours were constantly rejected. To avoid this inconvenience, which, if not attended to, would cause the seasons to shift in process of time through all the months of the year, an additional day is added to the month of February every fourth year; this fourth year is termed leap year, and is found by dividing the year of our Lord by 4; leap year leaves no remainder; other years are called the first, second, or third years after leap year, according as the remainder is 1, 2, or 3.

The Nautical Almanack, and *Robertson's Treatise on Navigation*, contain the best tables of the sun's declination, &c. &c.

Sun's apparent alt. l. l.	62° 09′ S.	
+ semidiam.—dip and refrac.	9 ½	
Correct alt.	62 18 ½	
Zenith dis.	27 41 ½ S.	
Sun's declination	12 45 S.	
Difference is lat.	14 56 ½ N.	or of the contrary name.

Example 3. Jan. 7, 1776, altitude of sun's lower limb at noon 87° 10′ S. height of the eye 30 feet, required the latitude.

Sun's apparent alt. l. l.	87° 10′	
+ semidiam.—dip.	10	refraction being inconsiderable.
Correct alt.	87 20	
Zenith distance	2 40 S.	
Sun's declinat.	19 13 N.	
Sum is latitude	21 53 N.	

Example 4. In the year 1778, July 30, the sun's meridian altitude lower limb was 84° 10′ N. required the latitude, the height of the eye being 30 feet.

Sun's apparent alt. l. l.	84° 10′
+ semidiam.—	10
Correct alt.	84 20
Zenith dist.	5 40 N.
Sun's declination	18 28 N.
Difference is lat.	12 48 N.

Example 5. Being at sea in the year 1777, close weather prevented the meridian observation of the sun, but the night proving clear, the northermost star in the square of the constellation of the *Great Bear* was observed to come to its least altitude 30° 10′. Required the latitude; the height of the eye being 20 feet.

Star's altitude apparent	30° 10′	
— dip and refraction	6 ½	
Correct altitude	30 03 ½	
Star's co-declination	27 03	N.
Sum is latitude	57 06 ½	N. by Prob. II.

Example 6. June 11, 1770, the sun's meridian altitude of the upper limb below the pole was observed to be 2° 08′. The height of the observer's eye being 16 feet; required the latitude.

Sun's apparent altitude upper limb	2° 08′	
— dip, refraction, and semidiam.	38	
Correct alt.	1 30	
Sun's declin. 23° 08′ N. its comp.	66 52	N.
Sum is latitude	68 22	N.

TO TAKE THE ALTITUDE OF THE MOON AT SEA.

The enlightened edge of the moon, or that edge which is round and well defined, must be brought in contact with the horizon, whether it be the upper or under edge; in other respects, the same method is to be used in taking the altitude of the moon as was directed for the sun.

Between new and full moons the enlightened limb is turned towards the west; and during the time from the full to the new moon, the enlightened limb is turned towards the east.

If that telescope, which shews objects inverted, be used, then the upper edge or limb of the moon will appear the lower, the left side will appear the right, and the contrary.

The wires of the telescope should be turned parallel to the plane of the instrument, as by keeping them in a perpendicular direction, they will serve at night as a guide for holding the sextant in a vertical position, which cannot otherwise at that time be readily ascertained.

The moon is generally bright enough to be seen by reflection from the unsilvered part of the glasses; if not, the telescope must be removed nearer to the plane of the instrument.

The observed altitude of the moon requires four corrections, in order to obtain the true altitude of her center above the horizon.

Correction 1. For the semi-diameter. This is to be found in the Nautical Almanac, page 7, of every month for every noon and midnight at Greenwich. If the lower limb was observed, add the semi-diameter thus found. If the upper limb was observed, it must be subtracted.

Correction 2. For the dip of the horizon to be subtracted.

Correction 3. For refraction. This is to be subtracted.

Correction 4. The moon's parallax in altitude This is to be added in the observed altitude. It is to be found in the Requisite Tables to the Nautical Almanac.

TO DETERMINE THE LONGITUDE AT SEA, BY TAKING THE ANGULAR DISTANCE BETWEEN THE MOON AND ANY CELESTIAL OBJECT.

The latitude is obtained at sea without difficulty, and with as much accuracy as is requisite for nautical purposes; but the motion of the earth on its axis prevents our ascertaining the longitude with the same facility; and hence it is that most methods of determining the longitude by celestial observation, consists in discovering the difference of apparent time between the two places under consideration.

The angular motion of the moon being much greater than that of any other celestial body, the observation of its place is much better adapted to discover small differences of time, than similar observations made with any other instrument. The only practical method of observing its place at sea, is that of measuring the angular distance between it and the sun and a fixed star.

GENERAL DIRECTIONS.

The most obscure, or rather, the least luminous of the two objects must be viewed directly, and the other must be brought by reflection in apparent contact with it.

The well-defined image of the moon must be always made use of for the contact, even though it should be necessary for that purpose to make the reflected image pass beyond the other.

In the night-time it is necessary to turn down one or more of the green screens, to take off the glare of the moon, which would otherwise prevent the star from being seen.

TO FIND THE DISTANCE BETWEEN THE MOON AND THE SUN.

The central distances of the sun and moon every three hours of time, at Greenwich, on such days as this method is practicable, are set down in the Nautical Almanac. From these distances you are to compute roughly the distance between their nearest limbs at the time of observation.

Mr. *Ludlam* says, the moon should be viewed directly through the unsilvered part of the horizon glass, but the sun by reflection; and, if it be very bright, from the unsilvered part of the glass.

If the sun be to the left hand of the moon, the sextant must be held with its face downwards; but with the face upwards, if the sun be to the right hand of the moon.

Set the index to the distance of the nearest limbs of the sun and moon, computed roughly as before; and, placing the face of the sextant agreeable to the foregoing rules, direct the telescope to the moon, putting the sextant into such a position, that if you look edge-ways against it, it may seem to form a line passing through the sun and moon, a circumstance that can be only obtained by practice, the parent of aptness; then give the sextant a sweep or swing round a line parallel to the axis of the telescope, and the reflected image of the sun will pass by the moon to and fro, so near that you cannot fail of seeing it.

The nearest edges, or limbs, may now be brought into exact contact, by moving the index, and then using the adjusting screw; observing, first, that on giving the sextant a motion round the axis of the telescope, the images of the sun and moon only touch at their external edges, and that the body of the sun must not pass over, or be upon the body of the moon. And, secondly, that the edge of the sun touch the round or enlightened edge of the moon. Then will the index point out the observed or apparent distance of the nearest edges of the sun and moon.

But the observed distance requires several corrections, before the true distance of the centers of the objects, as seen from the center of the earth, can be found.

Correction 1. Is the sum, or their semi-diameters, which is to be added, to give the apparent distance of the centers of the sun and moon.

The semi-diameter of the sun for every sixth day, and of the moon for every noon and midnight, at Greenwich, are to be found in the Nautical Almanac; from these their semi-diameters are to be computed at the time of observation, by the rules to be found in the same work.

Correction 2. Is to free the apparent distance of the effect of refraction and parallax, which will then be the true distance of the centers of the sun and moon, as seen from the earth.

For this purpose, two sets of tables, with directions how to use them, are to be found among the Requisite Tables to the Nautical Almanac; being a set of tables published for that purpose by the Board of Longitude. 8vo. 1781.

TO TAKE THE DISTANCE BETWEEN THE MOON AND SUCH STARS AS ARE SELECTED IN THE NAUTICAL ALMANAC, FOR THE PURPOSE OF FINDING THE LONGITUDE AT SEA.

The distance of these stars from the moon's center for every three hours at Greenwich, is given in the Nautical Almanac, from whence their distance from the enlightened edge may be roughly computed as before.

The star must be viewed directly; the moon is generally bright enough to be seen by reflection from the unsilvered part of the glass; the proper shade to take off the glare of the moon is soon found. When the star is to the left hand of the moon, the sextant must be held with its face upwards; but if the star be to the right hand of the moon, with its face downwards.

Set the index to the distance roughly computed, and placing the face of the octant by the foregoing rules, direct the telescope to the star. Then place the sextant so that, if seen edge-ways, it may seem to form a line passing through the moon and star, and give it a sweep round a line parallel to the axis of the telescope, and the reflected image of the moon will pass so near by the star, that you will see it in the field of the telescope; a proof that the sight is directed to the right star.

The enlightened edge of the moon, whether east or west, must then be brought into contact with the star, by moving the index. To know whether the contact is perfect, let the quadrant gently vibrate in a line parallel to the axis of vision, for the star should just graze the edge, without entering at all within the body of the moon; when this is the case, the index will shew the apparent distance of the moon from the star, which, when corrected, gives the true one.

Correction 1. For the semi-diameter of the moon. This may be found in the Nautical Almanac for every noon and midnight, at Greenwich; and from thence computed, by the rules there given, for the time of observation. If the observed or enlightened limb be nearest the star, the semi-diameter thus found is to be added; if the enlightened edge be the furthest from the star, then the semi-diameter is to be subtracted.

Correction 2. Is for refraction and parallax, to be found from the table as directed before for the sun and moon.

These corrections being properly made, you have the true distance of the moon's center from the star, as seen from the center of the earth. From this distance, and the time of observation, the longitude may be found.

The star to be observed is always one of the brightest, and lies in a line nearly perpendicular to the horns of the moon, or her longer axis; but if you have any doubt whether the sight be directed to the proper star, set the index to the supposed distance as before, hold the sextant as near as you can judge, so that its plane, seen edgeways, may coincide with the line of the moon's shorter axis, and moving it in that plane, seek the reflected image of the moon through the telescope. Having found the reflected image of the moon, turn the sextant round the incident ray, that is, a line passing from the moon to the instrument, and you will perceive through the telescope all those stars which have the distance shewn by the index; but the star to be observed lies in a line nearly perpendicular to the horns of the moon, there will, therefore, be no fear of mistaking it.

TO OBSERVE CORRESPONDING ALTITUDES OF THE SUN.

The basis of all astronomical observation is the determination of the exact time of any appearance in the heavens. By corresponding altitudes this time may be determined, without the apparatus of a fixed observatory; they are also useful in finding a meridian line, and may be easily and accurately made by a sextant.

For these observations it is necessary to be provided with a clock. These altitudes should be observed, in our latitude, at least two hours distant from noon. The best time is when the sun is on or near the prime vertical, that is, the east or west points of the compass.

About these times in the forenoon, take several double altitudes of the sun, write down the degrees, minutes, and seconds shewn on the arc, and also the exact time shewn by the clock at each observation; and let the different observations made in the forenoon be written one below the other in the order they are made.

In the afternoon, set the index to the same degree and minute as the last morning observation; note very exactly the time shewn by the clock when the sun is come down to the same altitude, and write down the time on the right hand of the last morning observation; proceed in the same manner to find the time by the clock of all the altitudes corresponding to those taken in the morning, and write down each opposite to that morning one to which it corresponds.

Take now the first pair of corresponding altitudes, add them together, and to half their sum add six hours; this being corrected for the change of the sun's declination between the morning and evening observations, you will have the time of solar noon derived from this pair of observations. Do the same for each pair, and take the mean of the times thus found from each pair, and you will have the exact time shewn by the clock at solar noon.

The time by the clock of solar or apparent noon being thus obtained, the time of mean noon may be had by applying the proper equation of time.

Or thus. Add 12 hours to the time of the afternoon observation, from which subtract the time of the forenoon one,* and add half the difference to the time of the forenoon or morning ob-

* This gives the whole interval between the observations, to which, if your clock is known to vary much in that time, you must add the clock's loss, or subtract its gain, during that interval, and half this corrected (if necessary) interval must be added to the time of forenoon, &c.

servation; this will give the time of apparent noon nearly.

Having this time nearly, it must be corrected by the table of equations for equal altitudes, on account of the sun's change in declination, in the interval between the observations; and you must also apply the equation of time found in the Nautical Almanac with contrary signs, subtracting when it is +, and adding when it is —.

Example. Equal altitudes taken June 1782.

East Azimuth.	h.	m.	s.	West Azimuth.	h.	m.	s.
a	8	55	46	c	3	8	44
b	8	57	41	b	3	6	48
c	8	59	27	a	3	4	58

h.	m.	s.	h.	m.	s.	h.	m.	s.
15	8	44	15	6	48	15	4	58
8	55	46	8	57	41	8	59	27
6	12	58	6	9	7	6	5	31
3	6	29	3	4	$33\frac{1}{2}$	3	2	$45\frac{1}{2}$
12	2	15	12	2	$14\frac{1}{2}$	12	2	$12\frac{1}{2}$

As the seconds differ add them together, and divide the sum by 3, by which you obtain a mean.

15
$14\frac{1}{2}$
$12\frac{1}{2}$
3)42(14

	h.	m.	s.		
Therefore,	12	2	14		the mean.
	0	0	0	8	equation for six hours.
	12	2	14	8	
		1	55	1	equation of time sub.
	12	0	19	7	clock near 20″ too fast for equal time.

For accurate *Tables of Equations to Equal Altitudes,* and the *Method of finding the Longitude at Sea by Time-keepers,* I refer the reader to Mr. *Wales's* pamphlet. 8vo. 1794.

MERIDIONAL PROBLEMS BY THE STARS, AND VARIATION OF THE MAGNETIC NEEDLE BY THE SUN.*

To fix a meridian line by a star, when it can be seen at its greatest elongation on each side of the pole.

Provide two plummets. Let one hang from a fixed point; let the other hang over a rod, supported horizontally about six or eight feet above the ground, or floor in a house, and so as to slide occasionally along the rod: let the moveable plummet hang four or five feet northward of the fixed one. Sometime before the star is at its greatest elongation, follow it in its motion with the moveable plummet, so as a person a little behind the fixed plummet may always see both in one, and just touching the star. When the star becomes stationary, or moves not beyond the moveable plummet, set up a light on a staff, by signals, about half a mile off, precisely in the direction of both the plummets. Near 12 hours afterwards, when the star comes towards its greatest elongation on the other side of the pole, with your eye a foot or two behind the fixed plummet, follow it with the moveable plummet, till you perceive it stationary as before; mark the direction of the plummets then by a light put up precisely in that line: take the angle between the places of the two lights with *Hadley's* quadrant, or a good theodolite; the center of the instrument at the fixed plummet, and a pole set up in day-light, bisecting that angle, will be exactly in the meridian seen from the fixed plummet.

* Mackenzie's Maritime Surveying, p. 47.

This observation will be more accurate if the eye is steadied by viewing the plumb-lines and star through a small slit in a plate of brass stuck upright on a stool, or on the top of a chair-back.

This problem depends on no former observations whatever; and, as there is nothing in the operation, or instruments, to affect its accuracy, but what any one may easily guard against, it may be reckoned the surest foundation for all subsequent celestial observations that require an exact meridian line. The only disadvantage is, that it cannot be performed but in winter, and when the stars may be seen for 12 hours together; which requires the night to be about 15 hours long.

To fix a meridian line by two circumpolar stars that have the same right ascension, or differ precisely 180 *degrees.*

Fix on two stars that do not set, and whose right ascensions are the same, or exactly 180 degrees different: take them in the same vertical circle by a plumb-line, and at the same time let a light be set up in that direction, half a mile, or a mile off, and the light and plummet will be exactly in the meridian.

In order to place a distant light exactly in the direction of the plumb-line and stars, proceed in the following manner. Any night before the observation is to be made, when the two stars appear to the eye to be in a vertical position, set up a staff in the place near which you would have the plummet to hang; and placing your eye at that staff, direct an assistant to set another staff upright in the ground, 30 or 40 yards off, as near in a line with the stars as you can. Next day set the two staffs in the same places; and in their direction at the distance of about a mile, or half a mile northward, cut a small hole in the ground, for the place where the light is to stand at night; and mark the hole so, that a person sent there at night may find it.

Then choose a calm night, if the observation is to be made without doors; if it is moon-light, so much the better, and half an hour before the stars appear near the same vertical, have a lighted lanthorn ready tied to the top of a pole, and set upright in the distant hole marked for it the night before; and the light will then be very near the meridian, seen from the place marked for the plummet. At the same time, let another pole, or rod, six or eight feet long, be supported horizontally where the plummet is to hang, six or seven feet from the ground, and hang the plumb line over it, so as to slip easily along it, either to one hand or the other, as there may be occasion, or tie a staff firmly across the top of a pole, six or seven feet long; fix the pole in the ground, and make the plumb-line hang over the cross staff. Let the weight at the end of the line be pretty heavy, and swing in a tub of water, so that it may not shake by a small motion of the air. Then shift the plumb-line to one hand or the other, till one side of the line, when at rest, cut the star which is nearest the pole of the world, and the middle of the light together: as that star moves, continue moving the plumb-line along the rod, so as to keep it always on the light and star, till the other star comes to the same side of the plumb line also; and then the plummet and light will be exactly in the meridian.

There are not two remarkable stars near the north pole, with the same right ascension precisely, or just a semi-circle's difference: but there are three stars that are very nearly so, viz. the pole star, ϵ in Ursa Major, and γ in Casiopeia. If either of the two last, particularly ε, are taken in the same vertical with the pole star, they will then be so very near the meridian, that no greater exactness need be desired for any purpose in surveying. At London γ is about $1'$ west of the meridian then; and ε much nearer it, on the west side likewise. Stars towards the S. pole proper for this observation are, γ, in the head of the cross; α, in the foot of the cross; and α, in the head of the phenix.

To find a meridian line by a circumpolar star, when it is at its greatest azimuth from the pole. By a circumpolar star is here meant, a star whose distance from the pole is less than either the latitude, or co-latitude of the place of observation.

1. Find the latitude of the place in which you are to observe the star.

2. Fix on a star whose declination is known, and calculate its greatest azimuth from the north, or elevated pole.

3. Find at what time it will be on the meridian the afternoon you are to observe, so as to judge about what time to begin the observation.

4. Prepare two plummets, and follow the star with one of them, as directed in page 427, until the star is stationary; then set up a light half a mile, or a mile off, in the direction of the plumb-line and star, and mark the place of the light in the ground, and also of the plumb-line.

5. Next day set up a pole where the light stood, with a flag flying at it: then with a theodolite, or *Hadley's* quadrant, set the index to the degree and minute of the star's azimuth from the north, found before; direct, by waving to one hand or the other, an assistant to set up a staff on the same side of the pole with a flag, as the pole of the world was from it, so as at the plumb-line these

two lines may make an angle equal to the azimuth of the star, and the plumb-line and staff will then be in the meridian.

When any star is descending, it is on the west-side of the pole of the world; while it ascends, it is on the east-side of it.

The pole of the world is always between the pole star and Ursa-Major: so that when Ursa-Major is W. or E. of the pole star, the pole of the world is W. or E. of it likewise.

To find the greatest azimuth of a circumpolar star from the meridian, use the following proportion.

As co-sine of the lat. : R :: S. of the star's distance from the pole : S. of its greatest azimuth.

The greatest azimuth is when the star is above the horizontal diameter of its diurnal circle.

On the N. side of the equator, the pole star is the most convenient for this observation; for the time when it is at its greatest elongation from the pole, may be known sufficiently near by the eye, by observing when ϵ, in Ursa-Major, and γ, in Casiopeia, appear to be in a horizontal line, or parallel to the horizon; for that is nearly the time. Or, the time may be found more precisely by making fast a small piece of wood along the plumb-line when extended, with a cross piece at right angles to the top of the upright piece, like the letter T; when the plummet is at rest, and both stars are seen touching the upper edge of the cross piece, then they are both horizontal. It is another conveniency in making use of this star, that it changes its azimuth much slower than other stars, and therefore affords more time to take its direction exact.

On the south side of the equator, the head of the cross is the most convenient for this observation, being nearest to the S. pole; and the time of its coming to the horizontal diameter of the diurnal circle, when it appears in a horizontal line with the foot of the cross, or the head of the phenix; and at the equator its greatest azimuth is one hour 16 minutes before or after that, as it is E. or W. of the meridian. The pole of the world is between the head of the cross and the last of these stars.

To find when any star will come to the meridian either in the south or north. Find the star's right ascension, in time, from the most correct tables, also the sun's right ascension for the day and place proposed; their difference will shew the difference between their times of coming on the meridian in the south, or between the pole and zenith; which will be after noon if the sun's right ascension is least, but before noon if greatest. Eleven hours 58 minutes after the star has been on the meridian above the pole, it will come to the meridian north of it, or below the pole.

Given the latitude of the place, and the declinations and right ascensions of two stars in the same vertical line, to find the horizontal distance of that vertical from the meridian, the time one of the stars will take to come from the vertical to the meridian, and the precise time of the observation.

CASE 1. *When the two stars are northward of the zenith, plate* 16, *fig.* 4.

Let Z P O be a meridian, Z the place of observation, H O its horizon, and Z P a vertical circle passing through two known stars, s and S; P the pole, E Q the equator, P s D and P S d circles of declination, or right ascension, passing through these stars: then is Z P the co-latitude of the place, P s and P S the co-declination of the two stars respectively, and the angle s P S the nearest distance of their circles of right ascension; VO the arc of the horizon between the vertical circle and the meridian; and d Q the arc of the equator between the star S and the meridian. When, in the triangle P Z s, the angles P Z s and s P Z, measured by the arcs V O and D E, are found, the problem is solved.

Solution 1. Begin with the oblique-angled triangle P s S, in which are given two sides P s and P S, the co-declinations of the two stars respectively, and the included angle S P s; which angle is the difference of their right ascensions when it is less than 180 degrees; but if their difference is more than 180 degrees, then the angle S P s is equal to the lesser right ascension added to what the greater wants of 360 degrees. From hence find the angle S s P by the following proportions:

As radius is to the co-sine of the given angle S P s, so is the tan. of the side opposite the required angle P s, to the tangent of an angle, which call M.

M is like the side opposite the angle sought, if the given angle is acute; but unlike that side, if the given angle is obtuse.*

Take the difference between the side P S, adjacent the required angle, and M; call it N. Then,

Sine N : sine M :: tan. of the given angle S P s : tan. of the required angle S s P, which is like the given angle, if M is less than the side P S adjacent to the required angle; but unlike the given angle, if M is greater than P S.

* Two arcs or angles are said to be like, or of the same kind, when both are less than 90°, or both more than 90°; but are said to be unlike, when one is greater, and the other less than 90°; and are made like, or unlike to another, by taking the supplement to 180° degrees of the arc, or angle, produced in like proportion, in place of what the proportion brings out.

2. Next in the oblique-angled triangle P s Z there are given two sides, P Z, the co-latitude of the place of observation; P S, the co-declination of the stars; and the angle P s Z opposite to one of them, which is the supplement of P s S, last found to 180°: from thence find the angle P Z s opposite the other side, by the following proportion:

Sine of P Z : sine P s Z :: sine P Z s, either acute or obtuse.

This angle P Z s, measured by the arc of the horizon V O, is therefore equal to the horizontal distance of the vertical of the two stars from the meridian. Let the direction of the vertical be taken by a plumb-line and distant light, as before directed, or by two plumb-lines, and marked on the ground. Next day, the degrees and minutes in the arc V O, may be added to it by *Hadley*'s quadrant, and a pole set up there, which will be in the direction of the meridian from the plumb-line.

3. Last, in the same triangle P s Z find the angle s P Z between the given sides, by the following proportions:

As radius is to the tan. of the given angle P s Z, so is the co-sine of the adjacent side P S, to the co-tan. of M.

M is acute, if the given angle and its adjacent sides are like; but obtuse, if the given angle and the adjacent sides are unlike.

As the co-t. of the side adjacent to the given angle P s, is to the co-t. of the other side P Z; so is the co-s. M, to the co-s. of an angle, which call N.

N is like the side opposite the given angle, if that angle is acute; but unlike the side opposite the given angle, if that angle is obtuse.

Then the required angle s P Z is either equal to the sum or difference of M and N, as the given sides are like or unlike.

The angle s P Z, thus found, added to s P S, and their sum subtracted from 180° will leave the angle d P Q, or the arc d Q that measures it, which is the arc of the equator the star S must pass over in coming from the vertical, Z V, to the meridian. Which converted into time, and measured by a clock or watch, beginning to reckon the precise moment that a plumb-line cuts both stars, will shew the hour, minute, and second, that S is on the meridian.

Find, by the right ascension of the star and sun, at what time that star should come to the meridian in the north, the night of the observation; subtract from it the time the star takes from the vertical to the meridian, and the remainder, corrected by the sun's equation, will be the time when the two stars were in the same vertical.

The lowest of the two stars comes soonest to the meridian below the pole, the highest of them comes soonest to that part of the meridian which is above the pole.

The nearer in time one of the stars is to the meridian when the observation is made, it is the better; for then an ordinary watch will serve to measure the time sufficiently exact. It is still more advantageous if one of them is above the pole, when the other is below it.

The nearer one of the stars is to the pole, and the farther the other is from it, the more exact will this observation be, because the change of the vertical will be the sooner perceived. For this reason, in north latitudes, stars northward of the zenith are preferable to those that are southward of it.

If the two stars are past the meridian when they are observed in the same vertical, then the arc d Q gives the time S took to come from the meridian to the vertical, and must be added to the time when that star was on the meridian, to give the time of the observation; and the arc of the horizon V O must be marked on the ground on the side of the vertical, contrary to what it would have been in the foregoing supposition; that is, eastward *below* the pole, and westward *above* it.

When two stars come to the vertical line near the meridian, it may be difficult to judge on which side of it they are at that time; for determining this, the following rules may serve.

The right ascension of two stars may be either each *less*, or each *more* than 180 degrees, or *one more* and the *other less*.

When the right ascension of each of the stars is *either less*, *or more*, than 180 degrees, they will come to the same vertical on the *east-side* of the meridian when the star, with the *greatest* right ascension is the *lowest*; but on the *west-side* of the meridian when it is *highest*.

When the right ascension of one of the stars is more than 180 *degrees, and that of the other less.*

If the right ascension of the highest is less than 180°, but greater than the excess of the other's right ascension above 180°, then they come to the vertical on the *east-side* of the meridian. But if the right ascension of the higher star is less than that excess, they come to the vertical on the *west-side* of the meridian.

If the right ascension of the higher star is *more* than 180 degrees, and that excess is *less* than the right ascension of the lower star, then they come to the vertical on the *west-side* of the meridian; but if the excess of the higher star's right ascension above 180° is *more* that the right ascension of the lower star, then they come to the same vertical on the *east-side* of the meridian.

CASE 2. *When the two stars are in the same vertical, southward of the zenith, plate* 16, *fig.* 3.

When two stars are observed in the same vertical line *southward* of the zenith, or toward the depressed pole, the operations and solutions are nearly the same as in Case 1. For let P Z O be a meridian, Z the place of observation, O H its horizon, Z V a vertical circle passing through the two stars S and s, P the pole, Q E the equator, P D and P d two circles of declination, or right ascension, passing through the two stars respectively; then is Z P the co-latitude of the place, P S and P s the co-declination of the two stars respectively, the angle S P s the difference of their right ascensions, the angle S Z Q, or O V the arc of the horizon which measures it, the distance of the vertical from the meridian; the angle s P Z, or d Q which measures it, the arc of the equator, which s must pass over from the vertical to the meridian, which converted into time, and measured by a watch, will shew when s is on the meridian; and subtracted from the calculated time that the star should come to the meridian, will, when corrected by the sun's equation, give the true time of the observation. S P s is the triangle to begin the solution with; and in the triangle s P Z, the angles s P Z and s Z P, when found, will give the solution of the problem, as in Case 1. For the supplement s Z B to 180° degrees is the angle V Z O, or its measure O V, the horizontal distance of the vertical from the meridian; the angle s P Z, or its measure d Q, is the equatorial distance of the vertical from the meridian; for all which the solution in Case 1, properly applied, will serve.

On the south-side of the zenith, when the *highest* of the two stars has the *least* right ascension, they come to the same vertical on the *east-side* of the meridian; but when the highest star has the *greatest* right ascension, they come to the same vertical on the *west-side* of the meridian.

PROBLEM. *To find the sun's amplitude at rising, or setting, and from thence the variation of the magnetic needle, with an azimuth compass.*

Make the needle level with the graduated circle in the box. Then, when the sun's lower edge is a semi-diameter above the horizon, take the bearing of its center, from the N. or S. whichever is nearer, through the sights making the thread bisect the sun's disk, and that subtracted from 90°, will be the sun's magnetic amplitude, or distance, from the E. or W. points by the needle

Next, calculate the sun's true amplitude for that day, by the following proportion:

As the co-sine of the latitude is to radius, so is the sine of the sun's declination at setting or rising to the sine of his amplitude from the W. or E.

Which will be N. or S. as the sun's declination is N. or S: and the distance in degrees and minutes between the true E. or W. and the magnetic, is the variation of the needle.

An easy and sure way to prevent mistakes, which the unexperienced are liable to in this calculation, is, to draw a circle by hand, representing the visible boundary of the horizon, and on it to mark the several data by guess; then, by inspecting the figure, it will easily appear how the variation is to be found, whether by addition or subtraction, and on which side of the north it lies, For example:

Plate 16, *fig.* 5. Suppose the variation was sought at sun-setting. Draw by hand a circle N W S E, to represent your visible horizon; in the middle of it mark the point C, for your station; from C, draw the line C W, to represent the true west; then on the north or south side of that line, according as the sun sets northward or southward of the true west, draw the line C ⊙, representing the direction of the sun's center at setting, and another line C w, for the magnetic west, either on the north or south side of ⊙, as it was observed to be, and at its judged distance. Then by observing the situation of these lines, it will easily occur whether the magnetic amplitude and true amplitude are to be added, or subtracted, to give the variation; and on which side of the true north the variation lies. In the present supposition, w ⊙ is the magnetic amplitude, and ⊙ W the true amplitude; ⊙ W therefore must be subtracted from ⊙ w, to give w W the distance of the one from the other; and n, the magnetic N. 90° from w, must be westward of N, the true north.

This method is sufficiently exact for finding the variation, but it is not exact enough for fixing a precise meridian line; because of the uncertainty of the refraction, and of the sun's center: but if the sun ascends, or descends, with little obliquity, the error then will be very little.

To find the sun's azimuth, and from thence to find the variation of the needle.

First, let the latitude of the place be exactly found. Next, let the quadrant be carefully adjusted for observation. Then, two or three hours before or after mid-day, take the altitude of the sun's center as exactly as possible, making the vertical wire of the telescope bisect the sun's disk; and, without altering the plane of the quadrant in the least, move the telescope vertically till you see some distant sharp object on the land, exactly at the vertical wire; and that object will be in the direction of the sun's azimuth when the altitude of its center was taken. If no such object is to be seen, let a pole be set up in that direction, about half a mile off, or as far as can be seen easily.

Next calculate the sun's azimuth by the following rule.

Add the complement of the latitude, the complement of the altitude, and the complement of the sun's declination to 90° together, and take the half of that sum, and note it down; subtract the complement of the declination from the half sum, and take the remainder; then take the complement arithmetical * of the sines of the complement of the altitude, and of the complement of the latitude, and add them together, and to them add the sines of the fore-mentioned half sum and remainder: half the sum of these four logarithms is the co-sine of half the azimuth required. Therefore, find by the tables what angle that co-sine answers to, double that angle, and that will be the sun's azimuth from the north.

If the sun's declination is S. in *north latitude*, or N. *south latitude*, in place of taking the complement of the declination to 90°, add 90° to it, and proceed as before.

In south latitudes the azimuth is found in the same manner; only, the sun's azimuth is found from the S.

* The complement arithmetical of a logarithm is found thus: begin at the left hand of the logarithm, and subtract each figure from 9, and the last figure from 10, setting down the several remainders in a line; and that number will be the arithmetical complement required.

Then, to find the variation, place your needle below the center of the quadrant, set it level, and find how many degrees the pole, or object, in the sun's azimuth, bears from the north by the needle; and the difference between that and the azimuth found by calculation, is the variation of the needle sought.

If the sun ascends, or descends with little obliquity, a meridian line may be fixed pretty exactly this way, because a small inaccuracy in the altitude of the sun's center will not be sensible in the azimuth. But, when the sun does not rise high on the meridian, this method is not to be relied on, when great exactness is necessary; for then every inaccuracy in latitude, altitude, and refraction, occasions severally a greater error in the azimuth. To mark the meridian line on the ground, place the center of a theodolite, or *Hadley*'s quadrant, where the center of the quadrant was when the sun's altitude was taken, and putting the index to the degree and minute of the azimuth, direct, by waving your hat towards one side, or the other, a pole to be set up, making an angle with the former pole placed in the azimuth, equal to the sun's azimuth found; and that last-placed pole will be in the meridian, seen from the center of the quadrant.

N. B. The sun's declination in the tables must be corrected by the variation arising from the difference of the time between your meridian and that of the tables; and also for the variation of declination for the hours before, or after noon, at which the sun's altitude was taken.

The two last problems are constructed and fully explained in *Robertson's*, *Moore's*, and other treatises on navigation; it is, therefore needless to be more particular here.

A COURSE

OF

PRACTICAL GEOMETRY*

ON THE GROUND,

BY ISAAC LANDMAN,

PROFESSOR OF FORTIFICATION AND ARTILLERY TO THE ROYAL MILITARY ACADEMY AT WOOLWICH.

It is as easy to trace geometrical figures *on the ground*, as to describe them on paper; there is, however, some small difference in the mode of operation, because the instruments are different. A rod, or chain, is here used instead of a scale; the spade, instead of a pencil; a cord fastened to two staves, and stretched between them, instead of a rule; the same cord, by fixing one of the staves in the ground, and keeping the other moveable, answers the purpose of a pair of compasses; and with these few instruments every geometrical figure necessary in practice, may be easily traced on the ground.

PROBLEM 1. *To draw upon the ground a strait line through two given points*, A, B, *fig.* 1, *plate* 27.

Plant a picket, or staff, at each of the given points A, B, then fix another, C, between them, in such manner, that when the eye is placed so as

* I have been obliged to omit in this Course, so liberally communicated by Mr. Landman, the calculations that illustrate the examples; this, I hope, will not in the least lessen its use, as this work will fall into the hands of very few who are ignorant of the nature and application of logarithms.

to see the edge of the staff A, it may coincide with the edges of the staves B and C.

The line may be prolonged by taking out the staff A, and planting it at D, in the direction of B and C, and so on to any required length. The accuracy of this operation depends greatly upon fixing the staves upright, and not letting the eye be too near the staff, from whence the observation is made.

Problem 2. *To measure a strait line.*

We have already observed, that there is no operation more difficult than that of measuring a strait line accurately; when the line is short, it is generally measured with a ten feet rod; for this purpose, let two men be furnished each with a ten feet rod, let the first man lay his rod down on the line, but not take it up till the second has laid his down on the line, so that the end may exactly coincide with that of the first rod; now let the first man lift up his, and count one, and then lay it down at the end of the second rod; the second man is now to lift up his, and count two; and thus continue till the whole line is measured. Staves should be placed in a line at proper distances from each other by Problem 1, to prevent the operators from going out of the given line.

When the line is very long, a chain is generally used; the manner of using the chain has been already described, page 204.

Problem 3. *To measure distances by pacing, and to make a scale of paces, which shall agree with another, containing fathoms, yards, or feet.*

In military concerns, it is often necessary to take plans, form maps, or procure the sketch of a field of battle, and villages of cantonment, or to reconnoitre fortified places, where great accuracy is not required, or where circumstances will not allow the use of instruments; in this case it is necessary to be well accustomed to measuring by the common pace, which is easily effected by a little practice; to this end measure on the ground 300 feet, and as the common pace is $2\frac{1}{2}$ feet, or 120 paces in 300 feet, walk over the measured space till you can finish it in 120 paces, within a pace or two.

Example. Let us suppose that we have the map of a country containing the principal objects, as the villages, towns, and rivers, and it be necessary to finish it more minutely, by laying down the roads, single houses, hills rocks, marshes, &c. by measuring with the common pace; take the scale belonging to the map, and make another relative to it, in the following manner, whose parts are paces. Let the scale of the map be 200 fathoms, draw a line AB, *fig.* 3, *plate* 27, equal to this scale, and divide it into four equal parts, AC, CD, &c. each of which will represent 50 fathoms; bisect AC at E, divide AE into five equal parts, A e, e f, f g, &c. each of which will be five fathoms; draw GH parallel to AB, and at any distance therefrom; then through the points of division A, e f, g h, E, C, &c. draw lines perpendicular to AB, and cutting GH, which will be thereby divided into as many equal parts as AB. Two hundred fathoms, at $2\frac{1}{2}$ feet per pace, is 480 paces; therefore write at H, the last division of GH, 480 paces, at I 360, at K 240, and so on. To lay down on the plan any distance measured in paces, take the number of fathoms from the line AB, corresponding to the number of paces in GH, which will be the distance to be laid down on the plan.

When plans, or maps, are taken by the plain table, or surveying compass, this is an expeditious method of throwing in the detail, and a little practice soon renders it easy.

A TABLE,

For reducing the common pace of $2\frac{1}{2}$ feet into feet and inches.

Paces	Feet	Inc.	P.	Feet	Inc.	P.	Feet	Inc.	P.	Feet	Inc.	P.	Feet	Inc.
1	2	6	16	40	0	31	77	6	46	115	0	61	152	6
2	5	0	17	42	6	32	80	0	47	117	6	62	155	0
3	7	6	18	45	0	33	82	6	48	120	0	63	157	6
4	10	0	19	47	6	34	85	0	49	122	6	64	160	0
5	12	6	20	50	0	35	87	6	50	125	0	65	162	6
6	15	0	21	52	6	36	90	0	51	127	6	66	165	0
7	17	6	22	55	0	37	92	6	52	130	0	67	167	6
8	20	0	23	57	6	38	95	0	53	132	6	68	170	0
9	22	6	24	60	0	39	97	6	54	135	0	69	172	6
10	25	0	25	62	6	40	100	0	55	137	6	70	175	0
11	27	6	26	65	0	41	102	6	56	140	0	71	177	6
12	30	0	27	67	6	42	105	0	57	142	6	72	180	0
13	32	6	28	70	0	43	107	6	58	145	0	73	182	6
14	35	0	29	72	6	44	110	0	59	147	6	74	185	0
15	37	6	30	75	0	45	112	6	60	150	0	75	187	6

Distances of a certain extent may be measured by the time employed in pacing them; to do this, a person must accustom himself to pace a given extent in a given time, as 600 paces in five minutes, or 120 in one minute; being perfect in this exercise, let it be required to know how many paces it is from one place to another, which took up in pacing an hour and a quarter, or 75 minutes. Multiply 75 by 120, and you obtain 120, the measure required.

This method is very useful in military operations; as for instance, when it is required to know the itinerary of a country for the march of an army, or to find the extent of a field of battle, an encampment, &c. It may be performed very well on horseback, having first exercised the horse so as to make him pace a given space in a determinate time, and this may be effected in more or less time, according as he is trained to walk, trot, or gallop the original space.

PROBLEM 4. *To walk in a strait line from a proposed point to a given object, fig. 2, plate 27.*

Suppose the point A to be the proposed point from whence you are to set out, B the given object; fix upon another point C, as a bush, a stone, or any mark that you can find in a line with B; then step on in the direction of the two objects B, C; when you are come within 10 or 15 paces of C, find another object D, between C and B, but in a line with them; it is always necessary to have two points constantly in view, in order to walk in a strait line.

This problem is of great use in measuring distances, and surveying by the pace; particularly in reconnoitring either on foot or on horseback. It may also be used when a battalion is ordered to take a position at the distance of 3 or 400 paces, and parallel to that in which it is standing; to effect this, let two non-commissioned officers, who are well exercised in the practice of this problem, step out from the extremities, or wings of the battalion, place themselves square with the front of the battalion, then fix upon two objects strait before them, and on a signal given, set out and step the required number of paces, then halt, thus becoming two guides for placing the battalion.

PROBLEM 5. *To place a troop* E F, *between two given points* A, B, *fig.* 4, *plate* 27.

To perform this, let two persons, each with a pole in his hand, separate about 50 or 60 paces from each other; and then move on till the poles are situated in the direction A, B, *i. e.* so that the person at C may see the point D in a direction with B, at the same time that the person at D sees C in a direction with the object A.

Method 2, *fig.* 5. To perform the same with one person. Take a staff C F, *fig.* 5, *plate* 27, three or four feet long, slit the top thereof, and place a strait rule, six or seven inches long, in this slit; place this instrument between the two points, and see if A and B coincide with E, then view D from B, and if A coincides therewith, all is right; if not, move the rule forward or backward till it is in the direction of D, E.

PROBLEM 6. *To raise a perpendicular from a point* C, *to the given line* AB, *fig.* 6, *plate* 27.

Set off two points E, D, in AB, equally distant from C; fold a cord in two equal parts, place a staff in the middle at F, fasten the two ends to the staves E and D; then stretch the cord tight, and the point F will be the required point, and the line C F will be perpendicular to AB.

PROBLEM 7. *From a given point* F, *out of the line* AB, *to let fall a perpendicular* C F, *fig.* 7, *pl.* 27.

Fold your cord into two equal parts, and fix the middle thereof to the point F, stretch the two halves to A, B, and where they meet that line plant two staves, as at E, D, divide the distance E D into two equal parts at C, and the line C F will be perpendicular to AB.

PROBLEM 8. *To raise a perpendicular* B C *at the end* B *of the line* B F, *fig.* 8, *plate* 27.

From D taken at pleasure, and with the length DB, plant a staff at the point A, in the direction F B; with the same length set off from D towards C, plant a staff C in the direction DA, and B C will be perpendicular to F B.

Method 2, *fig.* 9, *plate* 27. By the numbers 3, 4, 5, or any multiple thereof. Set off from C to D, in the direction CA, four feet or four yards, and plant a staff at D, take a length of three feet and five feet, or yards, according as the former measure was feet or yards; fasten one end of the cord at C, and the other at D; then stretch the cord so that three of these parts may be next to the point C, and five next to D; plant a staff at H, and C H will be perpendicular to C A.*

PROBLEM 9. *To draw a line* C F *parallel to the line* A B, *and at a given distance from it, fig.* 10, *plate* 27.

At A taken at pleasuse in A B, raise a perpendicular A C, then from B taken also at pleasure in A B, draw B F perpendicular to A B; make A C, B F, each equal to the given distance; plant staves at C and F, and in the direction of these two plant a third, E, and the line C E F will be the required line.

PROBLEM 10. *To make an angle* a b c *on the ground equal to a given angle* A B C, *fig.* 11 *and* 12, *plate* 27.

Set off any number of equal parts from B to C, and from B to A, and with the same parts measure A C; describe on the ground with these three lengths a triangle a b c, and the angle a b c thereof will be equal to the angle A B C.

PROBLEM 11. *To prolong the line* A B, *notwithstanding the obstacle* G, *and to obtain the length thereof, fig.* 13, *plate* 27.

At B raise the perpendicular B C, draw C D perpendicular to B C, and E D to C D, make E D equal to B C; then raise a perpendicular E F to E D, and E F will be in a strait line with A B.

The measures of the several lines A B, C D, E F, added together, give the measure of the line A F.

This problem is particularly useful in mining.

* Much trouble and time may be saved by using a cross-staff, before described; see p. 211. EDIT.

Problem 12. *To draw a line* B C, *parallel to the inaccessible face* b c *of the bastion* a, b, c, d, e, *in order to place a battery at* C, *where it will produce the greatest effect*, *fig.* 14, *plate* 27.

Place a staff at the point A, out of the reach of musket shot, and in a line with b c; from A draw A G, perpendicular to b A; set off from A to G about 375 paces, and at G raise G B perpendicular to A G, and produce this line as far as is requisite for placing the battery, *i. e.* so that the direction of the fire may be nearly perpendicular to one third of the face b c.

Problem 13. *To make an angle upon paper equal to the given angle* A B C, *fig.* 15 *and* 16, *pl.* 27.

Set off 36 feet from B to C, and plant a staff at C; set off also 36 feet from B to A, and plant a staff at A, and measure the distance A C, which we will suppose to be equal to 52 feet. Let D G be a given line on the paper; then from G with 36 equal parts taken from any scale, describe the arc D H, then with 52 parts from the same scale, mark off the distance D H; draw G H, and the angle D G H is equal to the angle A B C.

Problem 14. *To measure from the outside an angle* A B C, *formed by two walls* A B, C B, *fig.* 17, *plate* 27.

Lay off 30 feet from B to E in the direction A B, and plant a staff at E; set off the same measure from B to F in the direction B C, and measure F E, and you may obtain the measure of your angle, either by laying it down on paper, as in the last problem, or by calculation.

Problem 15. *To ascertain the opening of an inaccessible flanked angle* B C D *of the bastion* A B C D E, *fig.* 18, *plate* 27.

Place a staff at F, in the direction of the face B C, but out of the reach of musket shot; make

F G perpendicular to F B, and equal to about 250 paces; at G make G H perpendicular to G F, and produce it till it meets, at H, the direction of the face D C; place a staff at I in a line with D H, and the angle I H K will be equal to the required angle B C D, and the value of I H K may be obtained by Problem 13.

PROBLEM 16. *To ascertain the length of the line* A B, *accessible only at the two extremities* A, B, *fig*. 19, *plate* 27.

Choose a point O accessible to A and B, draw A O, O B, prolong A O to D, and make O D equal to A O; prolong B O to C, and make O C equal to B O, and C D will be equal to A B, the required line.

PROBLEM 17. *To let fall a perpendicular from the inaccessible point* D, *upon the right line* A B, *fig*. 21, *plate* 27.

From A and B draw A D, B D, let fall the perpendiculars A H, B F, upon the sides A D, B D, plant a staff at their intersection C, and another E, in the direction of the points C, D, and E C D will be the required perpendicular.

PROBLEM 18. *To find the breadth* A B, *of a river*, *fig*. 20, *plate* 27.

Plant a staff at C in a line with A B, so that B C may be about $\frac{1}{3}$ of the length of A B, draw a line C E in any direction, the longer the better. Set up a staff at D, the middle of C E, find the intersection F of E B, D A; draw D G parallel to B C; measure B F, F E; and you will find A B by the following proportion; as F E — B F : $\frac{1}{2}$ B C or D G :: B F : A B.

Inaccessible distances may be obtained in other modes, when circumstances and the ground will permit, as in the following problem.

PROBLEM 19. *Let it be required to measure the distance* AB *of the produced capital of the ravelin, from the head of the trenches* B, *to the angle* A, *fig*. 22, *plate* 27.

Draw BE perpendicular to AB, and set off from B to E 100 feet, produce BE to C, and make EC equal one eighth or tenth part of BE; at C raise the perpendicular CD, plant staves at E and C, then move with another staff on CD, till this staff is in a line with E and A. Measure CD, and you will obtain the length of AB by the following proportion; as EC : CD :: BE : AB; thus if BE = 100 feet, EC = 10 feet, and CD = 38 feet, then as 10 : 100 :: 38 : 380 feet, the distance of AB.

PROBLEM 20. *To measure the inaccessible distance* AB, *fig*. 23, *plate* 27.

Plant a staff at C, a point from whence you can readily see both A and B. By the preceding problems, find the length of CA, CB, make CD as many parts of CA, as you do CE of CB, and join DE; then as CD : DE :: CA : AB.*

PROBLEM 21. *To determine the direction of the capital of a bastion produced, fig*. 24, *plate* 27.

Upon the produced lines BD, BE, of the two faces CB, AB, place the staves E and D; find the lengths of EB and DB, by Problem 19, and measure DE. Then as the capital GB divides

* *An easy method of finding the distance of forts or other objects.*— If you go off at any angle, as 90, and continue in that direction until you bring the object and your first station under an angle of 63, the distance measured from the first station is equal to half the distance the object is from your first station. But should the ground not admit of going off at an angle of 90, go off at an angle of 45, and whatever distance in that direction you find the object and the first station under an angle of 106¼, is half the distance of the object and first station. See also pages 215 and 283 of these Essays.

the angle A B C, and its opposite E B D, into two equal parts, we have as D B : B E :: D F : E F, and as D B + B E : B E :: D E : E F, and consequently the point F, which will be in the direction of the capital G B produced.

PROBLEM 22. *Through the point* C *to draw a line* I K *parallel to the inaccessible line* AB, *fig.* 25, *plate* 27.

Assume any point D at pleasure, find a point E in the line AD, which shall be at the same time in a line with C B; from E draw E G parallel to D B, and through C draw G C F parallel to AD, meeting B D in F. Plant a staff at H in the line E G, so that H may be in a line with F A, and a line I, H, C, K, drawn through the points B C, will be parallel to AB.

PROBLEM 23. *To take a plan of a place* A, B, C, D, E, *by similar triangles*, *fig.* 26 *and* 27, *pl.* 27.

1. Make a sketch of the proposed place, as *fig.* 27. 2. Measure the lines AB, B C, C D, D E, E A, and write the measures obtained upon the corresponding lines a b, b c, c d, d e, e a, of the sketch. 3. Measure the diagonal lines A C, A D, and write the length thereof on a c, a d, and the figure will be reduced to triangles whose sides are known. 4. To obtain a plan of the buildings, measure B G, B F, G H, I K, F K, &c. and write down the measures on the sketch. 5. Proceed in the same manner with the other buildings, &c. 6. Draw the figure neatly from a scale of equal parts.

PROBLEM 24. *To take a plan of a wood, or marshy ground, by measuring round about it*, *fig.* 28 *and* 29, *plate* 27.

Make a rough sketch, *fig.* 29, of the wood, set up staves at the angular points, so as to form the circumscribing lines AB, B C, C D, and measure these lines, and set down the measures on the corresponding lines in the sketch, then find the value of the angles by Problems 13 and 14.

PROBLEM 25. *To take a plan of a river, fig.* 30 *and* 31, *plate* 27.

1. Draw a sketch of the river. 2. Mark out a line AB, *fig.* 30. 3. At C, upon AB, raise the perpendicular CH. 4. Measure AC, CH. 5. Measure from C to D, and at D make DI perpendicular to AB, and measure DI. 6. Do the same the whole length of AB, till you have obtained the principal bendings of the river, writing down every measure when taken on its corresponding line in the sketch, and you will thus obtain sufficient data for drawing the river according to any proportion.

PROBLEM 26. *To take a plan of the neck of land* A, B, C, D, E, F, G, *fig.* 32 *and* 33, *plate* 27.

Take a sketch of the proposed spot, divide the figure into triangles ABC, ACD, ADE, AEG, by staves or poles placed at the points A, B, C, D, E, F, G, measure the sides AB, BC, AC, CD, AD, AE, AF, FG, AG of the triangles, writing down these measures upon the corresponding lines ab, bc, ac, &c. then measure AH on AB, and at H raise the perpendicular HI, and measure its length; do the same at K, M, &c. writing down the measures obtained on their corresponding lines ah, hi, ik, kl, &c. Proceeding thus, you will ascertain a sufficient number of points for laying down your plan by a scale of equal parts.

PRACTICAL TRIGONOMETRY.

PROBLEM 27. *To ascertain the height of a building, fig.* 34, *plate* 28.

Measure a line FE, from the foot of the building, so that the angle CDA may be neither too acute nor obtuse; thus suppose EF=130 feet, place your theodolite at D, and measure the angle ADC=34° 56′.

Then as radius to tangent 34° 56′, so is EF= 130 feet, to AC.

Which, by working the proportion, you will find to be 89 feet 66 parts, or 89 feet 7.92 inches, to which adding four feet for DE, or its equal CF, you obtain the whole height 93 feet 7.92 inches.

PROBLEM 28. *To find the angle formed by the line of aim, and the axis of a piece of ordnance produced, the caliber and dimensions of the piece being known, fig.* 35, *plate* 28.

Suppose the line BI to be drawn through B, the summit of the swelling of the muzzle, and parallel to CD, the axis of the piece; the angle ABI will be equal to the angle AEC, formed by the line of aim AF, and the axis CD; then in the right-angled triangle AIB, we have the sides AI and BI to find the angle ABI, which we obtain by this proportion; as IB : AI :: radius, tangent of angle ABI equal to AEC required.

PROBLEM 29. *The elevation, three degrees, of a light twelve-pounder being given, to find the height to which the line of aim rises at the distance of* 1200 *yards, which is about the range of a twelve-pounder, with an elevation of three degrees, fig.* 35, *plate* 38.

The line of aim, which we suppose to be found by the preceding problem, forming with the axis of a light twelve-pounder 1° 24′, will make with the horizon an angle of 1° 36′; thus the height at the horizontal distance of 1200 yards will be the second side of a right-angled triangle, where the angle adjacent to the side of 1200 yards is 1° 36′, and may therefore be obtained by the following proportion; as radius to the tangent of 1° 36′, so is 1200 to 33 yards 1 foot 6 inches=FG.

PROBLEM 30. *The first embrasure* A *of a ricochee battery being direct, to find the inclination of the seventh embrasure* B, i. e. *the angle formed by the line of direction* B C, *and the breast-work* A B, *at the seventh embrasure* A B; *it is supposed that all the pieces are directed towards a point* C, *at the distance of* 1500 *feet, fig.* 36, *plate* 28.

The line of direction A C of the first embrasure is supposed to be perpendicular to the breast-work A B; therefore we have to find the angle A B C of the right-angled triangle B A C, in which the right angle is known A C = 1500 feet, and A B is determined by the size, the distance, and number of embrasures: thus, suppose the distance from the middle of one embrasure to another be 20 feet, this multiplied by 6, will be 120 feet, and equal to A B; then as A B to A C, so is radius to the tangent of angle A B C, 85° 26′.

PROBLEM 31. As the hurler D E, *fig.* 37, *plate* 28, is always perpendicular to the directing line of the gun, and as at least one end of it ought to be laid against the breast-work, it will make an angle A F D, which was found by the preceding problem to be 85° 26′; therefore, knowing the length D E of the hurler, and consequently its half D F, it will be easy to calculate the distance B F from the breast-work where the middle F of the hurler ought to be placed, upon the line of direction of the gun.

PROBLEM 32. *To ascertain the height of a building from a given point, from whence it is impossible to measure a base in any direction, the points* A *and* F *being supposed to be in the same horizontal line, fig.* 38, *plate* 28.

Measure the angles C E D, A E D, let C E D be equal 43° 12′, the angle A E D equal 2° 26′, and the height E F from the center of the instrument to its foot five feet; then in the right-angled triangle ADE we have AD=EF=5 feet, the angle DEA=2° 26′ to find DE, which may be obtained by the following proportion; as radius : co-tangent of angle DEA, so is AD to DE 118 feet; then in the right-angled triangle CDE we have DE 118 feet, angle CED 33° 12′ to find CD, which is found by the following proportion; as radius to tangent of angle CED, join DE to DC 77 feet, which added to EF 5 feet, is 82 feet, the height of the tower.

PROBLEM 33. *The distance* AC, 135 *toises, from the point* C, *to the flanked angle of the bastion being given, and also* AB, 186 *toises, the exterior side of the polygon, to find* BC, *fig.* 39, *plate* 28.

1. Find the angle B by the following proportion; as AB is to the sine of angle C, so is AC to the sine of angle B, 39° 8′; because it is plain from the circumstances of the case that B must be acute, and therefore angle BAC is also known. 2. BC is found by the following proportion; as sine angle C is to AB, so is sine of angle BAC : BC, 213 toises, three feet.

PROBLEM 34. *To find the height of the building* AC, *fig.* 40.

1. At B measure the angle FBC in the direction FB. 2. Set off any distance BD as a base. 3. Measure the angle BDC, CBD is the supplement of FBC, and BCD is the supplement of CBD+BDC.

Then as sin. $\angle$BCD : BD :: CDB : BC, BC being found, we have in the right-angled triangle FBC, the side BC and angle FBC, to find FC, which is found by this proportion; as radius to sin. angle FBC, so is BC : FC, FC added to AF, the height of the instrument, gives the height of the tower.

PROBLEM 35. *To ascertain the distance between two inaccessible objects*, C D, *fig.* 41, *plate* 28.

1. Measure a base AB, from whose extremities you can see the two objects C, D. 2. Measure the angles CAB, DAB, DBA, CBA, CBD. 3. In the triangle BAC we have the side AB and angles ABC, BAC, to find BC, which is found by the following proportion; as sine angle ACB : AB :: sine BAC : BC. 4. The angles CAB, CBA, added together and subtracted from 180, gives the angle ACB. 5. In the triangle ABD we have the angles DAB, ABD, and consequently ADB, and the side AB to find BD, which is found by the following proportion; as the sine of angle ADB is to AB, so is the sine of angle DAB : BD. 6. In the triangle CBD we have the two sides BC, BD, and the angle CBD, to find the angle CDB, and the side CD; to find CDB we use this proportion; as the sum of the two given sides is to their difference, so is the tangent of half the sum of the two unknown angles to the tangent of half their difference; the angle CDB being found, the following proportion will give CD; as sine angle CDB to BC, so is sine angle CBD to CD.

PROBLEM 36. *To draw a line through the point* B, *parallel to the inaccessible line* C D, *fig.* 41, *pl.* 28.

Find the angle BCD by the preceding problem, and then place your instrument at B, and with BC make an angle CBE equal BCD, and EB will be parallel to CD.

PROBLEM 37. *To ascertain several points in the same direction, though there are obstacles which prevent one extremity of the line being seen from the other, fig.* 42, *plate* 28.

1. Assume a point C at pleasure, from which the two extremities of the line AB may be seen.

2. Measure the distances C B, C A, and the angle A C B, and find the value of the angle C A B. 3. Measure the angle A C D, and then in the triangle A C D we have the side A C, and the two angles D A C, A C D, and therefore the third to find C D, and as sine angle A D C to A C, so is sine angle C A D to C D. 4. Set off C D, making with A C an angle equal the angle A C D, equal the measure thus found, and the point D will be in a line with A B; and thus as many more points, as G, may be found as you please; *in this manner* a mortar battery may be placed behind an obstacle, so as to be in the direction of the line A B.

Thus also you may fix the position of a richochee battery M, *fig.* 43, so as to be upon the curtain A D produced.

Problem 38. *To measure the height of a hill, whose foot is inaccessible, fig.* 44, *plate* 28.

1. Measure a base F G, from whose extremities the point A is visible. 2. Measure the angles A B C, A C B, A C D. 3. In the triangle A B C we have B C, and the angles A B C, A C B, to find A C; but as sine angle B A C is to B C, so is sine angle A B C to A C. 4. In the right-angled triangle A D C we have the side A C, and the angle A C D, to find A D; but as radius is to sine angle A C D, so is A C to A D, the height required.

Problem 39. *To take the map of a country, fig.* 45, *plate* 28.

First, choose two places so remote from each other, that their distance may serve as a common base for the triangle to be observed, in order to form the map.

Let A, B, C, D, E, F, G, H, I, K, be several remarkable objects, whose situations are to be laid down in a map.

Make a rough sketch of these objects, according to their positions in regard to each other; on this sketch, the different measures taken in the course of the observations are to be set down.

Measure the base AB, whose length should be proportionate to the distance of the extreme objects from A and B; from A, the extremity of the base, measure the angles EAB, FAB, GAB, CAB, DAB, formed at A with the base AB.

From B, the other extremity of the base, observe the angles EBA, FBA, GBA, CBA, DBA.

If any object cannot be seen from the points A and B, another point must be found, or the base changed, so that it may be seen, it being necessary for the same object to be seen at both stations, because its position can only be ascertained by the intersection of the lines from the ends of the base, with which they form a triangle.

It is evident from what has been already said, that having the base AB given, and the angles observed, it will be easy to find the sides, and from them lay down, with a scale of equal parts, the several triangles on your map, and thus fix with accuracy the position of the different places.

In forming maps or plans, where the chief points are at a great distance from each other, trigonometrical calculations are absolutely necessary.

But where the distance is moderate, after having measured a base and observed the angles, instead of calculating the sides, the situation of the points may be found by laying down the angles with a protractor; this method though not so exact as the preceding, answers sufficiently for most military operations.

PROBLEM 40. *The use of the surveying compass, fig.* 3, *plate* 15, *in determining the particulars of va-*

rious objects to be inserted in a plan, fig. 46, *plate* 28.

The first station being at A, plant staves at the requisite places, and then place the compass at A, directing the telescope to C; observe the number of degrees north AC, made by the needle and the telescope, or its parallel line drawn through the center of the compass, and mark this angle in your sketch, see *fig.* 46; observe and mark in the same manner the other angles, north AO, north AP, north AQ; then measure AC, and at the second station C, observe the angles north CM, north CQ, north CO, north CP, north CD, and so on at the other stations; in observing the angles, attention must be paid, when the degrees pass 180, to mark them properly in the sketch, in order to avoid mistakes in protracting.

PROBLEM 41. *To raise perpendiculars, and form angles equal to given angles by the surveying compass, fig.* 47, *plate* 28.

Let it be required to trace out the field work ADCB, at the head of a bridge.

The direction of the capital FE being given, describe the square ABCD in the following manner; place your compass at F, and as AB is to be perpendicular to FE, direct the telescope to E, and observe, when the needle is at rest, the number of degrees it points to; then turn the compass box upon its center, till the needle has described an arc of 90°, and place a staff in the direction of the telescope towards A, and AF will be perpendicular to FE; continue AF towards B, and make AF, BF, each equal 30 toises at E; in the same manner raise the perpendicular DC, and make ED, EC, each equal 30 toises; join DA, CB, and you have the square ABCD.

Make CG, DK, each equal to $\frac{1}{4}$ of AB, through G and K draw the line GK, set off GH, KI, each equal to $\frac{1}{4}$ of AB or CD, draw the lines of defence IC, HD, place the compass at I, and the telescope in the direction IH; observe where the needle points when at rest, and turn the compass till the needle has described an arc of 100, the value of the angle LIH of the flank, place a staff at L in the direction of the telescope, and at the same time in the line IH, which gives the length of the face DL and the flank IL; the face CM and flank HM are ascertained in the same manner.

Make BQ equal to $\frac{1}{6}$ of AB, and AR equal to $\frac{AB}{10}$, and draw DR, CQ, through Z the middle of AD, draw OZ, make RT equal to $\frac{1}{3}$ AB; at T and with TD, form an angle DTU, equal to 105°; plant a staff at U, so that it may be in the line ZO, and at the same time in the direction TU, which gives the flank TU and face OU.

Make QS equal $\frac{1}{3}$ AB, at S form an angle CSP equal to 120°, draw the line SP, meeting the river, and the lines O, U, T, D, L, I, H, will be the line of the tete de pont required.

THE USE OF THE PLAIN TABLE IN MILITARY OPERATIONS.

PROBLEM 42. *To take the plan of a camp, fig. 2, plate* 29.

Place the table at A, where you can conveniently see the greater part of the field, and having made a scale on it, fix a fine needle perpendicular to the table at the place that you fix upon to represent the point A; the fiducial edge of the index is always to be applied against the needle.

Turn the plain table so that the index may point to the object B, and be so situated as to take in the field; then plant a staff at C, in a line with B, point the index to the windmill F, and draw the indefinite line AF; then point it to K, the right wing of the cavalry K L, and draw AK, then to L, and draw AL, afterwards point the index to the steeple I, and then to the points M, O, P, N, H, E, G, then draw a line on the table parallel to the north and south of your compass, to represent the magnetic meridian.

Remove the plain table to C, planting a staff at C, measure AC, and set off that measure by your scale from A upon the line AC, and fix the needle at C; then set the index upon the line AC, and turn the table till the line of sight coincides with A, fasten the table, point the index to F, and draw CF, intersecting AF in F, and determining the position of the windmill F; from C draw the indefinite lines CM, CK, CL, &c. which will determine the points MKL, &c. draw KL and mn parallel thereto, to represent the line of cavalry.

Remove the table from C to D, setting up a staff at C, measure CD, and set off the distance on CB from your scale, place the needle at D, the index on CD, and turn the table till C coincides with the sights, and take the remarkable objects which could not be seen from the other stations.

Staves should be placed at the sinuosities of the river, and lines drawn at the stations A, C, D, B, to these staves, which will give the windings of the river.

Having thus determined the main objects of the field, sketch on it the roads, hills, &c.

PROBLEM 43. *To take a plan of the trenches of an attack, fig.* 1, *plate* 29.

The plan of trenches, taken with accuracy, gives a just idea of the objects, and shews how you may close more and more upon the enemy, and be covered from their enfilade fire, and also how to proceed in the attack without multiplying useless works, which increase expense, augment the labour, and occasion a great loss of men.

Measure a long line AB, parallel to the front of the attack DF, place the table at A, and set up a staff at B, point the index to B, and draw a line to represent AB, fasten the table, and fix a needle at the point A, direct the line of sights to the flanked angle of the ravelin C, and draw AC; proceed in the same manner with the flanked angles D, E, F, G, to draw the lines at the opening of the trenches, plant staves at H and R, from A draw a line on the table in the direction AH, measure AH, and set off that measure from your scale upon the line AH on the table.

Remove the plain table from A to B, set up a staff at A, lay the fiducial edge of the index against the line AB, and turn the table about till the staff at A coincides with the line of sight, then fasten the table, direct the sights to C, and draw BC, intersecting A at C; in the same manner ascertain the flanked angles D, E, F, G, draw a line in the direction BR, and set off the measure thereof from your scale.

Remove the table from B to R, and set up a staff at B where the plain table stood, lay the index upon the line corresponding with RB, then turn the table about till the line of sight is in the direction BR, screw the table fast, direct the sights towards P, and draw on the table the line RP, and by the scale lay off its measure on that line.

Remove the plain table from R to P, lay the index upon the line P R, and turn the table about till the line of sight coincides with R, screw the table fast, and draw a line upon it in the direction P Q; measure P Q, and take the same number of parts from your scale, and set it off on the line, and so on with the other station Q.

Having removed the table to the station S, and duly placed it with regard to Q, from the point S, draw the lines S V, S T, S U, setting off from your scale their lengths, corresponding to their measures on the ground.

Having thus taken the zigzags R P Q, S T W, and the parts U U W Z of the parallels, you remove the plain table to H, and proceed in like manner to take the zigzags H I K, &c. as above, which will represent on your plain table the plan of the attack required.

OF LEVELLING, *fig.* 48, *plate* 28.

Levelling is an operation that shews the height of one place in respect to another; one place is said to be higher than another, when it is more distant from the center of the earth than the other; when a line has all its points equally distant from the center, it is called the line of true level; whence, because the earth is round, that line must be a curve, and make a part of the earth's circumference, as the line A B E D, all the points of which are equally distant from the center C of the earth; but the line of sight A G, which the operation of levelling gives, is a right line perpendicular to the semi-diameter of the earth C A raised above the true level, denoted by the curvature of the earth, and this in proportion as it is more extended; for which reason, the operations which we shall give, are only of an apparent level, which must be corrected to have the true level, when the line of sight exceeds 300 feet.

Suppose, for example, that AB was measured upon the surface of the earth to be 6000 feet, as the diameter of the earth is near 42018240 feet, you will find B F by the following proportions.

* 42018240 : 6000 :: 6000 : B F equal to 0,85677 f. which is 10,28124 in. that is to say, between two objects A and F, 6000 feet distant from each other, and in the same horizontal line, the difference B F of the true level, or that of their distance from the center of the earth, is 10,28124 in.

When the difference between the true and apparent level, as of B F, has been calculated, it will be easy to calculate those which answer to a less distance; for we may consider the distances B F, b f, as almost equal to the lines A I, A i, which are to each other as the squares of the chords, or of the arcs AB, a b, because in this case the chords and the arcs may be taken one for the other.

Thus to find the difference f b of level, which answers to 5000 feet, make the following proportion; 6000 f. : 5000 f. :: 0,85677 : f b, which will be equal to 0,71399 f. or 8,56788 in.

The point F, which is in the same horizontal line with A, is said to be in the apparent level of A, and the point B is the true level of F; so that B F is the difference of the true level from the apparent.

Problem 44, *fig.* 49, *plate* 28. The above notions being supposed to know the difference of

* As the arc A B=6000 feet is but very small, it may be considered equal to the tangent A F, and in this respect, A F is a mean proportional between the whole diameter, or twice the radius B C, and the exterior part B F.

level between points B and A, which are not in the same horizontal line; then at A make use of an instrument proper to take the angle B C D, and having measured the distance C D, or C I, by a chain which must be kept horizontal in different parts of it on the ground A L V B, you may in the triangle C D B, considered as rectangular in D, calculate B D, to which add the height C A of the instrument, and calculate the difference of level D I, as we have shewn above.

But as this method requires great accuracy in measuring the angle B C D, and an instrument very exact, it is often better to get at the same end with a little more trouble, which is shewn by the following method.

PROBLEM 45. *Use of the spirit or water level, fig.* 50, *plate* 28.

Place the level at E, at equal distances from B and G, fix one station staff at B, the other at G; your instrument being adjusted, look at B, and let the vane be moved till it coincides with the line of sight; then look at the staff G, and let the vane be moved till it coincides with the line of sight H, and the difference in height shewn by the vanes on the two staves, will be the difference in the level between the two points B and G. Thus, suppose the vane at G was at 4 f. 8 in. and at B 3 f. 9 in. subtract one from the other, and the remainder 11 inches, will be the difference in the level between the two points B and G; you may proceed in the same manner with the other points; but more need not be said on this head, as I have already treated this subject very fully in the foregoing part of this work.

AN

ESSAY ON PERSPECTIVE;

AND A DESCRIPTION OF

SOME INSTRUMENTS,

FOR FACILITATING THE PRACTICE OF THAT USEFUL ART.

DEFINITIONS.

Definition 1. Perspective is the art of delineating the representations of bodies upon a plane, and has two distinct branches, linear and aerial.

Definition 2. Linear perspective shews the method of drawing the visible boundary lines of objects upon the plane of the picture, exactly where those lines would appear if the picture were transparent; this drawing is called the outline of those objects it represents.

Definition 3. Aerial perspective gives rules to fill up this drawing with colours, lights, and shades, such as the objects themselves appear to have, when viewed at that point where the eye of the spectator is placed.

To illustrate these definitions, suppose the picture to be a plate of glass inclosed in a frame PLN, *fig.* 1, *plate* 30, through which let the eye of the spectator, placed at E, view the object QRST; from the given point E, let the visual lines EQ, ER, ES, &c. be drawn, cutting the glass, or picture, in q, r, s, &c. these points of intersection will be the perspective representations of the original Q R S T, &c. and if they are joined, q r s t will be the true perspective delineation of the original figure. And lastly, if this out-line be so coloured in every part, as to deceive the eye of a spectator, viewing the same at E, in such a manner, that he cannot tell whether he views the real object itself, or its representation, it may be truly called the picture of the object it is designed for.

Definition 4. When the eye, or projecting point, is supposed at an indefinitely great distance, compared with the distance of the picture and the object to be represented, the projecting lines being then supposed parallel, the delineation is called a parallel one, and by the corps of engineers, *military perspective.*

Definition 5. If this system of parallel rays be perpendicular to the horizon and to the picture, the projection is called a plane.

Definition 6. If the parallel rays be horizontal, and the picture upright, it is called an elevation.

Definition 7. A right line E e, *fig.* 2, *plate* 30, from the eye, E, cutting the plane of the picture at right angles, and terminating therein, at e, is called the distance of the picture.

Definition 8. The point e, where this central ray cuts the picture, is called the center of the picture.

Definition 9. The point X, *fig.* 2, *plate* 30, where any original line cuts the picture, is called the intersection of that line.

Definition 10. The seat F, *fig.* 2, *plate* 30, of any point E upon a plane, is where a perpendicular from that point cuts the plane; thus, if a perpendicular E F, be drawn from any elevated point E, to the ground plane F G N at F, this point F is called the seat of the point E upon the ground plane.

Definition 11. Apparent magnitude is measured by the degree of opening of two radials passing through the extremes of bodies whose apparent magnitudes are compared; thus, the apparent magnitude of Q R, *fig.* 3, *plate* 30, to an eye at E, is measured by the optic angle Q E R. Hence it is evident, that all objects viewed under the same angle, have the same apparent magnitude.

Definition 12. The intersection G N, *fig.* 2, *plate* 30, of the picture with the ground plane, is called the ground line.

Definition 13. A plane passing through the eye, and every where parallel to the ground (or ground plane, as it is commonly called, because it is supposed every where flat and level) is called the horizontal plane, as E h n, *fig.* 2, *plate* 30.

Definition 14. The intersection of the horizontal plane with the picture is called the *horizontal line*; thus h e n, *fig.* 2, *plate* 30, is called the horizontal line, being the intersection of the horizontal plane E h e n, parallel to the ground plane F G N.

Definition 15. The center of any line is where a perpendicular from the eye cuts it.

Definition 16. A plane *i E F g h*, *fig.* 2, *no.* 2, *plate* 30, passing through the eye E, perpendicular to the ground, is called the *vertical plane*; and that part of the ground plan g f N which lies to the left hand, is called amplitudes to the left, and all that lying on the other side, amplitudes to the right, their measures being taken on the base line G F N, or its parallels, as depths are by f g, or its parallels.

Proposition 1. Parallel and equal strait lines appear less, as they are farther removed from the eye.

Let O P, Q R, *fig.* 3, *plate* 30, be two equal and parallel strait lines, viewed at the point E; Q R being the farthest off, will appear the least.

For, draw E Q and E R, cutting O P in r; then Q R and O r have the same apparent magnitude, *(definition* 11,*)* but O r is only a part of O P.

Corollary. Similar figures parallelly situated, appear less, the farther they are removed from the eye.

Proposition 2. All original parallel strait lines, P R, *fig.* 3, *plate* 30, which cut the picture at P, appear to converge to the same point O therein; viz. that point which is the intersection with the picture, and a line passing through the eye parallel to the original parallel lines.

For, since P R is parallel to E O Q, let R Q be parallel to O P, and therefore equal to it. By the last proposition, the farther R Q is taken from O P, the nearer it (the representation of the point R) approaches to the fixed point O, and the case is the same with any other line parallel to P R; and, therefore, if all the parallels be indefinitely produced, that is, at least till the strait lines measuring their distance become invisible to the eye, they will all appear to vanish together in this point, which, in consequence thereof, is called the *vanishing point* of all those parallels.

Corollary 1. Hence all original parallel planes, P R S P, intersecting the picture, as in P p, seem to converge to a right line therein, viz. that line O o, which is the intersection of the picture, with a plane Q O E o q, parallel to them all passing through the eye.

For, since p S, *fig.* 3, *plate* 30, is parallel to o q, therefore p S will appear to converge to its vanishing point o, by the foregoing proposition; and since both the points R, S, equally appear to tend to the respective points O o, therefore the line R S also appears to approach to the line O o; and this is the case with all planes parallel to P R S p, and therefore if they are produced till they become invisible, they will appear to meet in Oo, whence this line is called the vanishing of all those parallel planes.

Corollary 2. The representations of original lines pass through their intersections and vanishing points; and of original planes, through their intersections and vanishing lines.

Corollary 3. Lines parallel to the picture have parallel representations.

For, in this case the line E O, which should produce the vanishing point, never cuts the picture.

Corollary 4. Hence the representations of plane figures parallel to the picture, are similar to their originals.

Let q r s t u, *fig.* 4, *plate* 30, be the representation of the original figure Q R S T U, to an eye at E; then all the lines q r, r s, s t, &c. being respectively parallel to their originals Q R, R S, S T, &c. as well as the diagonals, t q, T Q, &c. therefore the inscribed triangle q t u, and Q T U, are similar to each other, and so of all the other triangles; and, therefore, the whole figure q r s t to Q R S T.

Corollary 5. The length of any line in the representation is to that of its respective original, as the distance of the picture to that of the original plane; for all the triangles E Q U, E q u, E Q R, E q r, &c. are similar; therefore, as E q is to q u, so is E Q to Q U; that is, as q u is to Q U, so is the distance of the picture to the distance from the plane Q U.

PROBLEM 1. *Having the center and distance of the picture given, to find the representation of any given point thereon, fig.* 5, *plate* 30.

From the eye E, to the center of the picture e, draw E e, and parallel to it Q d from the given point Q, cutting the picture in d; join e d, and draw E Q, intersecting e d in q, the perspective place of the original point Q.

For, e d is the representation of the original line d Q, indefinitely produced from its intersection q, till it appears to vanish in e; therefore q must be somewhere in this line, *pr.* 2, *cor.* 2; it must also be somewhere in E Q, and therefore in the point q, where they intersect.

PROBLEM 2. *The center and distance of the picture being given, to find the representation of a given line* Q R, *fig.* 6, *plate* 30.

Produce R Q, to intersect the picture in X, draw E V parallel to R Q X, cutting the picture in V, the vanishing point of the line R X; draw E R and E Q, cutting V X the indefinite representation of R X, in r and q, and r q is the representation of R Q.

This needs no demonstration.

PROBLEM 3. *Having the center and distance of the picture, to find the representation of an original plane, whose position with respect to the picture is given.*

Draw any two lines, except parallel ones, upon the plane, and find their vanishing points by the last problem; through these points a line being drawn will be the vanishing line required.

This is evident from *prop.* 2, *cor.* 1.

PROBLEM 4. *Having the center and distance of the picture given, to find the vanishing point of lines perpendicular to a plane whose representation is given, fig.* 7, *plate* 30.

Let P L be the vanishing line of the given representation, X its center, c that of the picture, and c E its distance; join E X, and perpendicular to it draw E f, cutting X c in f.

For, E P L is the original plane, producing the vanishing line P X L, and E f being a visual ray, parallel to all the original lines that are perpendicular to the plane E P L, or its parallels; f is therefore the vanishing point of all of them.

Corollary 1. When the vanishing line passes through the center of the picture; that is, when the parallel planes are perpendicular to that of the picture, the points X and e coinciding with E f become perpendicular to E e, or parallel to the picture, and therefore the lines will have parallel representations.

Corollary 2. If the original lines were desired to make any other angle with the original plane than a right one, it is only making x E f equal to it.

PROBLEM 5. *The center* e, *and distance* E e, *of the picture being given, and the vanishing point* f, *of a line parallel to* E f, *to find the vanishing line of planes perpendicular to that line whose parallel* E f *produces the vanishing point* f, *fig*. 7, *plate* 30.

Join f e, and make E X perpendicular to, cutting f e, produced in X, draw P L perpendicular to e x, and P L will be the vanishing line required.

The planes may form any angle instead of a right one, if that f E X be made equal to the same.

PROBLEM 6. *The center and distance of the picture being given, and the vanishing point of a line, to find the vanishing line of planes, perpendicular to the line whose vanishing point is given, fig*. 7, *pl*. 30.

From f, the given vanishing point, through e, the center of the picture, draw f e X, and through the eye E draw E f, perpendicular to which draw E X, cutting f e in X; make P X L perpendicular to X e f, and P L will be the vanishing line.

PROBLEM 7. *Having given the center and distance of the picture, the inclination of two planes, the* *vanishing line of one of them, and the vanishing point of their common intersection, to find the vanishing line of the other plane.*

Case 1. Let the inclination of the planes be a right angle, and let P L be the vanishing line, P the vanishing point of their common intersection. By Problem 5 find f, the vanishing point of lines perpendicular to the plane, whose vanishing line is P L; join P f, which is the vanishing line required.

For, since this last plane is perpendicular to the former, f will be the vanishing point of one line in it, *(prop.* 2*)* and P being the vanishing point of another, therefore P f is the vanishing line.

Case 2. When the inclination n e m, *fig.* 8, *plate* 30, is greater or less than a right angle. Let N x be the vanishing line of one plane, x the vanishing point of its intersection with the other plane; from the eye E draw E x, and perpendicular thereto, the plane N E M intersecting x N in N, and the picture in N f M, make the angle N E M equal to n e m the given inclination, join x M, which will be the required vanishing line.

For, the planes being parallel to the original planes by construction, M x, N x are their vanishing lines, *cor.* 2, *pr.* 2.

Problem 8. *To find the projection of any solid figure.*

Find the representation of any one of its faces by Problem 3, and of the others by the last; if any side be convex or concave, a number of points may be found therein by Problem 1, and curves drawn evenly through them will represent the curved superficies required; or squares, or other regular figures may be inscribed or circumscribed about the original figures, and these plain figures being projected by the foregoing methods, together with a few points therein, by which means the curves may be similarly drawn about or within these projected squares, &c. by this means also may tangents be drawn to the representations of all kinds of curve lines, in all kinds of situations.

GENERAL REMARK.

So far I have endeavoured to render the principles of perspective obvious by a mere inspection of the figures. For the young artist, whose mind seldom conforms to mathematical reasoning, may, in all the foregoing problems, suppose the plane of the picture placed upright upon that of the paper, which paper he may consider as the ground plane, the operator's eye E, being always in its proper situation with respect to the picture, as well as to the planes of the original objects. The data, or things required to be known before objects can be put into perspective, are their plans and elevations, see Defin. 5 and 6, which must be actually laid down by a convenient scale adapted to the size you mean your picture should be, which may be very easily accomplished; for you must remember, that the two extreme visual rays, that is, those which pass from the eye to the two opposite borders of the picture, must not make an angle less than *two-thirds*, or greater than *three-fourths* of a right one. With respect to the distance of the picture, it must be remembered, that objects cannot be seen distinctly nearer to a common eye than six inches, and therefore in the smallest miniature pieces the distance must exceed that quantity; the height of the eye should be about half the distance of the picture, and about the third part of the picture's whole height. For example, if the whole height of the picture be three, the distance from it should be two, and the height of the eye one. If the plan, when laid down by its proper scale, be bounded by a quadrangle G N c d, *fig.* 9, *no.* 2, *plate* 30, then the ground line of the picture is supposed to be placed upon the shortest side of it, and upright to its plane; therefore this side of the figure should be made exactly equal to the breadth of the picture, and the other two adjacent sides should, if produced, meet at the distance of the picture, as at F the foot of the observer, as is represented in *fig.* 9, *no.* 3. The problems already given are sufficient for all cases that can happen in putting objects in whatsoever position into perspective; and though they are perhaps solved in such a manner, as to give the clearest ideas of the genuine practice of perspective, yet others may possibly prefer some of the following methods, which are however easily deduced from the preceding system.

Example 1. Having given the center e of the picture P L N f, *fig.* 2, *no.* 2, *plate* 30, its distance e E 12 inches, height of E F six inches, to find the representation of a point T, whose depth in the plan is eight inches, and amplitude to the left five inches.

Method 1. Form f in f N, take f n = five inches, and having drawn T perpendicular to f N, make it equal to eight inches, then will T be placed in its proper situation, and n will be its seat on the picture; join T F, (F as usual being the foot of the observer's eye upon the ground plane, that is, in the present example, six inches perpendicularly below the eye E) cutting f N in t, draw t o perpendicular to f N, and join T E, intersecting t o in o, the representation of the point T required.

Otherwise, if the point T be situated on the ground plane, draw F T, cutting f N in t, and t s perpendicular to f N, cutting the visual E T in o, the point required.

If the point T has any elevation perpendicular over the same point, as at S, make T S equal to the height it should have, and draw E S, cutting the perpendicular o t s in s, the perspective point of S required.

Remark. This method is very convenient in some cases, as when there are many windows, &c. perpendicularly over one another, &c.

Method 2. In the horizontal line e E, *fig.* 2, *no.* 3, *plate* 30, from the center of the picture e, take e E equal to the distance of the picture; from the given point Q draw Q d perpendicular to the ground line G N, make d Q^2 equal to d Q, and join Q^2 E and e d, intersecting each other in q, the representation of the point Q.

Method 3. Perpendicularly over the center e, *fig.* 2, *no.* 4, *plate* 30, of the picture, take E e, equal to the distance of the picture, draw Q d perpendicular to G N, join E Q and e d, intersecting each other in q, the point as before.

Remark. Both these methods are in fact the same as that in Problem 1, as may be seen by comparing the figures with each other, being marked with the same letters for that purpose. It may be farther remarked, that both may be alternately used in the same piece, remembering to use that which you judge will give the bluntest intersection, that is, whichever makes the angle e q E the greatest.

And now supposing a clear knowledge both of the theory and practice to be obtained, it may not be amiss to shew how naturally a practice more elegant and simple may be deduced, viz. by supposing all the planes to coincide with the paper, or plane of the picture.

Thus, *fig.* 5, *plate* 30, if the triangles q e E, q d Q revolve round d q e till they fall into the plane P L N, no change takes place in any of the comparative distances E e, E Q, &c. and the point q preserves its situation on the picture, as in *fig.* 2, *no.* 4; and again, if in *fig.* 2, *no.* 4, e E be moved round on the point e, till it falls into the horizontal line at n, and d Q be moved similarly round, till it becomes, as in *fig.* 3, *no.* 3, parallel to e E, the point q still remains unaltered.

Method 4. Let m N f s r, *fig.* 2, *no.* 5, *plate* 30, be the ground line, f e E perpendicular to it, passing through the center of the picture e, and let f E^2 equal to f N, be the distance of the picture, N P being perpendicular to N F, and the representation of the point Q, whose distance from N r is r Q.

Draw E^2 Q, cutting N r in s, taking N m equal to Q r, and draw e m, cutting N P in o, draw o q parallel to N r, and s q to N o, intersecting each other in q, the point required.

Remark. The lines E^2 Q e m need not be drawn, but a dot made at s, where the ruler crosses n r and e m; a T square * may also be applied to the line N s r, and a dot made on its fiducial edge at s; then if the square be slid up till the fiducial edge crosses the mark at o, the point s will be transferred to q, the representation of the given point obtained without drawing any lines over the picture. (This method was first discovered by Mr. *Beck*, an ingenious artist, well known for many useful contrivances.) It is also more accurate

* See *fig.* 20, *plate* 31. This is a very useful article in drawing; a ruler *a*, is fixed as a square to *b*; there is also a moveable piece *c*. This ruler applied close to the side of a *true* drawing board, will admit of parallel lines being drawn, as well as oblique ones, with more ease and expedition than by the common parallel rulers. EDIT.

than the last method in general, as the intersections are more obtuse.

But nevertheless, when the two points in representation are projected very near together, it is the best way to work by the intersections and vanishing points of the original lines; for then having the full extent of the representation, its true directions may be ascertained very correctly. The vanishing points by this method are thus found; suppose for instance of the line I Q intersecting the ground line in I, in f e E^2 take e E equal to E^2 f, that is, equal to the distance of the picture; draw E V parallel to I Q, intersecting the horizontal line h o in V the vanishing point of I Q and all its parallels.

Example 2. To put any plane figure, as Q R S T, or M N O P, into perspective, *plate* 30, *fig*. 9.

This is, in fact, only a repetition of the last; for by finding the projections of the several points, as before, nothing remains but to join them properly, and the thing is done; thus the points m, n, o, p, projected from M, N, O, P, being joined, give m p o n, the representation of the quadrangle M P O N, &c. But if any line, as p o, happens to be very short, so that a small error in point o would considerably alter the direction of it, find the intersection and vanishing point of its original; or, if the vanishing point falls at too great a distance, as it very frequently does in practice, find the intersection E^2, and vanishing point Z of any other line passing through the point O, as the diagonal O M, and the truth of the projection may be depended upon. All the three last methods may be made use of in projecting the same figure; thus the point p, by the second method, that is, make e E in the horizontal line equal to E^2 k, the distance of the picture, draw P Y perpendicular to S X, and take Y k equal to Y P, join

E k and e Y, intersecting in p, which is a better intersection than the third method would give, viz. by drawing E P; but yet inferior to that in the fourth method.

Example 3. To put any solid body into perspective. Find the seats of all the points upon the ground plane, and project them as before; let G be the seat of one of the points, g its projection, produce e g to the ground line a E ^{2}, and make E ^{2} H perpendicular, and equal to the height of the given point, from its seat join H e, and it will cut the perpendicular g g ^{2}, in g ^{2}, the representation of the required point.

Or, by the *fourth method*, G ^{2} S being equal to to the distance of G from S X.

Make G ^{2} G ^{3} perpendicular and equal to the given height, draw G ^{2} e, cutting S U in e ^{2}, then e ^{2} g ^{2} parallel to S X, will cut the perpendicular g g ^{2} in the point g ^{2} required; or the point g may be transferred to g ^{2} by the T square, without drawing g g ^{2} or e ^{2} g ^{2}.

Proceed in this manner, till you have obtained all the requisite points in the figure, marking or numbering them as you proceed, or else correct them by strait or curved lines according to your original, by which means you may instantly see the connection of your work at all times without confusion.

Remarks. 1. Sometimes when there is a great number of small parts in a body to be perfectly made out, it may not be amiss to draw squares over the ground plane of the object, and find the seats of its several elevated points; then by turning these squares into perspective, the positions of the several points will likewise be found by inspection, and the horizontal row of squares will serve as a scale for the altitudes of bodies, whose seats lie in that row, or both plans and elevations may be used, as in *fig.* 10, *plate* 30.

2. Perspective may also be practised without having any recourse to ground plans; for, by taking the horizontal angles, the amplitudes of objects are to be ascertained, and by vertical ones their heights and depths in the picture; the angles thus taken may be entered in a table of this form,

Vert. ∠	Horizontal.		Vert. ∠
Up.	Left.	Right.	Down.

In protracting which angles you must make the distance of the picture radius and lay down the angles by a line of tangents adapted to that radius, which is therefore best done by means of the sector; or more expeditiously by the fourth method, page 478; for, if a protractor be fixed at E^2, *fig.* 9, *plate* 30, horizontal angles will cut the ground line S X in the same places, as if the lines forming them passed over the original points in the plan; and if the protractor be fixed at e, the same may be said of S U with respect to elevations..

Solid bodies may also be put into perspective, by drawing lines in particular directions, as from the center of a circle, or of concentric ones, and finding the representations of the same, raise perpendiculars of the proper heights, always supposing the bodies to be transparent, *fig.* 10, *plate* 30.

OF SHADOWS.

Luminous bodies, as the sun, moon, lamps, &c. are generally considered as points; but the artist takes the advantage of their not being perfectly so, by softening the extremities of his shadows, which are so softened in nature, and for this reason, which may be thus explained.

Let R S, *fig.* 11, *no.* 2, *plate* 31, be the radius of a luminous body, whose seat is rs, and center is S; from the opaque body O, draw x r, touching the extremities of both the bodies on both sides, by which means a penumbra, or semi-shade, q x n, will be formed on each side of the main shadow, which becomes extremely tender towards the outer extremity, and from thence gradually strengthens till it blends with the uniform shadow, which will, if the diameter of the luminous be greater than that of the opaque one, measured in the direction of their centers, converge to a point, as *n n*, *fig.* 11, *plate* 31.

N. B. The opaque body which casts the shadow is called the shading body; and those that are immersed in the shadow, are called shadowed bodies.

Problem 9. *To find the shadow of any object upon a plane.*

Through the luminous body draw planes touching all the illumined planes of the object, and the intersections of these planes with the given plane, will give the boundary of the shadow required.

Example 1. Let the luminous point be the sun, the plane of projection ARSB, *fig.* 12, *plate* 31, and the object the parallelogram ABCD.

The sun's rays, on account of his distance, may be supposed parallel, therefore the planes ORA, O S B, are parallel; and, therefore, since C D is parallel to B A, R S is parallel to it also, and the shadow a parallelogram, whose length is to the height of the object, supposed upright, as radius to the tangent of the sun's altitude; and its breadth as radius to the sine of the inclination of its rays, with B A the base of the parallelogram.

Therefore find the seat of one of the rays, as O A R, and make the angle A D R equal to the complement of the sun's altitude, and make the parallelogram B R, and the thing is done.

Example 2. Let the luminous body be a lamp placed at O, *fig*. 13, *plate* 31; the object, a parallelogram A B C D, standing upon the plane A R S D.

Having drawn O G perpendicular to the plane A R S D, and the rays O B R, O C S; from G, the seat of the lamp on the plane AS, draw G A R, G D S, intersecting the rays O B, O C, in R and S, and A R S D will be the shade required.

Example 3. Where the shadow from the sun, &c. passes over different objects.

Continue the sides C A D B to A′, A″, B′, B″, &c. *fig*, 14, *plate* 31, and where A G, B H, cut the body G H I K, draw K G A″, I H B, making the angles A′G A″, B H B′, equal to the inclination of the plane G I to G B, proceed in the same manner with every new plane M N; or if the object is curvilinear, tangents will always pass through the lines C A′ and D B respectively, except when they are perpendicular.

Example 4. Let a lamp O, *fig*. 15, *plate* 31, throw a shadow on the body R S U T, and let Q T U be the central line of the shadow from the parallelogram, AB, upon the ground; draw O B, G P, parallel to Q T, and from T to O P draw t P, touching the plane U s of the body; from P draw P S u, P w U, cutting the extremities of the shadow in U and u, and w U u S will be the shadow of AB upon the face U S; proceed in like manner with all the other illumined faces.

N. B. If t P never meets O P, it denotes the face of the body to be parallel to AB, and the shadow on that face to be a parallelogram.

The shadows, being thus ascertained, may be put into perspective by the foregoing rules.

PROBLEM 10. *To find the reflections of objects upon polished surfaces.*

Let fall a perpendicular upon the reflecting plane, to which draw a radial from the eye, as much below the horizontal line, as the real object appears to be above it.

Example 1. Let AB, *fig.* 18, *plate* 31, be any object placed on the water; from B draw B b, perpendicular to the surface Ab, which continue till the angles B E b and C E b are equal; that is, (E b being a horizontal line) till b c is equal to b B, and b C will be the reflection of AB.

Example 2. When objects are upright, the lines may be produced below the horizontal line, as much as the real ones are above it.

OF PARALLEL, OR MILITARY PERSPECTIVE.

In this kind of projections, the eye is supposed to be placed at an indefinite distance from the object in the diagonal, and looking down upon it in an angle of 45°, so that the top, one side, and one end, are seen under the same angle, and therefore appear in their true proportions with respect to each other; and therefore heights, lengths, and breadths must be laid down by the same scale, and all parallel lines made parallel, see *fig. A*, *plate* 30.

OF AERIAL PERSPECTIVE.

Before we can give rules for regulating the force of lights and shades in a picture, we must consider what degree of it the bodies themselves are endued with, according to their several positions with respect to the illuminating body.

Proposition 3. The intensity of light upon any plane is reciprocally as the square of the distance of that plane from the illuminating body.

Let A B C D, *fig.* 16, *plate* 31, be the shadow of the square a b c d upon a plane parallel to it, which projection will therefore be a square, and in proportion to a b c d as the square upon O A to that upon O a; therefore since the real quantity of light is the same as would be received upon A B C D, the intensity of it is reciprocally as the square upon A B to the square upon a b, or as square O A to square O a.

For example, if parallel planes are at the distance of one, two, and three feet from a luminous point, the intensity of light upon them would be one, one-fourth, one-ninth, &c.

Corollary. All parallel planes are equally illuminated by the sun at the same moment.

For, his rays being parallel, the squares a b c d, and A B C D are equal.

Proposition 4. If the sun's beams fall perpendicular upon one face AB, *fig.* 17, *plate* 31, of an object, and inclined upon another AC, the intensity of light on the faces, is as radius to the sine of the angle of incidence.

Produce AB to b, the quantity of light Ab receives is the same as would be received on AC if bA were away; therefore the brightness is as AC to Ab, that is, the brightness of A b, or AB is to that of AC as AC to Ab, or as radius to sine of the angle of incidence ACb.

Proposition 5. A plane uniformly enlightened, does not appear so to an eye in different situations.

For, as all bodies are porous, the little exuberances will have their light and dark sides, and the eye will view more of the former, as it is more nearly situated in a line with the rays of light, and more of the latter, the more it faces them.

The subject of this proposition is one great cause of the graduation of light upon the faces of buildings and other planes, and not altogether owing to a greater teint of air, as the artists call it, on that part which is the farthest off.

Remark. It is very necessary to observe, that transparent and polished bodies are not included among those mentioned in this proposition, for they seem most illuminated in that part which makes the angle of reflection equal to that of incidence; but if bodies of this kind are not flat, as water when just broken by small rippling waves, then the light is reflected from some part of almost every wave, and so is extended to a great space, but is strongest perpendicular under the luminary, and gradually decreases on each side.

The case is the same in the sky, which is brightest near the sun's apparent place, and graduates into a deeper azure as it retires farther off, and for a reason nearly the same; for the pellucid particles floating above us, having large interstices between them, act in the same manner as the rippling waves in disturbed water; and, therefore, the more obliquely the light strikes upon them, the more united their force will be to an eye situated in the proper angle of reflection.

Proposition 6. All shades and shadowing objects would be equally dark and indistinguishable, if they received no secondary or reflected light.

For, light is not visible of itself, but by striking upon other bodies renders them so, and these enlightened bodies serve as lights to bodies otherwise in shade, and such lights are called secondary or reflected ones, the chief of which is the sky.

Proposition 7. Every body participates of the colour of the light by which it is illumined; for, blue rays thrown upon a yellow body will produce a green; red rays, purple; and purple rays, that is, blue and red, black.

Corollary. Hence shadows are often observed green in the morning or evening, for the sky is always very green at those times compared with other times of the day, owing to the warm rays being more copiously reflected downwards by the sun's beams striking more obliquely on the atmosphere, which partly acts as a prism, and the shadows become more blue, as the sky becomes so; but clouds are of all colours, and as they are denser than the blue part of the sky, they throw stronger reflections, and cause many accidental teints in the shadows of bodies; therefore, as the shadow of every body is partially enlightened by all the bodies surrounding it, it must partake of the colours of all of them; and this is the grand source of harmony in painting, of which system the colour of the original light serves as a key, and is to be attended as nicely to in painting, as in music.

Proposition 8. Bodies partake more of the colour of the sky, as they are farther off.

For, the sky being only a body of air every where surrounding us, its natural colour supposed to be blue, the farther off any body is, the more of this blue air is intercepted between us and the body, and therefore the bluer it is, and that in proportion to its distance.

OF INSTRUMENTS FOR DRAWING IN PERSPECTIVE.

Various have been the methods used to facilitate the practice of perspective, as well for those who understand, as those who are ignorant of that art; and, though some have supposed that the warmth of imagination and luxuriance of fancy, which impels the mind to the cultivation of the fine arts, is not to be confined to mechanical modes, yet upon enquiry they will find, that the most able and accomplished artists are often obliged to have recourse to some rules, and to use some mechanical contrivances to guide and correct their pencil. So great is the difficulty, and so tedious the operation of putting objects in true perspective, that they trust mostly to their eye and habit for success; how well they succeed, we may decide from the portraits drawn by the best artists, and the different judgments formed concerning them. Mr. *Eckhardt* has well observed, that there is no artist who will be hardy enough to say, that he can delineate by the eye the same object twice with exactness, and preserve a just and similar proportion of parts in each. In one of the figures, we shall find some of the parts larger than in the other—both cannot be right: yet, supposing them perfectly the same, neither may be conformable to nature. Add to this, many situations of an object occur, which no eye, however habituated, can represent with accuracy.

On this account, I have a long time endeavoured to complete an instrument that should give the out-line of an object with accuracy. These Essays have now swelled so far beyond my intentions, that I must be as concise as possible. I must, however, acknowledge the valuable hints communicated by Mr. *Heywood*, and other ingenious men, The methods most generally in use are, 1. The camera obscura. 2. The glass medium or plane. 3. A frame of squares. The inconveniences and inaccuracies which attend these expedients, induced Sir *Christopher Wren*, Mr. *Ferguson*, Mr. *Hirst*, *My Father*, Mr. *Watt*, Mr. *Eckhardt*, *Pere Toussaint*, and others, to have recourse to different contrivances to remedy their defects; of which those by the Rev. Mr. *Hirst*, *My Father*, Mr. *Watt*, and Mr. *Eckhardt*, are undoubtedly the best; Mr. *Eckhardt*'s and Mr. *Hirst*'s vary but little from each other.

Those represented at *fig.* 1 and 2, *plate* 32, appear to me far superior to any that have been hitherto contrived; the object is delineated on an horizontal plane, the pencil B, may be moved in any direction, whether curved or strait, with the utmost freedom. By either, the artist may be sure of obtaining the measure of every part of the object with exactness; and this is performed without any loss of time. The instrument may be moved from any place, and brought back to the same with great exactness; and the outline may be formed either of a number of points, or one continued line, at the pleasure of the draftsman.

That represented at *fig.* 2, is the simplest of the two instruments: *fig.* 1, though more complex, merits, for its contrivances and motions, the attention of the mechanic, as well as the draftsman. They both move with facility in every direction, and the whole operation consists in looking through the sight C, which may be placed in any convenient situation, and moving the pencil B, so that the apex A, of the triangle may go over the object, whose outline will be delineated at the same time by the pencil B.

To lessen the expense, and render the instrument more portable, I have constructed an instrument somewhat similar to that represented at *fig.* 2, *plate* 32, but which moves only in a vertical plane, the board on which the drawing is made being in the same plane with the triangle.

To these may be added the parallel rule, and the perspective compasses. The distance of the rule from the eye, as it has no sights, must be regulated by a piece of thread tied to it, and held between the teeth.

Fig. 3, *plate* 32, is a pair of pocket brass perspective compasses, by Mr. *Jones*, that have been found very useful and convenient for taking readily the relative proportions of buildings landscapes, &c. and protracting them on the drawing. A, A, are the two legs, made of small tubes about six inches in length; B, B, are two sliding legs moving to different distances out of the tubes A, A; D is a sight piece with a small hole; E, E, are two steel points to take the sights by; F, F, are two more small steel points at the ends of the sliders to mark down the distance on the paper, after an observation; C is an arc with teeth fixed on one leg, by which, and the pinion G, the other leg is moved to the proper angle while observing. This arc is sometimes divided into degrees, and otherways subdivided by a set of figures, so as to give distances by inspection, &c. according to the pleasure of the purchaser. The sight C, turns down; the sliders B, B, go inwards; the arc takes off, and the whole packs into a small narrow case.

FINIS.

ADDENDA,

BY

THE EDITOR.

As this treatise is designed to comprehend a general collection of the most approved methods of surveying, I think the following method of surveying a large estate by Mr. *Emerson*, and the new method of surveying and keeping a field book by Mr. *Rodham*, as published in Dr. *Hutton's Mathematical Dictionary*, 2 vols. 4to. 1796, will be of real information to many surveyors; and, in my opinion, as deserving of practice as any other method I am acquainted with.

"TO SURVEY A LORDSHIP, OR LARGE ESTATE OF LAND.

"If the estate be very large, and contains a great number of fields, it cannot be done by surveying all the fields singly, and then putting them together; nor can it be done by taking all the angles and boundaries that inclose it. For in these cases, any small errors will be so multiplied as to render it very much distorted.

1. Walk over the lordship two or three times, in order to get a perfect idea of it, and till you can carry the map of it in your head. And to help

your memory, draw an eye draught of it on paper, to guide you, or at least of the principal parts of it.

2. Choose two or more eminent places in the estate for your stations, from whence you can see all the principal parts of it; and the fewer stations you have to command the whole, the more exact your work will be; and let these stations be as far distant from one another as possible; and they will be fitter for your purpose, if these stationary lines be in or near the boundaries of the ground, and especially if two lines or more proceed from one station.

3. Take what angles, between the stations, you think necessary, and measure the distances from station to station, always in a right line; these things must be done, till you get as many angles and lines as are sufficient for determining all your points of station. And in measuring any of these stationary distances, mark accurately where these lines meet with any hedges, ditches, roads, lanes, paths, rivulets, &c. and where any remarkable object is placed, by measuring its distance from the stationary line; and where a perpendicular from it cuts that line. And always mind, in any of these observations, that you be in a right line, which you will know by taking backsight and foresight, along your stationary line; which you must never omit. And thus as you go along any main stationary line, take offsets to the ends of all hedges, and to any pond, house, mill, bridge, &c. omitting nothing that is remarkable, and all these things must be noted down, for these are your *data*, by which the places of such objects are to be determined upon your plan. And be sure to set marks up at the intersections of all hedges with the stationary line, that you may know where to measure from, when

you come to survey these particular fields, which must immediately be done, as soon as you have measured that stationary line, whilst they are fresh in memory. By these means all your stationary lines are to be measured, and the situation of all places adjoining to them determined, which is the first grand point to be obtained. I would have you lay down your work upon paper every night, when you go home, that you may see how you go on.

4. As to the inner parts of the estate, they must be determined in like manner by new stationary lines. For, after the main stations are determined, and every thing adjoining to them; then the estate must be subdivided into two or three parts by new stationary lines; taking inner stations at proper places, where you can have the best view; and measure these stationary lines as you did the first, and all their intersections with hedges, and all offsets to such objects as appear; then you may proceed to survey the adjoining fields, by taking the angles that the sides make with the stationary line, at the intersections, and measuring the distances to each corner, from the intersections. For every stationary line will be a *basis* to all the future operations; the situation of all parts being entirely dependent thereon; and therefore they should be taken as long as possible; and are best to run along some of the hedges or boundaries of one or more fields, or to pass through some of their angles. All things being determined for these stations, you must take more inner stations, and continue to divide and subdivide; till at last you come to single fields; repeating the same work for the inner stations, as for the outer ones, till all be done. And close the work as oft as you can, and in as few lines as possible. And as it may require some

judgment to choose stations the most conveniently, so as to cause the least labour; let the stationary lines run as far as you can along some hedges, through as many corners of the fields, and other remarkable points, as you can. And take notice how one field lies by another; that you may not misplace them in the draught.

5. An estate may be so situated, that the whole cannot be surveyed together; bscause one part of the estate cannot be seen from another. In this case, you may divide it into three or four parts, and survey the parts separately, as if they were lands belonging to different persons; and at last join them together.

6. As it is necessary to protract or lay down your work as you proceed in it, you must have a scale of a due length to do it by. To get such a scale, you must measure the whole length of the estate in chains; then you must consider how many inches long the map is to be; and from these you will know how many chains you must have in an inch, and make your scale, or choose one already made accordingly.

7. The trees in every hedge row must be placed in their proper situation, which is soon done by the plain table; but may be done by the eye without an instrument; and being thus taken by guess, in a rough draught, they will be exact enough, being only to look at; except it be such as are at any remarkable places, as at the ends of hedges, at stiles, gates, &c. and these must be measured. But all this need not be done till the draught be finished. And observe in all the hedges, what side the gutter is on, and to whom the fences belong.

8. When you have long stations, you ought to have a good instrument to take angles with, which should be exact to a quarter of a degree at least;

and hardly any common surveying instrument will come nearer. And though the plain table is not at all a proper instrument to survey a whole lordship with, yet it may very properly be made use of to take the several small internal parts; and such as cannot be taken from the main stations; and is a very quick and ready instrument.*

Example. Walking over the lordship, I pitch upon the four stations A, B, C, D, *fig.* 1, *plate* 33, from which I can command the greatest part of it, there I set up marks. Then I measure along AB, which is a right line, and the boundary on one side of the land. In measuring along, I set down the distances measured, when I come at the corners of the fields *a*, *a*, *a*, *a*, where the hedges come in, and likewise where I cross the brook *b b*. Then having got to B, I set down the whole length of AB.

Next I measure from B to C, and in my way, I set down how far I have measured when I cross the hedges at *c*, *c*, *c*, *c*; and likewise where I cross the brook *b b* again. Thus I measure forward till I come at C, and then I set down the length of the stationary line B C.

After the same manner I measure along the stationary line CA, observing to set down the intersections with the hedges, as before; till I come at A, where I set down the length of CA. Then the three points, A, B, and C, are determined; and may be laid down in the plan; and all the foresaid points.

* The angles by this method require to be taken very correctly; and, as instruments are now constructed with an extraordinary degree of perfection, to a skillful observer the angles, however numerous, can be of no reasonable objection; for, the chain itself, and oftentimes the manner of using it, may create as many errors as might be found by taking a multitude of angles by the theodolite, &c.

Being come to A again, I go from A towards B, and in my way I survey every single field adjoining to the stationary line AB. To do which the shortest way, I take the angles at every intersection *a*, that the sides of each field make with the stationary line AB; and then I measure their lengths; by which every field is easily laid down. In the same manner I proceed from B to C, and measure every field adjoining to BC. And then I go to A, and measure every field in my way thither.

Next I go from A towards D, and set down, as before, all my crossing of the hedges; and the length AD, when I come at D. And in like manner I measure along DC, setting down all the crossings of the hedges as before, with whatever else is remarkable, as where a highway crosses at *d*.

Having finished all the main stations, we must begin to make inner stations. Therefore I take F and G for two stations, being in the lines AB and BC, the hedges from F to G running almost straight; then I measure from F towards G, and at *f*, I find a hedge going to the left, and going on to *g*, I find another hedge going to the right; and at *h*, I cross the burn. At *i*, there is an angle, to which I make an offset. Going on further, I come at a cross hedge *l*, going to the right; and then measure on to G, the end of the station. Now in going from F to G, we can take all the angles that the sides of the fields make with the stationary line FG, and then measure their lengths; by which these fields may be laid down on paper.

Then I take another inner station at I, and measuring from A to *o*, I come to the opposite corner of the field; then measuring on to *p*, I cross a hedge; then I proceed to my station I. Then I measure from I to F, and take an offset to *n*, where the hedge crosses the brook. Then I come to the corner of the last field at *m*; and then measure to the opposite corner at F, the other station. In your going from A to I, you may take the angles that the hedges make with your stationary line A I, and measure these hedges, and then they may be laid down. And the like in going from I to F.

All this being done, take a new station H, and measuring from B towards H, all the hedges lie almost in a right line. So going along we come at a cross hedge, and going further we come at a tree, in the hedge we measure along; going further we come at two other cross hedges; and a piece further we cross the brook; going on we come at a cross hedge; going on still we come to another cross hedge; all these hedges are to the left. Then going on still further, we have a windmill to the right; and afterwards a cross hedge to the left, and then we measure on to the station H. Then measuring from H towards C, we have a house on the left; and then go on to C. And the fields may be all surveyed as you go along B H and H C, and then laid down. And after this manner you must proceed through the whole, taking new stations, till all be done."

MR. JOHN RODHAM'S NEW METHOD OF SURVEYING, WITH THE PLAN OF THE FIELD-BOOK, *plate* 34.

"The field book is ruled into three columns. In the middle one are set down the distances on the chain line at which any mark, offset, or other observation is made; and in the right and left hand columns are entered the offsets and observations made on the right and left hand respectively of the chain line.

It is of great advantage, both for brevity and perspicuity, to begin at the bottom of the leaf and write upwards; denoting the crossing of fences, by lines drawn across the middle column, or only a part of such a line on the right and left opposite the figures, to avoid confusion; aud the corners of fields, and other remarkable turns in the fences where offsets are taken to, by lines joining in the manner the fences do, as will be best seen by comparing the book with the plan annexed to the field-book, as shewn in *plate* 34.

The marks called, *a*, *b*, *c*, &c. are best made in the fields, by making a small hole with a spade, and a chip or small bit of wood, with the particular letter upon it, may be put in, to prevent one mark being taken for another, on any return to it. But in general, the name of a mark is very easily had by referring in the book to the line it was made in. After the small alphabet is gone through, the capitals may be next, the print letters afterwards, aud so on, which answer the purpose of so many different letters; or the marks may be numbered.

The letter in the left hand corner at the beginning of every line, is the mark or place measured *from*; and, that at the right hand corner at the end, is the mark measured *to*: but when it is not convenient to go exactly from a mark, the place measured from, is described *such a distance* from *one mark* towards *another*; and where a mark is not measured to, the exact place is ascertained by saying, turn to the right or left hand, *such a distance* to *such a mark*, it being always understood that those distances are taken in the chain line.

The characters used, are ⌈ for *turn to the right hand*, ⌉ for *turn to the left hand*, and ∧ placed over an offset, to shew that it is not taken at right angles with the chain line, but in the line with some strait fence; being chiefly used when crossing their directions, and it is a better way of obtaining their true places than by offsets at right angles.

When a line is measured whose position is determined either by former work (as in the case of producing a given line, or measuring from one known place or mark to another) or by itself (as in the third side of a triangle) it is called a *fast line*, and a double line across the book is drawn at the conclusion of it; but if its position is not determined (as in the second side of a triangle) it is called a *loose line*, and a single line is drawn across the book. When a line becomes determined in position, and is afterwards continued, a double line half through the book is drawn.

When a loose line is measured, it becomes absolutely necessary to measure some line that will determine its position. Thus, the first line *a h*, being the base of a triangle, is always determined; but the position of the second side *h j*, does not become determined, till the third side *j b* is measured; then the triangle may be constructed, and the position of both is determined.

At the beginning of a line, to fix a loose line to the mark or place measured from, the sign of turning to the right or left hand must be added (as at *j* in the third line;) otherwise a stranger, when laying down the work, may as easily construct the triangle *h j b* on the wrong side of the line *a h*, as on the right one: but this error cannot be fallen into, if the sign above named be carefully observed.

In choosing a line to fix a loose one, care must be taken that it does not make a very acute or obtuse angle; as in the triangle *p* B *r*, by the angle at B being very obtuse, a small deviation from truth, even the breadth of a point at *p* or *r*, would make the error at B, when constructed, very considerable; but by constructing the triangle *p*B*q*, such a deviation is of no consequence.

Where the words, *leave off*, are written in the field-book, it is to signify that the taking of offsets is from thence discontinued; and of course something is wanting between that and the next offset."

Mr. Keith's improved parallel scale.

Mr. *Thomas Keith*, Teacher of the mathematics, has considerably improved the German parallel ruler, or that of Mr. *Marquois*, see pages 27 and 28. By making the hypothenuse, and the perpendicular line to it from the opposite angle, in the ratio of 4 to 1, and adding several scales, &c. its uses are considerably extended for drawing plans of fortifications, and other branches of the mathematics.

Fig. 2, 3, 4, and 5, *plate* 33, represent the two faces of the ruler, and the triangle of half the real dimensions.

The slider is a right-angled triangle, the perpendicular is divided into inches and tenths, and the base has three indices, and other divisions requisite to be used with the scale.

The ruler contains 16 different scales, which, by the help of the slider may be increased to 20, without guessing at halves and quarters. The figures on the one end shew the number of divisions to an inch, and the letters on the other end are necessary to exemplify its use. It likewise contains the names of the polygons, the angles at their centers, and a scale of chords by which all polygons may be readily constructed.

ITS USE AS A COMMON PARALLEL RULER.

To draw a line parallel to the slider. Lay the hypothenuse, or sloped edge of the triangle in the position you intend to have your line, place the scale against the base of the triangle, and draw a line along the slope edge, keep the scale fixed, and move the slider to the left or right hand, according as you want a parallel line above or below the other.

To draw a line parallel to the scale. Lay the scale in the position you intend to have your lines, and draw a line along the edge of it. Place the base of the triangle against this edge, the middle index standing at O on the scale, and make a mark against any one division on the perpendicular; turn the triangle the other side uppermost, and make a mark against the same division; join these marks, and this line will be parallel to the former. The same may be done by sliding the triangle, without turning it, if the lines are not required to be very long.

To draw a line parallel to the slider at a great distance. Draw a line along the slope edge of the triangle, and fix the scale as before, make a mark against 2 on the perpendicular; the index being at O, turn the triangle the other side uppermost, and make a mark against the same division, take away the triangle, and move the scale to these marks, apply the triangle again; proceed thus till you have got the proper distance, then draw a line along the slope edge, and it will be parallel to the former.

To draw a line parallel to the scale at a great distance. Draw a line along the edge of the scale, place the base of the triangle against that edge, the middle index being at O, and make a mark against 2 on the perpendicular; turn the triangle the other side uppermost, and make a mark against the same division; take away the triangle, and move the scale to these marks; proceed thus till you have got the proper distance, then draw a line along the scale, and it will be parallel as required.

ITS USE IN ERECTING PERPENDICULARS.

If a line is wanted perpendicular to the scale, apply the base of the slider to it, and draw a line from the scale along the perpendicular of the triangle; should a longer perpendicular be wanted, apply the perpendicular of the triangle to the scale, and draw a line along the base.

If a perpendicular is wanted to a line, which has been drawn along the hypothenuse of the triangle, keep the scale fixed, and apply the hypothenuse of the triangle to it, then draw a line along the perpendicular of the triangle.

ITS USE IN CONSTRUCTING POLYGONS

Having made choice of any one scale, take the side of your polygon from it. Take the degrees under the name of your polygon, and subtract them from 180; at each end of the above side of your polygon, make angles equal to half the remainder, and the distance from the intersection of the lines, which form the angles, to either end of the side of your polygon, will give the radius of its circumscribing circle.

Or, the three angles, and one side being given of a triangle, the radius, or remaining sides, may be found by trigonometry.

ITS USE IN DRAWING PARALLEL LINES AT ANY GIVEN DISTANCE, WITHOUT THE ASSISTANCE OF A PAIR OF COMPASSES.

If the side of your polygon was taken from the scale C. Move the slider from O to 1, 2, 3, &c. on the scale D, and you will draw a parallel line of the width of 1, 2, 3, &c. divisions on the scale C. This scale is 20 fathoms to an inch.

In a similar manner the scales G, L, and P, are to be used.

If the side of your polygon was taken from the scale C, *calling the divisions two each.* Move the slider from O, on the scale A, to 1, 2, 3, &c. on the same scale, and you will draw a parallel line of half the width of 1, 2, 3, &c. divisions on the scale C. This scale is 40 fathoms to an inch.

Similar instructions may be applied to the scales G, L, and P.

If the side of your polygon was taken from the scale C, *calling the divisions three each.* When the middle index stands at O, the divisions marked C on the slider will make straight lines with 5 and 6 on the scale D. By moving the slider to the right or left till the other divisions thereon make straight lines successively with 5, 6, &c. you will draw a parallel line of one-third of the width of 1, 2, 3, &c. divisions on the scale C. This scale is 60 fathoms to an inch.

Similar directions must be observed in using the scales G, L, and P.

If the side of your polygon was taken from the scale A. Move the slider from O, to 1, 2, 5, &c. on the scale B, and you will draw a parallel line of the width of 1, 2, 3, &c. divisons on the scale A. This scale is 10 fathoms to an inch.

Similar directions must be observed in using the scales E, I, and N.

If the side of your polygon was taken from the scale D. Move the slider from O, to 2, 4, 6, &c. on the scale B, and you will draw a parallel line of the width of 1, 2, 3, &c. divisions on the scale D. This scale is five fathoms to an inch.

In making use of H, M, and Q, similar directions must be observed.

Note. There is no absolute necessity for always moving the slider from O, and it may be used either side uppermost. Any one of the three indices may likewise be made use of, &c.

To render the scale still more perfect, Mr. *Keith* has made the following additions.

1. The divisions on the scale A, and the fourth scale from A, have been subdivided for the purpose of constructing sections, &c. Ten of the divisions on the scale A make one inch; if, therefore, you call these divisions six each, the small divisions on the same scale, nearest to the left hand, will be one each; if you call the large divisions on the scale A, five, four, or three each, then the sets of smaller divisions in a successive order from the left hand division above mentioned, towards the right, will be one each.

2. Five of the divisions on the fourth scale from the edge make an inch; if, therefore, you call these divisions 3, 5, 7, 9, or 11 each, then each of the sets of small divisions, from the left hand towards the right, will be subdivided into parts of one each. Thus you have a scale divided into 2½, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, and 60 equal parts to an inch, exclusive of the scales on the three other edges of the ruler.

3. The line P on the slider stands for polygons; R, radius; the figures in this line being the radii

of the several polygons under which they stand, the side being 180 toises in each. If the side of your polygon be different from 180 toises, say, as 180 toises are to the radius under the number of sides your polygon contains, so is the side of your polygon to its radius. *Ex.* What is the radius of an octagon, the exterior sides being 120 toises? as 180 : 235 · 18 :: 120 : 156 · 78 Ans. The remaining lines are to be read thus—The English foot is to the French foot as 107 is to 114, or as 1 is to 1.065, a toise six French feet, a fathom six English feet. The English foot is to the Rhynland foot as 9715 is to 10000; the Rhynland foot is to the French foot as 1033 is to 1068; the Rhynland rod is 12 Rhynland feet.

It may not be amiss to remark, that either the scale or the slider will erect a perpendicular instantaneously, for the long divisions across the scale make right angles with the edges.

OF THE GUNNER'S CALLIPERS, OR COMPASSES.

This article is generally included in the magazine case of instruments for the military officer, or engineer; is a very useful mathematical instrument in the artillery service, and, as its description was omitted by our late author, I have taken the opportunity of inserting some account of it here.

The principal uses of this instrument are to take the diameters of common shot, the bore or caliber of a piece of ordnance, estimate the weight of shot, quantity of powder, &c. for guns of given dimensions, and other particulars in practical gunnery.

Fig. 6 and 7, are the representations of the two faces of the callipers marked A, B, C, D. They consist of two thin flat brass rulers moving on a joint, curved internally to admit the convex figure of a ball, whose diameter is to be taken by the points at the end A; these points are of steel, to prevent much wear. The rulers are made from six to 12 inches in length from the center, according to the number of lines and tables to be engraved upon them. The usual length for pocket cases is six inches, the scales upon which I shall now describe.

On *Fig*. 6, ruler A, is contained, 1. A scale of inches, divided into tenths, and continued to 12 inches on the ruler B.

2. A table shewing the quantity of powder necessary for charging the chambers of brass mortars and howitzers.

3. On the ruler B, is a line marked *Inches*, beginning from the steel point, for giving the diameters of the calibers of guns in inches.

4. A line marked *Guns*, contiguous to the preceding, shewing the nominal pounders, or weight of shot for the respective bores of the guns in inches.

5. On the semicircular head of the rule is a semicircle divided into degrees, figured in contrary directions, to measure angles by, and give the elevation of cannon, &c.

6. Next to the preceding, is a circular scale marked *Shot Diameters*, which, with the chamfered edge marked *Index*, shews the convex diameter in inches of a shot, or other object placed between the points. The quadrant part of the joint prevents this being represented in the figure.

On the other face of the callipers, C and D, *fig*. 7, are engraved,

7. on the ruler C, a table marked *Brass Guns*, shewing the quantity of powder necessary for the proof and service charges of brass guns.

8. The line of lines, marked *Lin.*

9. Two circular scales on the head of the ruler, marked *Shot*, shewing the weights of iron shot, as taken by the points of the callipers.

On the ruler D, besides another corresponding line of lines,

10. A table shewing the quantity of powder necessary for proof and service charges of iron guns, from ½ to 42 pounders.

11. To these are sometimes added various figures of a circle, cube, &c. with numbers.

12. A table of the weights and specific gravities of a cubic foot of various metals, ivory, wood, waters, &c.

EXPLANATION OF THE LINES AND TABLES.

1. The line of inches, graduated contiguous to the exterior edges of the sides A and B, *fig.* 6, when opened to a straight line, makes a measure of 12 inches and tenths for the purpose of a common rule.

2. The table shewing the quantity of powder necessary for mortars and howitzers. This table is adapted for both sea and land. Thus, by inspection merely is shewn that a 13 inch brass mortar at sea requires 30 pounds of powder; by land only 10 pounds. Mortars and howitzers under 10 inches have only the land quantities inserted.

3. The line of inches for concave diameters. It commences from the steel point at 2 inches, and continues on to 10 inches, and is subdivided to halves and quarters. When used, the legs are placed across each other, and the steel points brought to a contact with the internal concave surface of the gun, at the diametrical, or greatest

possible distance; the callipers then being taken out, and inspected where in the scale one external edge of the rule is upon the other scale, that division will give in inches and quarters the caliber of the gun required.

4. The line shewing the weight of shot for the bores. Adjoining to the preceding line is the one marked *Guns*, proceeding from $1\frac{1}{2}$ to 42 pounds.

It is therefore evident, that at the same time the caliber in inches is given, the weight of the shot is also given by inspection on the scale, by the side of the ruler crossing both; and, from the weight of the shot, the caliber in inches. Thus $4\frac{1}{4}$ inches shews a 9-inch shot, and *vice versa.*

5. The semicircular graduation on the joint-head. The degrees are figured to 180 contrary-ways, so as one set to be the complement of 180 to the other; and they are used to lay down or measure angles, find the elevation of cannon, &c.

First. *To lay down any angle, suppose* 30 *degrees.* Open the legs till the chamfered edges cut 30, and two lines drawn against the outside edges will be at the inclination, or angle of 30 degrees, tending to a proper center. If a center point be desired, cross the legs till the other chamfered edge cuts 30; the outside edges will then be at the angle of 30 degrees, and the place of their intersection the angular point.

Secondly. *To measure an entering or internal angle.* So apply the outward edges of the rulers, that they may exactly coincide with the legs or sides of the given angle. The chamfered edge on the semicircle on the outward set of figures will point out the degrees contained in the given angle.

Thirdly. *To measure a salient, or external angle.* Place the legs of the callipers over each other, and make their outside edges coincide with the legs or sides of the angle to be measured; then will the chamfered edge cut the degree required on the inner semicircle, equal to the angle measured.

Fourthly. *To determine the elevation of a cannon or mortar.* A rod or stick must be placed into the bore of a cannon, so as to project beyond its mouth. To the outward end of this stick, a string with a plummet must be suspended; then with the legs of the callipers extended, so as to touch both the string and rod, the chamfered edge will cut the degrees equal to the complement of 90° of the true angle of the elevation of the cannon, figured on the outer semicircle. So that if the edge had cut 30 degrees, the true elevation would have been 60° the complement to 90.

6. The circular line of inches on the head next to the preceding, marked *Shot Diam.* This scale extends from 0 to 10 inches, and is subdivided into quarters. The chamfered edge marked *Inches*, is the index to the divisions.

To measure by this scale the convex diameter of shot, &c. Place the shot between the steel points of the callipers, so as to shew the greatest possible extent; the chamfered edge, or index, will then cut the division shewing the diameter of the shot in inches and quarters.

N. B. The diameter given by this scale gives the shot rather less than by the scale No. 4, on account of a customary allowance of what is called *Windage.*

7. The table marked *Brass Guns*, to shew the quantity of powder necessary for proof and service charges of brass guns. This table is arranged for the size of the piece, called either *Heavy*, *Middle*,

or *Light*, into their respective columns. *Ex.* Suppose it is required to know the quantity of powder for a proof and service charge of a heavy 42 pounder. Under 42 are 31 lb. 8 oz. for the proof charge, and 21 lb. for the service charge.

8. The line of lines, marked *Lin.* This is a line of equal parts, and for various occasions may be so used. For other problems, being the same as usually laid down upon the sector, I must refer the reader to the description of that instrument already given in this work.

9. Two circular scales on the joint head for giving the weight of iron shot, marked *Shot.* These scales are engraved upon the chamfered part of the head of the callipers; they are, in fact, but one continued scale, only separated for the advantage of being more conspicuous. They shew the weights of iron shot from ½ to 42 pounds. The diameter of the shot is to be taken by the points of the callipers, and then by an index engraved on the ruler is pointed out the proper weight.

10. The table shewing the quantity of powder necessary for proof and service charges of iron guns, marked *Iron Guns.* An inspection of the figure, or the callipers themselves, shew the table formed into three columns. The first, marked *Iron Guns*, shewing the weight of the shot; the second, *Proof*, shewing the quantity of powder for the proof charge; the third, *Service*, the quantity requisite for a service charge; and all these adjusted from ½ to 42 pounders.

11. The mathematical figures. The most vacant part of the callipers are sometimes filled up by six mathematical figures, with numbers annexed to each to assist the learner's memory, and indicating as follows.

1. A circle with two diametrical lines, about which are the numbers 7. 22, and 113. 355; they both are the proportion of the diameter to the circumference, the latter being nearer the truth than the former. The following examples may be therefore readily worked by them.

Required the circumference of a circle to any given diameter.

As 7 : 22 :: given diameter : the circumference required.

Or 113 : 355 :: diameter : the circumference more exactly.

And the converse of these.

As 22 : 7 :: circumference : the diameter.

As 355 : 113 :: circumference : the diameter more exactly.

2. A circle with an inscribed and circumscribed square, and inscribed circle to this smaller square. To this figure are annexed the numbers 28. 22. 14. 11. denoting that the larger square is 28, the inscribed circle is 22. The area inscribed at the square in that circle 14, and the area of the smaller inscribed circle 11.

Both the proportion of the squares and circles are in the proportion of 2 to 1; and from them the area of any circle may be found, having its diameter given. For example.

Required the area of a circle whose diameter is 12.

Now the square of 12 is 144.

Then as 28 : 22 :: 144 : 113.1, the area.

Or as 14 : 11 :: 144 : 113.1.

3, Represents a cube inscribed in a sphere; the affixed number $89\frac{1}{7}$ shews that a cube of iron inscribed in a sphere of 12 inches in diameter, weighs $89\frac{1}{7}$ pounds.

4, Represents a sphere inscribed in a cube, with the numbers 243 affixed; it is to shew the weight in pounds of an iron globe 12 inches in diameter, or a globe inscribed in a cube, whose side is 12 inches.

5, Represents a cylinder and cone, the diameter and height of which are one foot. To the cylinder is affixed the number 364,5, the weight in pounds of an iron cylinder of the above dimensions. The cone shewn by the number 121,5 that it is the weight of the same base and height, and is one-third of that of the cylinder. Cones and cylinders of equal weight and bases are to one another as 1 to 3.

6, Represents an iron cube, whose side is 12 inches, to weigh 464,5 pounds. The figure of the pyramid annexed to the cube, the base and height of which, if each 12 inches, denotes the height to be one-third of the cube's weight, viz. 154⅚.

Remark. Globes to globes are as the cubes of their diameters; cubes to cubes, as their cubes of the length of their sides; of similar bodies, their weights are as their solidities. Hence the dimensions and weight of any body being given, the weight or dimensions of any other similar body may be found.

EXAMPLES.

1. *The sides of a cube af iron being two feet, required the weight.*

The sixth figure shews that a cube of iron, whose sides are each 1 foot, weighs 464,5. Therefore,

As 1 (the cube of 1 foot) : 464,5 :: 8 (the cube of 2 feet) : 3716, the weight of an iron cube whose sides are 2 feet.

2. *The diameter of an iron shot being six inches, required the weight.*

By the fourth figure a one foot iron ball weighs 243 pounds, and 6 inches $=,5$ the $\frac{5}{10}$ of a foot.

Therefore, as 1 (the cube of 1 foot) : 243 (the weight) :: ,125 (cube of ,5 or $\frac{5}{10}$) : 30,375 pounds, the weight required.

Another rule. Take $\frac{1}{8}$ the cube of the diameter in inches, and $\frac{1}{8}$ of that eighth, and their sum will be the weight required in pounds exactly. Or, the weight of a four inch shot being nine pounds, the proportions 64 : 9 may be used with equal exactness.

3. *The weight of an iron ball being given, to find its diameter.*

This rule is the converse of the preceding, the same numbers being used; as 243 (the weight) : 1 (cube of 1 foot;) or, as 9 pounds : 64 (cube of 4.)

Another rule. Multiply the weight by 7, and to the product add $\frac{1}{9}$ of the weight, and the cube root of the sum will be the diameter in inches. Thus, an iron ball of 12 pounds weight will be found to be 4,403 inches. The methods of this rule are too evident to require worked examples.

4. The following example will make up a sufficient number, by which the learner may know how to apply readily any number from the figures just desbribed.

A parcel of shot and cannon weighing five tons, or 11200 *pounds is to be melted, and cast into shot of three and five inches diameter, and weight of each sort to be the same; required the number there will be of each.*

1st. Half of 11200 is 560 pounds.

2d. Cube of 3 inches, or cube $\frac{1}{4}$ of a foot $=\frac{1}{64}$.

Then, as 1 (cube of 1 foot) : 243 (the weight of a 1 foot ball) :: $\frac{1}{64}$: 3,79, weight of a 3 inch iron shot.

N. B. In this proportion 64 divides 243 on account of the fraction.

Now 5 inches $=\frac{5}{12}$ of a foot, its cube $\frac{125}{1728}$.

Therefore, as 1 : 243 :: $\frac{125}{1728}$: 17,67, weight of an iron 5 inch shot.

Dividing 5600 by 3,79 gives 1504, the number required of the 3 inch shot.

Dividing 5600 by 17,57 gives 313, the required number of the 5 inch shot.

A table of specific gravities and weights of bodies is added, or not, at the pleasure of the purchaser. It is not essential to the general uses of the callipers, although many curious and useful problems relative to the weights and dimensions of bodies may be obtained from it in the most accurate manner.*

The quantity of lines placed upon the callipers may be increased or arranged at pleasure. The following 19 were the greatest number that I ever knew of being placed upon them.

1. The measures of convex diameters in inches. 2. The measures of concave diameters in inches. 3. The weights of iron shot from given diameters. 4. The weight of iron shot proper to given gun bores. 5. The degrees of a semicircle. 6. The proportion of Troy and Averdupoise weight. 7. The proportion of English and French feet and pounds. 8. Factors useful in circular and spherical figures. 9. Tables of the specific gravity and weights of bodies. 10. Tables of the quantity of powder necessary for proof and service of brass and iron guns. 11. Rules for computing the number of shot or shells in a finished pile. 12. Rules concerning the fall of heavy bodies. 13. Rules for the raising of water. 14. The rules for shooting with cannon

* See Mr. Adams's Lectures, five vols. 8vo. a new and improved edition of which is now in the press, and under my correction and augmentation.

or mortars. 15. A line of inches. 16. Logarithmic scales of numbers, sines, versed sines, and tangents. 17. A sectoral line of equal parts, or the line of lines. 18. A sectoral line of plans or superficies. 19. A sectoral line of solids.

GUNNER'S QUADRANT.

Fig. 8, *plate* 33, is a representation of a quadrant used for elevating a cannon, or mortar, in the most expeditious manner. The bar A, is placed in at the mouth; the index B, brought to the arc till the bubble of the spirit level settles in the middle. The angle is then read off to minutes upon the arc by the nonius at C.

GUNNER'S PERPENDICULAR.

Fig. 9, *plate* 33, is a representation of a small level and perpendicular. It is used to find the center line of a piece in the operation of pointing it to an object, or to mark the point for a breech hole, &c. The spirit level A, determines the position on the gun, and the spring index point B, serves to mark the necessary points upon the surface to obtain the line by.

SHOT GAUGES.

Are a set of brass rings, all connected to one center, with holes suitable to the diameters, or pounders of iron shot, from four to 42 pounders; being all respectively marked, and are too evident to need a description here.

ERRATA.

Page 6, line 2, Preface, *for* desighed *read* designed.
—— 89, — 2, Note, *for* desiderata *read* desideratum.
—— 380, — 11, *for* plate 28 *read* plate 31.
—— 500, — 14, *for* By making, &c. *read* By making the base of the triangle and a perpendicular line to it, drawn from the opposite angle, in the ratio of 4 to 1.

A LIST

Of the principal Instruments described in this Work, and their Prices, as made and sold by W. *and* S. Jones, *Holborn, London.*

PLATE I.

	l.	s.	d.
Partial, or complete magazine cases of instruments, as represented in the plate, from 1l. 18s. to —	10	10	0

PLATE II.

Parallel rulers, *fig.* A, B, C, D, and E, according to the length and mounting, from 2s. 6d. each, to	2	12	6
N. B. Of *figs.* A, or B, or C, or D, or E, one of them is included in the cases above, to order.			
Improved ruler, F G H — — — — — —	1	11	6
Ditto I K L — — — — — — — —	1	16	0
Ditto represented at M, from 10s. 6d. to — —	0	16	0

PLATE III.

Fig. 1. Marquois's parallel scales — — — —	0	9	0
——2 and 3. Protractors, wood or ivory, from 2s. to	0	7	0
——4. Sectors, according to the length and materials they are made of, from 2s. 6d. to —	3	13	6
——7. Brass proportional compasses — — —	2	2	0
Without the adjusting rod, *fig.* A, *plate* 1	1	11	6
——9 and 9 *a.* Elliptical, sector, and beam compasses	3	13	6
——10. Beam compasses from 1l. 4s. to — —	4	4	0
——11. Sisson's beam compasses — — — —	3	3	0
——12. Triangular compasses — — — — —	1	1	0
Ditto represented at *fig.* N, *plate* 1 — —	0	13	0
——13. Calliper and beam, small size — — —	1	11	6

PLATE X.

Bevel, &c. rulers, *figs.* 4, 12, 15, 16, 17, and 18, various prices from 10s. 6d. to — — — —	3	13	6

PLATE XI.

Fig. 1. Suardi's geometric pen — — — — —	4	4	0
Ditto, with a great variety of wheels, &c. to	7	7	0

Fig. 2.	Elliptical drawing board — — — —	2	12	6
——3.	Elliptical brass compasses — — — —	2	12	6
——4.	Parabolic drawing machine — — — —	2	12	6
——5.	Cyclograph for drawing circles of large radii	4	14	6
——6.	Spiral machine — — — — — —	4	14	6
——7.	Protractor, perpendicular, &c. — — —	5	5	0
	PLATE XIV.			
Fig. 1.	King's surveying quadrant with sights —	1	18	0
——2.	Portable surveying brass cross and staff —	1	18	0
——3.	Brass pocket box cross and staff — —	0	18	0
——6.	Improved ditto, from 1l. 11s. 6d. to — —	3	3	0
——4.	Optical squares — — — — — —	1	1	0
——5.	Ten inch common theodolite and staves —	4	14	6
——7.	Best universal pocket theodolite — — —	10	10	0
	PLATE XV.			
Fig. 1.	Common circumferentor and staves — —	4	4	0
——2.	Improved ditto 4l. 14. 6d. to — — —	5	15	6
——3.	Surveying compass with telescope — —	2	2	0
——4.	Six inch pocket common theodolite and staves	3	3	0
——5.	Six inch theodolite by rack work and staves	12	12	0
	Ditto without rack work, common — —	8	8	0
	PLATE XVI.			
Fig. 1.	Second best 7 or 8 inch theodolite and staves	22	1	0
——2.	Very best improved ditto, ditto — — —	31	10	0
	N. B. For large theodolite see the frontispiece, from 80 l. to — — — —	300	0	0
	PLATE XVII.			
Fig. 1.	Plane table and staves in a box — — —	4	4	0
——2.	Beighton's improved ditto — — — —	6	6	0
——3.	Spirit level of the best kind with telescope and staves, according to the adjustments and finishing, from 8l. 8s. to — —	12	12	0
	Common ditto, without compass, from 3l. 13. 6d. to — — — — — —	6	6	0
——4.	Six inch circular protractors 1l. 18s. to —	2	10	0

Fig. 5.	Ditto by rack work	4	14	6
	Common circular brass protractors 6s. to	0	18	0
——6.	Perambulator, or measuring wheel	6	6	0
——8.	Way wiser for carriages 6l. 6s. to	15	15	0
——9.	Station staves, the pair	2	12	6
——10.	Pocket spirit levels, from 10s. 6d. each, to	1	11	6
——11.	Improved universal ditto	1	11	6

PLATE XIX.

Fig. 1.	Hadley's quadrant, ebony and brass, 2l. 12s. 6d. to	3	13	6
——4.	Best metal sextant, 8 to 10 inches radius	13	2	6
	Brass stand and counterpoise for a ditto	4	4	0
	Second best ditto 8l. 8s. to	10	10	0
——11.	Pocket box sextant from 2l. 2s. to	3	3	0
——12.	Artificial horizon complete	2	2	0

PLATE XXII.

Fig. 2.	Feather edged 12 inch box scales, each	0	2	6
	Ditto ivory, each	0	8	0

PLATE XXXI.

Fig. 19.	Pantagraph, 2 feet, best	4	14	6
	Common ditto, 12 to 18 inches, from 1l. 4s. to	3	13	6

PLATE XXXII.

Fig. 1.	Perspective machine	6	6	0
——2.	Ditto, ditto	5	5	0
——3.	Perspective compasses	1	18	0

PLATE XXXIII.

Fig. 2, 3, 4, and 5.	Keith's improved parallel scales in a case	0	10	6
	Ditto in wood and ivory, or all ivory, 1l. 5s. to	2	2	0
——6 and 7,	Gunner's callipers, from 2l. 12s. 6d. to	6	6	0
——8.	Gunner's quadrant	2	12	6
——9.	——— level and perpendicular	1	16	0

For farther particulars see W. and S. Jones's general catalogue.

PLATES

TO THE

GEOMETRICAL

AND

GRAPHICAL ESSAYS,

BY THE LATE

GEORGE ADAMS,

MATHEMATICAL INSTRUMENT MAKER TO HIS MAJESTY, &c.

THE SECOND EDITION,

CORRECTED AND ENLARGED BY

WILLIAM JONES,

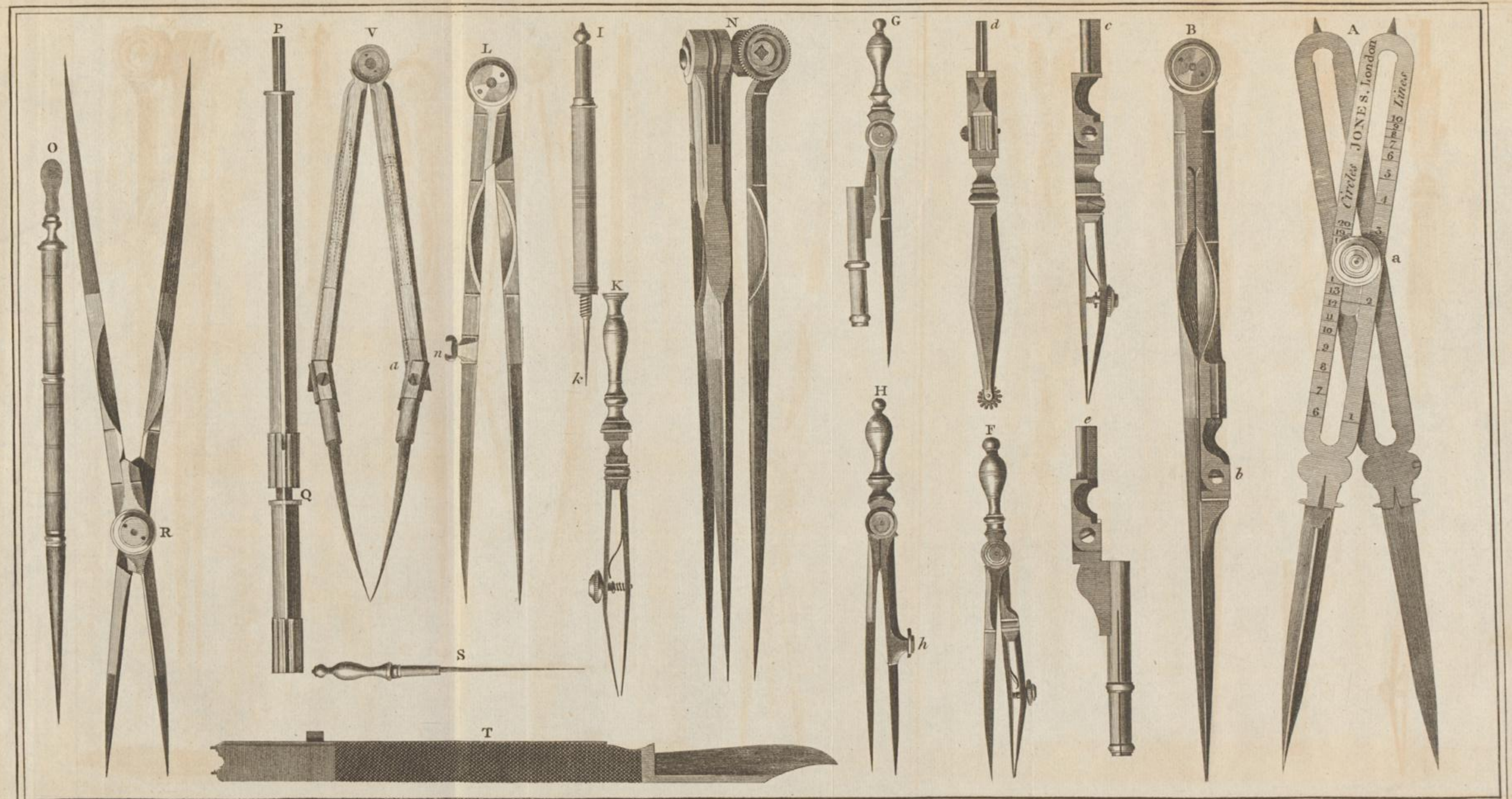
MATHEMATICAL INSTRUMENT MAKER.

LONDON:

PRINTED BY J. DILLON, AND CO.

AND SOLD BY W. AND S. JONES, OPTICIANS,
HOLBORN, LONDON.

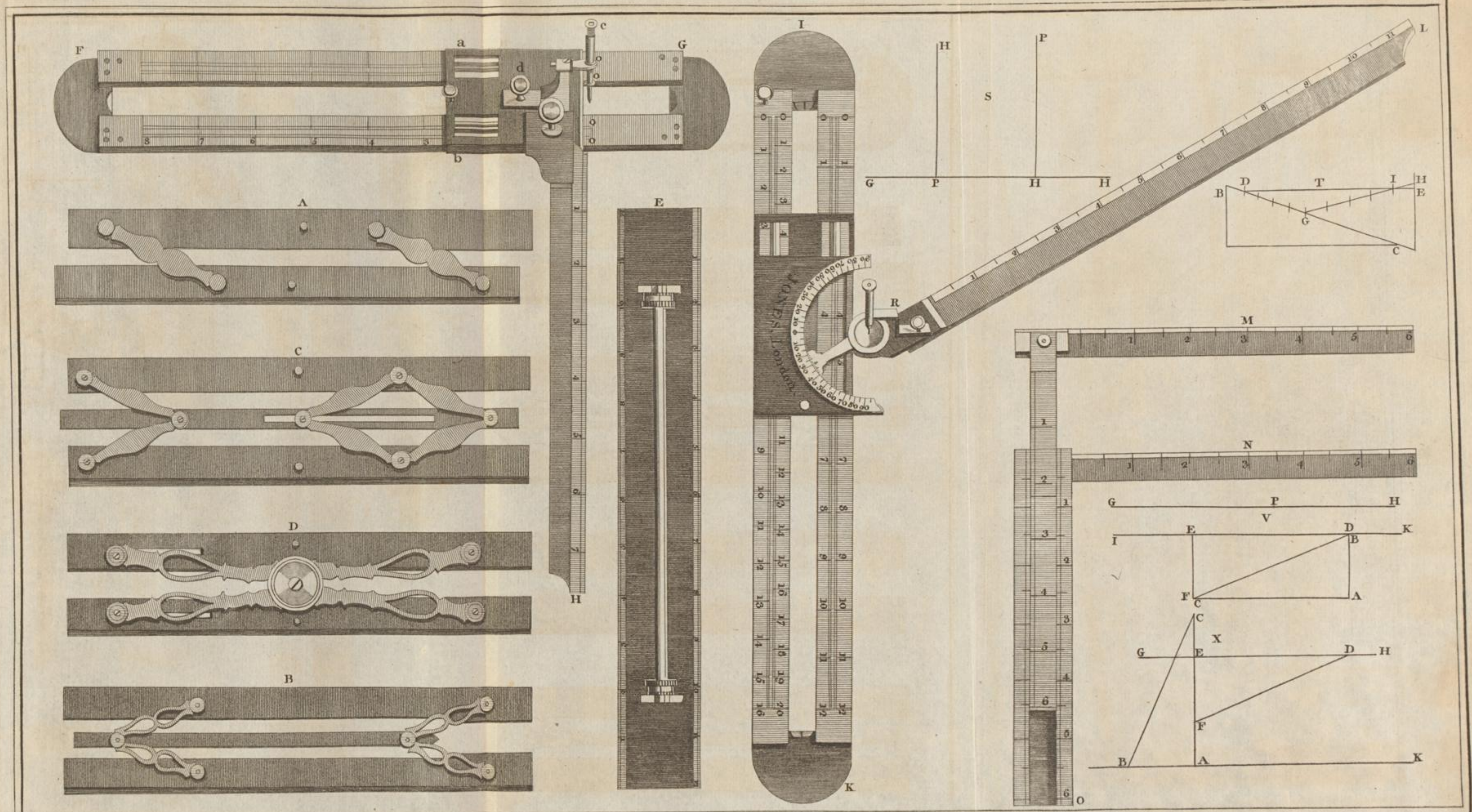
1797.



T. Milne delin.t

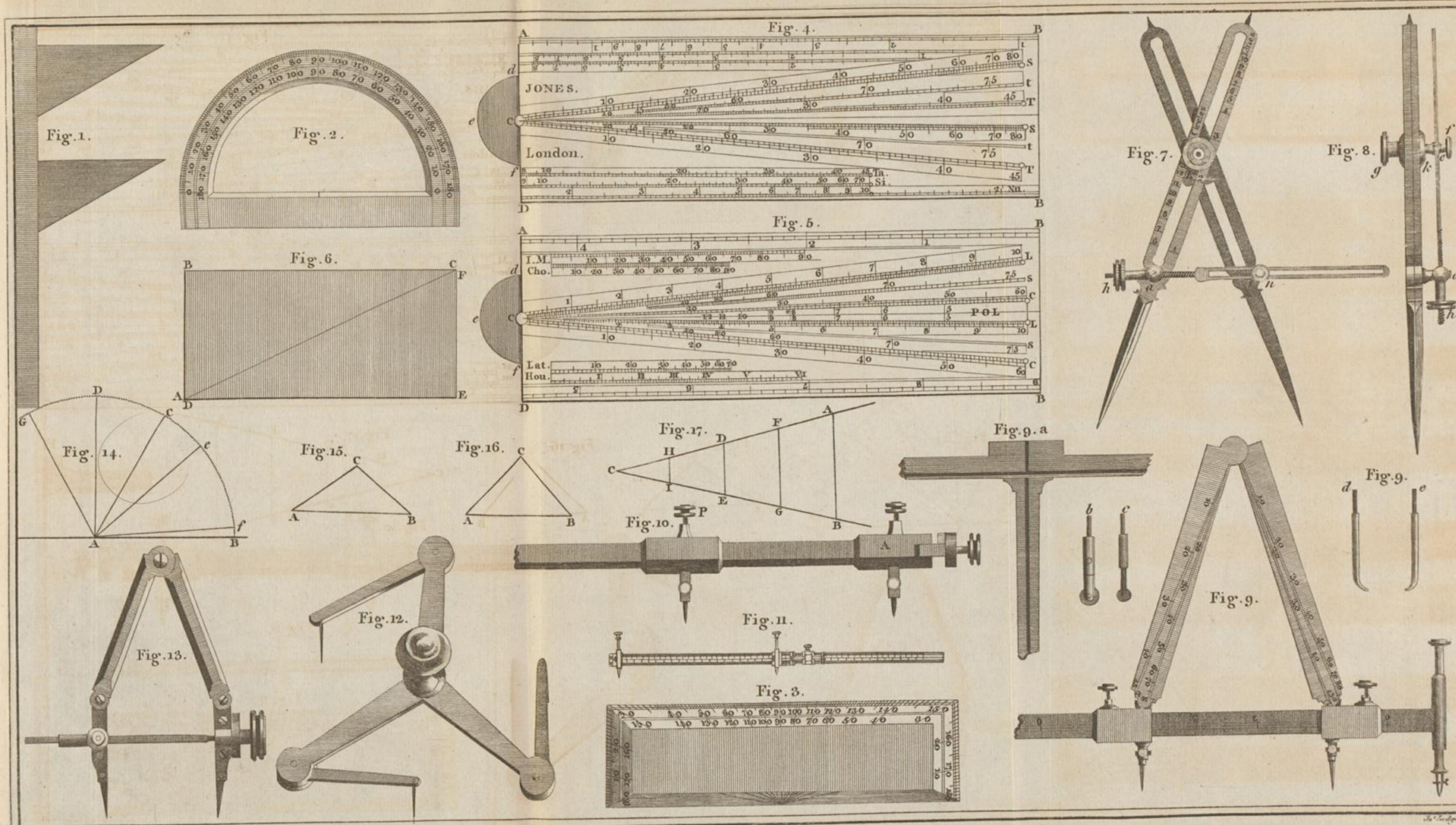
Jn.o Lodge sc.

London. Printed for & Published by W. & S. JONES, 135 Holborn, as the Act directs, December 1st 1796.

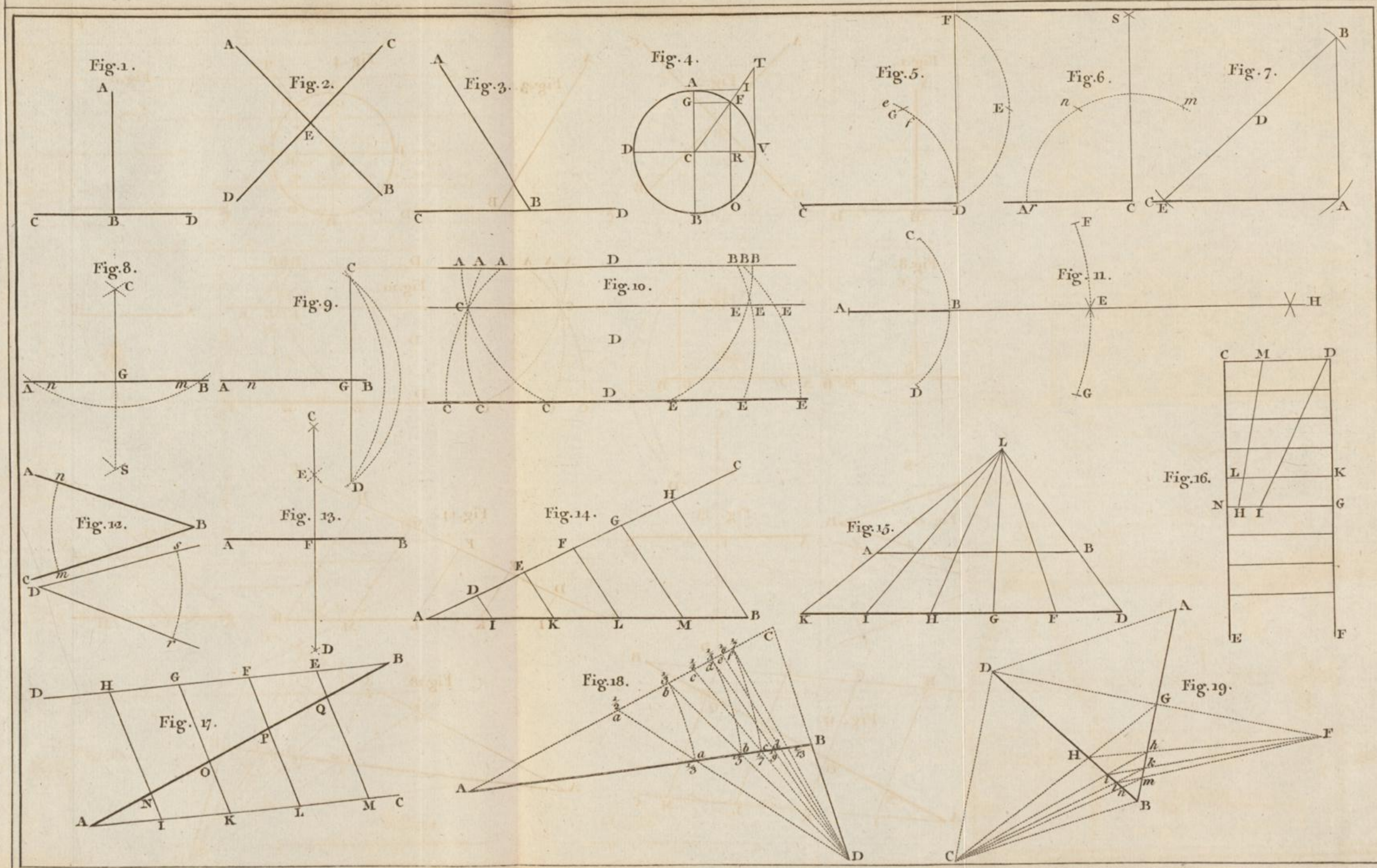


T. Milne delin. Jas. Lodge Sculp.

London. Printed for & Published by W and S. Jones, 135, Holborn, as the Act directs, 1st. Jany. 1797.

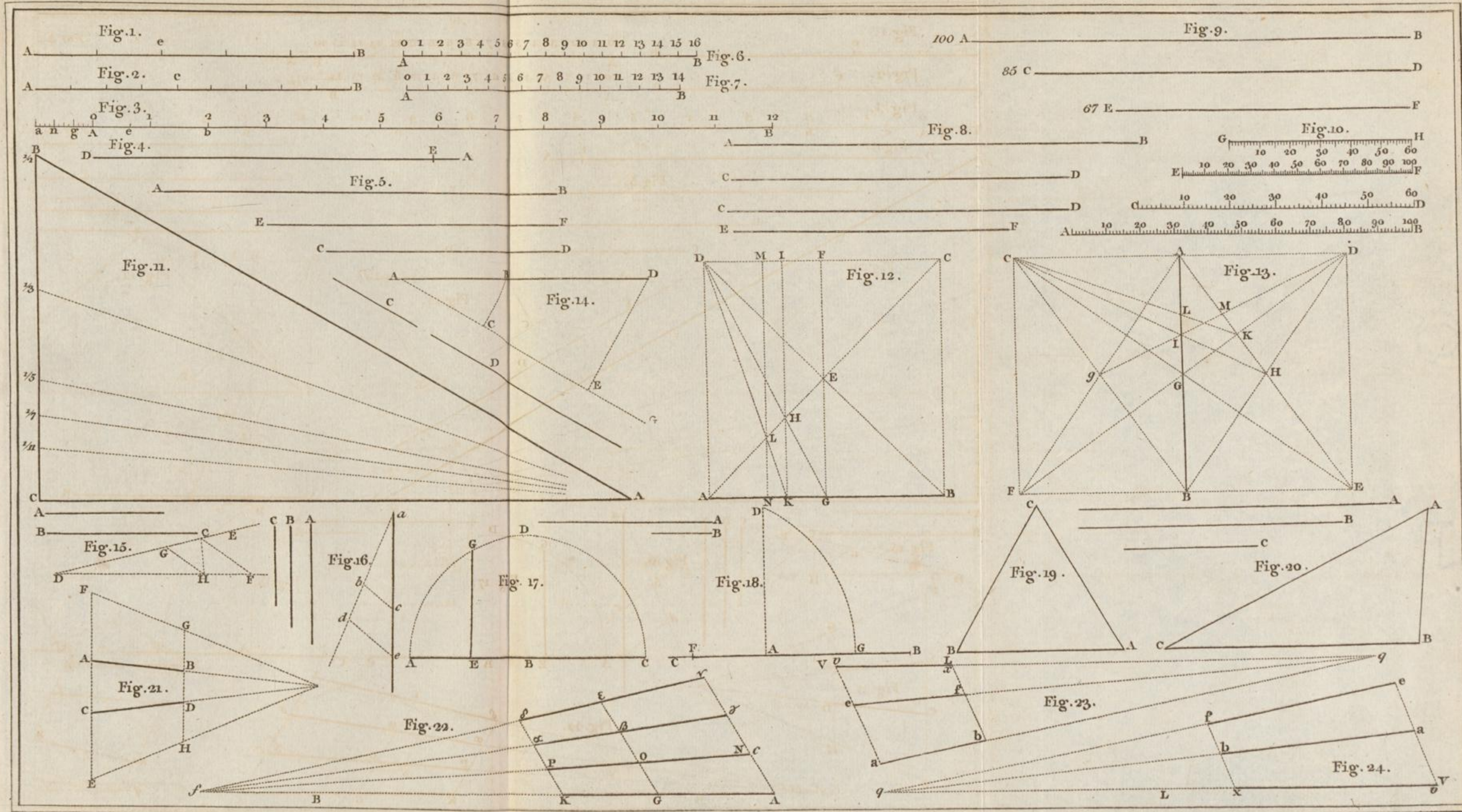


London. Printed for & Published by W. and S. Jones, 135, Holborn, as the Act directs, 1st. December, 1796.



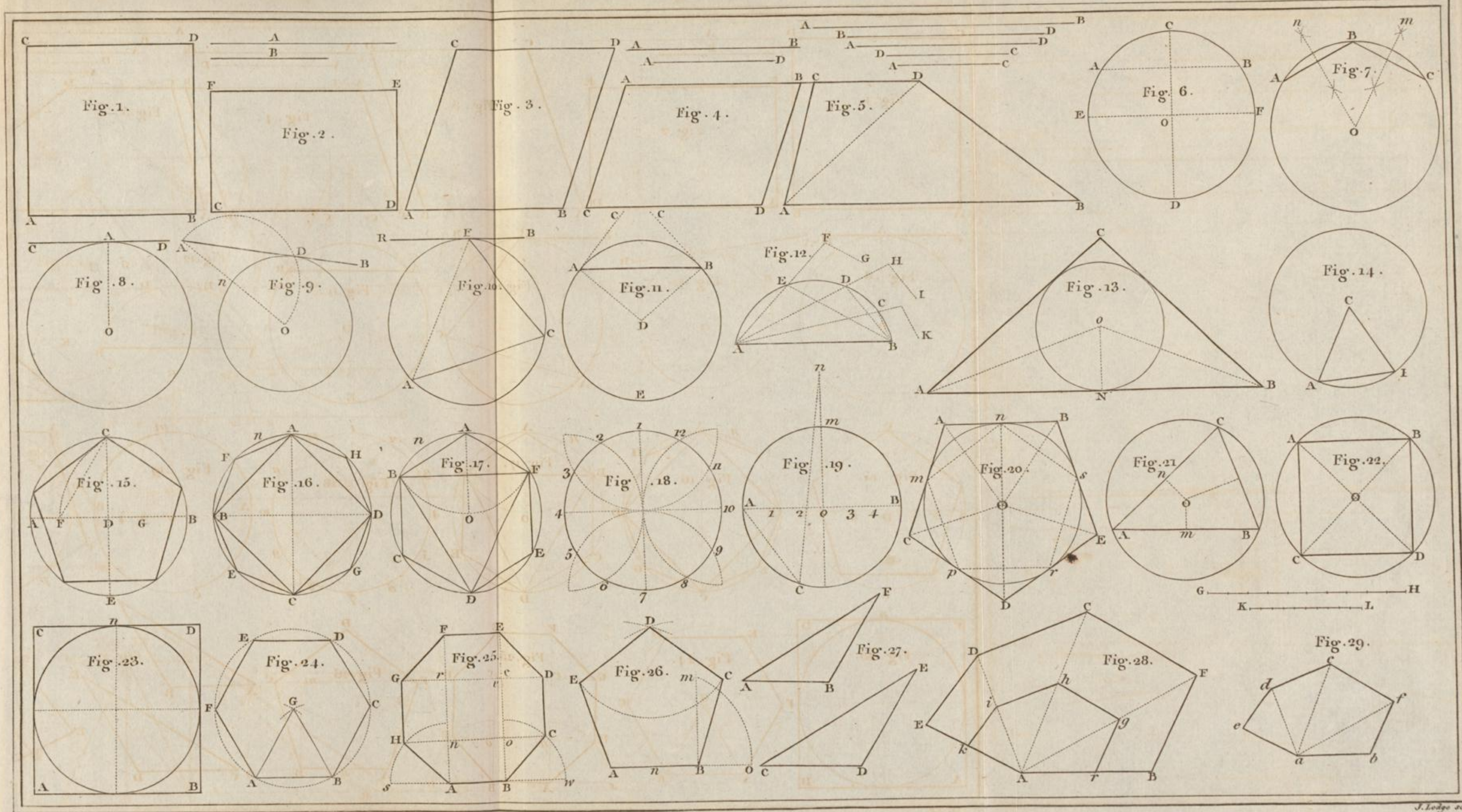
T. Milne delin. J. Lodge sc.

London. *Printed for & Published by* W. and S. Jones, *135 Holborn, as the Act directs 1st December 1796.*



T. Milne del. Jn.o Lodge sc.

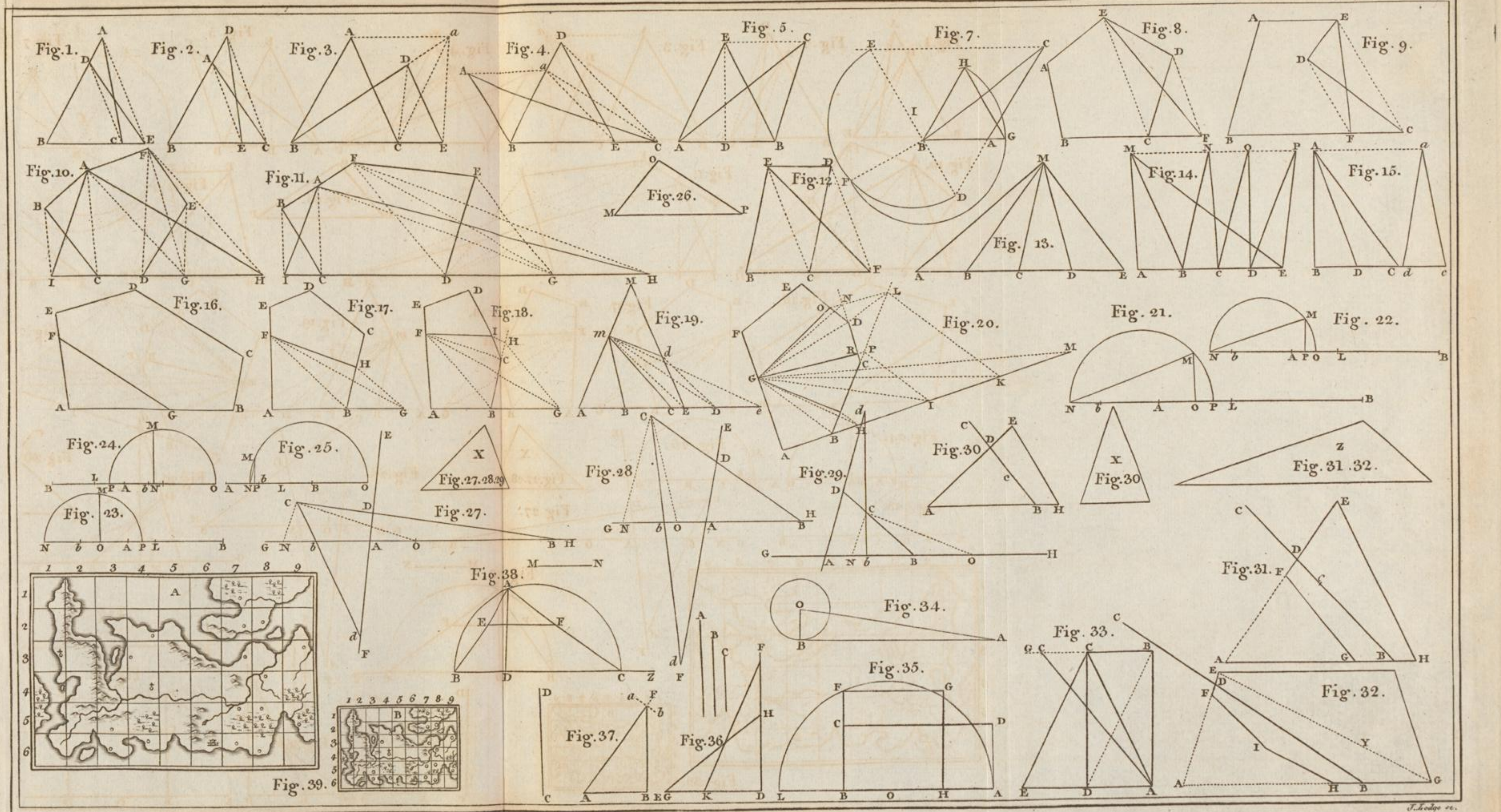
London. Printed for & Published by W. and S. Jones, 135 Holborn, as the Act directs, 1.st December 1796.



T. Milne del.

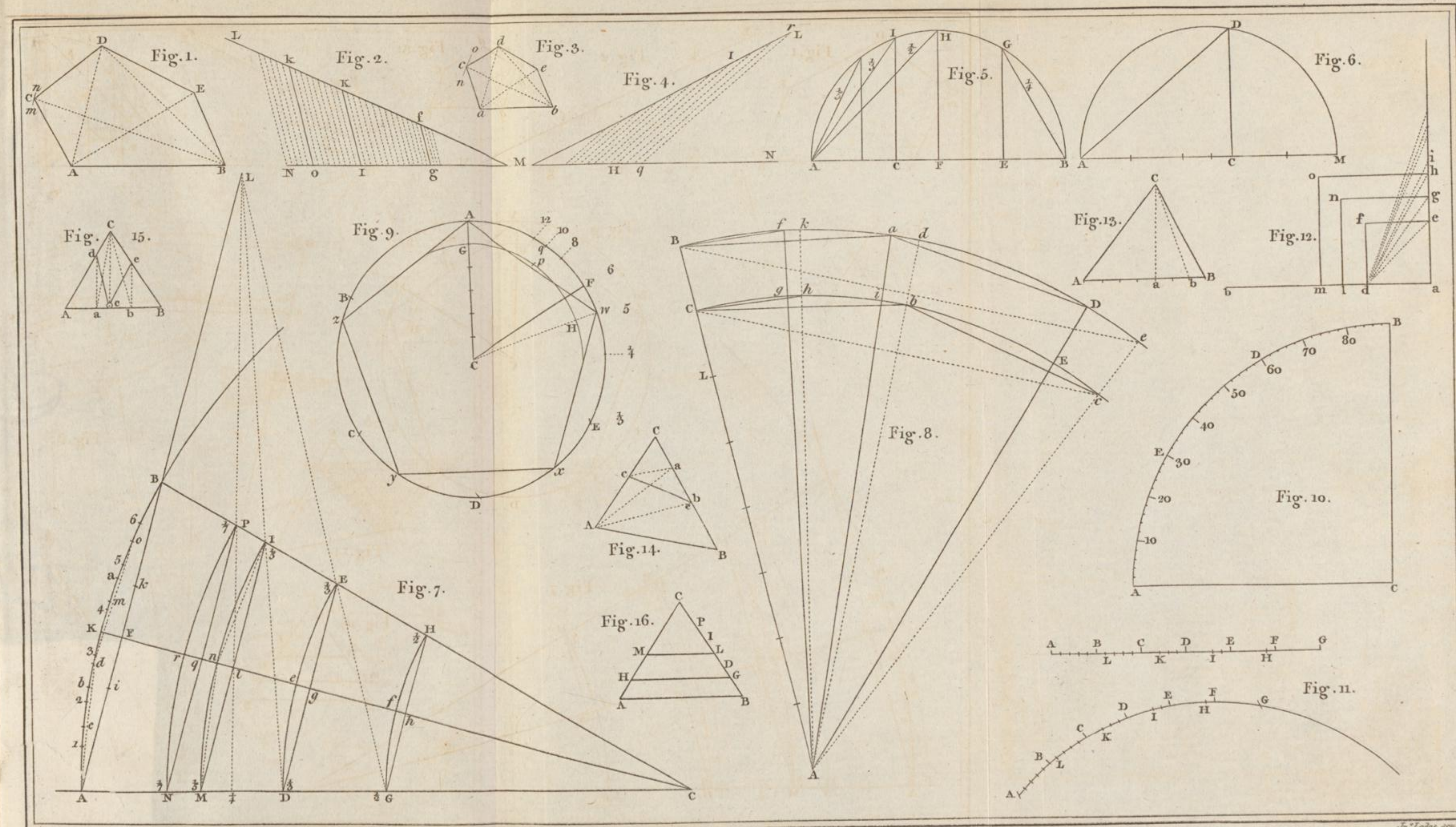
London. Printed & Published by W. and S. Jones, 133 Holborn, as the Act directs 2d. December 1796.

J. Lodge sc.



J. Lodge sc.

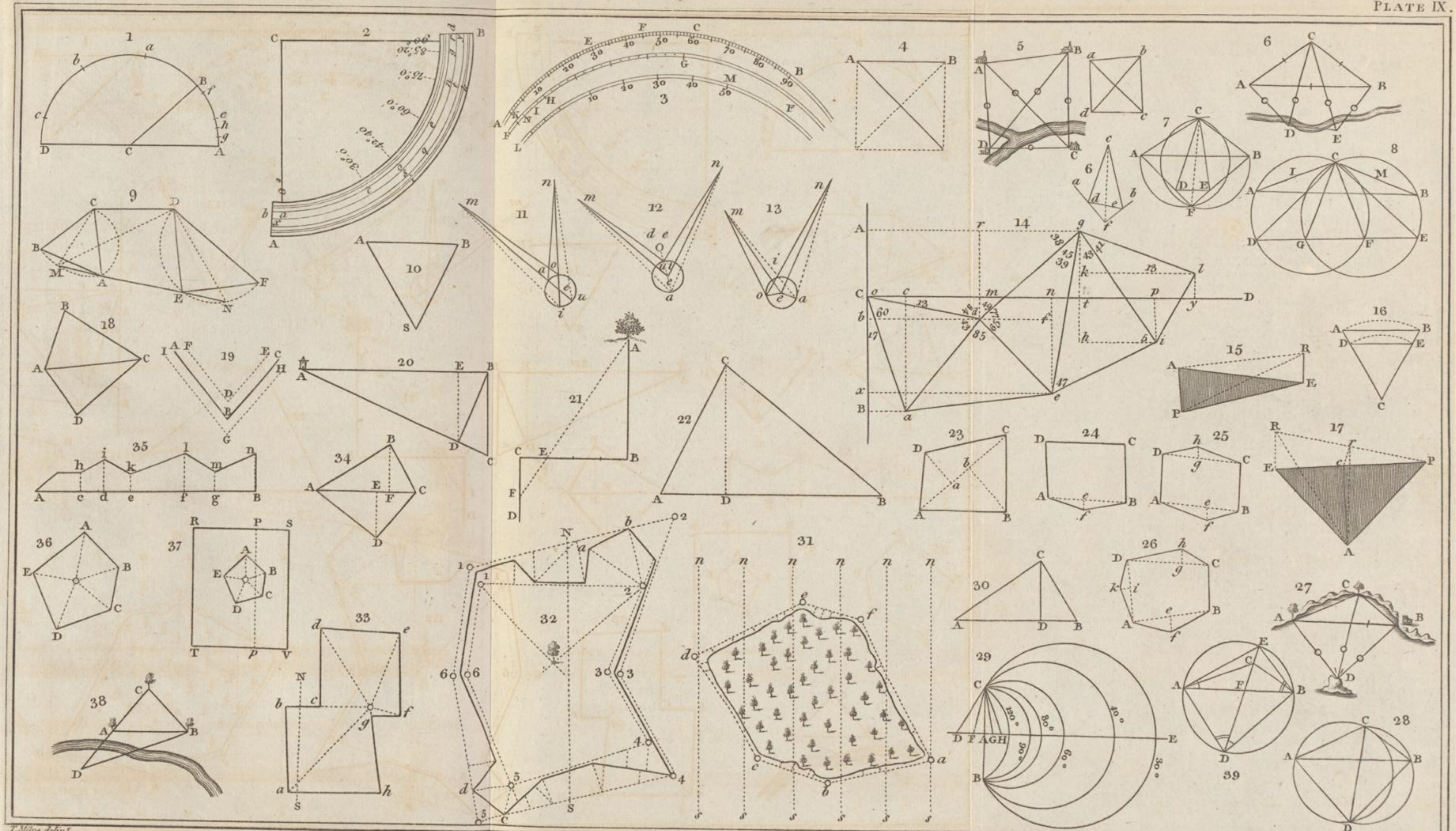
London. Printed for & Published by W. and S. Jones, 135 Holborn, as the Act directs, 1st December 1796.



Jn.o Lodge sculp.

London. Printed for & Published by W. and S. Jones, 135 Holborn, as the Act directs 1st December 1796.

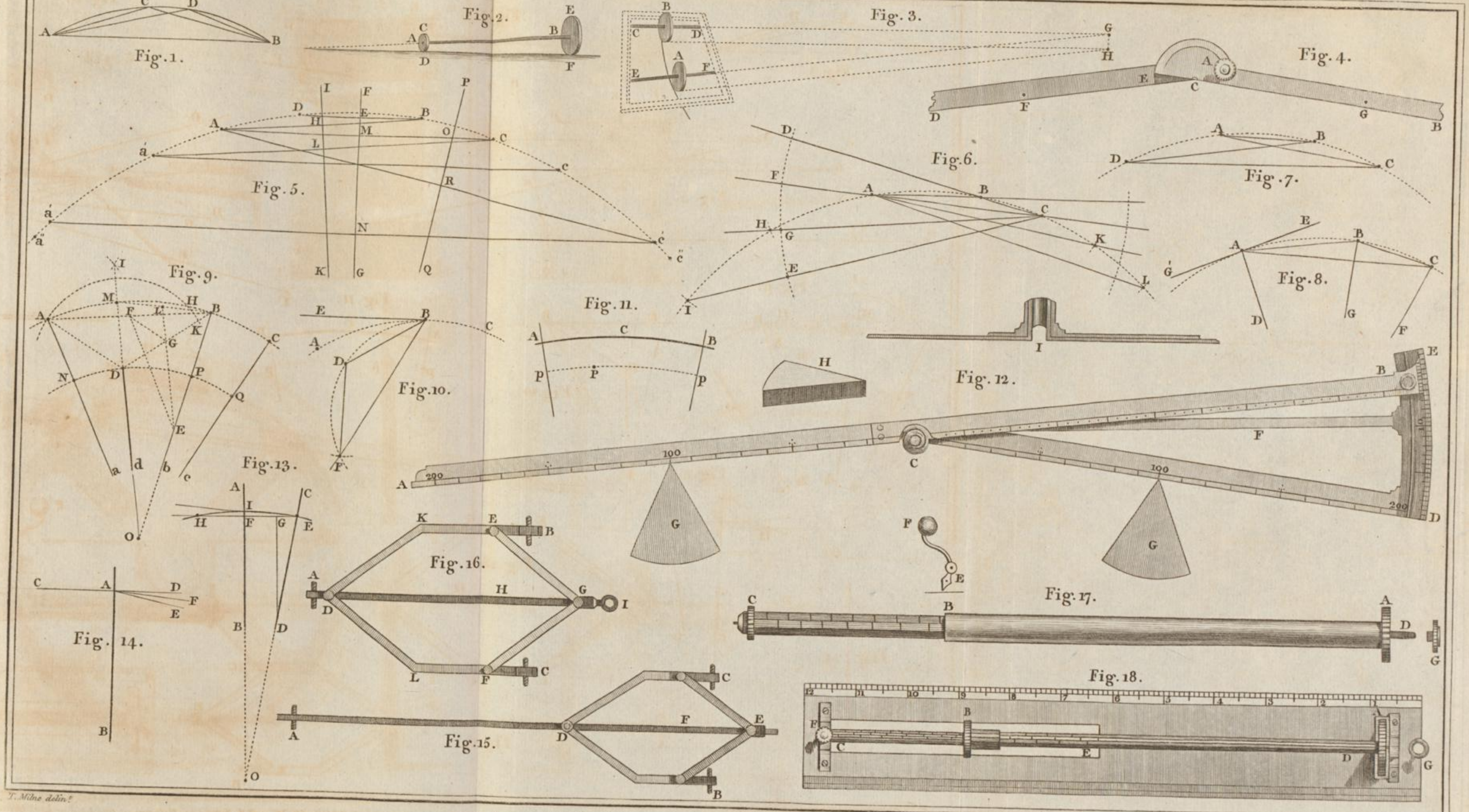
PLATE IX.



T. Milne delin.

London. Printed for & Published by W. and S. Jones 135 Holborn, as the Act directs, 1st December 1796.

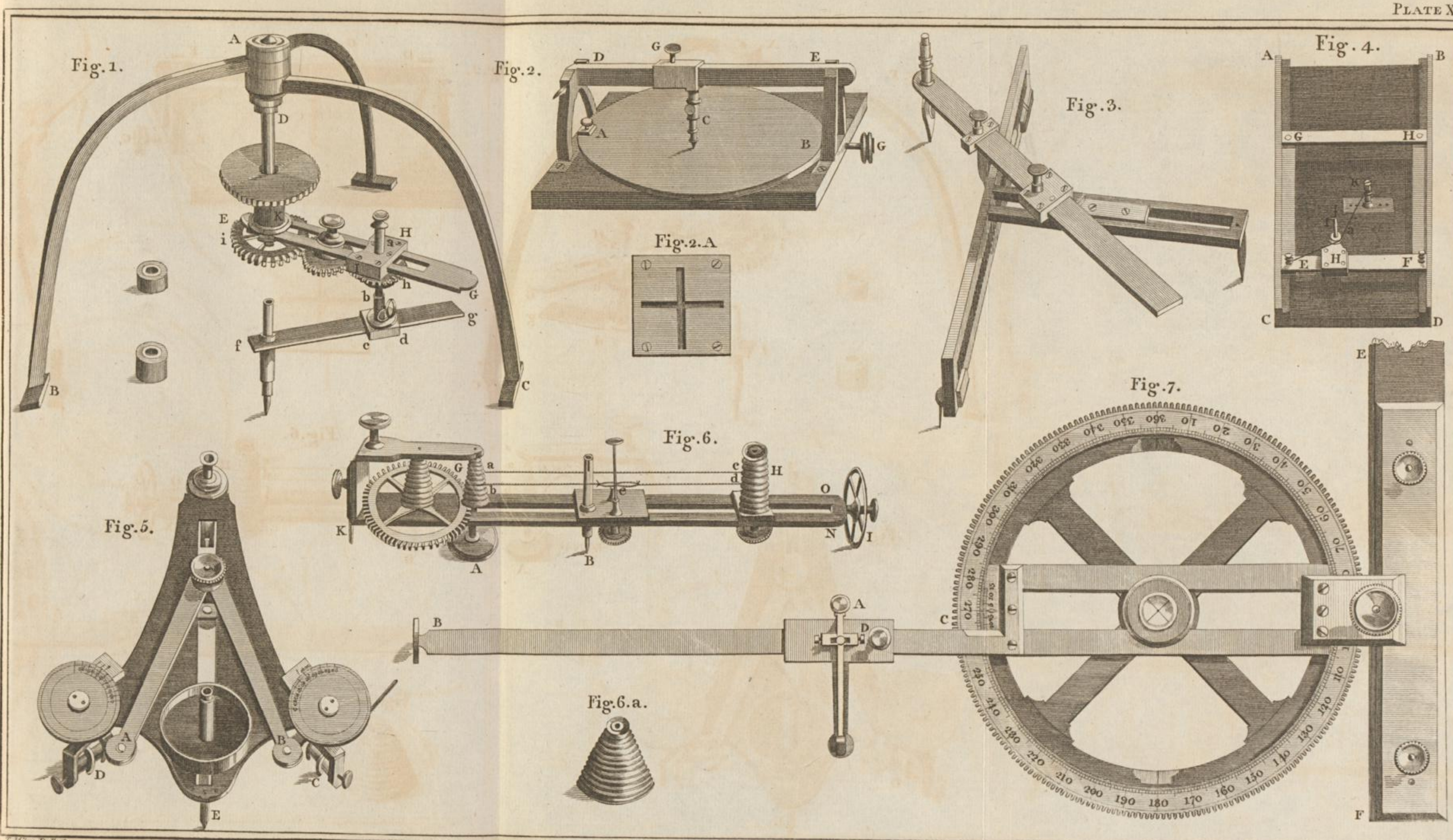
Jn. Lodge sculp.



T. Milne delin.

London, *Printed for & Published by* W. and S. Jones *135 Holborn, as the Act directs 1st December 1796.*

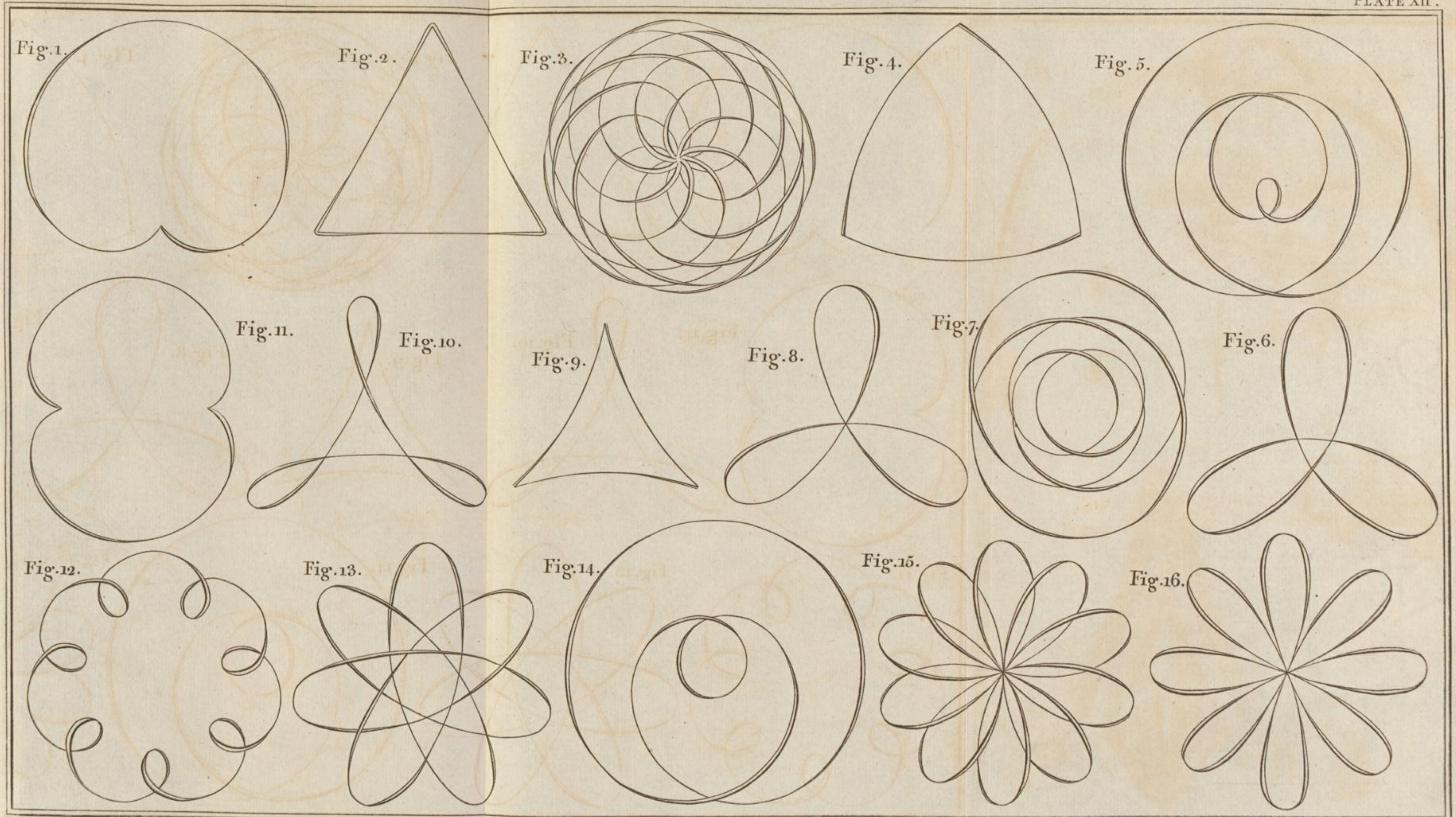
Jn. Lodge sculp.



T. Milne Delin.t Jn.o Lodge sculp.

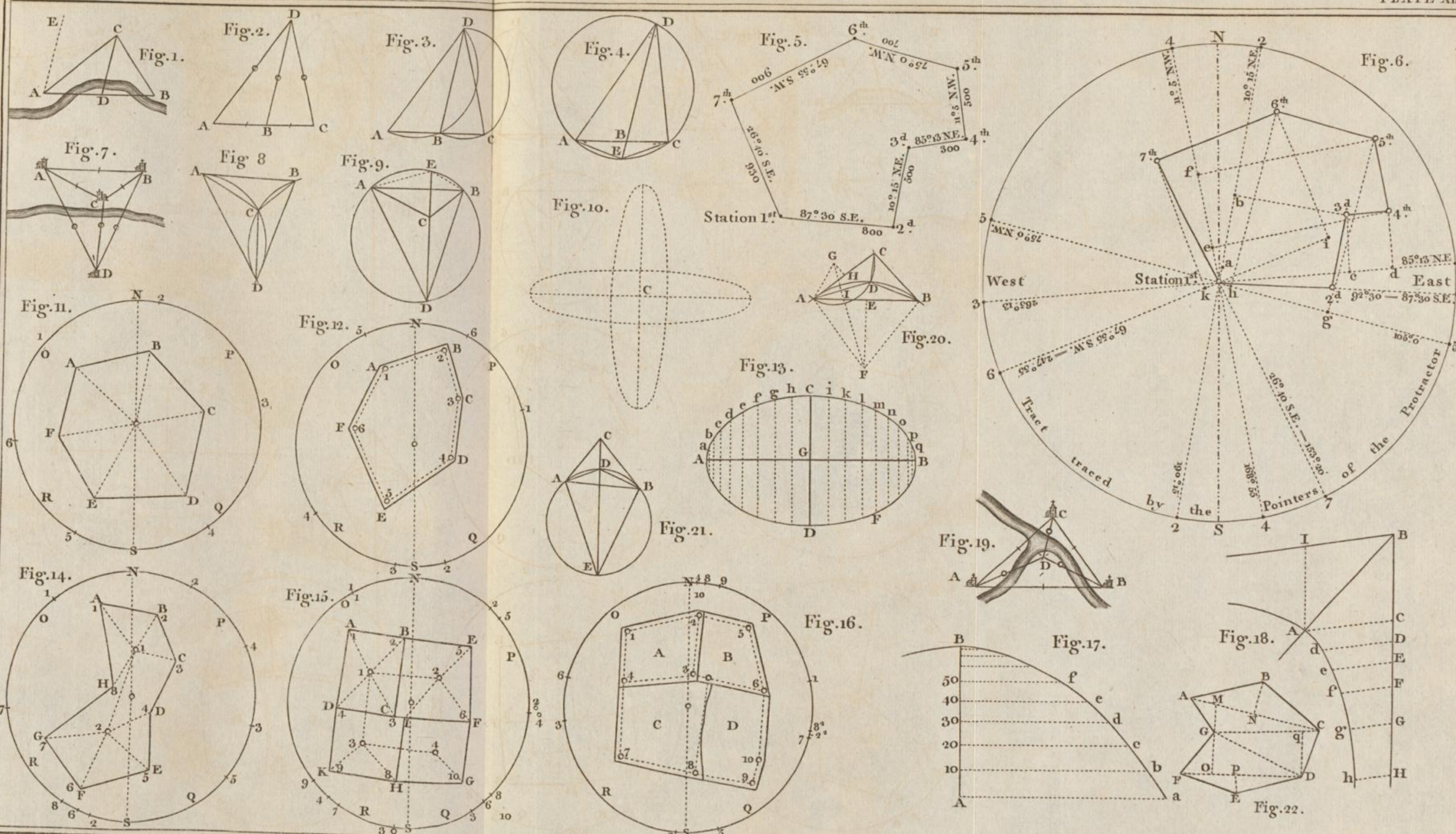
London. Printed for & Published by George Adams, N.o 60 Fleet Street, as the Act directs. June 1 1791.

PLATE XII.



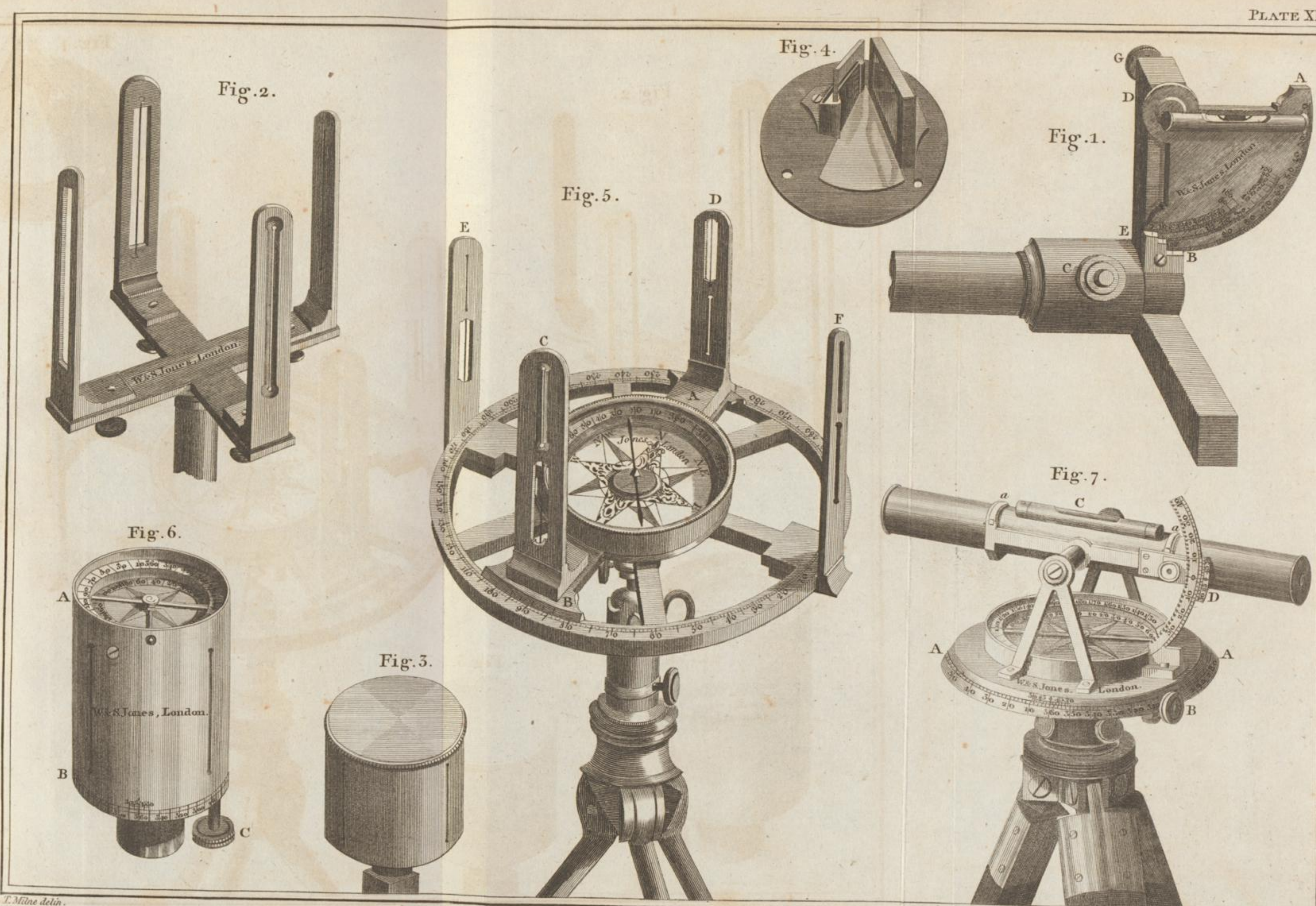
Jn.º Lodge sculp.

London. *Printed for & Published by* George Adams, *Nº 60 Fleet Street, as the Act directs, June 1 1791.*



T. Milne delin. London. Printed for & Published by George Adams, No. 60 Fleet Street. as the Act directs. June 1.1791. Jn. Lodge sculp.

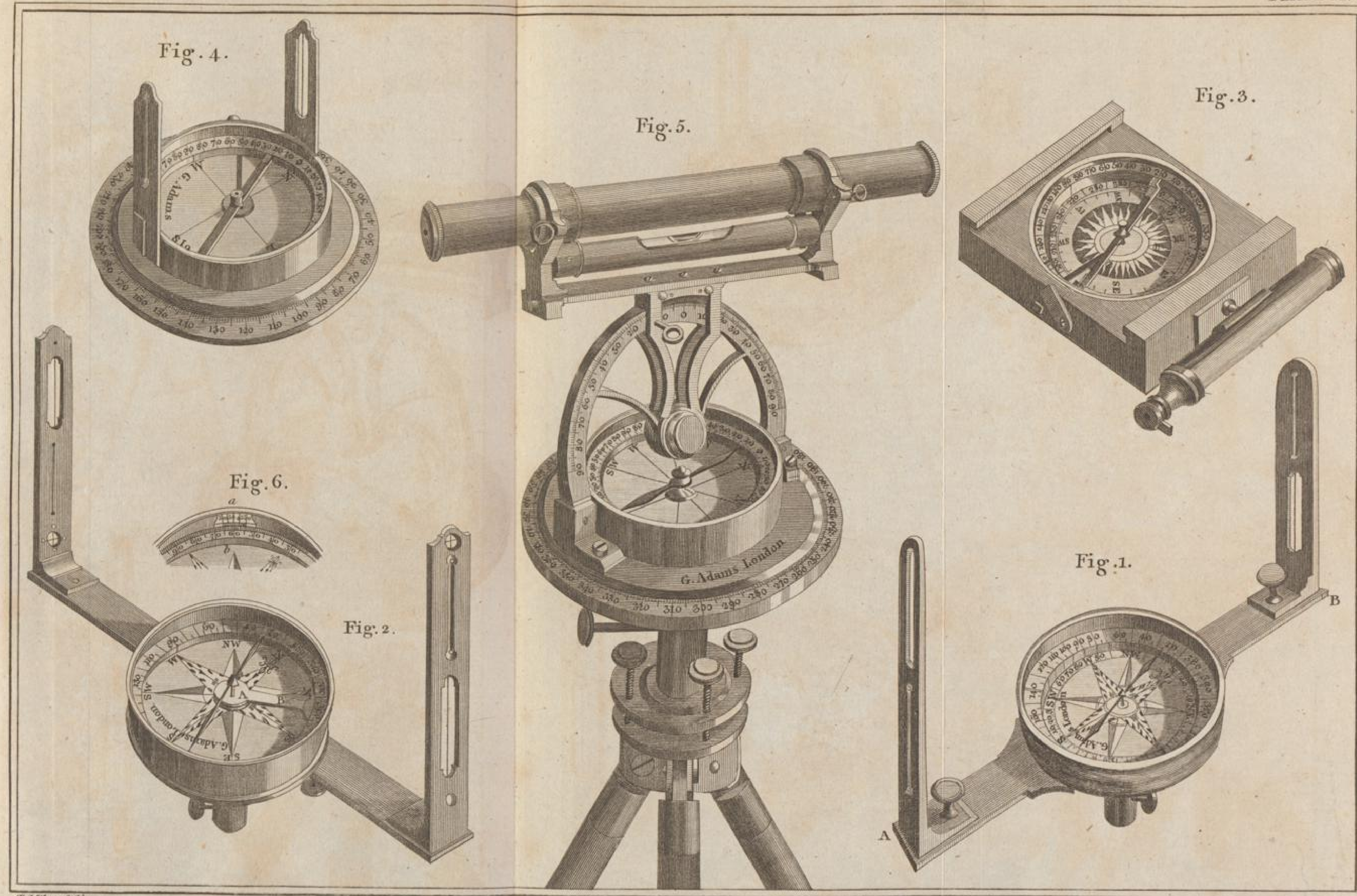
PLATE XIV.



T. Milne delin.

London. Printed for & Published by W. & S. Jones, 135 Holborn, as the Act directs. Jan. 1797.

Jn. Lodge sc.

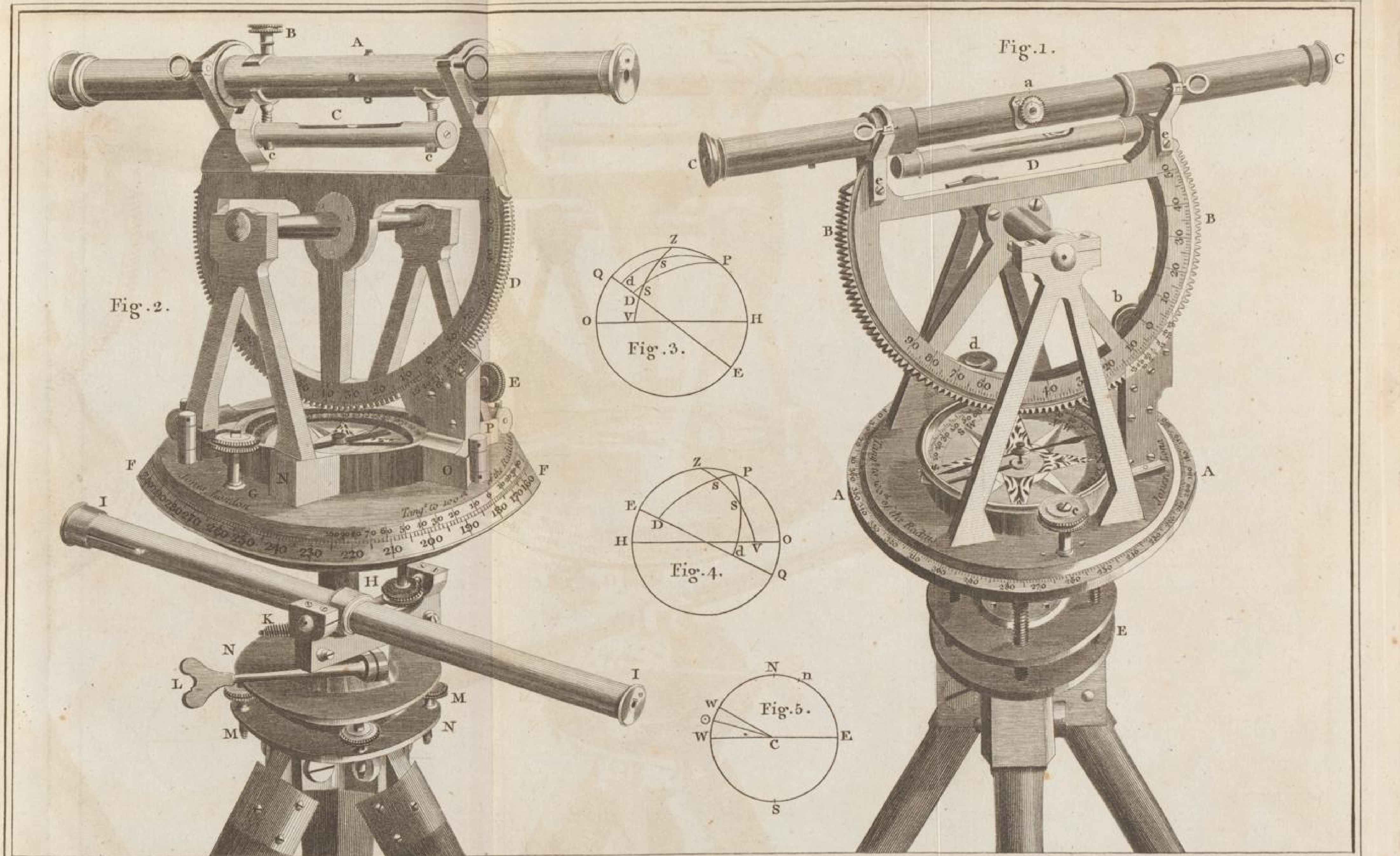


T. Milne delin.

London. Printed for & Published by W. and S. Jones, 135 Holborn, as the Act directs, 3rd March 1797.

Jnº Lodge sc.

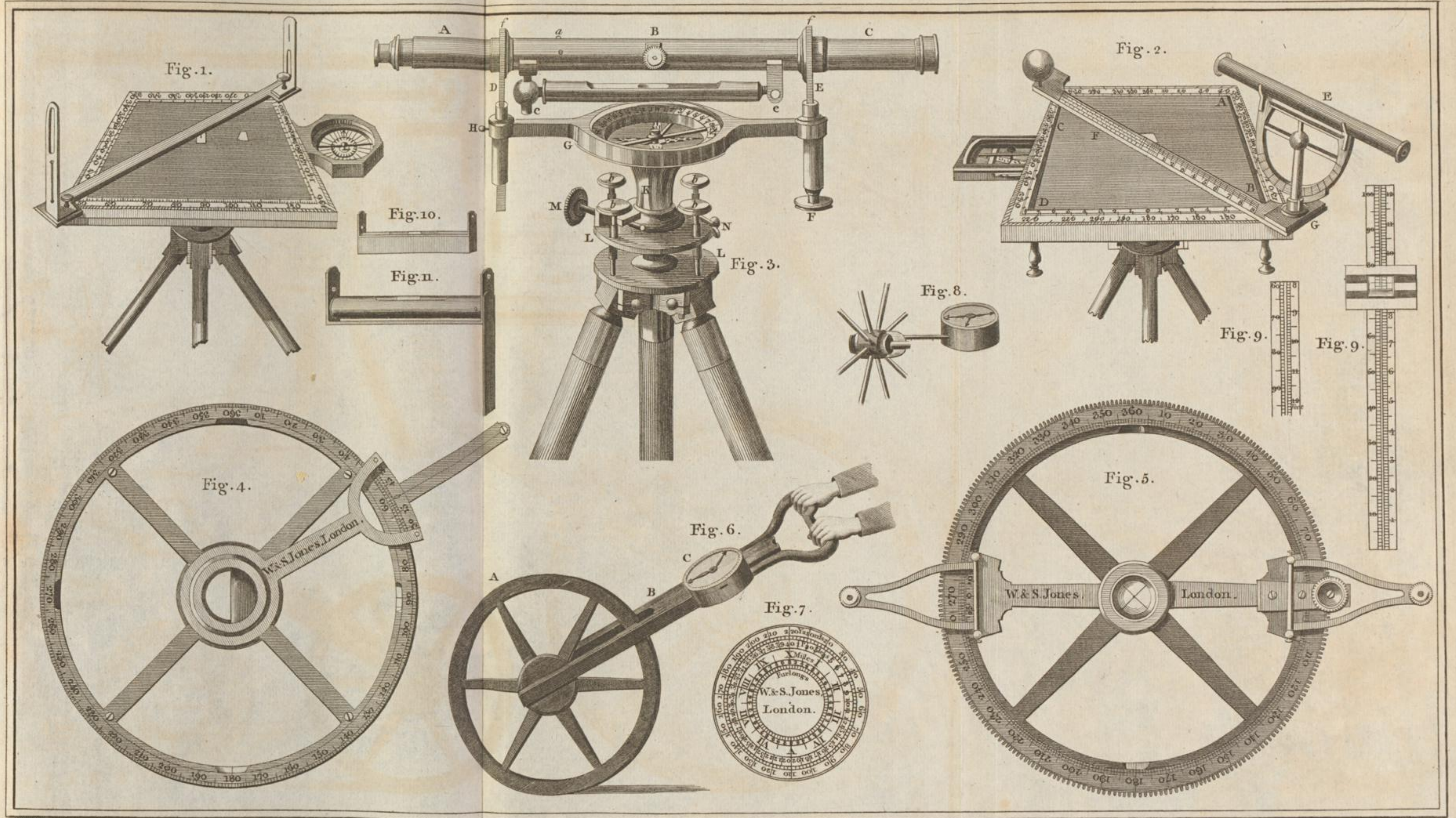
PLATE XVI.



T. Milne delin.

Jn. Lodge sc.

London. Printed for & Published by W. & S. Jones, No. 153 Holborn, as the Act directs, March 27th 1797.

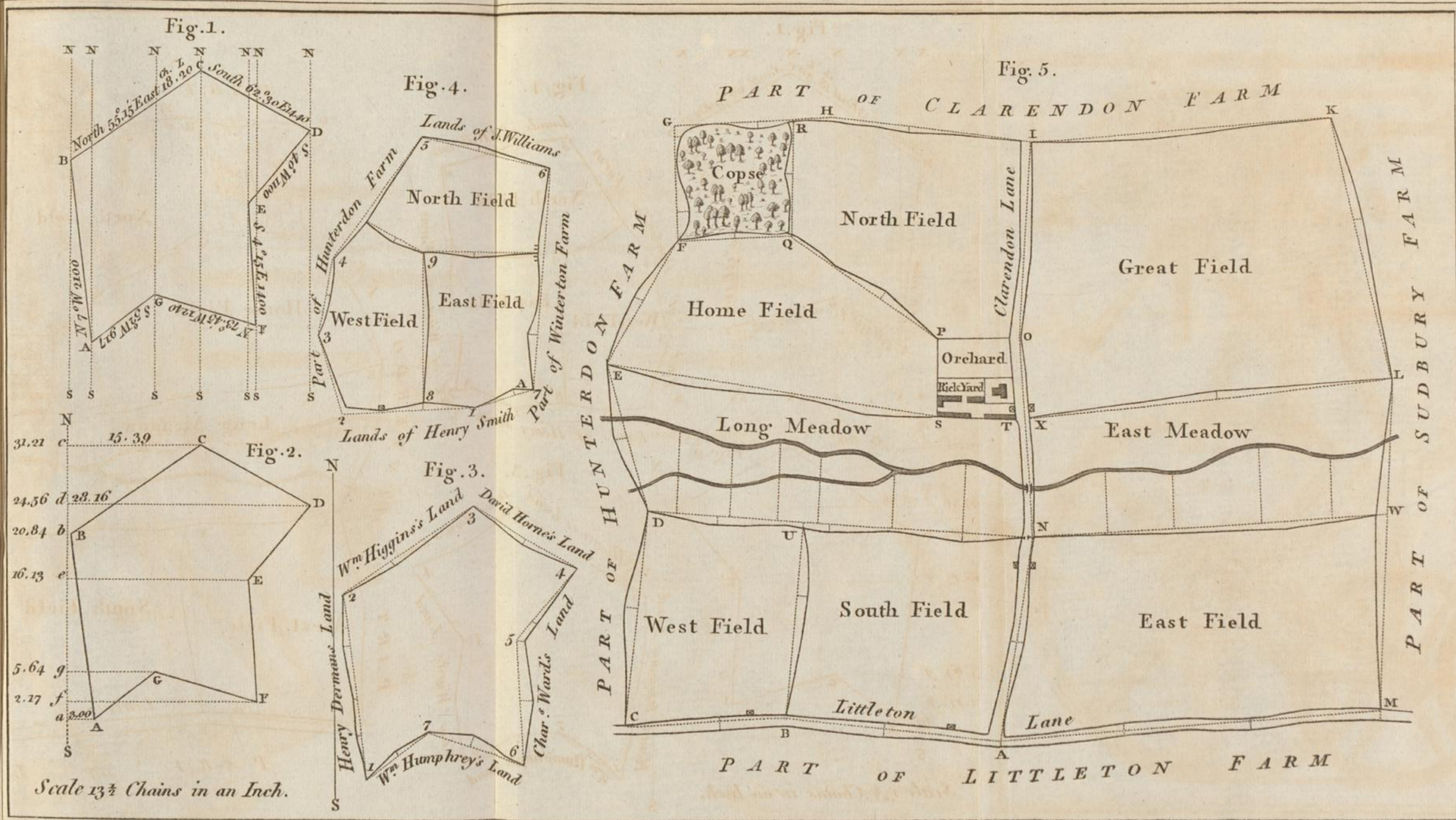


T. Milne delin.

London. Printed for & Published by W. & S. Jones 135 Holborn, as the Act directs, 3rd March 1797.

Jn.o Lodge sculp.

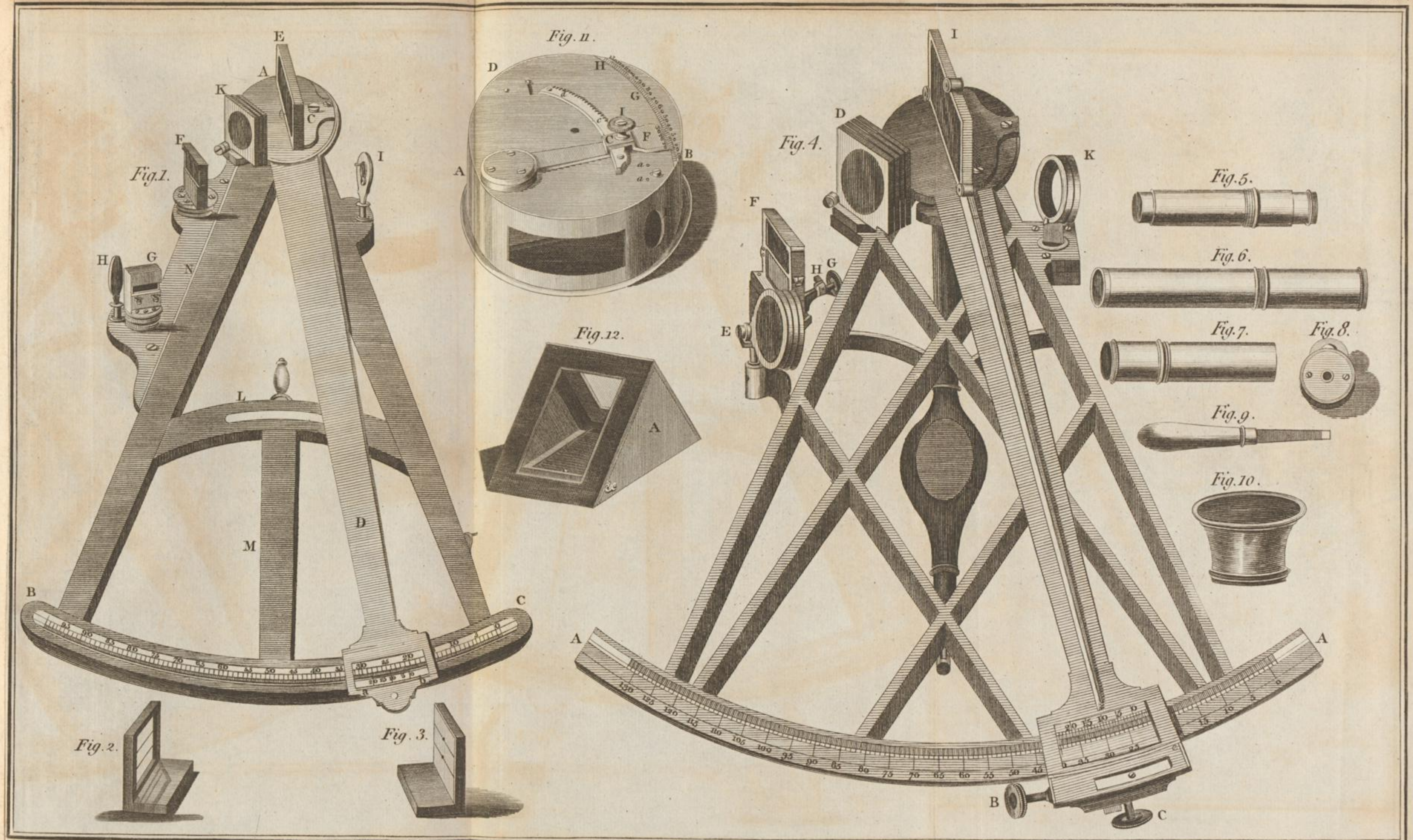
PLATE XVIII.



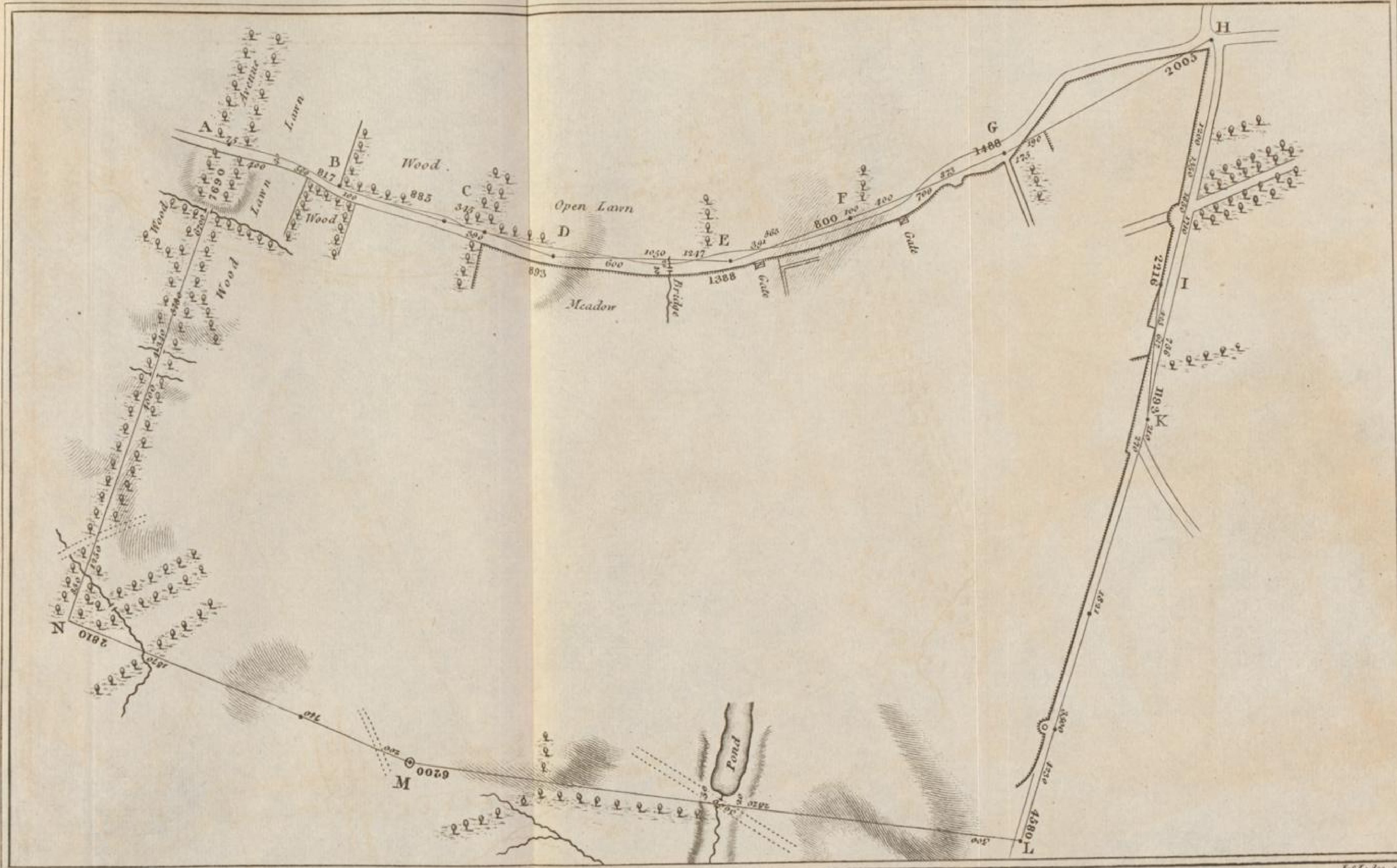
T. Milne del.

London. Printed for & Published by George Adams. No. 60 Fleet Street, as the Act directs June 1. 1791.

Jn.o Lodge sc.

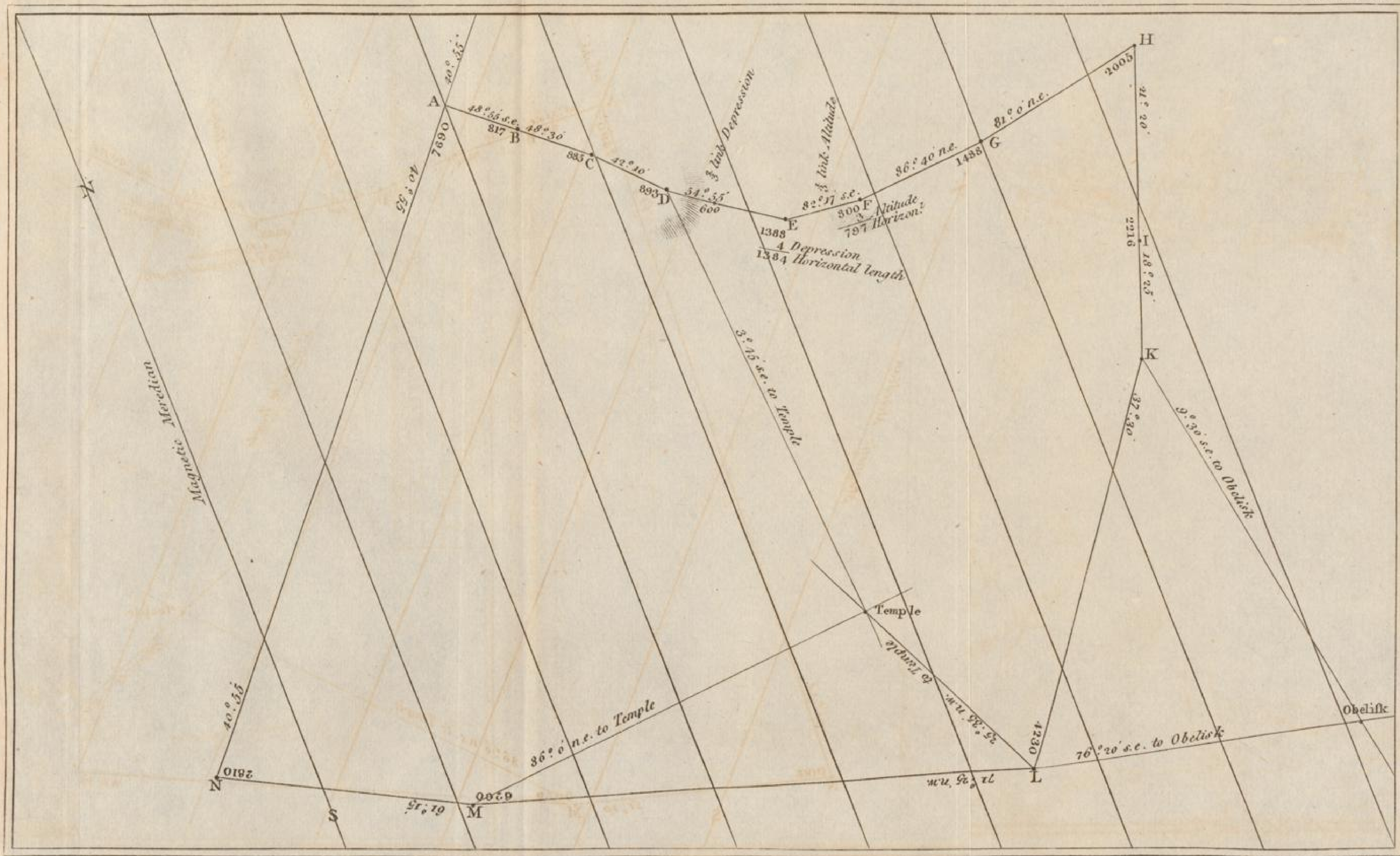


London. Printed for & Published by W. and S. Jones, 135 Holborn, as the Act directs, 3rd March 1797.



Jn.o Lodge sc.

London. Printed for & Published by George Adams, No. 60 Fleet Street as the Act directs. June 1.1791.



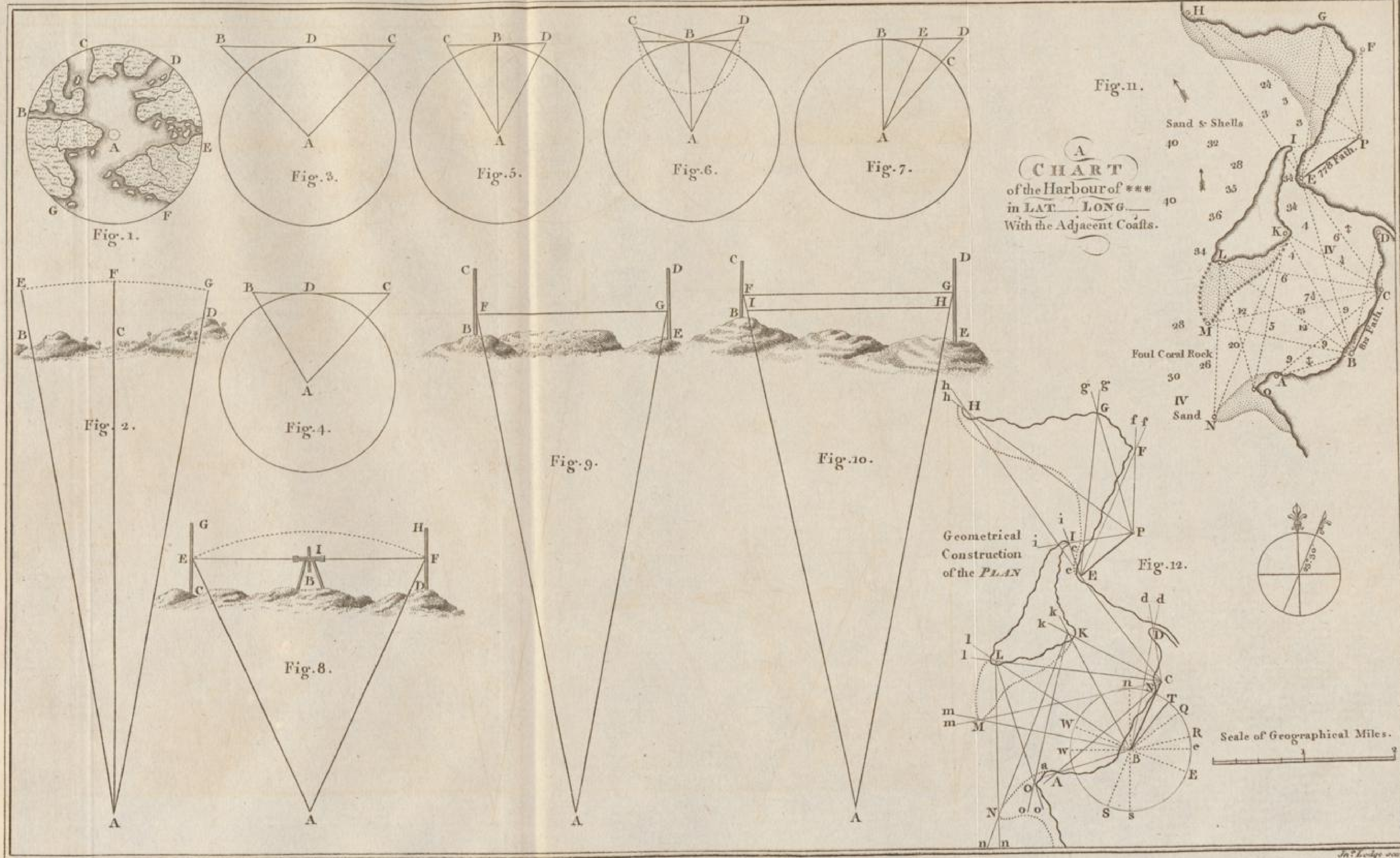
London. Printed for & Published by George Adams. No. 60 Fleet Street as the Act directs. June 1. 1791.

Jn. Lodge sc.

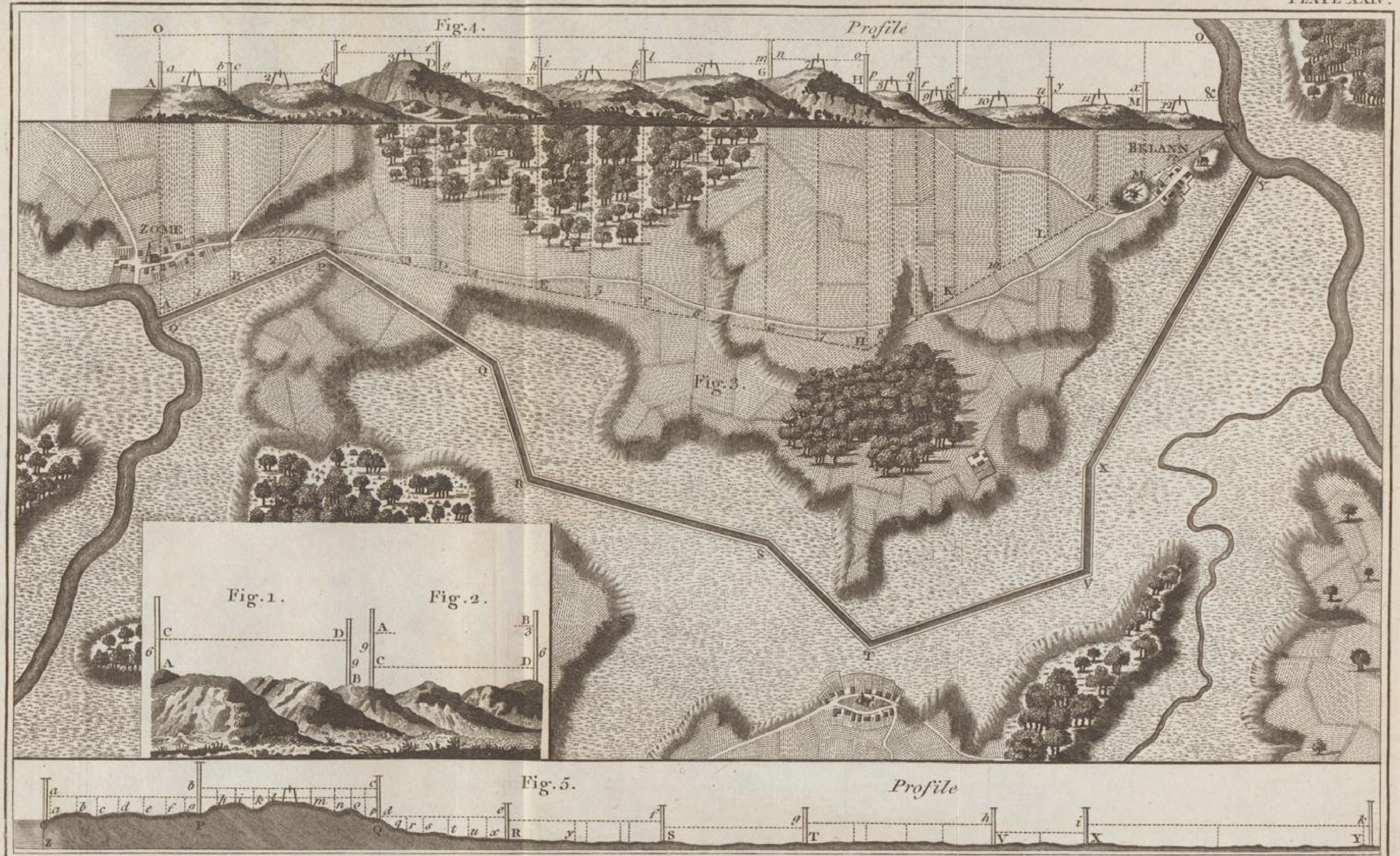
T. Milne delin.t

London Printed for & Published by W. and S. Jones, 135 Holborn, as the Act directs 1.st December 1796.

Jn.o Lodge sc.



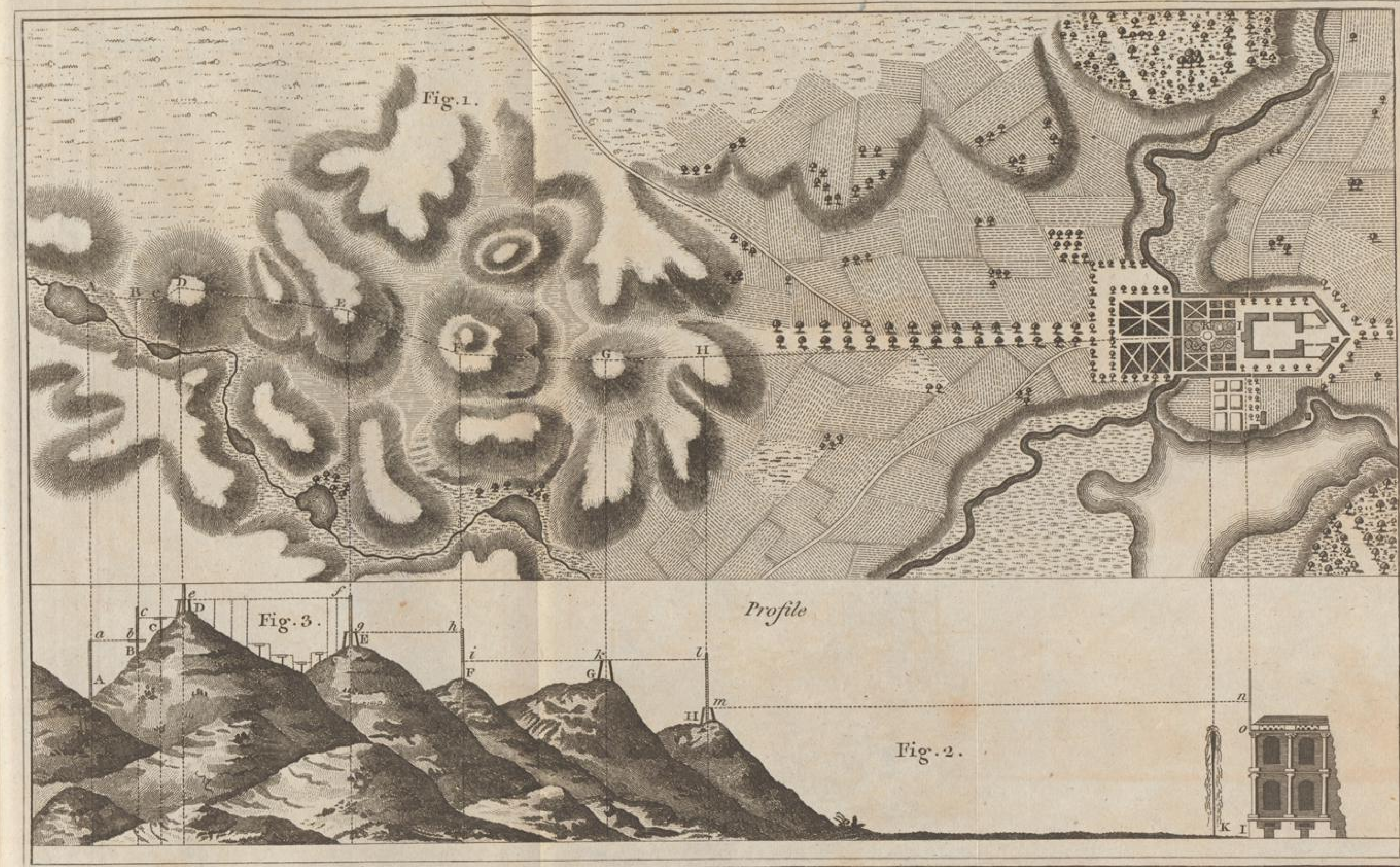
T. Milne delin.
Jno. Lodge sculp.
London, Printed for & Published by W. and S. Jones, 135 Holborn, as the Act directs, 1st December 1796.



T. Milne delin.

J. Lodge sc.

London. Printed for & Published by W. and S. Jones, 135 Holborn, as the Act directs, 1st December 1796.



T. Milne delin.

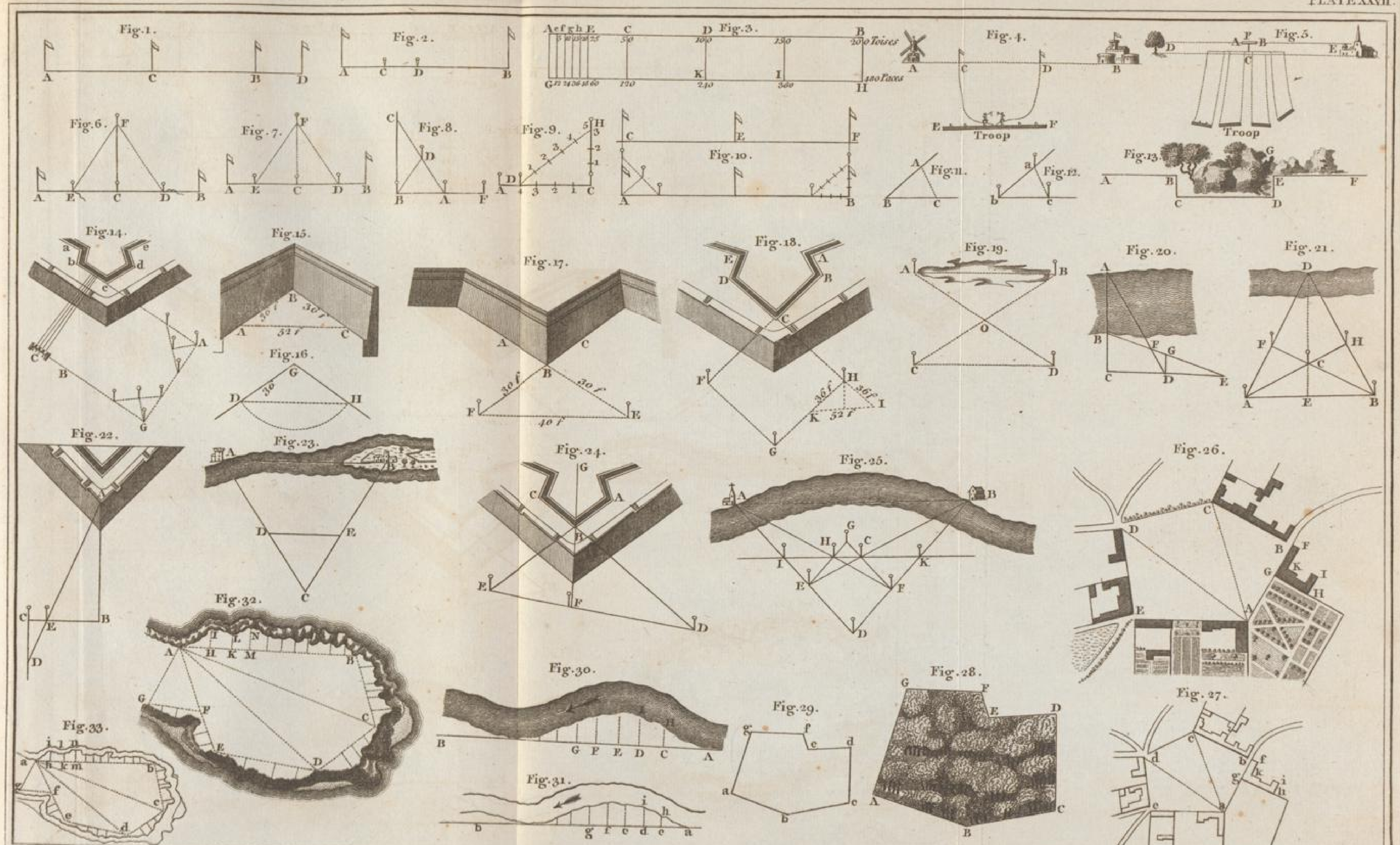
Jn. Lodge sc.

London. Printed for & Published by George Adams, No. 60 Fleet Street, as the Act directs June 1. 1791.

T. Milne delin

Jn. Lodge sc.

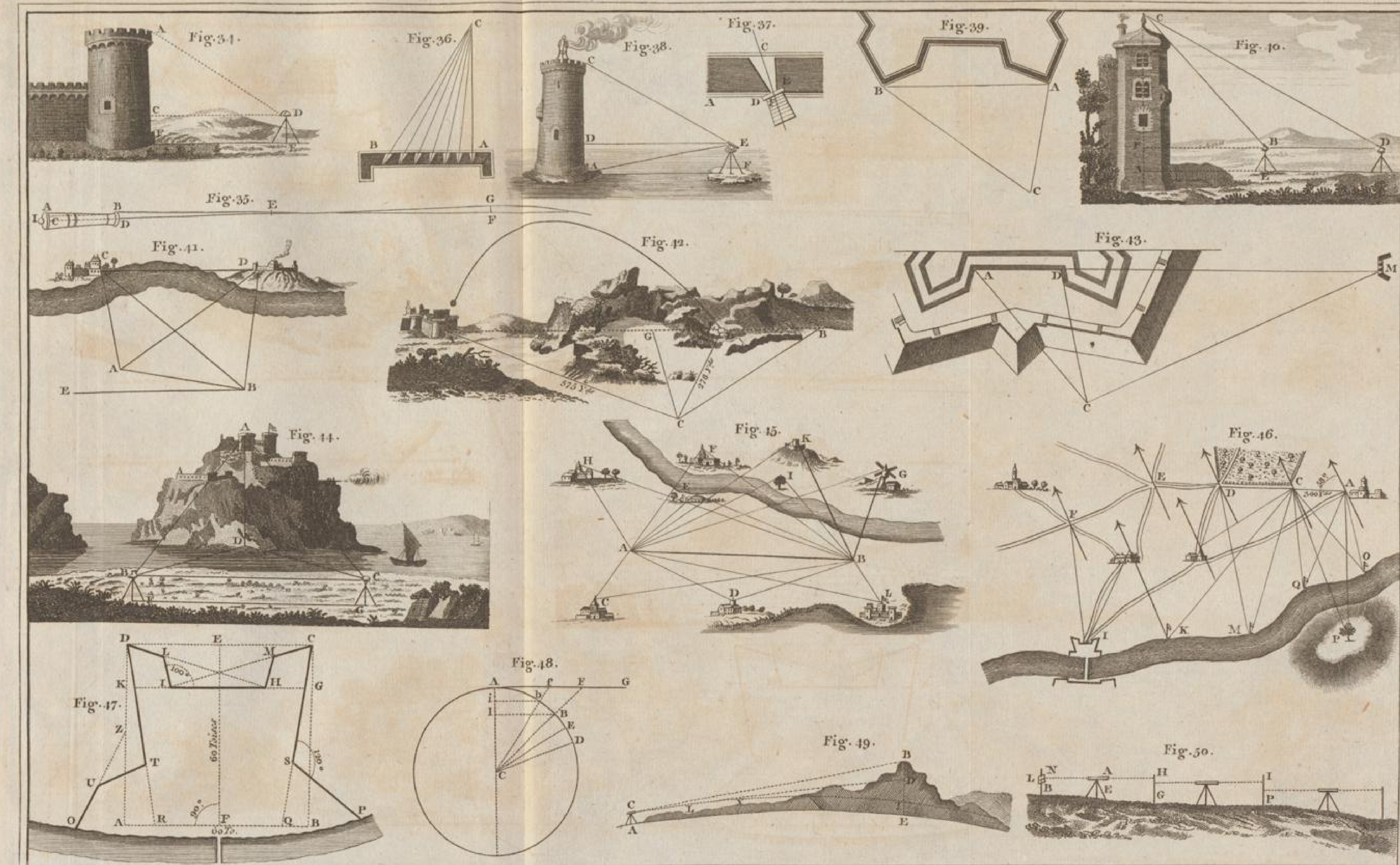
London. Printed for & Published by George Adams, No. 60 Fleet Street. as the Act directs. June 1. 1791.



T. Milne delin.

J. Lodge sc.

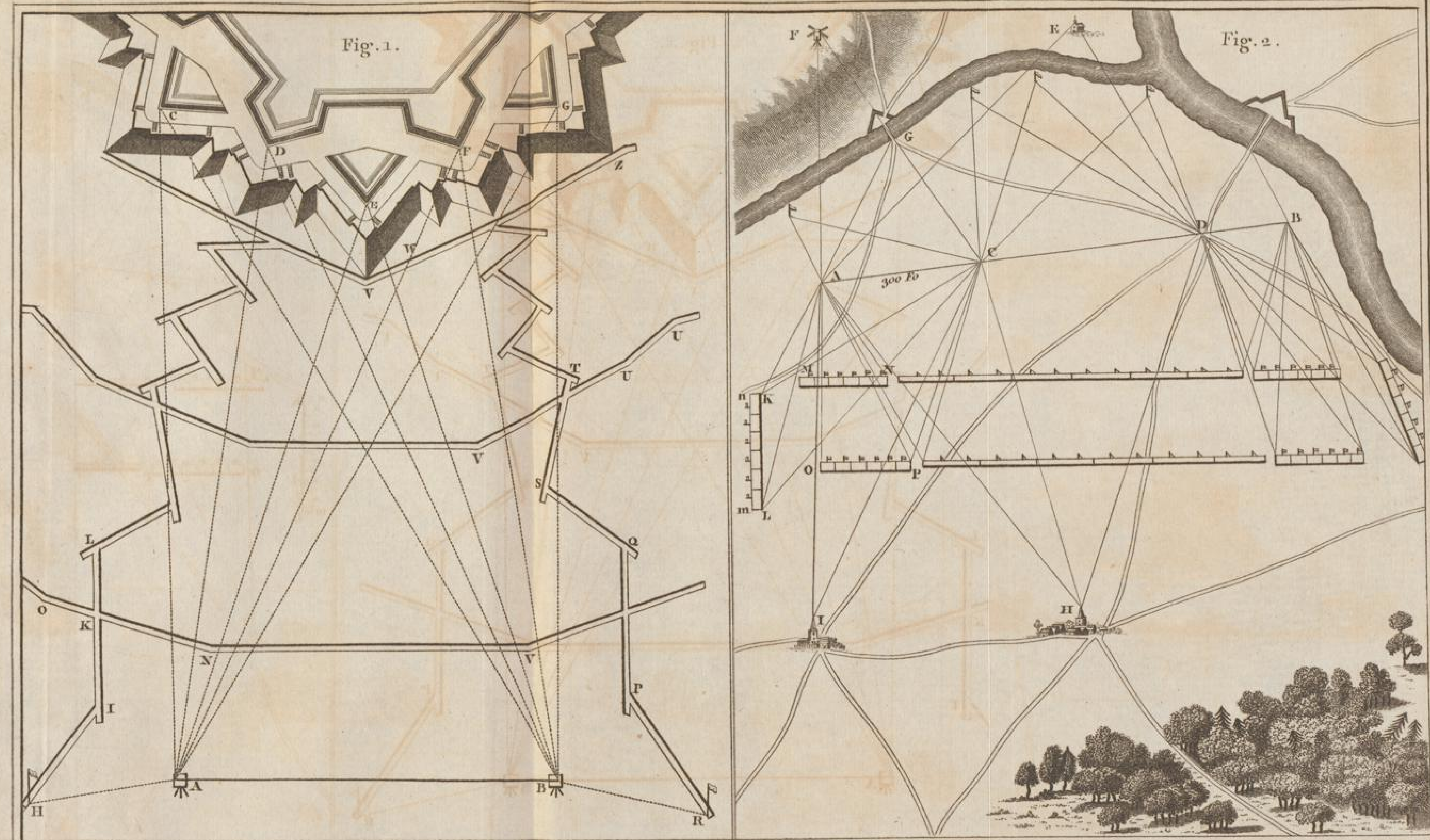
London. Printed for & Published by George Adams. No. 60 Fleet Street, as the Act directs. June 1. 1791.



T. Milne delin.

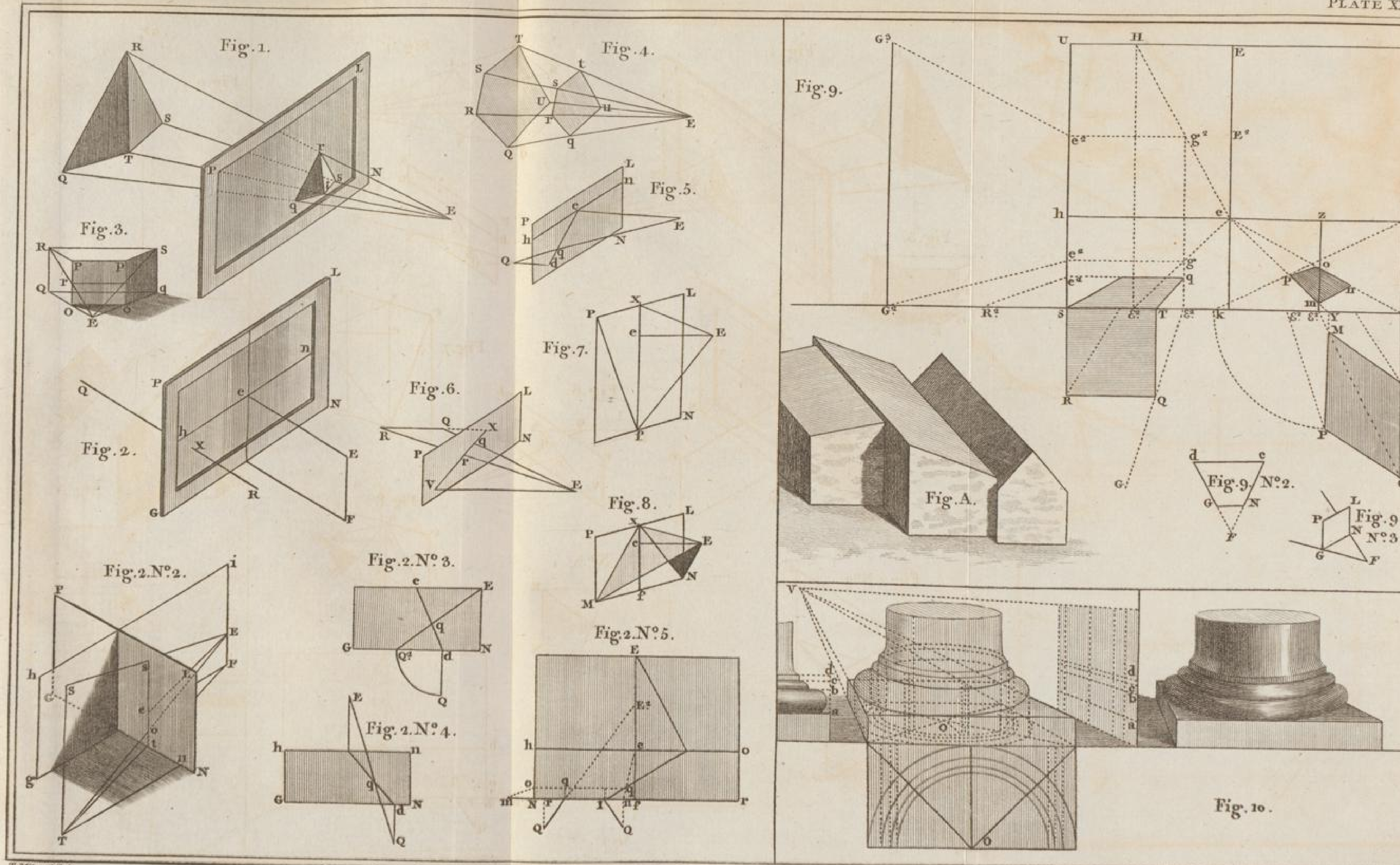
London. Printed for & Published by George Adams. No. 60 Fleet Street, as the Act directs. June 1. 1791.

J. Lodge sc.



T. Milne delin. Jno Lodge Sc.

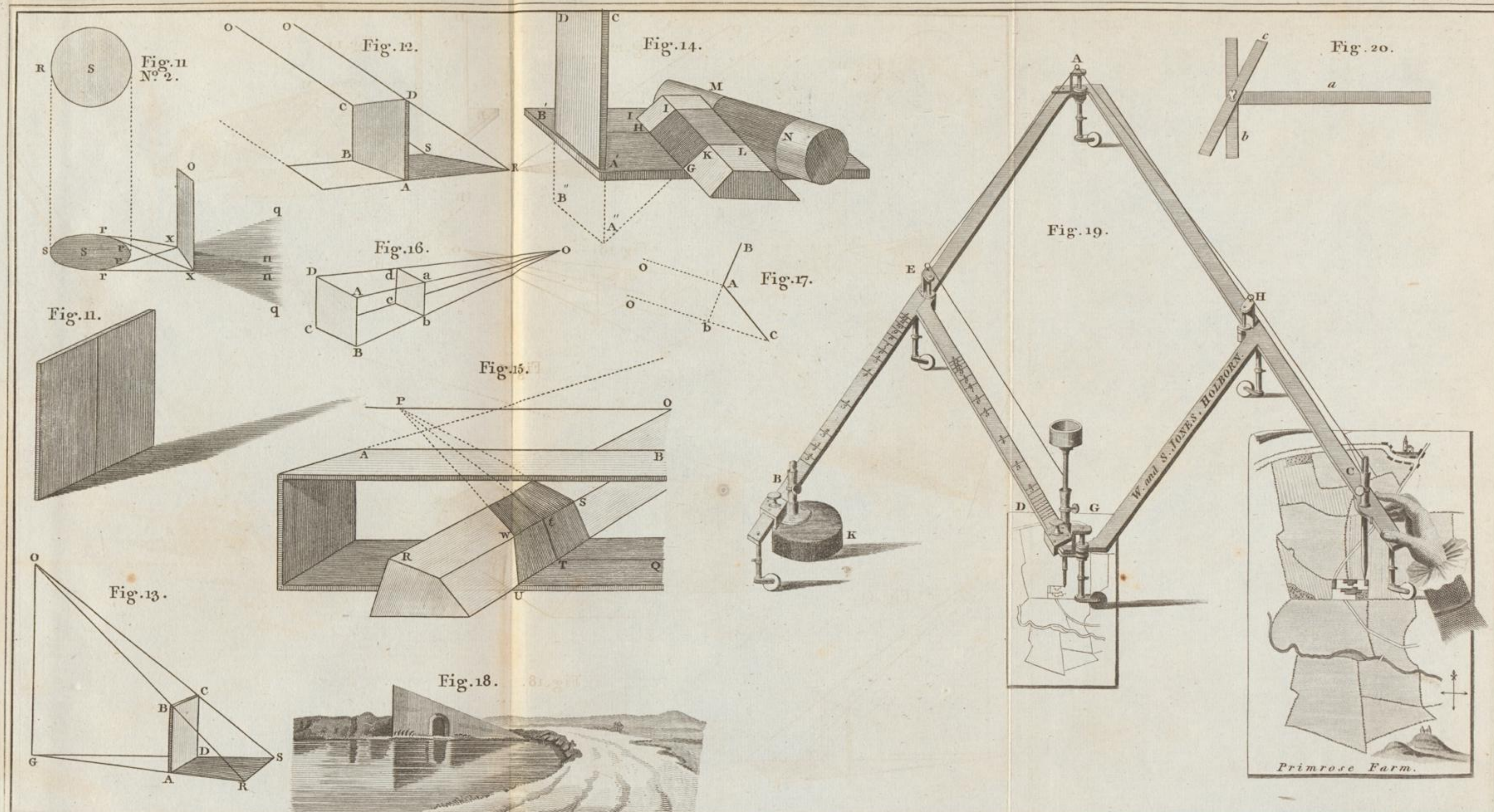
London. Printed for & Published by George Adams, No. 60 Fleet Street, as the Act directs June 1st 1790.



T. Milne delin.

London, Printed for & Published by George Adams N.° 60 Fleet Street, as the Act directs, June 1. 1791.

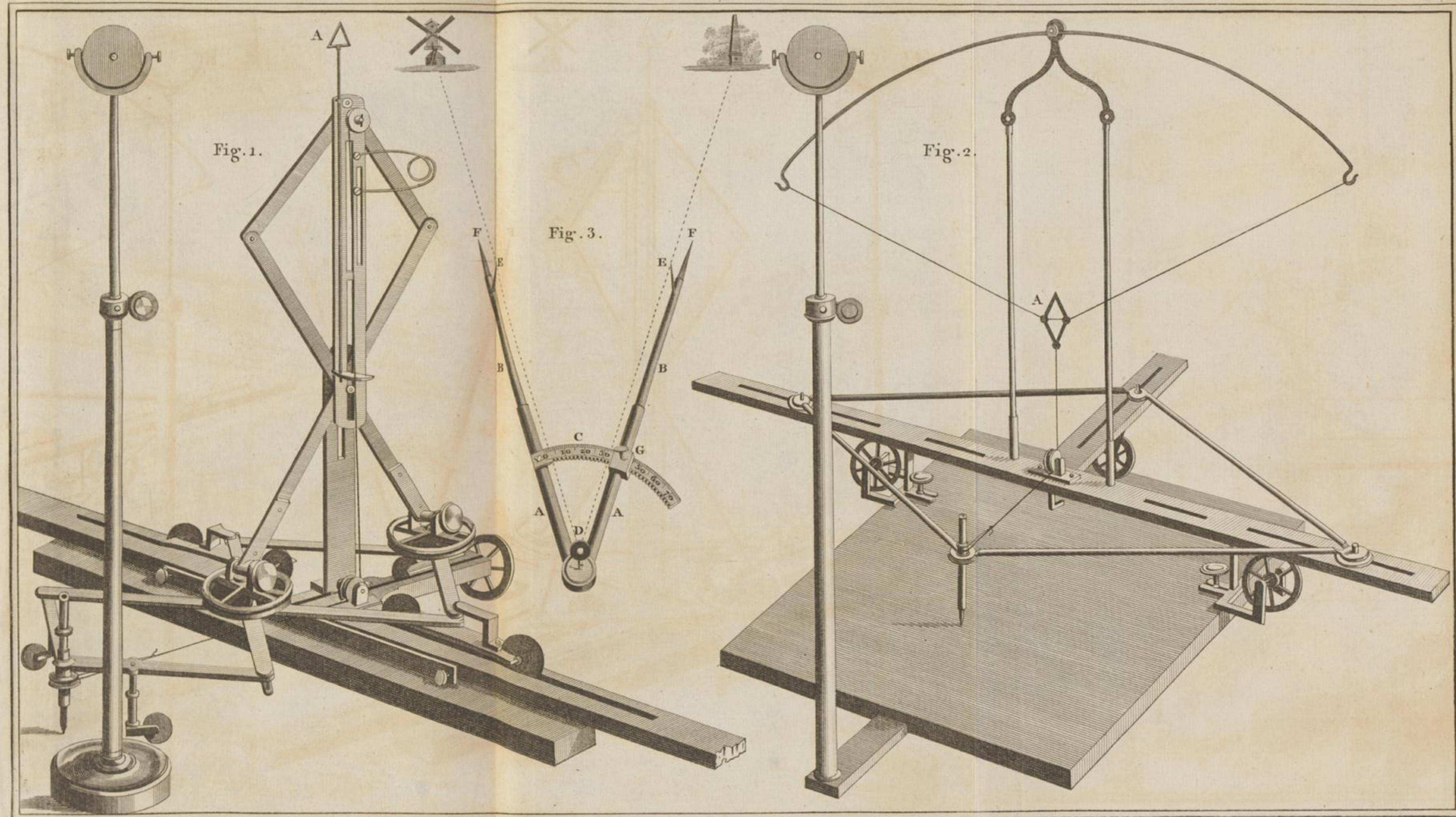
Jn.° Lodge sculp.



Milne delin.t

Jn.o Lodge sculp.

London. Printed for & Published by W. and S. Jones, 135 Holborn, as the Act directs, 3rd March 1797.

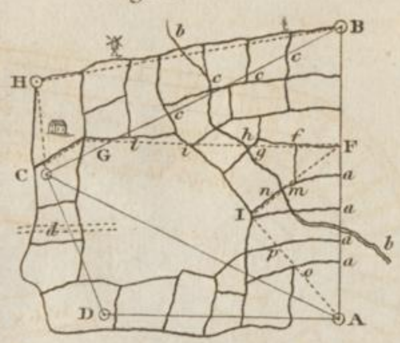


T. Milne delin.t

Jn.o Lodge sculp.

London. Printed for & Published by W. and S. Jones, 135 Holborn, as the Act directs, 1st December 1796.

Fig. 1.



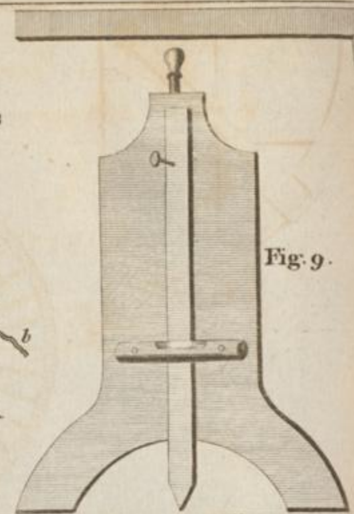


Fig. 9.

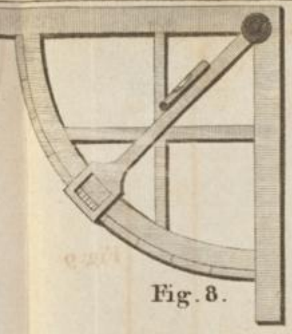
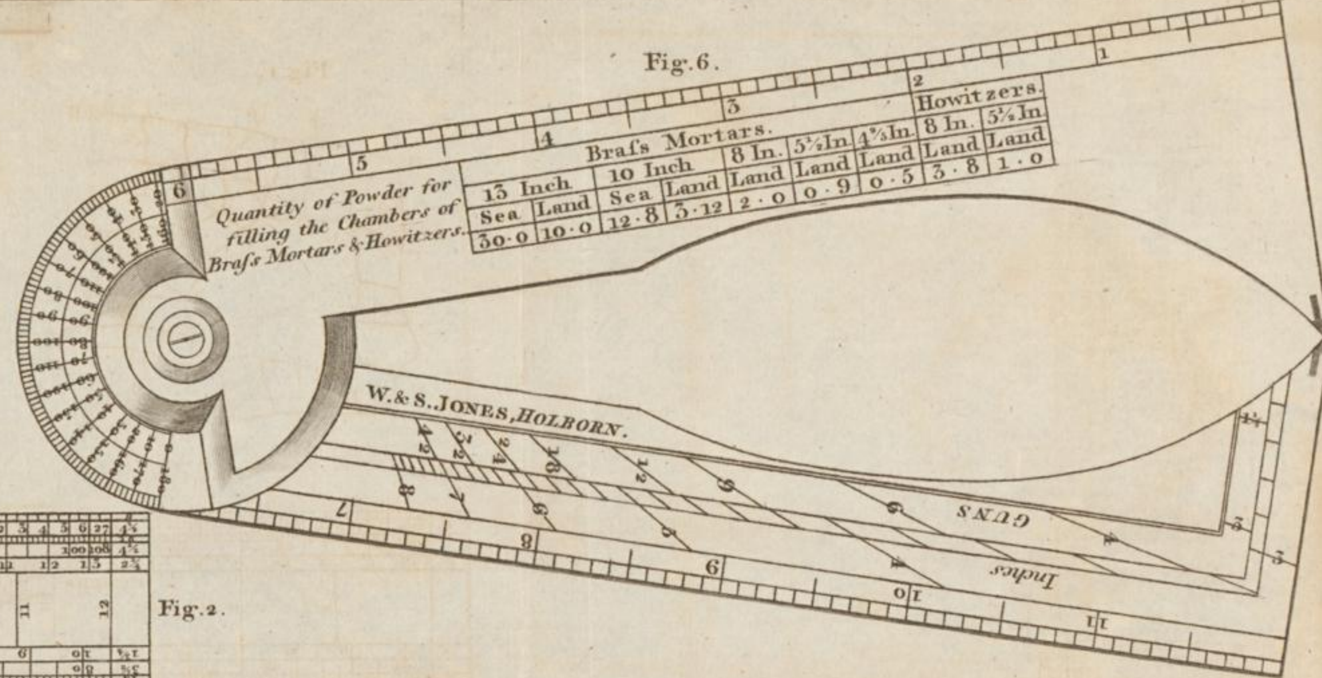


Fig. 8.

Fig. 6.



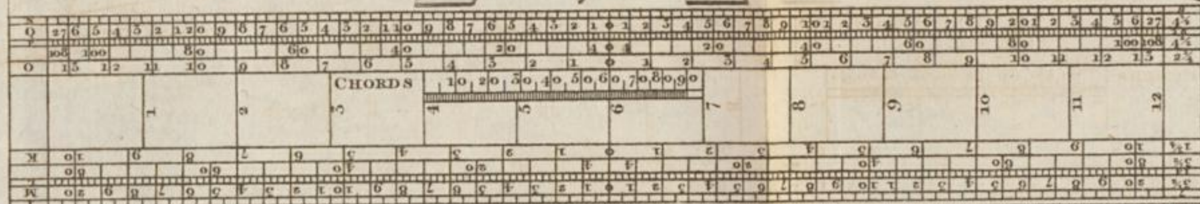


Fig. 2.

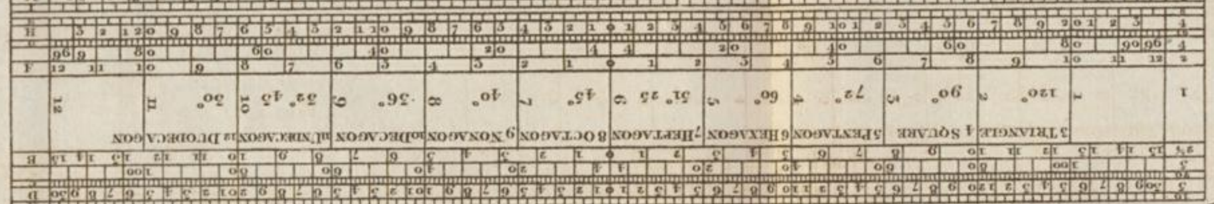


Fig. 3.

Fig. 7.

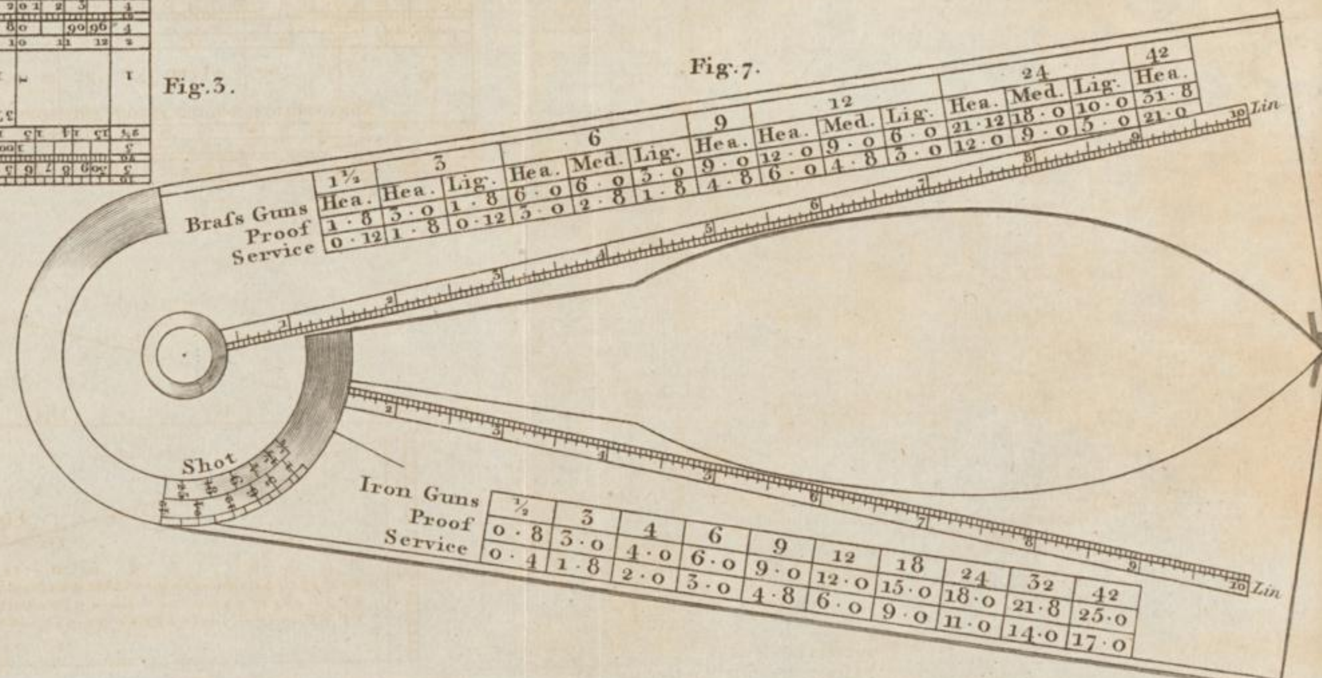


Fig. 4.

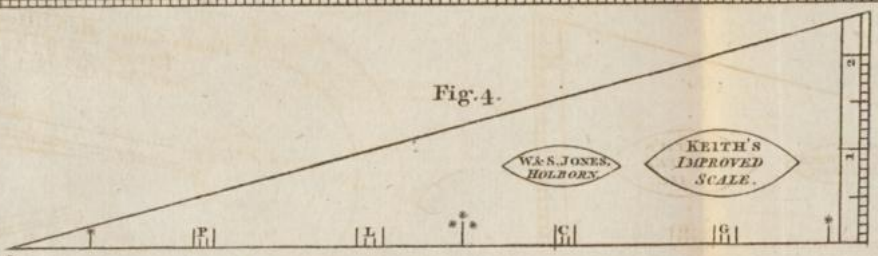
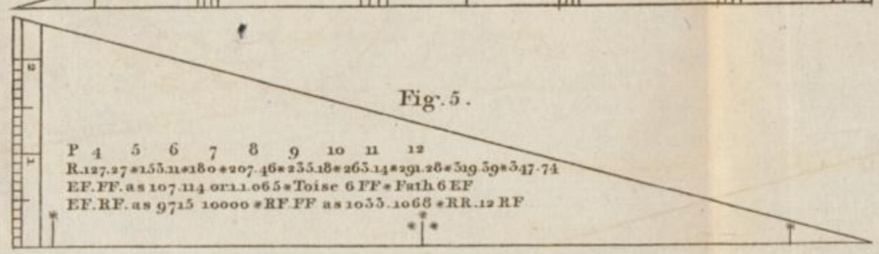
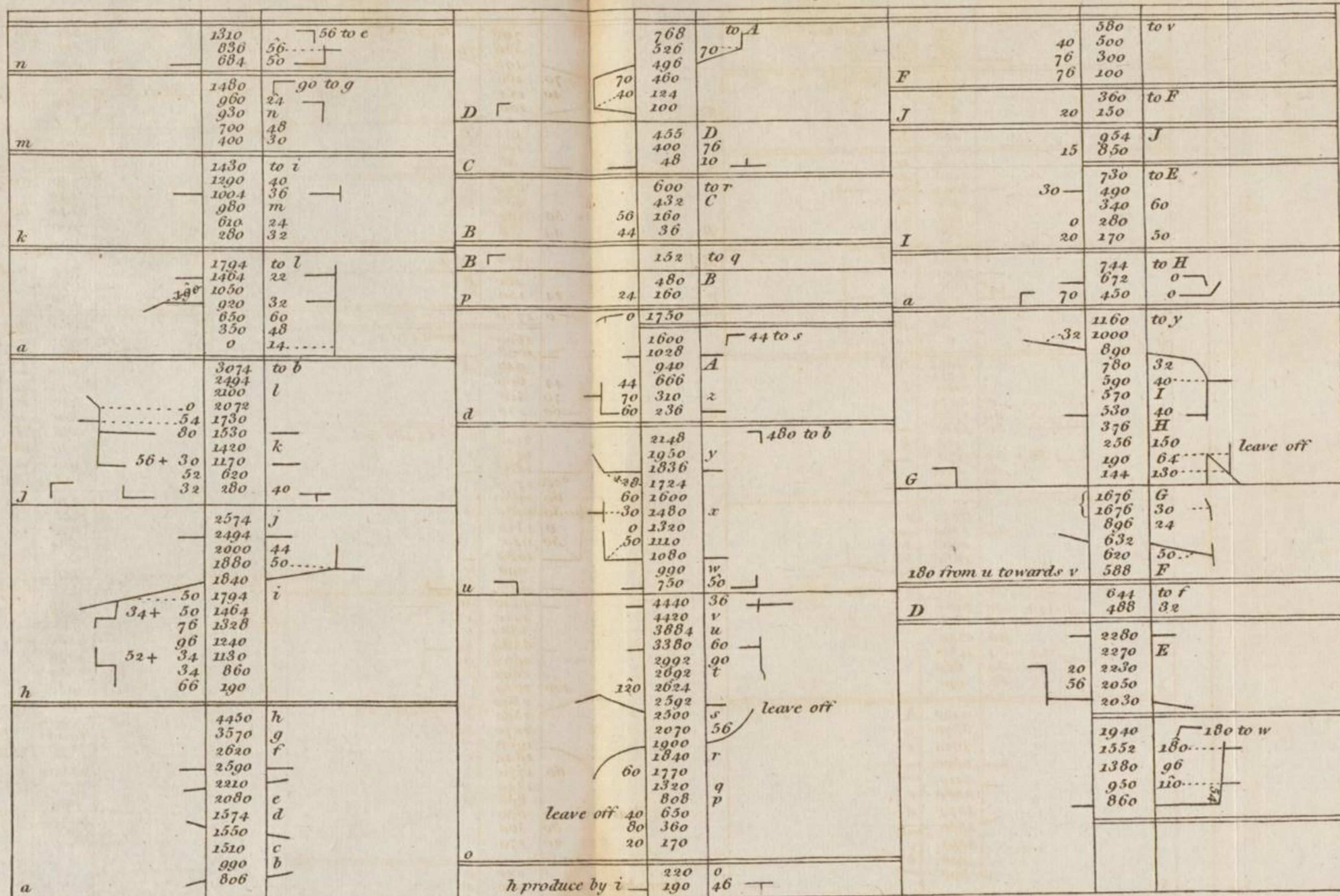


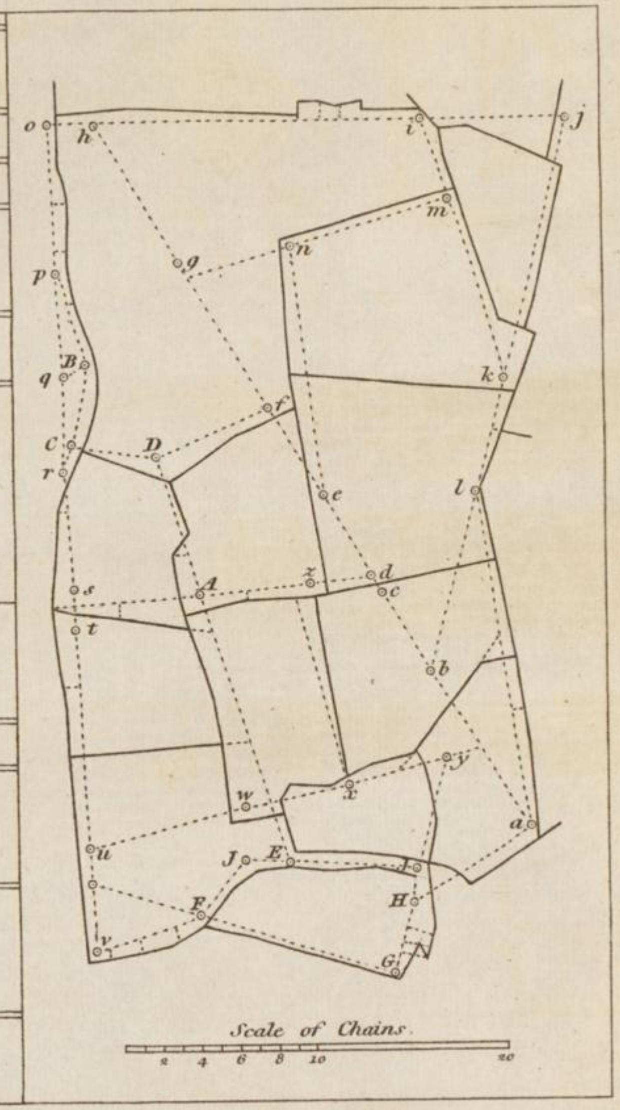
Fig. 5.



New Plan of a Field Book, by Mr. John Rodham.



Plan from the foregoing Field Book.



Lodge sc.

London. Printed for & Publish'd by W. and S. Jones, 135 Holborn, as the Act directs, 3rd March 1797.