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## GBOETHME AND MRIGOTOMEMME,

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## AN EASY AND COMCIBE RZSTEM

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 PUBLISHED BY JAMBE I. CUTLER AND 00.

Sames 1. Culler \& Co. Printero.
1829.

DISTRICT OF NEW-HAMPSHIRE, to voit : District Clerk's Office.
$\overline{\overline{L \cdot S}}$ BE IT REMEMBERED, That on the fifth day of

L.S.October, A. D. 1828, in the fifty-third year of the Independence of the United States of A merica, James Halz, of the said District, has deposited in this Office the Title of a Book, the right whereof he claims as author, in the words following, to wit :
" Elements of Geometry and Trigonometry, with an easy and concise System of Land Surveying. By James Hale.
In conformity to an Act of the Congress of the United States, entitled "An Act for the Encouragement of Learning, by securing the Copies of Maps, Charts and Books, to the Authors and Proprietors of such Copies, during the times therein mentioned :" and also to an Act entitled an Act supplementary to an Act, entitled, An Act for the encouragement of Learning, by securing the Copies of Maps, Charts and Books to the Authors and Proprietors of such Copies during the times therein mentioned; and extending the benefits thereof to the Arts of Designing, Engraving, and Etching, Historical and other Prints.

> CHARLES W, CUTTER, Clerk.

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## PRINEAOTE :

THE principal design of the following work is to present to the $F$ student the common principles of Field Surveying, in a concise, attractive, and intelligible manner. It does not pretend to much originality of matter, but every thing extraneous or foreign to the subject is omitted. It was supposed a work of this descrip, tion would be profitable to the youth of this country, who wish to improve their minds, by extending their knowledge of the Mathematics, beyond the rules of common Arithmetic. It is hoped the execution of the work is such, that, it will be a convenient, and useful assistant, to the practical Surveyor.

The work is divided into two parts.
Part I.-Contains Geometry and Trigonometry, with various explanations, and the use of these sciences, as connected with Surveying.

Part II.-Treats wholly on Surveying, and contains particular directions for taking the Survey of Fields, differently situated. Rules for calculating their Area, Geometrically or Arithmitically, and directions for laying out and dividing land.

While treating on Surveying, instead of prolix and abstruse demonstrations, frequent references are made to the system of Geometry and Trigonometry in the first part of the work. It was thought, that shewing existing analogies would, more effectually, elicit the intellectual powers of the student.

Should this little volume meet the approbation of a candid and enlightened public, whose opinion is ever heard with respect, the author will consider himself amply rewarded for his exertions to promote useful education ; but should it be otherwise, the satiofaction, taken in its compilation, cannot be considered of small account.

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## PARTI. ELEMENTS OF GEOMETRY.

## SEOTNON I。

## DEFINITIONS.

1. Geometry is a Science, by which the measure and properties of Magnitude are determined.
2. A Point is considered as a mark only, without any regard to dimensions.
3. A Line has length, but not breadth.
4. A Right Line is the shortest that can be drawn between two Points.
5. A Superficies or Surface has length and breadth, but no thickness.
6. The measure or content of a Surface is called an Area. Fig. 1.
7. Parallel Lines are such as are equally dis-

B tant from each other as A. B.-C. D. Fig, 1.


Fin2.
8. An Angle is the opening between 2wo Lines, which begin at a Point, and recede from each other. At B is an Angle formed by the opening of the Lines BA and BC. Fig. $\mathcal{L}$.
9. If a Right Line CD, fall upon another Right Line AB, so as to incline to neither side, but make the Angles on each side dual, then those Angles are Right Angles, and the Line CD Perpendicular to AB, Fig. 3.


Fig. 3.


D
10. An Obtuse Angle is greater than a Right Angle, as ADC, and an Acute Angle is less than a Right Angle; as CDB. Fig. 4.


Note - When an Angle is expressed by three letters, the middle letter represents the Angular Point. A Right Angle contains 90 degrees.

Fig. 5.
11. A Triangle is afigure bounded by three Lines ; as ABC. Fig. 5.

12. The most Natural Division of Triangles is into two kinds, viz:-That of Right Angled Triangles, and Oblique Angled Triangles.
18. A Right Angled Triangle bas one Right Angle; as ABC. Fij. 6.
14. A Triangle, constructed in any other manner, is an Oblique Triangle: as ABC. Fig. 5 or 7.

15. In a Right Angled Triangle, the longest side is called the Hypothonuse, and the other two, the Legs or Base, and Perpendicu-
lar. In Oblique Triangles, any side may be called the Base, and the other two, the Legs or Sides.
16. The Height of a Triangle is a Perpendicular Line, falling from any Angle to its opposite Side. AD is the Perpendicular Height of the Triangle ABC. Fig. 8.

17. If the Perpendicular fall without the Triangle, the Base must be continued to determine its Length. CE is the Perpendicular height of the Triangle, ABC ; the Base being continued to E. Fig. 9.


Fi. 10.
18. If one foot of the Dividers be Exed at the Point C , and being open to a certain extent the other foot be carried round, the space comprehended is called a Circle, the Curve Line thereby described is the Circumference or Periphery of the Circle, and the Point $\mathbf{C}$ its Centre. Fig. 10.

19. The extent in the Dividers, being the length of the Line $G$ D, is Semidiameter or Radius. Whence it is manifest, from the construction that all Radii of the same Circle are equal. Fig. 10.
20. The Diameter of a Circle is a Right Line drawn from one side of the Circumference, through the Centre, to the other side, dividing the.Circle into two equal parts, called Semicircles; a AB, or DE. Fig. 10.
21. Art Arch, or Arc, is any part of the Circumference of a Circle; as DF, or AGE. Fig. 10.
22. A Chord is a Right Line, drawn from one end of an Arch
to the other end, and is the measure of the Arch. FG is the Chord of the Arch FAG. Fig. 10.

Note.-The Chord of an Arch of 60 Degrees is equal, in length, to the Radius of the Circle.
23. A Segment of a Circle is the Space, or Area, comprehended between a Chord and the Circumference; as FAGF. Fig. 10.
24. A Quadrant is one quarter of a Circle; as BCD. Fig. 10
25. A Sector of a Circle is a part thereof contained between two Radii, and an Arch less than a Semicircle ; as FCD, or F CE. Fig. 10.
26. The Complement of an Arch is what it wants of 90 De grees, or a Quadrant. FD is the Complement of the Arch AFD. Fig. 10.
27. The Supplement of an Arch is what it wants of $180 \mathrm{De}-$ grees, or a Semicircle. BDF is the Supplement of the Arch FA. Fig. 10.
28. The Circumference of every Circle is supposed to be divided into 360 equal parts, called Degrees ; each Degree into 60 equal parts, called Minutes; and these into Thirds, \&c.
29. The measure of an Angle is the Arch of a Circle contained between two Lines which form the Angle, the Angular Point being the Centre; thus the Angle DCF is measured by the Arch D F. Fig. 10. Hence, an Angle is greater or less, according to the opening of the Lines which form it, without jegarding their length.

Fig. 11.
30. A Square is a Figure bounded by four equal sides, and having four Right Angles. Fig. 11.

Fig. 12.
31. A Parallelogram or Oblong Square is bounded by four Sides, the opposite ones being equal, and the Angles Right. Fig. 12.

Fig. 13.
32. A Rhombus is an inclined Square, having its Angles Oblique. Fig. 13.

33. A Rhomboides is an inclined Parallelogram,: having its Angles Oblique. Fig. 14.

Fig. 14.


B
Fig. 15.
A
34. A Trapezoid is a part of a Mriangle, cut by a Line Parallel to its Base, having two Parallel sides; though of unequal length. Fig. 15.


B
35. The Perpendicular Height of a Rhombus, Rhomboides, or Trapezoid is a Line drawn from one of its Angles to its opposite side, thus the dotted lines AB , in the three last figures, represent their Perpendicular Height.

Fig. 16.
36. A Trapezium is a Figure of four unequal Sides. Fig. 16.

37. A Diagonal is, a Line drawn between two opposite Angles ; as the Line AB. Fig. 16.
38. Figures, consisting of more than four Sides, are called Polygins, ; if the Sides be equal to each other, they are called Regular Polygons ; if unequal, Irregular Polygons. They are sometimes named from the number of their sides. One of five sides is called a Pentagon; of six a Hexagon; of seven a Heptagon ; of sight an Octagon, \&c.

## SNOTNON IR

## GEOMETRICAL PROBLEMS.

PROBLEM I. To bisect or divide into two equal parts, a given Right Line AB. Fig. 17.


With any distance in the Dividers more than half the given Line, with one foot in A, describe an Arch above and below the Line; with the same distance, and one foot in B, describe Arches crossing the former ; draw a line through the intersection of those arches crossing AB ; then $\mathrm{AE}=\mathrm{EB}$.

Fig. 18. D

PROBLEM II. To erect a PerFendicutar from the end, or any part of a given Line AB. Fig 18.


With any distance, set one foot of the Dividers on the Point from which the Perpendicular is to be erected, as at $\mathbf{C}$, and describe an arch GEF ; set off the same distance from G to C, and from E to F ; upon E and F as Centres, describe two Arches at $\mathbf{D}$; from their intersection to the point $\mathbf{C}$ draw $\mathbf{C D}$ a Perpendicular.

Another method. Lay the Centre Point of the Protractor on the Point C, with the Arch upwards, and the edge exact'y on the Line AB; at 90 degrees, on the Arch of the Protractor, make a Point in the paper ; from which to the Point $C$, draw the Perpendicular.

PROBLEM III. From a point A, to drop a Perpendicular on a given Line BC.
Fig. 19.


From any Point as at $\mathbf{F}$ in the given Line, with the distance from F to A , describe an Arch ADE ; make $\mathrm{DE}=\mathrm{AD}$; from the Point A, in a direction towards E, draw the Perpendicular.

Fig. 20.
PROBLEM IV. Through a given point $D$, to draw a line $C D$, parallel to a given line $A B$. Fig. 20.


From any Point in the given line, as at E, with the distance from $E$ to $D$, and one foot of the Dividers in $E$, describe the Arch DF ; with the same distance, and one foot in D , describe EC; make EC=DF; through the Points $C$ and $D$ draw CD , which will be parallel to AB .

PROBLEM V. To draw a line CD, parallel to a given line $\boldsymbol{A B}$, at a given distance.
 Fig. 21.

Take the given distance in the Dividers, and, from any two Points, in the given Line, as at E and F , describe two Arches; draw the Line CD, touching the extremities of these Arches.

Note. In the practice of surveying, much depends on Parallel Lines being drawn with accuracy ; the Points made on paper should be as small as possible.

Fig. 22.
PROBLEM VI. To make a Mri- B——O angle of three given Lines as BO. BN. NO. Fig. 22.


## GEOMETRY.



Draw BN, from $B$ to $N$; from $B$, with the length of the Line BO, describe an Arch as at $\mathbf{O}$; and from N , with the Line NO, describe another Arch, intersecting the former ; from the intersectio a draw the Lines BO and NO, and the Triangle is completed.

Fig. 23.


Open the Dividers to any convenient distance, and with one foot in $\mathbf{B}$, describe the Arch, fg ; with the same distance and one foot in D describe HL ; make HL equal to fg; through the Point I draw LD, and the Angles will be equal.

PROBLEM VIII. Tomake an Acute Angle at the Point B on the Line AB ; suppose of 38 Degrces. Fig. 24.


Take 60 Degrees, in the Dividers, from the line of Chords on the Scalef (see note def, 22) with one foot in B describe an Arch from the Line A B; from the same Scale, take 38 Degrees, , and lay it on the Arch from $\mathbf{C}$ to $\mathbf{E}$; through $\mathbf{E}$ draw $E B$, and the $A a_{1}$ gle at $B$ will consist of 38 Degrees. Or, lay the Centre Point of the Protractor on B, with the edge on the Line AB, and torn the Arch upwards or downwards, as the Angle is to be made ; prick off the number of Degrees, contained in the required Angle, and draw EB,

## PROBLEM IX.

To make an Obtuse Angle, suppose of 115 Degrees. Fig. 25.

Fig. 25.


Describe the Arch CE by the last Problem; set of 90 Degreer from $C$ towards $E$, from which point set off the excess above 90 , siz : 25 to E; from E to B draw a Line, and the Angle at B will contain 115 Degrees.

The construction of this by the Protractor is manifest, from the directions in the last Problem.

PROBLEM X. To measure a given Angle, Fig. 24 or 25.
Describe an Arch by Problem 8; and if the Angle be Obtuse, take a certain part of the Arch in the Dividers, measure this digtance on the line of Chords; find the number of Degrees contained in the remaining part of the Arch; add them together, and their sum will be the measure of the Angle. Or lay the Centre of the Protractor on B, with the edge on BC, and, turning the Arch of the Protractor in the proper direction, the number of Degrees contained in the given Angle, is seen, on the Arch, over the Line BE.

Nore. If the Lines, which form the Angle, are not of a sufficient length to admit of either of these operations, they must be continued, to a proper distance.

PROBLEM XI. To make a Square, the length of one side being given. Fig. 26.


Draw AB, the given length; from $\mathbf{B}$ erect a Perpendicular to $\mathbf{G}$ of the same length; from $A$ and $C$ as Centres, with the same distance in the Dividers, describe Arches ax $D$; fiom their intar section draw Lines to $C$ and $A$. a Parallelogram. Fig. 27.

Fig. 27.

## PROBLEM XII. To make



Draw AB, equal to the longest Side of the Parallelogram; on $B$ erect a Perpendicular to $\mathbf{C}$, the length of the shortest Side; from $C$, with the length of the longest Side, describe an Arch at $D$. and from $A$, with the length of the shortest side, describe an Arch intersecting the other ; from which Intersection draw Lines to $G$ and $A$.

PROBLEM XIII. To describe a Circle, which shall pass through any threegiven Points, not in a Straight Line. Fig. 28.


Let the three given Points be ABC ; draw Lines from $A$ to $B$, and from B to C ; Bisect those Lines by Problem I. The Point D, where the Bisecting Lines eross each other, will be the Centre of the Circle, the Radius of which, is the distance from $\mathbf{D}$ to any of the given Points.

Note. By an application of this Problem, the Centre of a Cirde may be found, when the whole, or a part only, of the Circumfer ence is given.


## EEONION ITH.

## TRIGONOMETRY.

Trigonometry is a Science, by which the Sides and Angles of Triangles are measured. This may be performed by Geometry, with a Scale, Dividers, and Protractor ; or by Calculation, with the assistance of Logarithms ; it may likewise be performed by the

Dividers and the Lines of Numbers, Sines and Tangents, on Gunter's Scale.

The Geometrical method only is here given. This will be' found the readiest method, its application to surveying the most easy, and it is sufficiently exact, if proper care be taken in drawing the Figures. Trigonometry is not often necessary in Surveying ; but cannot be entirely dispensed with, as will be hereafter shewn. It is of two kinds, Rectangular and Oblique.

As Trigonometry is confined to the measuring of Triangles, it may be proper, here, to introduce the following remarks, to inform the learner of some of the properties of a Triangle, and the Proportions which exist between them.
Remark I. The three Angles of any Triangle, when added together, amounts to 180 Degrees.-Hence, if one Angle of a Triangle be known, subtract it from 180, and the remainder is the sum of the other two ; and if two Angles be known, their sum being subtracted from 180, the remainder is the other Angle.

Remark II. In every Right Angled Triangle the two Acute Angles are equal to 90 Degrees; therefore, if one Acute Angle be substracted from 90, the remainder is the other Angle.

Remari III. In every Right Angled Triangle the Square of the Hypothenuse is equal to the sum of the other two sides.-Hence, the Hypothenuse of a Right Angled Triangle mey be found, by having the sides, thus; The Square Root, of the sum of the Squares, of the Base and Perpendicular, will be the Hypothenuse.-Having the Hypothenuse, and one Side, given to find the other; The Square Root, of the difference of the Squares, of the Hypothenuse and the given Side, will be the required Side.

Remark IV. Triangles, having the same height, and standing on the same, or equal Bases, are of equal Area.-This remark applies to laying out land, in the form of a Triangle.

Remark V. Triangles, having the same Height, but different Bases, are in proportion to each other as their Bases.-This remark applies to the division of a Triangle.

Remari VI. Equiangular Triangles have the same proportion to each other as exists between the Squares of their Homologous, or like Sides ; that is, if the Angles of two Triangles be respectiveIy equal to each other, then as the Area of one Triangle, is to the Square of its longest side, so is the Area of the other Triangle to the Square of its longest side ; and so of the other sides. This remark applies to the division of a Trapezoid or Triangle, by Linen parallel to their sides.

## RECTANGULAR TRIGONOMETRY.

By this, are measured the Sides and Angles of Right Angled Trir aagles.

## CASE 1.

The Angles and Hypothenuse given, to find the Base and Porpendicular.

In the Triangle ABC, suppose the Angle A $38^{\circ} 30^{\prime}$ consequently the Angle $\mathrm{C}_{\mathbf{2}} 51^{\circ} \quad 30^{\circ}$ (see $\mathrm{Re}-$ mark 2.) and the Hypothenuse 30 parts (as feet,rods or chains) required AB and BC. Fig. 29.

Fig. 29.


Note. Sides and Angles, which are given, are marked thua, those required thas, 0 .
Draw the Line AB at pleasure; at A, make an Angle of $38^{\circ}$ 30 (by Prob. 8, Sec. 2.) and draw AC, in length 30, from a Scale of equal parts (see the description of Gunter's Scale, Part. 2. Sec. 1.) from C, drop a Perpendicular on AB, and the Triangle is completed. Measure AB , and BC from the same Scale that AC wa taken, and the answer will be, AB 23, 5 and BC 18, 6.

[^0]
## CASE II.

The Base and Angle given, to find the Perpendicular and $\mathrm{Hy}_{\boldsymbol{y}}$ pothenuse.

Fig. 30.
In the Triangle ABC, the Angle $A$ is $33^{\circ}{ }^{40}$, consequently the Angle C $56^{\circ} 20^{\prime}$, and the Base AB 86; required the Hy pothenuse $A C$, and the Perpendicular BC. Fig. 30.


Draw AB, in length 86, from a Scale of equal parts ; at $B$ erect a Perpendicular at pleasure (by Problem 2. Sec. 2.) and at A make an Angle of $33^{\circ} 40^{\circ}$, and draw $A C$ which will intersect $B C$ in $C$, and the Triangle is completed. Measure AC and BC from the same Scale that AB was taken from, and the answer will be AC, 103, 3 and BC, 57, 3.

## CASE III.

The Base and Perpendicular given, to find the Angles and Hypothenuse.

Fig. 31.
In the Triangle ABC there is given AB 64, and BC 4!9, to find the Angles A and C, and the Hypothenuse AC. Fig. 31.


Draw the line $A B$ in length 64 from a Scale of equal parts; at $B$ erect a Perpendicular, on which lay 49 , from $B$ to $C$, from the same Scale; join the Points $A$ and $C$ by drawing AC, and the Triangle in constructed. Measure AC, from the same Scale, and it will be B 2

80, 6. Measure the Angles (by Problem 10, Sec. 2.) and the Angle at A will be $37^{\circ} 30^{\circ}$, and the Angle at $\mathrm{C}, 52^{\circ} 30^{\circ}$.

Having found the Angle at A, the Angle at B may be found by nubtracting $37^{\circ} 30^{\prime}$ from $90^{\circ}$. (See Remark 2.)

The Hypothenuse AC may be found by the Square Root, withont constructing the Triangle, thus; AB, 64, squared is 4096 , and BC, 49, squared is 2401; their sum is 6497, the Square Root of which in 80, 6, nearly'for AC. (See Remart 3.)

## CASE IV.

The Base and Hypothenuse given, to find the Angles and Porm pendicular.

In the Triangle ABC, there is given the Base AB 40, and the Hypothenuse AC 50, to find the Angles A and C, and the Perpendicular BC. Fig. 32.


Draw the Line AB, in length 40 ; on $B$ erect a Perpendicular an indefinite length ; then take 50 in the Dividers, with one foot in A , cross the Perpendicular in $\mathbf{C}$; draw a Line from $\mathbf{A}$ to $\mathbf{C}$ and the Triangle is constructed. Measure BC on the same scale that AB: and $A C$ were taken, and the answer will be 30 . Measure the $A n-$ gles $A$ and C, with a Protractor or Line of Chords; the Angle at A is $37^{\circ}$ and the Angle at $\mathbf{C}, 43^{\circ}$.

BC may be found by the Square Root, thus; the Square of AB. being subtracted from the Square of $A C$, the remainder willibe the Square of BC. (See Remark 3.)
$A C 50 \times 50=2500$
AB $40 \times 40=1600$

## Case V.

The Angles and Perpendicular given, to find the Base and Hypothenuse.*

Fig. 33.
In the Triangle ABC suppose the Angle A $40^{\circ}$, consequently the Angle C $50^{\circ}$, and BC 170, to find AC and AB. Fig. 33.


Draw a Line at pleasure ta represent $A B$; on $B$ erect a Perpendicular to C , in length 170 ; at $\mathbf{C}$ make an Angle of $50^{\circ}$; the Angular Line drawn from 6 will intersect the Line $A B$ in $A$, which completes the Triangle. Measure AB and AC on the same Scale of equal parts that $B C$ was taken, and the answer will be, $A B 202,6$, and $A C$ 264, 5.

## CASE VI.

The Perpendicular and EIypothenusa given, to: find the Angles
Fig. 34.

In the Triangle ABC there is given BC 306, and AC 370. to find the Angles $A$ and $C$, and the Base AB. Fig. 34.


Draw a Line at pleasure to represent the Base $A B$;: $B$ erect - Perpendicular in length 306; with the Line AC 870 in the Di-. viders, and one foot in $\mathbf{C}$, cross the first drawn Line in $\boldsymbol{A}_{i_{,}}$dram:
the Line $\mathbf{A C}$ and the Triangle is completed. Measure $A R$ on the same S : ale from which BC and AC were taken, and the answer will be 208. Measure the Angles A and $\mathbf{C}$ with a Protractor or Line of Chords, and the Angle at A will be $55^{\circ} \mathbf{4 8 ^ { \prime }}$, at $\mathbf{C} \mathbf{3 4}{ }^{\circ} \mathbf{1 2}$.

The operation to find AB by the Square Root must be obvioua from Case 4.

## OBLIQUE TRIGONOMETRY.

By this, are measured the Sides and Angles of Oblique Triasglea.

## CASE I.

Twoo Angles and one Side given, to find the other Sideai Fig. 35.

In the Triangle ABC the Angle at $\triangle$ is $48^{\circ}$, at ${ }^{6} 60^{\circ}$, and the Side AB is 200 ; required the Sides AC and BC. Fig. 35.


Draw the Line $A B$ in length 200; on the Point $A$ make an Angle of $48^{\circ}$, at B make an Angle of $60^{\circ}$; the intersection of the Lines at C, forming the Angles constitutes the Triangle. Measure the required Lines on the same Scale from which AB was takon, and the answer will be AC 182, and BC 156.

## CASE II.

Two Sides and an Angle opposite to one of them given, to find the other Augles and Side.

In the Triangle $A B C$ given AB 240, the Angle A $46^{\circ} 30^{\circ}$ and BC 200, to find the Angle C, being acute, the Angle B, and the Side AC. Fig. 36.

$$
\text { Fig. } 36 .
$$



Draw $A B$ in length 240 ; at the Point $A$ make an Angle of $\mathbf{4 6}^{\circ}$ 30 , and draw AC indefinitely ; with BC 200, in the Dividers, and one foot in B , describe the Arch CD, intersecting AC in $\mathbf{C}$ and D ; draw BC and AC, and the Triangle is constructed. If the required Angle had been Obtuse, the Lines should have been drawn from $\mathbf{B}$ to $\mathbf{D}$, and from $\mathbf{D}$ to $\mathbf{A}$. Measure the Line $\mathbf{A C}$ on the same Scale from which AB and BC were taken, and the answer will be 263, 7. Measure the required Angles with a Protractor or Line of Chords. Angle at $\mathbf{C} \mathbf{6 0} \mathbf{3 0}$. Angle at $\mathbf{B} 73^{\circ}$.

## CASE III.

Two Sides and a contained Angle given, to find the other Angles and Side.

Fig. 37.
In the Triangle ABC there isgiven the Side AC: 75, 4, the Side AB 85, 6, and the included Angle at $\Lambda \mathbf{3 6} 6^{\circ} \mathbf{4 0}$, to find the Angles B and C, and the Side BC. Fig. 37.


Draw the Line $A B$ in length 85, 6; at the Point $A$ make an Angle of $36^{\circ} \cdot 40^{\circ}$ and draw $A C$, on which set 75, 6 from $A$ to $C$, and draw BC and the Triangle is constructed. Measure the Side BC on the same Scale from which the other Lines were taken, and the answer will be 51,5 . Angle at $\mathrm{C}, 82^{\circ} 30^{\prime}$. $\mathrm{B}^{\prime} 60^{\circ} 50^{\circ}$.

## CASE IV.

The Sides given to find the Angles.
Fig. 38.
In the Triangle ABC there is given, $A B, 64, A C, 74$, and $B C$ 34, to find the Angles ABC. Fig. 38.


The construction of the Triangle is plain from Problem 6. Sec. 2. Measure the Angles, with a Protractor or Line of Chords.

## SEOMNOSN IV.

Application of Trigonometry to ascertaining the Heights and Distances of objects.

## HEIGHTS.

The most convenient instrument for taking Heights is a Quadrant, being a quarter of a circle, the Arch of which is divided into $90^{\circ}$; the Degrees being divided into halves and quarters, if the size of the instrument will admit. It has two straight Sides, which meet at a Point at the top of the instrument ; from which Point, a small weight is suspended by a horse hair, or a fine silk thread, sufficiently long to vibrate freely below the Arch.
An Angle of Altitude is taken by the Quadrant in the following manner.
The instrument being held, or, which is much better, set on a ataff, perpendicularly, that the weight may swing freely; turn that part of the Arch next your eyc, marked $90^{\circ}$; look along the Side to the top of the object ; the Degree, then cut by the hair or thread, will be the Angle of Altitude.

PROBLEM I. To ascertain the Height of a Perpendicular cbject, on a i.corizontal IPlane.

Fig. 39.

Let BC represent a tree, or. any Perpendicular object, the Height of which is required. Fig. 39.


At one station as at $A$, take an Angle of Altitude as before directed to C ; the top of the object, and measure the distance from this place to the foot of the object. The necessary requisites are then obtained, which in this example are as follows, viz.

Angle at A $52^{\circ}$.
Distance AB 60 feet.
The learner will perceive, that this is an application of Case 2, of Rectangular Trigonometry; the Angle and Case being given, to find the Perpendicular Construct the Triangle as there directed and BC will be 76, 8 feet; to which must be added the Height of the top of the instrument from the Plane or ground.

PROBLEM II. T, find the Height of a Perpendicular object, which is inaccessible, on level ground.

Fig. ${ }^{4} 0$.

Let DC represent a Tower, tho Height of which is required; but eannot be approached, by reason of a trench, nearer than B. Fig. 40.


At B, take an Angle of Altitude to $\mathbf{C}$; measure any convenient distance backward to $\boldsymbol{A}$; at $\boldsymbol{A}$ take another Angle of Altitude to $\mathbf{C}$, and the necessary requisites are acquired, viz.

Angle at B55 ${ }^{\circ}$.
Distance AB 87 feet.
Angle at $A 3^{\circ}$.
Draw the Line $A B$ in length 87 ; from $B$ make an Angle of $55^{\circ}$, and from A, an Angle of $37^{\circ}$; trom C, the Point of intersection of
the Lines which form these Angles, let fall the Perpendicular CD, and that will be the Height of the object, the Line AB being continued to D ; the Line CD, being measured on the same Scale from which AB was taken, the answer will be 138,8 feet, to which must be added the Height of the observer as before.

As the Mensuration of Heights is not particularly connected with the science of Surveying, its further prosecution will be omitted.

## DISTANCES.

A Circumferentor or any of the instruments, used in surveying, for taking the bearing of Lines, are proper to be used for finding the contained Anglen between Lines, necessary for ascertaining Distances.

Fig. 41.
PROBLEM I. Let BC represent a Boundary Line of a Field, which cannot be measured by reason of a bog or river ; an object at $\mathbf{C}$ is visible from B, and from A; the line BA being measured is 10 rods, the Angle CBA is found to be $62^{\circ}$, and the An-
 gle BAC 76 ${ }^{\circ}$. Required BC. Fig. 41.

This is an application of Case I. Oblique Trigonometry; if the Triangle be constructed as there directed, the length of the Lipe BC will 14, 5 rods.
It may be proper here to inform the learner the easiest method, by which contained Angles, as those at B and A, (See Fig. 41.) are found, when Distances, and likewise the Area of ground are to be ascertained.
Set up the Circumferentor at the Angular Point, as at B, and take the course or bearing of the Lines BC and BA which form the Angle; then
Rule 1. If the two feft hand letters, which express the Courses, N. $45^{\circ}$ E. $\mathrm{N} .45^{\circ} \mathrm{W}$. grees of both Courses together; their sum will be the contained Angle,

Rule 2. If the left hand letters be alike, and the right hand letS. $70^{\circ} \mathrm{E}$. ters alike, subtract one Course from the other, the reS. $30^{\circ}$ E. (mainder will be the contained Angle.

Rule 3. If the left hand letters be unlike, and the right hand N. $64^{\circ}$ E. S. $85^{\circ}$ E. letters alike, add both Courses together and subtract their sum from 180, the remainder will be the contained Angle.
Rule 4. (If the left hand letters be unlike, and the right hand N. $60^{\circ} \mathrm{W}$. $\{$ letters unlike, subtract one course from the other, the S. $20^{\circ} \mathrm{E}$. remainder from 180, and the last remainder will be the contained Angle.
For an application of these Rules take the preceding Problem. To find the Angle at B; the course of the Line BC'is $\mathrm{N} .34^{\circ} \mathrm{W}$; of BA, $\mathbf{8} .84^{\circ} \mathrm{W}$. The third Rule applies to this case.
$118 \quad 62^{\circ}$ Angle at B
To find the Angle at A. By reversing the course of BA it becomes $\mathbf{N} .84^{\circ} \mathbf{E}$. for $\mathbf{A B}$; and the course of the Line $\mathbf{A C}$ is $\mathrm{N}: 8^{\circ}$ E. The second Rule applies to this case.

$$
\text { subtract }\left\{\begin{array}{l}
N .84^{\circ} \mathrm{E} . \\
\mathbf{8}^{\circ} \mathrm{E} .
\end{array}\right.
$$

$76^{\circ}$ Angle at $\mathbf{A}$.

PROBLEM II. Suppose B and $C$ to be two corners of a field, or any objects, the distance between which, cannot be measured with a Chain; but from a station at $A$, the distance may be measured to C, which is 70 rods, \& to $B$ which is 82 rods, and the Angle at $A$, found by the bearing of the two Lines, is $56 .{ }^{\circ}$ Required the length of the Line BC. Fig. 42.


Here are two Sides and a contained Angle given, to find the other side. It is performed by Case 3, Oblique Trigonometry, to which the learner is referred.

Answer, BC, 72 rods.

PROBLEM III. Suppose ABC a Triangular piece of ground, which, by an old Survey, is found to be thus; AB 260, AC, 160, and BC 150 rods. The bound at $\mathbf{C}$ is destroy-
 ed, and no remnants of the Lines AC and BC are to be found ; the Line AB only remaining. What Angles must be set off from A and $B$ to run new Lines exactly where the old ones were? Fig. 43.

Here are the Sides given to find the Angles. It is performed by Case 4, Oblique Trigonometry, or Problem 6. Sec. 2.

Ans. $\left\{\begin{array}{l}\text { Angle at A } 32^{\circ} \\ \text { Angle at B } 34^{\circ}\end{array}\right.$
The Point of Intersection, of the Lines AC and BC, will be the place for the bound at $\mathbf{C}$.

## PART IT.

## SURVEYING.

## SNCMHRTN Ti

## A brief description of some of the Instruments, used in Surveying.

## CIRCUMFERENTOR.

This Instrument is a Circular Box, covered with a glass lid, generally about five or six inches in diameter, in the centre of which is a steel pin, on which is placed a needle, which, being constructed with a magnetic power, always points nearly to the North and South points of the Horizon, when the Instrument is Horizontal, and the needle at rest. On the North and South points of the Box, is an index, to the ends of which arescrewed perpendicular brass sights. In each sight is a large and small aperture, one over the other ; the small aperture in one, being opposite the large one, in the other. In the middle of the large opertures, is placed a horse hair, or fine silk
thread. The Instrument has a Socket, in which, being placed the head of a staff, it is supported while used. To the Socket is generally a Ball, that the Instrument may be readily fixed in a Horizontal position.

The Circle is divided into $360^{\circ}$, marked under, or at, the ends of the Needle, thus; from the North to the East into 90, and from the North to the West into 90 ; from the South to the East into 90 , and from the South to the West into 90. By this Instrument the Course or Bearing of the Boundary Lines of a Field are determined.

To ascertain whether the Needle be correct, and in good order for use, set the Compass in some place, where it is not affected by Iron or Steel ; when the Needle is at rest, apply to one end of it a piece of Iron or Steel, to attract it from its place; then remove the Iren or Steel to a distance, and if the Needle settles at the same Point as before, in may depended on as correct.

Other Instruments, governed by the Needle, are sometimes used, as the Theodolite, Plain Table, Semicircle, \&c. As they are, in many respects, similar to the Circumferentor, a description of them is omitted.

## THE CHAIN.

A Four Pole Chain consists of 100 Links, each Link betiag $\mathbf{7 , 9 2}$ Inches in length, but the Chain, commonly used in New-England, is two Rods in length, consisting of 50 Links. In the middle of the Chain and at every ten Links, is, usually, a piece of Brass. By this Instrument, the Distances of the Boundary Lines of a Field are measured.

Distances, in this country, are generally stated in Rods and Links, in Deeds, and other Instruments, where a description of Land is necessary.

It may be proper here to observe, that Inclined Surfaces, as the. sides of hills, are measured Horizontally, and not on the Plane or Surface of the hill. To effect this, in ascending a hill, the hinder end of the Chain must be raised, Perpendicularly, over the Stick, left by the forward Chainman, till the Chain is in a Horizontal position; at which time, the forward Chainman must place his Stick in the ground, at the end of the Chain, the Chain being straightly drawn. The Perpendicular position may be determined by a Plummet and Line. But in descending a hill, the same must be observed by the forward Chainman, respecting the point where he must place his Stick.

## PROTRACTOR.

This Instrument is a Semicircle, usually made of brass, and 4 or 5 Inches in Diameter, the Arch of which is divided into 180 Degress, and numbered both ways. It is used with a Scale to delineate, or draw a Map or Plan, of a piece of land, from the Field Book.

## GUNTER'S SCALE.

This Instrument is a Rule two feet in length; it is generally made of wood. On one side are Lines of Numbers, Sines, and Tangents, by which the different statements in the Cases of Trigonometry may be solved; also Lines of Sine and Tangent Rhumbs, Versed Sines, \&c., the use of which it is unnecessary here to describe. On the other side is a Line of Chords, for measuring and laying off Angles, and answers the purpose of a Protractor. At the left end are two Scales of equal parts, one of an Inch, and the other of half an Inch; at one end of the large Scale is an Inch, divided into ten equal parts; at the other end of the small Scale is half an Inch, divided also into 10 equal parts; both of which are Diagonally divided, by Lines drawn slantwise across the Scale. This part of the Scale is used for taking Distances with the Dividers, for the purpose of drawing a Plan, and is thus performed. If it be required to draw a Plan of 20 Perches to an Inch, then the extent of one Inch, in the Dividers, will represent 20 Perches, and one Division on the Diagonal Inch, 2 Perches; and, proceeding downwards, on the Diagonal Line, each Division is two tenths of a Perch.

A thorough knowledge, of the Instruments here described, cannot be obtained without some practice, and instruction from persons acquainted with their use.

## STOKNON IT:

Introductory Problems, for Reducing the Measures used in Surveying.

The usual Measure of land is the Acre: $\mathbf{4 0}$ Square Rods make a Rood, and 4 Roods, or 160 Square Rods, Perches, or Poles, make an Acre.

PROBLEM I. To reduce Two Rod Chains to Rods and Decimal Parts.
Multiply the Chains by 2 for the Rods, and the Links by 4 for the Decimal. If the Links exceed 25, add one to the Rods, and multiply the remainder of the Links by 4 for the Decimal. If the Links do not exceed 2, a Cipher must be prefixed to the left hand.

1. In 19 Chains 21 Links, how many Rods, \&c.?
$19-21$
$2 \quad 4$

Ans. 38,84 Rods.
2. In 15 Chains 27 Links how many Rods, \&c.? Ans. 31,08 Rods.

PROBLEM II. To reduce Two Rod Chains to Four Rod Chains.

Divide the Chains by 2, to which annex the Links if any. If the given Chains be an odd number, call the remainder 50 Links, which must be added to the given Links.

In 17 Two Pole Chains 42 Links, how many Four Pole Chains and Links?

| 2) $17-42$ |
| :---: |
|  |

PROBLEM III. To reduce Four Rod Chains and Links to Rods and Decimal Parts.

Muiltiply the Chains and Links by 4, the Product will be Rods and Hundredths.

In 13 Chains and 64 Links, how many Rods and Decimal Parts?
13-64

4
Ans. 54,56 Rods.
PROBLENI IV. To reduce Rods and Links to Four Rod Chains and Links.

Divide the Rods by 4, to the Quotient anmex the Limks, adding thereto ${ }_{2} 5$ for every Unit in the remainder.
In 53 Rods 17 Links, how many Chains and Links?
4) $53-18$

Ans. 13 Cha. 42 Links.

PROBLEM V: To reduce Square Chains to Acres.
Divide the Chains by 10, or which is the same, cut off the Right hand figure; the Quotient will be Acres and Decimals: Thus 846 Square 4 Pole Chains make 84, 6 Acres.-Multiply the Decimal by 4, and cut off from the Right hand of the Product, one figure ; the figure at the left will be Roods; multiply the figure cut off by 40 , cutting off as before, and the figures at the left will be Rods.

PROBLEM VI. To reduce Square Rods to Acres.
Divide by 160 for the Acres, and the remainder by 40, if it exceed that number, for the Roods, or Quarters of an Acre; the last remainder will be Square Rods.

In 656 Square Rods how many Acres?
Ans. 4 Acres 16 Rods.

## SEOMNON MMP

## To calculate the Area of Plain Rectilinear Figures and Circles.

PROBLEM I. To find the Area of a Square.
Multiply the length of one Side by itself; the Product is the Area.

How many Acres in a Square piece of land, the length of one Side being 40 Rods?

$$
\text { Ans. } 40 \times 40=1600 \div 160=10 \text { Acres. }
$$

PROBLEM II. To find the Area of a Parallelogram. Multiply the longest by the shortest Side.
How many Acres in a pieee of land, 63 Rods long and 28 broad?

Ans. $63 \times 28=1764 \div 160=11$ Acres 4 Rods.
PROBLEM III. To find the Area of a Right Angled Triangle.

Multiply the Base by half the Perpendicular, or the Perpendicular by half the Base, the Product is the Area; or, multiply the Base and Perpendicular together, and half the Product is the Area.

The Base of a Triangle being 45 Rods its Perpendicular 17, required the Area.
$45 \times 8,5=382,5$ Rods $=2$ Acres 1 Rood 22,5 Rods. Or $45 \times 17=765 \div 2=382,5$ Rods.

PROBLEM IV. To find the Area of an Oblique Triangle.

From the Angle opposite the longest Side, drop a Perpendicular to that Side ; then multiply the Base by half the Perpendicular ; or, proceed in other respects as directed in the last Problem, for the Area.


Note. Without drawing the Perpendicular its length is thus obtained. Place one foot of the Dividers in the Angular Point, opposite the Base, extend the other, so that when describing a small Circle, ea, it will just touch the Base, and neither go the least above or below it ; that distance in the Dividers, measured on the same Scale, by which the Triangle was constructed, is the length of the Perpendicular.

Rule 2. If the three Sides of a Triangle be given, its Area may be thus cbtained:-
From half the sum of the three Sides, subtract each Side severally, then extract the Square Root of the continued Product of the half sum and three remainder for the Area.
The three Sides of a Triangle are severally 20,24 and 30 Rods; required the Area.


## 37 half sum

Then $37 \times 17 \times 13 \times 7=57239=239,8+$ Rods Area.
Note. To survey a Field which lies in the form of a Square, Parallelogram or Triangle, in order to determine its Area, no instrument is necessary, but the Chain, provided the Lines and Bounds be known.-To survey a Square pigce of Ground, measure
one Side only.-To survey a Parallelogram, measure one of the long, and one of the short Sides.-To survey a Triangular piece, measure the three Sides, or the Base and Perpendicular only, if it be a Right Angled Triangle, and calculate their Area by the preceding Problems.

PROBLEM V. To find the Area of a Rhombus or Rhomboides.

Multiply one Side by the Perpendicular height, the Product is the Area. (See Def. 32, 33, and 35, Part. 1, Sec. 1.)
PROBLEM VI. To find the Area of a Trapezoid. (See Def. 34.)

Multiply half the sum of the two Parallel Sides by the Perpendicular Height, the Product is the Area.

PROBLEM VII. To find the Area of a Trapezium. (Sec Def. 36 and 37.)
Multiply the Diagonal by half the sum of the two Perpendiculars, falling from the Angles, opposite the Diagonal, to the Diagonal; or, half the Diagonal, by the sum of the Perpendiculars, and the Product is the Area; or, half the Product of the sum of the Perpendiculars, multiplied by the Diagonal, is the Area.

Fig. 45.

In the Trapezium ABCD, the Diagonal BD is $\mathbf{2 6 , 4}$ Rods, the:Perpendicular Am 8 Rods, and Cn 10,6 Rods; what is the Area? Fig. 45. Ans. 245,52 Rods.

## D



It is required to calculate the Area by the several methods, mentioned in the Rule.

The method of finding the Area of any Irregular figure, of more than four Sides, will be given in the next Section.

PROBLEM VIII. Circles. The Diameter and Circumference of a Circle being given.

Muthiply half the one, by half the other, the Product is the Area; or, thultiply the Square of the Diameter by 0,7854 , and the Product is the Area; or, multiniy the Square of the Circumference by 0,07958 , and the Product is the Area.

If the Area be given to find the Diameter or Circumference ;Divide the Area by 0,7\&54, and the Square Root of the Quotient is the Diameter; or, divide the Area by 0,07958 ; and the Square Root of the Quotient is the Circumference.
Note. As 113: 355: : the Diameter of a Circle to the Circumference; and, As 355: 113: : the Circumference of a Circle to the Diameter.

A gentleman gave his son 10 Acres of ground, to be located in a large meadow, if he would enclose it with a fence; the son, knowing that it could be enclosed by the least fence, in the form of a Circle, located it accordingly; what was the length of his fence?

Ans. $141,79+$ Rods.

## SEOTNON PVo

Directions for taking the Survey of a Field, with the method of keeping a Field Book; also for drawing a plan of the same, and finding its Area by Geometry.

## EXAMPLE I.

Let the Figure ABCDE, with the annexed Field Book, represent a Field to be Surveyed, and its Area determined. Fig. 46.

$$
\text { Fig. } 46 .
$$

> FIELD BOOK.

AB. N. $19^{\circ}$ E. 108 Rods. BC. S. $77^{\circ}$ E. 91 " CD. S. $27^{\circ}$ E. 61 "

4 Links.
DE. S. $61^{\circ} 15^{\prime}$ W. 113
Rods 13 Links.
EA. N. 62 ${ }^{\circ} 15^{\prime}$ W. 59 Rods.


The Survey of the Field is supposed to begin at Station A. Having set the Compass, direct the Sights to an object at B, and
note its Course, which place in the first line of the Field Book; N. $19^{\circ} \mathrm{E}$. Let the Line be measured, and place the Distance, 108 Rods, at the end of the Course, which completes the first Line of the Field Book. At B, direct the Sights to an object at C, and place its Course in the second line of the Field Book; S. $77^{\circ}$ E. Let the Line be measured, and enter the Distance, 91 Rods, at the end of the Course, which completes the Second Line of the Field Book. In the same manner, proceed from one Corner of the Field to another, until you arrive at the first Station A, which completes the Survey.

## To Protract or draw a Plan of the Field.

On some convenient part of the paper, draw a Meridian on North and South Line, which, in this Survey, is the first dotted Line, on the left of the Plan. Make a Point in this Line, as at A, for the first Station. On this Point, lay the Centre Point of the Protractor, with the edge on the Meridian Line, and turning the Arch to the right, the upper part of the paper being considered as North, make a Prick or Point, in the paper, at the Side of the Arch, at $19^{\circ}$, counting from the upper Angle of the Protractor. From the first Station, through this Point, draw a blank Line, with a Protracting Pin or Dividers, and lay thereon, from a Scale of equal parts, the Distance, 108 Rods, from A to B, and draw the Lines AB. Through B draw a Line, Parallel to the Meridian Line. On the Point B, lay the Centre of the Protractor, the edge being on the Parallel line; and turning the Arch to the Right, because the Course is Easterly, prick off the Course $77^{\circ}$, counting from the Lower Angle of the Protractor, because the Course is Southerly; draw and lay off the Line BC as before directed. Proceed in the same manner to draw Parallel Lines, through each Station, and lay off the remaining Sides of the Field.

Note. The Links must be reduced to the Decimal of a Rod.

If the Survey and Protraction be accurately made, the end of the closing Line will come exactly to the first Station; should this not be, very nearly the case, and the Protraction accurate, a Re-survey must be taken.

To find the Area.
Divide the Field into three Triangles, or one Triangle, and one

Trapezium, by the Lines EC and AC. Number the Triangles 1, 2, 3, and measure them as directed in Problem 4, and 7, of the last Section, and calculate their Area in the following manner.
 Area. 3467,43
8727,45
vided by 160, will give 96 Acres 34 Rods.
The first Area is obtained by multiplying the Perpendicular, by half the Base. The second Area, by multiplying the sum of the two Perpendiculars, by the Diagonal AC, and taking half the Product.

## EXAMPLE II.

The following Field Book and its corresponding figure represent a Field Surveyed and protracted as above directed, the ealculations being made in Chains and Links.

Fig. 47.


## FIELD BOOK.

|  | N. | 56 | 15 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| BC. | N. | $26^{\circ}$ | 30' | E. |  |
| CD. | S. | $71^{\circ}$ | $30^{\prime}$ | E. |  |
| DE. | S. | $26^{\circ}$ | $30^{\prime}$ | E. |  |
|  | S. | $45^{\circ}$ | $0^{\prime}$ | W. | 8.47 |
|  | S. | $63^{\circ}$ | 30' | W. | 13.44 |
| H | N. | $76^{\circ}$ | ${ }^{\prime}$ | W. | 24.73 |
|  |  |  |  |  |  |

Area, 1909,11 Chains, or 190 Acres 3 Roods 25 Rods.
Having protracted this Field divide the Map into its appropriate number of Triangles, and calculate their Area as directed in the last Problem. The number of Triangles into which a Field is divided will always be two less than the number of its Sides.

Notr. The Links operate as Decimal parts of a Chain.

## EXAMPLE III.

In the preceding Examples, the corners of the Field are supposed to be visible from one to the other, or the Courses of the Lines known; but it is frequently the case, that the Surveyor cannot avail himself of either of these advantages; in such cases, the following method will be found accurate, easy and expeditious.

Fig. 48,


The Survey of the Field ABCDE, (See Fig. 48.) is begun at Station A. The corner at $B$ cannot be seen from A, nor is the Course of the Line known. A Course $S .3^{\circ}$ E. is taken from $A$ towards B, represented by the dotted line Af; then "Measure the nearest Distance Bf, from the Corner, to the Random Line Af,"* which may be called the Stationary Distance, and in this Examples is 38 Links. The length of the Random Line, to this place, will be the true Distance of the Line AB, which is 44 Rods. To calculate the true Course of the Line AB, use the following Proportion. As the length of the Line run, 44 Rods, is to $57,3 \dagger$ so is the Stationary Distance 38 Links, to the Variation, necessary to run the true Course. This must be added to, or subtracted from, the Course of the Random Line, as the case may require, and the true Course is obtained; thus,

> Rods Deg. Links

As 44 : 57,3: : $38: 1^{\circ} 58^{\prime} 7^{\prime \prime}$ or $2^{\circ}$ nearly.
Subtract $2^{\circ}$ from the Course of the Random Line, and it will leave $1^{\circ}$; therefore, S. $1^{\circ}$ E. 44 Rods must be entered as the first Line of the Field Book. $\ddagger$ The course of the Line BC is known ; N. 76 ${ }^{\circ}$ W. its distance is taken to 2 , on the bank of a pond, from whence the corner at C , on the opposite bank, can be seen. Measure the Lipe 2, 3, and take its bearing N. $4^{\circ}$ E. 8 Rods; take the bearing of 3 ! $\mathrm{S} .80^{\circ} \mathrm{W}$. there is then, in the Triangle 2 C 3 , the Angles, known by the Bearings, and one Side given, to find 2 C . This is performed by Case 1. Oblique Trigonometry. An account of the Triangle must be inserted in the Field Book, under the Second Line; and before the plan is drawn, the Distance 2 C must be added to 24 Rods, the Distance of B 2, and the Distance from B to $C$ will be obtained. The Corner at $D$ is a Tree, standing on a high ledge of rocks. The length of the Lines CD, and DE, cannot. be measured with a Chain. Take the Course of the Line, from C to $D$, which note in the Field Book; then take the Course and Distance of a Line from C to $E ; N 2^{\circ} W 35$ Rods; then take the Course of the Line from $E$ to $D, S .40^{\circ} \mathrm{W}$. which insert in the Field Book. The Course and Distance, of the Line from E to A, is obtained in a similar way, as that from $\mathbf{A}$ to $\mathbf{B}$, and is $\mathbf{S} .88^{\circ} \mathbf{E}$. 42 Rods.

[^1]
## FIELD BOOK.

AB. S. $1^{\circ}$ E 44 Rods.
BC. N. $76^{\circ}$ W. 24 "
2.3. N. $4^{\circ}$ E. 8 "

3C. S. $80^{\circ} \mathrm{W}$.
CD. N. $40^{\circ} \mathrm{W}$.

CE. N. $\mathbf{2}^{\circ}$ W. 35 "
ED. S. $40^{\circ} \mathrm{W}$.
EA. S. $88^{\circ}$ E. 42 " To first Station.
Area, 11 Acres, 3 Roods, 18 Rods.

## To Plotract this Field.

In the first place, complete the Triangle 2 C 3, by finding the contained Angles, at 2 and 3, (See Problem 1, of Distances, Sec. 4, Part 1.)

$$
\text { Add }\left\{\begin{array}{lllll}
2 & \mathrm{C}, & \mathrm{~N} . & 76^{\circ} & \text { W. } \\
\text { 2. } & 3 . & \mathrm{N} . & 4^{\circ} & \mathrm{E} .
\end{array}\right\} \text { By Rule } 1 .
$$

$80^{\circ} \quad$ Angle at 2.
Subtract \{3.2. S. $4^{\circ} \mathbf{W} . \quad$ By reversing the By Rule 2. $\left\{\mathbf{3 C .}\right.$ S. $80^{\circ}$ W. bearing of 2,3 .
$76^{\circ} \quad$ Angle at 3.
By Contracting the Triangle, the Distance 2C, across the Pond, is found 19 Rods; which makes the Line BC 43 Rods.The plan may then be drawn as directed in the preceding example, by laying off the Courses from Parallel lines; or, more accurately, by finding the number of Degrees, in the contained Angles, made by the Bearing of the Lines. Under the Problem, last refered to, will be found the Rules for fiading these Angles.
Draw the Line AB, according to the directions before given, for laying of the first Course and Distance. Reverse the Course AB. and it will be $\mathrm{N} . \mathbf{1}^{\circ} \mathrm{W}$. and the Course of BC is $\mathbf{N} . \mathbf{7 6}^{\circ} \mathrm{W}$. of course, the Angle at $\mathbf{B}$ is $75^{\circ}$, found by the second Rule. Make an Angle at B of $75^{\circ}$, and draw BC in length 43 rods. Having plotted to $\mathbf{C}$, the third Station, reverse the last Course, which must always be done to find the quantity of the Angle by these rules,* and find the Angle ECB, and lay off the Line CE; next find the quantity of the Angles DCE, and DEC, and lay them off from each

[^2]end of the Line CE. The Intersection of these lines, at the Point D, represents the Corner on the Ledge. The Length of the Lines CD and DE, if required, may be measured in the Dividers on the Scale. Lastly, find the Quantity of the Angle AEC; or AED, and draw EA. Divide the plan into three Triangles, or one Triangle and one Trapezium ; and calculate its Area as before directed.

This methed of Protracting a Field is preferable to that of doing it by Parallel Lines; it being difficult to draw them with perfect accuracy. An attention to the Courses will show in what Direction, the Angle is to be made. If there be an external Angle to the field, the quantity of the Angle, as found by these Rules, will be without the Field, and the Lines mast be drawn accordingly.

## EXAMPLE IV.

Fig. 48.


The survey of the Field ABCD was begun at $A$. The Lines $A B, B C$, and $C D$ were surveyed according to the directions in the preceding examples. The Line DA passes through the Border of 2 Swamp thickly covered with bushes and other impediments, to avoid which, a Course and distance were taken fromDto e, thence to $f$, and from thence to $A$.

## FIELD BOOK.

| AB. | N. $15^{\circ}$ |  | E. 13 | 34. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| BC. | S. $60{ }^{\circ}$ | 30 | E. 18 | 85. |  |
| CD. | South |  | 10 | 00. |  |
| De. | S. $35^{\circ}$ |  | W. 4 | 00. | Without the |
| ef. | N. $67^{\circ}$ |  | W. 17 | 90. | Fith |
| $f$ A. | N. $24^{\circ}$ |  | W. 3 | 30.) |  |

## Area, 22 Acres, 9 Rods.

Having Protracted the Field according to the Survey, draw a Line from the third to the first Station, and calculate the Area of that part only contained north of this Line. The Course and Distance of this Line, $N .72^{\circ} \mathrm{W} .20^{\circ} 87$ Links, may easily be ascertained on the plan.

To avoid impediments of this kind it is often practised to make an Offsett, as from D, at Right Angles from the Line DA, and keeping that Course till directly opposite the bound at A; but in following this method the Course of the Line DA ought to be known. It is likewise sometimes practised to consider such, as Closing Lines, and find their Course and Distance by the Protraction ; but this method cannot be depended on, and prevents the detection of any error, which may have been committed in the survey.

To ascertain at what part of a survey an error was committed without the trouble of an entire resurvey, take a Course from every corner of the Field while performing the survey, to some elevated object therein, as at E, or, from as many corners as the object is visible, and insert the Courses on the left side of the Field Book, opposite the Stations from which they were respectively taken. In protracting, lay off each Course as they were taken; so far as these Lines intersect, or meet in one Point, all is right; but if one Line diverge from the point of Intersection, an error must have been committed on the Line preceding the Station, from which the diverging Line of Intersection was taken; so that by going to this part of the Field, the error can be readily corrected ; care must however be taken, that no error be committed in Protracting, or taking the Courses of Intersection.

## EXAMPLE V.

To survey a Field bytaking Offsets, when the Boundary Lines are very irregular. See Fig. 50.

Offsets are Perpendicular Lines, measured from the Stationary Distances to the Angular Points of the land.

Fig. 50.


In the Figure, the marked Lines represent the Boundary of the Field, and the dotted Lines, those Stationary Distances, from which Offsets are taken. From $\boldsymbol{A}$ to $\boldsymbol{B}$, the Field is bounded by a brook. Take the Course and Distance from $\boldsymbol{A}$ to $\boldsymbol{B}$. From this Line, measure the Offsets to the several Angles, at Right Angles from the Line; noticing, in the Field Book, at what part of the Line they are taken; as at $a, c, d, c$, \&c. Proceed in the same manner round the Field, or at every place, where it is thought most convenient to take Offieta.
FIELD BOOK.


## To Protract the Field.

Draw the Stationary Lines according to the Directions in either of the preceding examples. At Right Angles from these Lines, and at the proper places, according to the Field Book, lay off the several Offsets, by Perpendicular Lines, to the Right and Left, as they were taken. Connect the ends of the Perpendiculars by Lines, which will represent the Boundary of the Field.

## To find the Area.

Find the Area within the Stationary Lines, as before taught; then of the small Triangles, Trapezoids, \&c., between these and the Boundary Lines; add the contents of those without the Stationary Lines, to the Area; and from this sum subtract the Contents of those within ; the Remainder will be the Area of the Field.
The Area within the Stationary Lines is 2412 Rods. The fig. ure, Aiba, is an Oblique Triangle, the Base, Ab, 20 Rods, 20

Links, and the Perpendicular, ia, 5 Rods 2 Links, known by the Field Book. Its Area is 52,8 Rods. The figure, bhe, is a Right Angled Triangle. By Subtracting $A b$ from $A c$, known by the Field Book, we have the Base bc, 8,6 Rods, and the Perpendicular $h c$ is $\mathbf{4 , 4}$ Rods. Its Area is $\mathbf{1 8 , 9}$ Rods. The figure $h g d c$ is a Trapezoid. Subtract $A c$ from $A d$, the Remainder 5,32 Rods is its Perpendicular Height $c d$, and the half sum of its Parallel Sides (See Problem 6. Sec. 3. Part 2.) $h c$ and $g d$ is 3,2 Rods. Its Area is 17 Rods. The next figure, in Course on the Plan, is a Trapezoid. Its Area is 14,2 Rods; the next figure is a Triangle. Its Area is 14,2 Rods. All the above figures are without the Stationary Line AB. The Sum of the Areas, of the two remaining figures is 131 Rods, within the Stationary Line. Therefore 117,1 Rods, the Sum of the External Areas, being added to 2412 Rods, make 2529,1 Rods, from which Subtract 131, and the Remainder 2398,1 Rods $=14$ Acres, 3 Roods, 38 Rods is the Area of the Field.

The Student will perceive, that some Decimals are omitted in the preceding calculations.
In Practical Surveying it is recommended to adopt the method here presented of taking Offsets, where the Field is bounded by short lines, not only on account of the ease of Surveying, Protraction and Calculation, but the quantity of ground can be ascertained with greater certainty than by the usual method. It must be admitted that the Area of any Field, arithmetically computed, from the measure taken on the ground, will be more accurate than that which is obtained from Geometrical Projection.

## EXAMPLE VI.

To Survcy a Field from one Station, within the Field, from whick all the Angles can be seen. See Fig. 51.

Take the Course and Distance, from the Station, to each Aagle of the Field.


## FIELD BOOK.

From Station to A. N. $\mathbf{2 0}^{\circ}$ W. 8 Chaing 70 Links.


Area, 25 Acres, 3 Roods, 14 Rode.

## To Protract the Field.

Draw a Meridian Line as N. S. Sefect any Point in this Line as at $d$ for the Station; from which lay of the several Courses and Distances; conrect the ends of these Distances by Lines, as AB, BC, \&cc., which will represent the Boundary of the Fietd.

## To find the 1 rea.

Find the Area of the several Triangles, into which the Field is divided by the Stationary Lines; their Sum is the Area of the Map, in Square Chains and Links, which reduce to Acres.

Field Books to Exercise the Learner in plotting Fields and finding their Area.

No. I.
Rods.

| 1. | S. $1^{\circ}$ | W. 29.4 |
| :---: | :---: | :---: |
| 2. | $85^{\circ}$ | W. 31. |
| 3. | S. $6^{\circ}$ | W. 10.1 |
| 4. | N. $88^{\circ}$ | W. 68. |
| 5. | North | 64.2 |
| 6. | East | 8. |
| 7. | N. $1^{\circ}$ | E. 792 |
| 8. | East | 65.4 |
| 9. | South | 81. |
| 10. | East | 19.1 |

Area, 65 Acres, 2 Roods, 10 Rods.

No. II.

> Ch. L.
> 1. N. $75^{\circ} \quad 0^{\prime} \quad$ E. 13.70
> 2. N. $20^{\circ} 30^{\prime}$ E. 10.30
> 3. East 16.20
> 4. S. $33^{\circ} 30^{\prime}$ W. $\mathbf{3 5 . 3 0}$
> 5. S. $76^{\circ} 0^{\prime}$ W. 16.
> 6. North
> 7. S. $84^{\circ} 0^{\prime}$ W. 11.60
> 8. N. $53^{\circ} 15^{\prime}$ W. 11.60
> 9. N. $36^{\circ} 45^{\prime}$ E. 19.20
> 10. N. $22^{\circ} 30^{\circ}$ E. 14.
> 11. S. $76^{\circ} 45^{\prime}$ E. 12.
> 12. S. $15^{\circ} \quad 0^{\prime}$ W. 18.85
> 13. S. $16^{\circ} 45^{\prime}$ W. 10.12
> Area, 110 Acres, 2 Roods, 23 Rods.

No. III.
R. L.

1. "S. $7^{\circ}$ W. 16.
2. S. $1^{\circ}$ E. 91.
3. S. $1^{7}$ W. 40.
4. N. $88^{\circ}$ W. 79.
5. N. $\mathbf{1}^{\circ}$ W. $\mathbf{3 7 . 2 0}$
6. N. $34^{\circ}$ E. 84 .
7. N. $12^{\circ}$ W. 17.
8. N. $6^{\circ}$ W. 18 .
9. N. $15^{\circ}$ E. 35.5
10. S. $88^{\circ}$ E. 5 .
11. S. $10^{\circ}$ E. 43.
12. N. $74^{\circ}$ E. 29.7

Area, 61 Acres, 2 Roods, 20 Rods.

No. IV.
Ch. L.

1. S. $40^{\circ}$ W. 17.50
2. N. $45^{\circ}$ W. 22.25
3. N. $36^{\circ}$ E. 31.25
4. North 13.50
5. S. $81^{\circ}$ E. 46.50
6. S. $8^{\circ}$ W. 34.25
7. West
32.50

Area, 207 Acres, 3 Roods, 33 Rods. ogle

It is recommended to the Student to perform the Calculations to each Field Book in this Section in both measures; for which purpose it will be necessary to reduce those given in Rods and Links to Chains and Links, and those given in Chains and Links to Rods and Decimal Parts. This will greatly improve his calculation, and render both measures familiar.

## STETNON V.

## The method of obtaining the Area of a Field by Calculation, without drawing a Plan, called Rectangular Surveying.

## TRAVERSE TABLE.

This Table, at the end of the volume, is calculated to show, how far North or South, East or West, the end of any Stationary Line, in a Survey, is from the beginning of it. Northings or Southings of Lines are called difference of Latitude, or, simply, Latitude; Eastings or Westings are called Departure, Longitude, or Meridian Distance.

## To find the Latitude and Departure, for any Course and Distance.

The Course, when less than $45^{\circ}$ is found at the top; but when more than $45^{\circ}$, at the bottom of the page ; and the distance in the right or left hand column. Under the Course, if itbe less than $45^{\circ}$, or over it, if it be more, and against the Distance, the Latitude and Departure are found; each column, being marked at the top and bottom of the page. If the Distance exceed 40 Rods or Chains, take two or more numbers which, added together, will be equal to the Distance, and find the Latitude and Departure for each of those numbers; add the several Latitudes together, and their sum will be the whole Latitude;' and in the same manner find the Departures; or it is sometimes more convenient, to find the Latitude and Departure by Multiplication, as in the second Example.

When the Distáance is in Chains and Links, or whole Numbers and Decimals, find the Latitude and Departure for the Chains, or whole Numbers and then for the Links or Decimals, removing the

Decimal Point in the Table, to the Left according to the given Decimal.

## EXAMPLES.

1. Required the Latitude and Departr:e, the Course and Distance being N. $23^{\circ} 15^{\prime}$ W. 36 Rods, or Chains.

Under $231-4^{\circ}$, and against 36 , is 33,08 for the Latitude, and 14,21 for the Departure.
2. Required the Latitude and Departure, the Course and Disn tance being S. $62^{\circ} 30^{\circ} \mathbf{E .} 90$ Rods.

Over $621-2$, and against 30, is 13,85 Latitude and 26,61 Departure, these multiplied by 3 give $\mathbf{4 1 , 5 5}$ for the Southing, and 79,83 for the Easting.
3. Required the Latitude and Departure, the Course and Distance being N . $21^{\circ} \mathrm{W}$. 67,6 Rods, or 67 Chains 6 Links.

For 40 Lat. 37,34 27. 25,21


The following Field Book describes a Field Surveyed according to the directions in Section 4; the Area of which is requifed by calculation.

| $40^{\circ}$ | W. 70 | Rods. |
| :---: | :---: | :---: |
| N. $45^{\circ}$ | W. 89 |  |
| N. $36^{\circ}$ | E. 125 | " |
| North | 54 | " |
| S. $81{ }^{\circ}$ | E. 186 | " |
| S. $8^{\circ}$ | W. 137 | " |
| West | 130 |  |

In the first place, rule a sufficient number of Lines each way, as in the following Example, and set down the No. of the Station in the first Column, and the Courses and Distances in the second; then find, by the Traverse Table, the Northing and Southing, Easting and Westing of each respective Course and Distance, and set them in their proper Columns, marked at the top, N. S. E. W.
acres 3 roods, and 33 rods.

I MTdWVXG

Tho Southing and Westing against the first Course, are found by adding the Latitude and Departure, severally, for 40 and 30 to; gether. The Northing and Westing, of the second Course, by muls
tiplying the Latitude and Departure of 40 by 2, and adding to the Products the Latitude and Departure of 9. These will serve as examples for the other Courses.

Having proceeded thus far add up the Northings, Southings, Eastings and Westings; and, if the Survey be truly taken, the sum of the Northings and Southings will be equal, as also the Eastings and Westings. If these differences are large, so as much to exceed one Rod, in a Survey of one hundred Acres, and no particular Course and Distance, where a mistake is suspected, can be altered to rectify them, a Re-Survey ought to be taken. But if the differences be small, the Northings and Southings, and the Eastings and Westings may be balanced, by Subtracting one half of the difference from the largest column, and adding it to the least; and let the Additions and Subtractions be divided among the several Courses, according to their length; or, regard may likewise be had to the situation of the land, for, in general, the Course and Distance, of some lines of a Field may be so accurately taken, as not to admit of the probability of error; while other Lines may be attended with so many obstructions, or difficulties, as render it almost impossible to avoid error.

In Example 1, the upper numbers, against each Course, are the Northings, \&c , as taken from the Traverse Table. These being 2dded, the Southings are found to exceed the Northings by ,34, and the Eastings to exceed the Westings by ,20. They are balanced by taking, 17 from the Southings, and adding, 17 to the Northings ; and by taking, 10 from the Eastings, and adding, 10 to the Westings. Thus, add to the first Northing, 08 and to the second , 09 ; and Subtract from the first Southing ,02, from the second ,05 and from the third ,10. Subtract from the first Easting, 02, and from the second , 08 ; and add to the first Westing ,01, to the second , 08 , to the third, 02 , and to the fourth, 04 . The lower numbers, in these columns, then represent the Latitude and Departure of each Course, as balanced, and, being added, equal each other.

In the next place, form a Column of Meridian Distances, the upper numbers, in which, show the sum of the Distances at each end of every Line, from the first Meridian, or from a North and South Line, passing through the Station where the Calculation begins. This Column is formed by selecting such a place, in the Colymn of Eastings, as will admit of a continual addition, of 损 4 Hstings and Subtraction of the Westings, without fupquing, opt hiforescoming to the same Station at which, thei Gajculation, begt of wit

 upper number, in the column of Meridian, Ristanges, opposite this

 or "Weating to be added, therefore, set down 146 , 0 , for the next

Meridian Distances to this number, add the Easting against the 5 th Course, and the sum is 330,54 , for the next Meridian Distance; to which add the same Easting again, and the sum is 514,18 ; from this sum, Subtract the next Westing 19,10, and the remainder is 495,08; Subtract the same Westing again, and the remainder is 475,98; from this, Subtract the next Westing, 130,04, the remainder is 345,94 ; Subtract the same Westing again, the remainder is 215,90 ; from this, Subtract the next Westing, and the remainder is 170,90 ; Subtract the same Westing again, and the remainder is 125,90 ; Subtract the next Westing and the remainder is 62,95 ; Subtract the same Westing again, and 00,00 remains, which shows that the Additions and Subtractions are truly made.

Having finished this Column, make a Column of half Meridian Distances, by dividing the upper numbers in the preceding column, by 2. If the last Decimal, in the number to be divided, is an odd number, take half the greatest even number, and omit the remainder. The process, in forming this Column, is so simple, that no farther explanation is necessary.

The work being thus prepared, multiply the numbers in the Column of half Meridian Distances, by the respective Northings or Southings standing against them. Place the Products by those multiplied by the Northings in the Column of North Areas, and those multiplied by the Southings in the Column of South Areas; add up these Columns, and subtract the less from the greater, the remainder will be the Area of the Field, in Square Rods, or Square Chains and Links, whichever measure was used in the Survey.

## REMARKS.

This method of calculation divides a Field, in connexion with some of the adjoining ground, into Right Angled Triangles, Trapezoids, Squares or Parallelograms, the Areas, of which, are obtained according to the different Problems, for finding the Area of the respective figures The Northings or Southings being one side of a Triangle, Square or Parallelogram, or the height of a Trapezoid. The upper Meridian Distances are the other side of the Triangles, the sum of the two opposite Sides of a Square or Parallelogram, or the sum of the two Parallel Sides of a Trapezoid ; of course, it is easy to perceive; that multiplying the respective Northings or Southings, by one half of the upper Meridian Distances, will be the Area of the respective figures, whether Triangles, Trapezoids, Squares, or Parallelograms.

As some land is included in the calculation, which does not belong to the Field, and some, both within and without the Field, is
frequently calculated twice over, and sometimes oftener ; therefore, by subtracting the lesser Areas from the greater, the true content of the Field is obtained.

If, instead of making a Column of half Meridian Distances, the upper numbers in the Column of Meridian Distances, were multiplied by the respective Northings or Southings standing against them, the difference of the sum of these Products would be double the Area of the Field, as in the second Example.

| $\mid$ | Cuurses and Distances in Rods. | N. | S. | L. | w. | \|M. D. $\mid$ | N. A. | S. A. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | N. $75^{\circ} \mathrm{E}$. 54,8 | 14,2 |  | 52,9 |  | $\left\lvert\, \begin{aligned} & \text { \| } \\ & 2885 \\ & 2385\end{aligned}\right.$ | 3341,26 |  |
| 2 | N. 20 1-2 ${ }^{\circ}$ E. 41,2 | 38,6 |  | 14,4 |  | $\left\|\begin{array}{\|c\|} 302,6,6 \\ 317,0 \end{array}\right\|$ | 11680,36 |  |
| 3 | East. 64,8 |  |  | 64,8 |  | $\left\|\begin{array}{\|l\|l\|} \hline 3816,8 \\ 44 \end{array}\right\|$ |  |  |
| 4 | S. $331.2{ }^{\circ} \mathrm{W} .141,2$ |  | 117,7 |  | 77,9 | $\left.\begin{array}{\|l\|} \hline 368,7 \\ 200,8 \end{array} \right\rvert\,$ |  | 43395,99 |
| 5 | S. $76^{\circ} \mathrm{W} 64$ |  | 15,5 |  | 62,1 | 2028,7 |  | 3544,85 |
| 6 | North, 36 | 36,0 |  |  |  | $\left.\begin{array}{\|l\|} \hline 166,6 \\ 166,6 \end{array} \right\rvert\,$ | 5977,60 |  |
| 7 | S. $84^{\circ}$ W. 46,4 |  | 4,9 |  | 46,1 | ${ }_{74,4}^{120,5}$ |  | 590,45 |
| 8 | N. $5311.4^{\circ}$ W. 46,4 | 27,8 |  |  | 37,2 | 37,2 $\mathbf{0 0 , 0}$ | 1034,16 |  |
| 9 | N. $363-4^{\circ} \mathrm{E}$ E6,8 | 61,5 |  | 46,0 |  | 46,0 92,0 | 2829,00 |  |
| 10 | N. $221.2{ }^{\circ} \mathrm{E}$ E 56 | 51,7 |  | 21,4 |  | $\begin{aligned} & 1134,8 \\ & 13,8 \end{aligned}$ | 5862,78 |  |


| 11 | S. $763-4^{\circ}$ E. 48 |  | 11,0 | 46,7 |  | $\begin{aligned} & 181,5 \\ & 228,2 \end{aligned}$ |  | 1996,50 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | S. $15^{\circ}$ W. 48,4 |  | 41,9 |  | 11,2 | $\begin{aligned} & 217,0 \\ & 205,8 \\ & \hline \end{aligned}$ |  | 9092,30 |
| 13 | S. 16 3-4 ${ }^{\circ}$ W. 40,5 |  | 38,8 |  | 11,7 | $\begin{aligned} & \hline 194,1 \\ & 18,4 \end{aligned}$ |  | 7531,08 |
|  |  | 229,8 | 229,8 | 246,2 | 246,2 |  | 30745,16 | $\begin{aligned} & 66151,1 ; \\ & 30745,16 \end{aligned}$ |
|  |  |  |  |  |  |  |  | $3{ }^{35406,0}$ |
|  |  |  |  |  |  | Area in Rods. |  | 1770,3 |

It will be observed, that in this Example, the Columns of Latiitude and Departure are carried to but one place of Decimals, which can generally be done without making a difference of, perhaps; one Square Rod, in 20 Acres of Land, and the Additions, Subtractions, and Multiplications are greatly diminished.


E 2

The preceding is calted the Pennsylvania method of catcuration. Another system of Rectangular Surveying, differing but little from this, and frequently used by practical Surveyors, is that adopted by Mr. Flint in his "Treatise on Surveying," a specimen of which is given below.

| 8u! $_{\text {8u, }}^{\text {u! mody }}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -088'008(\% |  |  |  |  |  |  |  |  |  |
| :188'8 nac9:2ns \|etre:'s! |  |  |  | $\left\|\begin{array}{l} 86^{\prime} 96 \\ m^{\prime}, 1 \sigma^{2} \end{array}\right\|$ | $\begin{aligned} & 86 ' 9 a \\ & \mp 6^{\prime} 97 \end{aligned}$ | $\left.\begin{aligned} & 68^{6} 61 \\ & 96^{6} 61 \end{aligned} \right\rvert\,$ | $\left\|\begin{array}{l} 68^{\prime} 61 \\ 88^{6} \mathrm{RI} \end{array}\right\|$ |  |  |
| 1899'11\% |  | ¢ 99 'ze | 86'9\% |  | $18{ }^{\prime} \mathrm{I}$ | 200\% |  |  | 8 |
| 1898'991 |  | 88'6t | 29's\% |  | 81'\% | 38\% |  | 00't's ocs s | 2 |
| 328t'Ls |  | 66'90 | 9988\% |  | II'I | $\begin{aligned} & 961 \\ & 9 \% \% 1 \end{aligned}$ |  |  | 9 |
| 1798'81 |  | 08'68 | ゅ®'\%\% |  | $\begin{aligned} & 84.5 \\ & 060 \end{aligned}$ | 8t |  |  | 9 |
| HLI'988 |  | 99'88 | 98'91 |  | $2 \mathrm{I}^{\text {' }}$ | $\begin{array}{l\|} \hline 60.01 \\ 00^{\prime} 01 \\ \hline \end{array}$ |  | 90'01 'TS ol 'S | T |
| 30\%'t |  | $98 \times 91$ | 69'91 |  |  | $86^{\circ}$ |  |  | 8 |
|  | 18888 | $2 I^{\text {c }}$ | $21^{\text {' }}$ |  | $21^{\text {c }}$ |  |  | 88'61 'S \% \%- ${ }^{\text {- }}$ | 8 |
| $3017{ }^{\text {c }}$ \% 1 |  | 80'9x | 00'00 | $\begin{aligned} & 86^{\circ} 96 \\ & \mathbf{n}^{2} 2 \% \end{aligned}$ |  | $\begin{aligned} & 9 \boldsymbol{y}^{\prime} \\ & \hline 0^{\prime} \end{aligned}$ |  | $80^{\circ} \mathrm{LE}{ }^{\circ} \mathrm{M}$-68 ${ }^{\circ} \mathrm{E}$ | I |
| $\begin{aligned} & \text { seavy } \\ & \text { qinos } \end{aligned}$ | $\left\|\begin{array}{l} \operatorname{sez} \mathbf{y} \\ \text { unan } \end{array}\right\|$ | $\begin{gathered} \text { don } \\ \text { don } \\ \hline \end{gathered}$ | $\begin{array}{\|c\|} \hline 109 \\ \text { darin } \\ \hline \end{array}$ | $\cdots$ | '31 | S | N |  | ${ }^{\circ} \mathrm{N}$ |

-II TIdWVXG

Frion the Traverse Tabbe take the Northings, Southings, Eastings, and Weatings, and balance the Columns as before directed, ester -rthich, instead of forming a Column of Meridian Distances
proceed to form a Departure Column, the numbers, in which, show how far the end of each Side of the Field is East or West of the Station, where the Calculation begins. This Column is formed by a continual addition of the Eastings and subtraction of the Westings; or by adding the Westings and subtracting the Eastings. See Example 4.

In this Survey the Calculation begins at the Second Station; the Easting against this Course, $\mathbf{1 7}$ is the first number to be placed in the first Departure Column; to this add 16,52 the next Easting, which make 16,69 for the next Departure; to this add , 17 the next Easting which make 16,86 the next Departure ; to this add 5,58 and 22,44 is the next Departure; to this add 1,11 and 23,55 is the next Departure ; to this add 2,12 and 25,67 is the next Departure ; to this add 1,31 and $\mathbf{2 6 , 9 8}$ is the next Departure. From this subtract 26,98 the Westing and 00,00 remains to be set against the remaining or first Course.

In the next place form a second Departure Column, the numbers in which, like the upper numbers in the Column of Meridian Distances, in the Pennsylvania method, show the sum of the Distances at each end of every Line from the first Meridian, or from a North and South Idiae passing through the Station where the Calculation begins.

The number which begun the firet Departure Coilumn must be set against the same Course to begin the second Departure Column; to which add the second number, in that Column, for the second is this; for the third, add the second and third; and for the fourth, add the third and fourth, and so on till the Columa be completed

The first nnmber to be placed in the second Departure Column is, 17 ; to this add $16 ; 69$ which make 16,86 for the second number ; to 16,69 add 16,86 which make 33,55 for the third namber; to 16,86 add 22,44 which make 39,30 for the fourth number; to 22,44 add 23,55 for the next mumber; to 23,55 add 25,67 for the nexi number; to $\mathbf{2 5 , 6 7}$ add $\mathbf{2 6 , 9 8}$ for the next number; and to 26,98 add 00,00 for the upper number.

The werk being thus prepared, multiply the several numbers in the second Departure Column by the Northings or Southings standing against them respectively; place the products of those multiplied by the Northiags in the Column of North Areas, and of those multiplied by the Southings in the Column of South Areas; add up these two Columns and subtract the less from the greater, the remainder will be double the Area of the Field in Square Rods, or Square Chains and Links, according to the measure used in the Survey.

The following Calculation and Corresponding figure are added to demonstrate the preceding systems of Rectangular Surveying. In the following Example, the two Columns of Departure and the Meridian Distances according to the Pennsylvania method are inserted, that the student may perceive the similarity of the results, occasioned by the different methods of calculation.

Fig. 52.


S

|  |  |  | $\dot{V}$ <br> ‘d © <br> N $j^{0} \mathrm{man}_{\mathrm{S}}$ s 10 wns | ' 8 I'V | $899 \text { єә. }$ | $!n^{6} \% 8$ |  | n's | an'sm1 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 00'\%\% | $\left\lvert\, \begin{aligned} & 00000 \\ & 0 n^{\prime} \geqslant 8\end{aligned}\right.$ | 00'78 | 00'00 | 00'¢8 |  |  |  | 00'78 ${ }^{\text {2so }} \mathrm{M}$ | 9 |
| 0000'989\% |  | 00'\%8 | $\left\|\begin{array}{l}00 \times 891 \\ 100^{\prime} \times 91\end{array}\right\|$ | 30'891 | 00'ャ8 |  | $\therefore$ | 00'\#s |  | 00'ts yrnos | 9 |
| \$809'8888 |  | 80662 | $\left\|\begin{array}{l}000^{\circ} 891 \\ 176 \times 91\end{array}\right\|$ | 97'891 | 00't8 |  | $\boldsymbol{T C S}^{\mathbf{C}} \mathbf{6}$ | $80^{6} 67$ |  | 00\%0s 'rioll ${ }^{\circ}$ | $\pm$ |
|  | 1 | 9才'69 |  | $\underset{\sim}{6} \times 11$ | 95'tL |  | 00608 |  |  | 00'08 ${ }^{\text {aseg }}$ | 8 |
|  | 1-5tz't201 | $8 \% 68$ |  | 20: 59 | 97'ty |  | 98\%\% |  | 8L'İ |  | \% |
|  | 3290'L121 | cóoI |  | $116 \%$ | [11'06 |  | $\begin{aligned} & 1108 \\ & z: 100 \\ & \hline \end{aligned}$ | $\cdots$ | $\begin{aligned} & 58112 \\ & 59512 \end{aligned}$ |  | I |
| -8อ̇V 'S | 'вәд̈ $^{\text {¢ }} \mathbf{N}$ | I W $\cdot \mathrm{H}$ | $\cdot \mathbf{T} \cdot \mathbf{}$ | ${ }^{\text {ded }}$ - | ${ }^{\text {da }}$ ( 1 | - $\mathbf{M}$ | ' I | 's | - | sчu! 78 ' 10 u jsit 20 asino? |  |

- A MdWVX'

In this urvey the Meridian Line N. S. passes through the first Station on he west Side of the Map. The North Area, standing against th first Course, is the Area of the Right Angled Triangle A2B, and is obtained by multiplying the Half Meridian Distance, 10,05 , being half the length of the Line 2B, by 71,35; the Northing, being the ngth of the Lire A2. This is according to the rule for finding the Area of a Triangle. See Problem 3d. Sec. 3d.

Without usirg the Column of Half Meridian Distances, it is easy to see the esult would be the same, to multiply the Northing by 20,11. the second Departure and upper Meridian Distance, as seen in 7th and 8th Columns, and take half the Product, excepting the trifling difference which arises frem the last Decimal's being an odd number. This Area is without the Field, and is to be subtracted from the South Areas.

The North Area standing against the second Course is the Area of the Right Angled Trapezoid 2BC3 and is thus obtained. The length of the Line $3 \mathrm{C}, 4 \mathrm{4}, 46$ is the sum of the Eastings made by the first and second Courses, as is evident from the manner of obtaining this number in the first Departure Column ; to this add 20,11 the Easting of the first Course, being the length of the Line 2B, and the sum of the two Parallel Sides of the Trapezoid is obtained 64,57, which is the second number in the second Departure Column, and the second upper Meridian Distance ; the half of this sum 32,28, the second number in the column of Half Meridian Distances, is multiplied by the length of the Line 23, which is 31,73 , the Northing of the second Course (See Problem 6. Sec. 3.) and the Prodact 1024,2444 is placed for the second number in the column of North Areas. This Area is likewise without the Field, and is to be subtracted from the South Areas.

Against the third and sixth Courses there is no Area. These Courses being one East and the other West, there is no Northing or Southing to be multiplied into them; they can be used only in forming the Departure Columns or Meridian Distances.

The South Area against the fourth Course is the Area of the Trapezoid 1ED3, and is thus obtained. The length of the Line $1 \mathrm{E}, 84$ is the sum of the Eastings made by the four first Courses which may be seen by a careful examination of the Example and Figure; this number is the fourth in the first Departure Column, to which add the length of the Line 3D, 74,46, the third number in the first Departure Column, being the sum of the Eastings made by the three first Courses, and their Sum 158,46 is the two Parallel Sides of the Trapezeid, and is the fourth number in the second Departure Colurnn, and the fourth upper Meridian Distance, the half of which is 79,23 the fourth number in the Column of Half Meridian Distances. This Sum multiplied by 49,08 the length of the Line 3,1, the Southing made by the fourth Course, the Product

3888,6084 is the Area of the Trapezoid, and is the number to be placed in the Column of South Areas, opposite the fourth Course.
The South Area against the fifth Course is the Area of the Parallelogram IEFA, and is thas obtained. The fifih Course, being due South, it is evident the Sum of the Eastings will remain the same, of course the length of the Line AF, 84, is the same as that of $1 \mathrm{E}, 84$; these, being added, make 168 the Sum of the two longest Sides of the Parallelogram, the Half Sum of which is 84,00 for the fifth number in the Column of Half Meridian Distances, which being multiplied by 54 , the length of the Line A1, the Southing made by the fifth Course, the Product 4536 is the Area of the Parallelogram, and is the number to be placed in the Column of South Areas opposite the fifth Course.
By inspecting the Figure and attending to the preceding illustrations, it will be seen that the land contained in the two North Areas is without the Boundary Lines of the Field ; and that the two South Areas include the whole of the Field, and what was included in the North Areas: Therefore, if the North Areas be subtracted from the South Areas, the remainder will be the Area of the Field.

To draw a Plan of the foregoing Field from the several Latitudes and Meridian Distances.

On some convenient part of the paper draw the Meridian Line NS. Set the Northing of the first Line from A to 2, the Northing of the second from 2 to 3 . Set the Southing of the fourth Line from 3 to 1. From these Points draw Lines of sufficient length Perpendicular ta the Meridian. On these Lines set the Meridian Distances or Departures of the respective Stations, viz. from 2 to B 20,11, from 3 to $\mathbf{C} 44,46$, (See 1st Departure Column, from 3 to $D$ 74,46, from 1 to E 84, and from A to F 84. From one of these last points to another draw the Boundary Lines of the Field.

## SHOMTON VI。

## LAYING OUT AND DIVIDING LAND.

PROBLEM I. To lay out any number of Acres, or Acres and Rods, in the form of a Square.

Reduce the Area to Square Rods, the Square Root of which i: the Side of the Square, in Rods, or Roods and Decimals.

It is required to lay out 810 Acres in the form of a Square.
810 Acres $=129600$ Rods the Square Root of which is $\mathbf{3 6 0}$ Rods.

PROBLEN II. To lay out any number of Acres in the form of a Parallelogram, one side being given.

Divide the Content in Rods, by the given Side in Rods, the Quotient will be the required Side.

What Distance must be measured from each end of a Line, 29 Rods in length, that the Parallelogram may contain 3 Acres 0 Roods 27 Rods?

$$
\begin{aligned}
& \text { A. Rods. Rods. } \quad \text { R. Links. } \\
& 3-27=507 \div 29=17-12 \text { Ans. }
\end{aligned}
$$

PROBLEM III. To lay out any number of Acres in the form of a Parallelogram, the length of which is to exceed the Breadth, by a yiven Proportion.
Divide the Area in Rods, by the Proportion between the length and breadth; the Square Root of the Quotient will be the shortest Side.
It is required to lay out $\mathbf{3 1}$ Acres $\mathbf{4 0}$ Rods, twice as long as broad.
A. R. Rods.
$31-40=5000 \div 2=2500$, the Square Root of which is 50 Rods, for the shortest Side ; the longest, found by Problem 2.

PROBLEM IV. To lay out any number of Acres in the form of a Triangle, being confined to a certain Base.
Divide the Area in Chains or Perches, by half the Base, the Quotient is the Perpendicular.

What is the Perpendicular Height of a Triangle, to contain 100 Acres, the Base being $\mathbf{4 0}$ Chains?

$$
\begin{aligned}
& \text { Acres. Chains. } \\
& 100 \times 10=1000 \div 20=50 \text { Chains, Ans. }
\end{aligned}
$$

The Perpendicular may be erected on any part of the Base, and Lines run from its extremity to the ends of the Base, will lay out the Triangle.

If the given Base be so situated that a Perpendicular of sufficient length cannot be erected, the Base must be continued, for the purpose only of erecting the Porpendicular. (See Def. 17. Sec. 1. Part 1.)

## DIVIDIEG mant.

## EXAMPLE I.

To divide a Triangle by a Line proceeding from any of the Angles, to the opposite Side, which may be called the Base.

Measure the Base, then say, as the Area in Rods or Chains of the whole Triangle, is to the Length of the Base, so is the Area of the part to be divided off, to its part of the Base. (See Remark 5. Sec. 3. Part 1.)

Fig. 52.


The Triangle ABC, Fig. 7, contains 25 Acres 16 Rods. It in required to divide off 9 Acres 1 Rood by a Line, proceeding from C so AB, which is 138 Rods in length.

25 Acres 16 Rods= 4016 Rods.
9 Acres 1 Rood= 1480 Rods.
As 4016 : $138:$ : $1480: 50,85$ Rods, Ans.
This distance, 50 Rods 20,5 Links may be measured from 8 to $D$, or from $\mathbf{A}$ to $E$, and Lines run from $C$ to $E$, or $D$; and CBD, or CAE will contain the number of Acres and Rods, required to be divided off.

## EXAMPLE II.

## To Divide a Trapezoid.

It is required to divide the Trapezoid $A B C D$, into two equal; parts, by a Right Line, FE, Parahtel to AD, or BC. Required, by Calculation, the Distance of the Point F, from B or A. Fig. 88.

Firg. 53,
*


## FIELD BOOK.

> AB. South, 30 Rods.
> BC. N. $80^{\circ}$ W. $60 \quad$ "
> CD. N. $391^{\circ}$ W. 45,5 "
> DA. S. $80^{\circ}$ E. 89,4 "
> Ares, 13 Acres, 3 Roods, 7 Rods, $=2207$ Rods.

In the first place, the Triangle AGD, of which the Trapezoid hay be considered a part, must be completed, by finding the length of BG and CG. To find BG, say, as the difference between AD and BC , is to AB , so is $\mathbf{B C}$ to $\mathbf{B G}$.

Diff AB. BC. BG.
As 29,4: $30:$ : $60: 61,22$
To find CG.
Diff. CD. BC. CG.
As 29,4: 45,4:: 60 : 92,65
In the Triangle CBG, the three Sides are given to find the Area, by Problem 4. Rule 2. Sec. 3. Part 2. Area, 1809,85 Rods. Add the Area as now found, to one half the Trapezoid, and it will be 2913,35 Rods, for the Area of EFG. Then (by Remark 6. Sec. 3. Part 1.) say,

CBG. Sq. of BG. EFG. Sq. of FG.

$$
\text { As } 1809,25: 8747,88:: 8913,35: 6033,02 .
$$

The Square Root of $\mathbf{6 0 3 3 , 0 2}$ is 77,67 , for $F G$, from which subtract BG, as found above, and the remainder, $\mathbf{1 6 , 4 5}$ Rods, is the Distance from $B$ to $F$ required. If this Distance be subtracted from 30, the length of $A B$, the remainder, 13,55 , is the Distance from $A$ tọ $F$.
By an application of the last statement of this Example, a Triangle may be divided by a Line Parallel to either of its Sides.

Note. The two first statements may be demonstrated by the principles of the Rule of Three.

## EXAMPLE III.

To take off any given kumber of Acres from an irregular 'Field. Fig. 54.


Let ABCD \&c. represent a Field, containing 11 Acres, from which it is required to cut off a piece as DEFGHD; that shall contain 5 Acres, by a Line from $\mathbf{D}$, to the opposite Side AF. Required the Point H, Geometrically.

Draw the Line DG, which you may judge to be near the Dividing Line. Find the Area of the Part DEFG, which, suppose may want 140 Rods of 5 Acres. This is to be set off in the form of a Triangle, the Base of which is to extend from G. towards A. Draw the Line DI, the Base GI being of sufficient length, that the Triangle DIG may contain more than 140 Rods; find the Area of this Triangle, which, suppose to be 340 Square Rods, as likewise the length of the Base GI; on the Scale, whioh; suppose to be 8 Rods. Then (by Example 1, of this Section) say ;

DIG. GI. DGH GH.
As 340 : $8: 1$ 140: 8 Rods 78 Links.
Measure this Distance, viz. 3 Rods 7,3 Lhnks, from $\mathbf{G}$ to $H$, and draw the Jine DH for the Dividing Line,

## EXAMPLE IV.

The Parallelogram ABCD, Fig. 55 , represents a piece of land, owned in common by two men, which is to be divided into two equal parts, by a Right Line, proceeding from the end of a stone wall at $G$, to the Line BC. Required BH, by calculation.

B
Fig. 55.
H C


Measure the whole piece, and find its Area, noting the Distance from A to E, and take the Courses and Distances of the wall, from $\mathbf{E}$ to $\mathbf{G}$.

## FIELD BOOK.

| AB. North | 160 Rod |  |
| :---: | :---: | :---: |
| BC. East |  | Area 100 Acres. |
| D. West | 160 |  |
| AE. | East 64 | Rods. |
| EF. | N. $24^{\circ} \mathrm{W} .60$ | - |
| FG. | N, 612 ${ }^{\circ}$ E. 40 | " |

Calculate the Area of the part GFEABG, by Rectangular Surveying.
The Course and Distance, of the closing Line BG, aríe not known by the Survey; it is, however, plain that the Southing of the Line BG must be the difference, between the Southing of the two Lines on the stone wall, (their Course being reversed;) and the Northing of the Line AB; therefore, place as much Southing for BG as will balance the Northing, and asmuch Easting as will balance the Westings.

'GロษVT LV NOILVTNDTVO

100 Acres $=16000$ Rods.
Half to be taken off $\mathbf{8 0 0 0}$
Area as now found 5939,1
Remainder 2060,9
This Remainder is to be contained in the Triangle BGH. Calculate in the next place the Area of the Triangle GBC.

The Northing and Westing of the Line GB will be the same, as the Southing and Easting of the same Line, in the preceding Calculation; the Southing and Weating of the Line CG are found by balancing the columns as before.


- By Example 1, of this Section.

GBC BC BGH BH
As 3275,5:100::2060,9:62,91 Rods, which determines the Point $H$ in BC.

It is recommended to the learner to reduce the Field Notes in
this Example to Chains and Links, and perform the Calculation a second time.
In the business of Surveying, and particularly that of Dividing Land, the practitioner will frequently find opportunities for the exercise of his judgment, in the application of the principles of Geometry and Trigonometry ; it being difficult to give particular rules for all the various ways, in which it is sometimes required to divideFields.

If an estate, or large tract of land, is to be divided among a number of persons, an accurate Survey, and Plan, of the whole must first be made, and its Area ascertained by Geometry, which is, in such cases, the most usual method, or by calculation; after which, the different Divisions may be made on the Map, according to the directions of the owners, or persons authorised to make the Division.

The work will be more accurate, if the Map be drawn on a large Scale.
If Roads, Brooks, or any remarkable objects, intersect the Boundary Lines, the places of such Intersection should be inserted in the Field Book of the first Survey.

In such Surveys, the nearer the sum of the Areas, of the several Divisions, agree with the whole Area, the more accurate is the work.

## 

## THE MAGNETIC NEEDLE.

This Instrument, notwithstanding its utility, is subject to some irregularities, particularly its annual Variation, and local Attraction. The number of Degrees, which the Needle differs from a true Meridian Line, either East or West, is called the Variation of the Needle. This is different in different places, and is not the same, at any place, for two successive years. Its variable motion is more rapid in some years than in others, which renders it not reducible to any precise rules. The Calculations in a Survey are not affected by this irregularity of the Needle, the Variation being the same in every part.

The local Attraction is the effect, which any iron substance has upon the Needle, when near, by attracting it from the place, in which it would naturally settle. As the earth, in some places, contains, near its surface, minerals, which attract the Needle from its
true Course ; if, in such cases, proper care be not used, a wrong Course will be taken, or a crooked Line run. To guard against error, in this and oifer cases, the Surveyor should take Backsights at every Station, and if the Compass shoold not reverse correctly to the last Station, the Line mast be continued by the Index, or Sights, without regarding the Needle, till removed fom the influence of the Attraction.

## DIRECTIONS FOR RUNNING A LINE.

If it be required to set up Stakes, on a straight Line, between two Bounds, which cannot be seen from each other, the following Directions may be depended on as accurate. Set up the Compass at one of the Bounds, and take a Course, which you may judge will direct exactly to, or near, the other Bound. Send an assistant forward with a Staff, or Stake, for an object, as far as the object can be distinctly seen, and cause him to place the Stake, exactly in the Line of your Course. Let the Chainmen now commence measuring on the Line you run, and, as they proceed, cause a Stake to be placed at every 20 Rods, or oftener, if they cannot be seen from each other. Leave an assistant at thé Bound, or first Station, with a Stake for a Back-sight object, and set up the Compass at the place, where the other Assistant's Stake stood, he proceeding forward; settle the Needle to the same Course, with which you started, and if it reverse correctly to the last Station, all is right. Place again the forward Assistantin the Line, and the other at the second Station, and thus proceed with the Random Line, till you are opposite the other Bound ; then measure the nearest distance between the Random Line and the Bound, which may be called the Stationary Distance," and the place, from whence this Stationary Distance is taken, is the termination of the Random Line. The Calculation for removing the Stakes on the true Line is thus made. As the length of the Random Line is to the Stationary Distance, so is 20 Rods, to the Distance the first Stake must be moved. Suppose the Random Line 160 Rods, and the Stationary Distance 40Links ; then,
R. L. R. L.

As 160:40: $: 20: 5$ the Distance which the first Stake must be moved ; of course the second must be moved 10 , and the third 15 ; and so adding 5 Links at each Stake, until the whole are moved, in the direction the Stationary Distance was taken.

[^3]Norx.-All the Stakes, and particularly those used as objects, should be straight, and stand perpendicularly.

## ANOTHER METHOD.

As the Magnetic Needle has some irregularities, and, at the best, is very difficult to be adjusted in exactly the same position, at a number of different Stations, many skilful Surveyors have adopted the following method to determine the position of a straight Line, between two known Boundaries.
The Surveyor should be furnished with three Stakes, called Plumb Stakes, about two inches wide with straight edges, on each of which should be suspended a Plumb or Weight, in an aperture near the top, by a line about two feet in length, by means of which they can be easily adjusted in a Perpendicular position. Set one of these Stakes at the Boundary from whence the Line is to proceed, and place the others so as to form a straight Line, which may be judged to direct at, or near, the other Bound. Let the Stake at he Bound be moved forward as far as it can be distinctly seen, keeping them in a straight Line. At every 10 or 20 Rods, as may be necessary, place a Stake, called a Tally Stake, exactly in the Line. Proceed in this manner, by moving the hindmost Stake forward, till you arrive opposite the Bound where the Line is to end. The Stationary Distance must then be measured, and the Staken moved as before directed.

## QUESTIONS.

1. A Wheel has 6 Felloes. What proportion does the Chord of one of its Felloes bear to the diameter of the Wheel? See Note Def. 22, Sec. 1, Part 1.

Ans. As 1 to 2.
2. A piece of land, containing 18 Acres, 2 Roods, 16 Rods, is sold for $\$ 20,20$ per Acre. What is the amount of it ?

Ans. 8375,72.
3. Suppose a Field, measured by a Chain 3 Inches too long, or
by one 33 feet 8 Inches, is found to contain 41 Acres, 1 Rood, 33 Rods. What is the true Area of the Field?

Ans. 42 Acres 13 Rods.
To solve a question of the foregoing kind, whether the Chain be too long or too short, use the following proportion.

As the Square of the length of the true Chain, in inches, is to the Area, as found by the Chain made use of, so is the Square of the length of this Chain, to the true Area of the Field.
4. Suppose a Territory to contain 43264 square miles. If this quantity of land be laid out in a Square, what will be the length of one of its Sides?

Ans. 208 miles.
5. In surveying a piece of land, I measured North $10^{\circ}$ East 80 Rods, thence South $80^{\circ}$ East 60 Rods. How far was I then on a direct Line from the first Station?

Ans. 100 Rods.
6. What is the difference between a Field, 28 Rods long by 20 broad, and two others, each of half the dimensions?

Ans. 1 Acre 3 Roods.
7. Required the dimensions of a Parallelogram, containing three Acres, and bounded by 104 Rods of straight fence?

Ans. 40 Rots by 12.
8. How much less fence will it take to enclose 10 A'cres, in the form of a circle, than of a square !

Ans. 18,21 Rods.



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| 7 | 7700024 | 699027 | 699031 | 699034 |
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| 98391125 | 891129 | 890133 | 890137 |
| 10) 990139 | 990143 | 989,148 | 98115210 |
| 1119391531 | 108915810 | 0881631 | 1087167 |
| 1211381671 | 1183172 | 187177 | 18618312 |
| 1312371811 | 12871871 | 2361921 | 123519313 |
| 1413361951 | 13362011 | 3351071 | 138421314 |
| 151435 ? 33 1 | 14952151 | 4842231 | 148322315 |
| 161534 ? 23 | $158423)$ | 532236 | 158124316 |
| 1710333 37 | 163324416 | 6312511 | 168025917 |
| 181732 P 31 | 1781253,17 | 7392661 | 177927418 |
| 191332231 | 13802 731 | 379231 | 187328919 |
| 2011931273 | 1979237 | 73296 | 7730 |
| 2120301292 | 2078301 | 277310 | 763 |
| 22217030 | 2177316 | 176325 | 74335 |
| 232278320 | 22763302 | 27.3402 | 227335023 |
| 242377334 | 2375344 | 237455 | 237236524 |
| 252476348 | 24743592 | $247370{ }^{2}$ | 247138 |
| 26257536 | 2573373 | 2571385 | 257039626 |
| 2726 74, 7 | 26723872 | 2670399 | 266941127 |
| 2822733190 | 27714022 | 2769415 | 276712628 |
| 292372404 | 2870416 | 286342 | 36614129 |
| 302971418 | 29694312 | 2967443 | 96545630 |
| 3113070 | 3063445 | 3066458 | 3064,472 |
| 32316944 | 3167459 | 6547 | 163487 |
| 333263450 | 32664743 | 3264483 | 326250233 |
| 343367473 | 33654883 | 3363503 | 336051734 |
| 353466487 | 3464502 | 3462517 | 34595 |
| 363565501 | 3563517 | 3560532 | 355854 |
| 373664515 | 3662531 | 3659547 | 365756 |
| 3763529 | 3761545 | 3758562 | 37565 |
| 39336254 | 3860560 | 3857576 | 3855 |
| 403961557 | 3959574 | 395659 | 53 |
| m. | Dem. | Don. Lat. | n. |
| 2 , | 313.41 Jre. | B11-2 $\overline{\text { Dra. }}$ | $\overline{811-4} \overline{\mathrm{D}}$ |


|  | 91-4 D.e. | $\frac{1.2}{\text { leez. }}$ | 93-4 D9.e. |
| :---: | :---: | :---: | :---: |
| 109016 | 093016 | 099017 | 099017 |
| 2193031 | 197032 | 197033 | 197934 |
| 3296047 | 296048 | 296050 | 296051 |
| 4395063 | 395064 | 395066 | 394068 |
| 5494078 | 493080 | 493083 | 493085 |
| 6593094 | 592096 | 592099 | 591102 |
| 7691110 | 691113 | 690016 | 690119 |
| 8790125 | 790129 | 789132 | 7881 |
| 9) 389141 | 888145 | 883149 | 387152 |
| 10 988(156 | 9871161 | 9861160 | 16910 |
| 111036178 | 1086177 | 10351132 | 108418611 |
| 121135188 | 1184193 | 1134198 | 118320312 |
| 1331239203 | 1283209 | 123221 | $1231 \cdot 2213$ |
| 141333219 | 13822251 | 1331231 | 133.323714 |
| 151432235 | 1480241 | 1479243 | 147325415 |
| 161530250 | 15792571 | 157326 | $15772711^{6}$ |
| 17167926 | 678273 | 1677231 | 167529317 |
| 13177823 | 7772891 | 1775297 | 17743051 |
| 19.1877297 | 1875305 | 1374314 | 1873332 |
| 201975318 | 19743121 | 197333 | $19713393 n$ |
| 2112074329 | 2073338 | $2{ }^{1} 134$ | 20703.56 |
| 22/2173344 | 2171354 | 217036 | 2163373 |
| 232272360 | 2270370 | 2263330 | 226733 |
| 24.2370375 | 2369386 | 2367396 | 2365106 |
| 25\%2469391] | 2467402 | 2466413 | 2464123 |
| 36\|2568407 | 2566418 | 2564429 | -25 $62440{ }^{2} 6$ |
| 272667422 | 2665434 | 2663448 | -26614.57 77 |
| 23-2766438 | 2764450 | 2762463 | $276{ }^{2} 174$ |
| 29:23 34454 | 2362466 | 2360479 | 2353491 |
| 30,29 33439 \| | $2961432 \mid$ | 29591495 | 295751330 |
| $31 / 3062485$ | 13060498 | 30575 12 | 305515 |
| 323161501 | 3158514 | 315652 | 315454 |
| 3313250516 | 32575309 | 325554 | \|32 $52 / 55933$ |
| 343358.532 | 3356547 | $3353>61$ | 335157634 |
| 353457548 | 3454563 | 345257 | 3449593 |
| 363556563 | 3553579 | 355159 | 3548610 |
| 373654579 | 3652595 | 3649611 | 364762737 |
| 333753594 | 3251611 | 3748627 | 374564 |
| 393852610 | 3849627 | 3847844 | 3844660 |
| 403951626 | 3948643 | 3945660 | 67840 |
| De.in Lat. | Dan: Lat. | Den. limat. | $1{ }^{1}$ |
| 81 Neg. | 3034 Deg. | $\overline{0}_{0} 1-2$ De | 4 |







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|  |  | 07 | 28 | 079 |  |  |  |  |  |
|  | 380 | 10 |  | 10 |  | 10 | 3 |  |  |
|  | 48 | 129 | 48 |  | 48 | 13 |  |  |  |
|  | 58 | 15 | 57 | 158 | 57 | 160 | 57 |  |  |
|  | 67 | 181 |  | 18 |  |  |  |  |  |
|  | 773 | 207 | 77 | 210 | 771 | 214 | 77 |  |  |
|  | 86 | 23 | 86 | 237 | 86 | 241 |  |  |  |
| 10 | 96 | 259 |  |  | 9 | 267 |  |  |  |
|  | 106 | 28 | 10 | 28 | $\overline{10}$ | 294 |  |  |  |
|  | 159 | 311 | 11 | 316 | 115 | 321 | 11 |  |  |
| 13 | 25 |  | 1254 | 342 | 12 |  | 12 |  |  |
|  | 1352 | 36 | 351 | 36 |  |  |  |  |  |
|  | 4 |  | 1447 | 39 | 14 | 401 |  |  |  |
| 16 |  | 41 |  | 4 | 15 |  |  |  |  |
|  | 164 | 440 | 1640 | 447 | 16 | 45 |  |  |  |
|  | 1739 | 46 | 173 | 473 | 17 |  |  |  |  |
| $19$ |  |  |  |  | 183 |  |  |  |  |
| 20 |  |  |  |  | 19 |  |  |  |  |
| $\overline{21} \mid$ |  | 54 |  | 55 | 20 |  |  |  |  |
|  |  | 56 |  | 57 | 21 |  |  |  |  |
|  | 2 | 595 | 221 | 60 | 221 |  | 22 |  |  |
|  |  | 621 | 2315 | 63 | 231 | 64 | 231 |  |  |
|  |  |  | 2412 |  |  |  |  |  |  |
|  |  | 67 |  | 68 |  |  |  |  |  |
|  |  | 699 | 26 | 710 |  |  |  |  |  |
|  |  | 72 |  |  |  |  |  |  |  |
|  | , | 751 |  |  |  |  |  |  |  |
|  | 2893 | 77 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  | 8 |  |  |  |  |  |  |  |
|  |  | 85 | 3184 | 86 | 318 |  |  |  |  |
|  |  |  | 3280 | 89 | , 32 |  | 3272 |  |  |
|  |  |  |  | 9 |  |  |  |  |  |
|  |  | - |  |  | 34 |  |  |  |  |
|  | 53 | 95 | 3570 |  | 35 |  |  |  |  |
|  | 71 | 98 | 36 | 10 | 36 | 10 |  |  |  |
|  | 37 | 100 | 376 | 02 |  | 1 |  |  |  |
|  | 386 | 10 | 385 | 052 | 3855 | 10 | 50 |  |  |
|  |  |  |  |  |  |  |  |  |  |
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| Lat. Dep | , Ue" | Lat. ${ }^{\text {D }}$ Dep. | Dep. |
| $\overline{1} 0 \overline{94} \overline{034}$ | $\overline{094} 0$ | 0940 |  |
| 2183068 |  | 1870 | 187 |
| 328210 | 28110 | 281 | 2811 |
| 4376137 | 35 | 37 | 374 |
| 5 470 | 469 | 46817 |  |
| ${ }^{6} 564205$ | 2 | 5622 | 561 |
| 7) 658238 | 657 | 65624 | 655 |
| 75227 |  | 28 | 748 |
| $9{ }^{9} 846308$ | 84431 |  | 2 |
| 109403 | 938346 | 93735 | 93: |
| 111034376 | 10 32381 | 10 |  |
| 121128410 | 1126415 | 12442 | 1124 |
| 131222.445 | 122045 | 121845 |  |
| 141316479 | 313485 | 31149 |  |
| 151410513 | 1407519 | 40552 | 403 |
| ${ }^{61504} 547$ | 150155 | 49956 | 19 |
| 711597581 | 159 | 59 |  |
| 181691616 | 16836 | 1686 | 1683 |
| 9178565 |  |  | 1777 |
| 201187968 | 1876692 |  |  |
| ¢1919 71 | $19 \overline{70} 7 \chi^{\prime \prime}$ |  | + |
| ${ }^{220667} 75$ | 76 | 2361 77 |  |
| 23'21 61787 | 2158796 | 215 58 |  |
| ¢422 5582 | 2252 |  |  |
| 25234985 | 2345 | 8 |  |
| 2624 43889 | 2439 | 34591 | 31 |
| :772537 92 | 33 |  |  |
| ${ }^{2} 863195$ |  |  |  |
| 292725.99 | 00 | - |  |
| 3028191026 | 2815103 | 81 c 10 | 2805106 |
| 312913106. | 29081073 | 304 108 |  |
| 3230071094 | 300211 | 2997112 | 2992:11 |
| 3331011129 | 30 | 91] 115 |  |
| 3431951163 | 3190117 | $318{ }_{\text {E }}^{11} 91$ | 117 |
| 3532831197 | 32841211 | 32781220 | 327112 |
| 3633831231 : | \|337712 | 372126 | 336612 |
| 3734771265 | 347112 |  |  |
| 38835711300 | 35651315 | 35591331 | 355413 |
| 3936651334 | $36: 591350$ | 35531366 |  |
| 403759136 | 37 | 37 471401 | 37 |
| - Dep. Lat. | Dep. | D | 10 p |
| 70 Deg. | 1693.4 |  | $\overline{69} \frac{1.41 \mathrm{leg}}{1 .}$ |


| $\frac{2}{\theta_{0}} \text { Lat. Deg. Dep. }$ | Dati- Dpi. | Lat. Dep. | Lat. Dep. $\frac{w_{2}}{2}$ |
| :---: | :---: | :---: | :---: |
| 0930 | 090036 | 093037 | 093037 |
| 187072 | 186072 | 186 | 186 |
| 280108 | 284109 | 2 | 279 |
| 373143 | 378145 | 37214 | 372 |
| 5467179 | 466181 | 18 | 64 |
| 560215 | 559217 | 558 | 557 |
| 654251 | 652254 | 651 | 650 |
| 8747287 | 74629 | 744 | 743 |
| $9{ }^{9} 840 \mid 323$ | 839 | 837 | 836334 |
| 10934 | 93236 | 93 |  |
| 10273 | $10<0$ 39c | 1023403 | \%2 |
| 121120430 | 1118435 | 1117440 | 15 |
| 131214466 | 121247 | 1210476 |  |
| 14130750 | 130550 | 513 |  |
| 151400538 | 139854 | 1396550 |  |
| $16.1494{ }^{16} 7$ | 149158 | 148958 |  |
| 11.158760 | 158461 | 158262 |  |
| 181680645 | 167865 | 6 |  |
|  | 177168 | 1768 696 <br> 1861  <br> 733  | O |
| 21 1961 | 1957761 | 1954 | 950788 |
| 205478 | 2050797 | 304780 |  |
| 2147824 | 214483 | 84 |  |
| 2241860 | 32378 | 2338880 | 5 |
| 233489 |  | 91 | 5 |
| 263427 | 2423942 | 241995 |  |
| 372521 | 2516979 | 2512 | 27 |
| 2826141003 | 210101 | 260510 |  |
| 7071039 | 270310. | 26981063 | 2694107529 |
| 3028011075 | ${ }^{7} 96108$ | 2791110 | 2786111230 |
| W12894 1111 | 2889112 | 2884113 | 3 |
| 3229871147 | 11 | 29771 | 2972118638 |
| 3330811183 | 30761196 | 307012 |  |
| 3431741218 | 3169123 | 316312 |  |
| 3532681254 | 262 | 56 |  |
| 3633611290 | 3355130 | 33501319 |  |
| 3734541326 | 34331341 | 34431356 | 37,137137 |
| [3835481362 | 35421377 | $3536 \mid 13$ |  |
| 3641139 | 36514 | 62914 | 322144535 |
| 10373414 | 37 | 37221466 | 3715148240 |
| Lat | $\overline{\text { Dep. }}$ I 1 at. | Dep. L,at. |  |
| 69 | 683.4 1)eq. | 68 1-2 Deg. | $68 \mathrm{f}-4 \mathrm{lder} \cdot \square$ |







|  |  |  |  |  | ${ }^{27}$ 1-2 Deg. |  |  |  |  |  |
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|  |  |  |  |  |  |  | 046 |  |  | 047 |
|  | , | 891 | 178 | 8092 | 17 |  | 09 | 177 |  |  |
|  | 267 | 136 | 267 | 7137 | 26 |  | 13 | 26 |  |  |
|  | 356 | 618 | 356 | 183 | 35 |  | 18 | 35 |  |  |
|  | 5 | 227 | 445 | 22 | 44 |  | 231 | 442 |  | 23 |
|  | 535 | 272 | 53 | 27 | 5 |  | 27 | 53 |  |  |
|  | 624 | 31 | 6 | 321 | 62 |  | 32 | 61 |  | 32 |
|  | ${ }^{81} 713$ |  | 711 | 366 | 71 |  |  | 70 |  | 372 |
|  | 9802 |  | 800 | 4 | 79 |  | 416 | 7 |  | 4 |
|  | 891 | 454 | 8 | 45 | 88 |  |  | 8 |  | 466 |
|  | 98 | 499 | 97 | 50 | 97 |  | 5 | 97 |  |  |
|  | 121069 | 54 | 1067 | 54 | 106 |  | 55 | 10 |  | 5 |
|  | 131158 | 590 | 1156 | 595 | 115 |  | 00 | 115 |  |  |
|  | 141247 | 636 | 1245 | 641 | 124 |  |  | 123 |  | 5 |
|  | 51337 | 681 | 1334 | 68 | 133 |  | 9 | 13 |  | 9 |
|  | 61426 | 726 | 1422 | 73 | 1419 |  |  | 141 |  |  |
|  | 71515 | 772 | 1511 | 778 | 150 |  | 8 | 150 |  |  |
|  | 81604 | 817 | 1600 | 82 | 159 |  | 3 | 15 |  | 3 |
|  | 9)1693 | 863 | 1689 | 870 | 1685 |  | 7 | 1681 |  | 8 |
|  | 01782 | 90 | 17 | 916 | 177 |  | 231 | 177 |  | 3 |
|  | 187 |  |  |  |  |  |  | 185 |  | 7 |
|  | 2196 | 999 | 1956 | 1007 | 195 | 1016 | 16 | 1947 |  |  |
|  | 2049 | 1044 | 2045 | 105 | 204 | 106 | 62 20 | 203 |  |  |
|  | 42138 | 1090 | 21341 | 1099 | 212 | 10 | 2 | 21 |  |  |
|  | 2228 | 1135 | 22231 | 1145 | 2218 | 15 |  | 221 |  |  |
|  | 2317 | 1180 | 23111 | 1190 | 230 | 201 |  |  |  |  |
|  | 2406 | 1226 | 2400 | 123 | 23 | 2 |  |  |  |  |
|  | 2495 | 1271 | 24891 | 128 | 248 | 29 |  | 24 |  |  |
|  | 9258 | 1317 | 25781 | 1328 | 2572 | 13 |  |  |  |  |
|  | 26 | 1362 | 26671 | 1374 | 2661 | 13 |  |  |  |  |
|  | $\overline{2762}$ | $\overline{1407}$ | $\overline{2756}$ | 1419 | 275 | 43 |  |  |  |  |
|  | 2851 | 1453 | 28451 | 1465 | 283 | 14 |  |  |  |  |
|  | 2940 | 1498 | 29341 | 1511 | 292 | 5 |  |  |  |  |
|  | 3029 | 1544 | 302311 | 1557 | 301 | 57 |  | - |  |  |
|  | 3119 | 158 | 31121 | 1603 | 310 | 6 |  |  |  |  |
|  | 328 | 1634 | 320016 | 1648 | 319 | 6 |  |  |  |  |
|  | 397 | 1680 | 32891 | 169 | 328 | 17 |  | 3274 |  |  |
|  | 3386 | 1725 | 337817 | 1740 | 3371 | 175 |  |  |  |  |
|  | 34751 | 1771 | 346717 | 1786 | 3459 | 180 |  | 351 \|18 | 1816 |  |
|  | 3564 |  | 355618 | 1831 | 3548 |  |  | 54018 | 1862 |  |
|  |  | Lat. |  |  |  |  |  |  |  |  |

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 72361130923561319235013302344134027 $8244913572443136824371379,2431 / 138928$ $0|26241454| 26171466|261114772605| 148930$


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| 1087050 | 0 ¢ 6050 | 086051 | 086051 |
| $2{ }_{2} 173100$ | 173101 | 17210 | 172 |
| 3 263150 | 259151 | 25 | 2.53 |
| 343200 | $34620 \pm$ | 34520 | 344 |
| 5433250 | 432 512 | 431254 | 430 |
| 6520300 | 518302 | 517305 | 516 |
| 7606350 | 605353 | 603355 | 602 |
| 8693400 | 691403 | 789406 | 6831409 |
| 977945 | 77745 | 875457 | 773 |
| (10) 866500 | 864504 | 86250 | 859, 511 |
| (T1) 9535 | 90055 | 9435 | 945 |
| 121039600 | 103760 | 1034609 | 1031 |
| 131126650 | 1123655 | 1120660 | 117 |
| 141212700 | 12097051 | 120671 | 03 |
| 151299750 | 1296756 | 1292761 | 897 |
| 16138680013 | 1382805 | 1379812 | 818 |
| 17147285011 | 1469856 | 465863 | 1461869 |
| 18155990 | 155590 | 1551914 | 4792 |
| L9 16459501 | 1641957 | 163796 | 633 |
| 20.173 .2100011 | 1723100 | 1723101 | 1719102380 |
| 318191051 | $1 \overline{314} 10$ 5s | $180310{ }^{\text {bj }}$ | 180.51074 |
| 92190.511091 | 19001108 | 18961117 | 189111 |
| $23 / 1992115$ | $1987^{11531151}$ | 19821167 | 1977117623 |
| 2430761200 | 20731203 | 2363121 | 2063122 |
| 25816512502 | 21691259 | 2154126 | 214912 |
| 263353130312 | 22461310 | 22401320 | 2234113 |
| 3723331359 P | 23321360 | 23261370 | 2320138 |
| 332425140312 | 24191411 | 24131421 | 240614 |
| 2925114592 | 25051461 | 2493147 | 24921 |
| 325.591503 | 35931511 | 2585152 | 2578153430 |
| 31235355) | $25 \overline{73} 15 \overline{62}$ | $2 \overline{71} 1157$ | $266415 \overline{85} 31$ |
| 322771160 | 27611612 | 27571624 | 27501636 |
| $33.2353,16.5023$ | 23.511662 | 2343167 | 233516 |
| 34.23441700 | 29371713 | 293317 |  |
| 35303117503 | 30231763 | 3016177 | 3003,179035 |
| 353118180 | 31101814 | $310218 \cdot 27$ | 3094184136 |
| 3732041850 | 31981864 | 328318 | 189,189237 |
| 3332911909 | 32831914 | 3719 | 326619 |
| $393377119.50 \mid 3$ | 33:69 1965 | 33.691979 | 3352,19 |
| 4034642000 | 31.553 | 34472030 | 34382045 |
| $\bigcirc$ | Dep. Tat. | $\overline{\text { Dep. }}$ Leat. | Dep. 1. Lat |
| $60^{\circ} \mathrm{Dtg}$. |  | 59.1-2 Deg. 4 | $159 \mathrm{I}-4 \mathrm{Dex}:$ |




| $\stackrel{\text { Lat. Dep. }}{\text { Lit. }}$ | Lat. Dep. | Lat. Dep. | $\text { Lat. }\left.\right\|^{\text {Dep. }} \text { \| }$ |
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| $\overline{1} 084$ | 084 | 083 | 08. |
| 2168109 | 16711 | 167110 | - 111 |
| 3252163 | 251164 | 250166 | 249 |
| 4335218 | 3351219 | 334221 | 333 |
| 5419 2.72 | 41827 | 417 | 416 |
| $6{ }^{5} 503327$ | 5023 | 5001331 | 499 |
| 7587381 | 58 | 5 | 582 |
| 8671436 | 66943 | 6674 | 665 |
| 9755490 | 7 | 750497 | 7 |
| 10839545 | 8365.4 | 83455 | 831 |
| 11 $\overline{923} 599$ | $9 \% 603$ | 9176 | 5 61111 |
| 121006654 | 1004658 | 1001662 | 99866712 |
| 1310997031 | 1087713 | 1084718 | 108172213 |
| 14.1174762 | 117176 | 16777 |  |
| 151258817 | 5482 | 251828 | 12478 |
| ${ }_{16} 613428.71$ | $13.38,877$ | 13348831 | 133088916 |
| $171426192{ }^{1}$ | 1422932 | 1418938 | 141304417 |
| 181510980 | 150598 | 5019 | 14 |
| 1915931035 | 5.89104 | 1049 | 1580105619 |
| 2016771089 | 16.731097 | 668110 | 1663111120 |
| -17 $\overline{1761 / 1144}$ | $1756 \overline{151}$ | $7701110 y$ | 1746116721 |
| 184511.9318 | 18401206 | 18351214 | 1829122822 |
| 231929125311 | 19231261 | 1918126 | 19121 |
| 24-201311307 | 30071316 | 0 | 19 |
| 0971362 | 20911371 | 20851380 | 207913 |
| 21811416 | 21741426 | 21631435 | 216214 |
| 22641471 | 22531480 | 22514490 | 224 |
| 33431525 | 23421535 | 335515 | 232315 |
| 23215.79 | 34251590 | 24181601 | 2411161 |
| 3025161634 | 35091645 | 250 | 49416 6 |
| $\overline{31} 26001683$ | $25 \overline{92} 11700$ | 35851711 | 257817 |
| [322684 17.43 | 2676175 | 1766 | 266117 |
| ${ }_{63} 27631797{ }^{\text {a }}$ | $37601809 \mid$ | 275218.21 | 274418 |
| 34235118.52 | 2343186 | 3351877 | 2327188 |
| 351906 | 29271919 | 29191932 | 391019 |
| 3630 191961 | 33111974 | 30021987 | 39 9322 |
| 37310320.15 | 3094202. | 30.8500 | 0762 |
| 383131872070 | 31 | 169 |  |
| 3932712124 | 3262913 | 3252.31 | 2 |
| 40335521.79 | 33:45219: |  |  |
| - Di. Dep Lat. |  |  |  |
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|  | Lat. Dep | 6, 1-4 Deg. |  |  |  |  |  |  |
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|  | $\overline{081} 059$ | 08 | 059 | 08 | 059 |  |  |  |
|  | 162118 | 161 | 118 | 161 | 119 |  |  |  |
|  | 243176 | 42 | 177 | 帾 | 17 |  |  |  |
| 4 | 324235 | 323 | 237 | 32 | 23 | 32 |  |  |
|  | 405294 | 403 | 296 | 402 | 297 | 401 |  |  |
|  | 485853 | 484 | 355 | 48 | 357 | 48 |  |  |
|  | 66411 | 565 | 414 |  |  | 5 | 419 |  |
|  | 647 470 | 645 | 473 | 64 | 476 | 641 | 479 |  |
|  | 728 | 726 | 532 | 72 | 53 | 721 |  |  |
| 10 | 809 | 806 | 591 | 80 |  |  |  |  |
| 11 | 890 64 | 857 | 65 | 85 | 654 |  | 6581 |  |
|  | 971 | 968 | 710 |  | 714 |  |  |  |
|  | 0527 | 1048 | 769 | 10 | 77 | 104 | 77 |  |
|  | 113382 | 1129 | 8 | 11 | 833 | 11 | 83 |  |
|  | 121488 | 1210 | 887 | 120 | 892 | 12 | 89 |  |
|  | 1294940 | 1290 | 946 | 128 | 95 | 12 | 95 |  |
|  | 1375999 | 1371 | 100 | 136 | 0 | 13 | 017 |  |
| 18 | 1456105 | 14521 | 1064 | 14 | 1071 | 14 | 1077 |  |
| 19 | 15371117 | 1532 | 12 | 152 | 1130 | 15 | 1137 |  |
| 20 | 16.181176 | 1618 | 1183 | 1608 | 119 |  | 11972 |  |
| $\overline{21}$ | 1699 | 169 | 42 | 1688 | 1249 |  |  |  |
|  | 178012 | 177 | 01 | 17 | 1309 | 17 |  |  |
| 23 | 18611352 | 1855 | 1360 | 184 | 13 |  |  |  |
|  | 1942 | 1935 | 19 | 192 |  |  |  |  |
|  | 20231469 | 2016 | 78 | 201 | 48 |  |  |  |
|  | 21031528 | 2097 | 1537 | 209 | 54 | 20 |  |  |
|  | 21841587 | 2177 | 1597 | 2170 | 16 | 21 |  |  |
|  | 651646 | 2258 | 1656 | 2251 | 16 |  |  |  |
|  | 23461705 | 233 | 715 | 2331 | 725 | 23 |  |  |
| 30 | $2427 \mid 1763$ | 2419 | 1774 | 2412 | 178 |  |  |  |
|  | 1822 | $\overline{2500}$ | 183 |  |  |  |  |  |
| 32 | 1881 | 2581 | 189 |  |  |  |  |  |
|  | 26701940 | 2661 | 1951 | 26 | 1963 |  |  |  |
|  | 1998 | 2742 | 2010 | 27 | 022 |  |  |  |
|  | 2057 | 282 | 0 | 28 | 2082 |  |  |  |
|  | 29122116 | 2903 | 2129 | 289 |  | 288 |  |  |
| 37 | 29932175 | 2984 | 2188 | 2974 | 2201 | 296 |  |  |
|  | $3074 \mid 2234$ | 30.64 | 224 | 30 | 22 | 304 |  |  |
|  | 31552292 | 314 | 230 | 313 | 232 | 312 | 3 |  |
| 40 | 3236.2351 | 3226 | 2365 | 321 | 2379 | 320 | 239314 |  |
|  | $\overline{\text { Dep. }}$ Lat. |  |  |  |  |  |  |  |
| 㐭 | 54 | 633-4D | Deg. | 581.21 | Dep. |  |  |  |





|  | $-\frac{41}{\text { Lat. }^{\text {Deg. }}} \\| \frac{\text { Dep. }}{1}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  | 151131 | 15013 |  |  |
|  | 226197 | 19 | 22519 | 224200 |
|  | 30226 | 26 | 2 |  |
|  | 37732 | 33 |  |  |
|  | 4531394 | 45139 |  |  |
|  | 528459 | 5 | 24 |  |
|  | 60452 | 52 | ¢ |  |
|  | 67959 | 67759 | 67 |  |
| 10 | 6 | 75265 |  |  |
|  | 83072 | 8277 | 824 |  |
|  |  |  | 89979 |  |
|  | 981 |  |  |  |
|  | 1057 | 10539 | 10 | 10 |
|  | 113 | 112 | 11 |  |
|  | 105 | 120310 | 119810 | 11 |
|  | 111 | 1278112 | 127 |  |
|  | 1358118 | 1353118 | 134811 |  |
|  | 14341247 | 1428125 | 1423125 |  |
|  |  | 150 |  |  |
|  |  | 15 | $\overline{1573}$ | 15 |
|  | 1443 | 16541451 | 1648145 | 1641 |
|  |  | 172915 |  |  |
|  | 1811157 |  | 17 |  |
|  | 188716 | 188 | 1872165 | 18 |
|  | 1962170 | 19551714 | 194717 |  |
|  | 20381771 | 2030 | 202217 |  |
|  | 1837 | 4 | 20 |  |
|  | 2189190 | 21801912 | 17219 |  |
|  |  | 22 | 2247 |  |
|  |  |  |  |  |
|  | 24152099 |  |  |  |
|  | 24912165 | 24812176 | 24722187 |  |
|  | 2566223 | 25 | 254622 |  |
|  | 26412296 |  |  |  |
|  |  |  |  |  |
|  | 279224 | 2782 | 27 71;2452 |  |
|  | 仡 | 2857250 | 2846 |  |
|  | 0 | 293225 | 292 |  |
|  | 301926 | 30072637 |  |  |
|  |  |  |  |  |
|  | 49 Deg. |  |  |  |



| $\text { 芸 } \frac{43 \text { Deg. }}{\text { Let. } 1 \text { Dep. }}$ | $\frac{131-4 \text { Dug. }^{1+1}}{\text { Lat }}$ | Lat. Dog. Litat | $\left\|\frac{439-4 \mathrm{Deg} .}{\mathrm{La}_{\mathrm{a}}-1.1} \mathrm{t}\right\|$ |
| :---: | :---: | :---: | :---: |
| $\bigcirc 07306$ | 078 069 | 073 069 | 079 |
| 2146136 | 146 | 1451 | 144138 |
| 3219205 | 219206 | 2182 | 217 |
| 293.273 | 291274 | 29027 | 289277 |
| 366341 | 364843 | 3631 | $\begin{array}{llll} 3 & 61 & 3 & 46 \\ 4 & 3 & 4 & 1 \end{array}$ |
| 439.409 | 437411 |  | $\begin{aligned} & 433 \\ & 5 \end{aligned}$ |
| 7 <br> 8 <br> 8 <br> 58554 | $\begin{array}{ll}510 & 480 \\ 583 & 548\end{array}$ |  |  |
| 85855546 9658614 | 583 <br> 65648 <br> 6817 |  | $\begin{aligned} & 578 \\ & 650 \end{aligned}$ |
| 10731682 | 728685 | 72568 | 72 |
| 11804750 | 801754 | 79875 | 795 |
| 878818 | 874822 | 870 | 86783012 |
| 13951887 | 4789 | 94389 | 93989913 |
| 141024.955 | 1020959 | 101696 | 101196814 |
| 1510971023 | 10931028 | 108810 | 1084163715 |
| 1611701091 | 11651096 | 116111 | 15611 |
| 1712431159 | 1238116 | 1233117 | 1228117617 |
| 1813161228 | 13111233 | 13061239 | 1300124518 |
| 1913901206 | 13841302 | 13781308 | 1372 |
| 2014631364 | 14571370 | 14511377 | 14 |
| 2115361432 | 1530 | 15 | 15171352121 |
| 22, 160915 | 160215 | 15961 | 91 |
| 2316821569 | 675 15 | 168815 | 1661159023 |
| 341755163 | 174816 | 174116 |  |
| 2518281705 | 182117 | 181317 |  |
| 23619021773 | 1894178 | 188617 | 1878 |
| 2719751841 | 1967 | 959 | 7 |
| ${ }^{2} 4819$ | 203919 | 202119 | 2023193628 |
| 292121197 | 21121987 | 2104 | 20 |
| 3021942 | 218520 | 620 | 216720 |
| 3122672114. | 255821 | 22492 | 2239214431 |
| -3233 40218 | 233121 | 232122 | 231222 |
| 3324132251 | 24822 | $339422{ }^{\prime}$ | 238422 |
| 248723 | 476233 | 246623 | 245623 |
| 3525602387 | 254973 | 253924 | 252824 |
| -36263324 55 | 262224 | 26112478 | 260124 |
| ${ }_{37} 2706252$ | 2695253 | 2684,25 | 267325 |
| 3827792592 | 2768260 | 42756261 | $6^{6} 274529$ |
| ${ }^{39} 92852.2660$ |  | 298926 |  |
|  | Dep. Lat. | Dep. Lat. | Dep. Lat. |
| 47 De 3 . | 46.34 Deg . | 46 1.2 Deg | $481-4$ Des. |


| \% | $441.4 \mathrm{D}_{0}$ | $441-2 \mathrm{D}_{\text {cg }}$ | 45 DO 8. |
| :---: | :---: | :---: | :---: |
| Lat. ${ }^{\text {Dop. }}$ | Lat. Dep. | $\frac{L_{a}}{0} 1{ }^{\text {dep }}$. | Latit ${ }^{\text {D }}$ Dep. |
| $1{ }^{1} 072069$ | 072076 | 071 | 071071 |
| $2{ }^{2} 144139$ | 143146 | 143140 | 141 |
|  |  | 214 2140 210 | 212 |
| 5 560 347 | 358 | 35785 |  |
| 6432417 | 430415 | 428 | 424 |
| 504486 | 501 | 499 | 495 |
| 575556 | 573558 | 571561 | 566 |
| $9{ }^{6} 4762$ | ${ }^{645} 628$ |  |  |
| 10.719695 | 71669 | 71370 | 707 |
| $117{ }^{11} 791764$ | ${ }^{788} 768$ | 7885781 | 778 |
| ${ }^{12} 8638834$ |  | 856 | 848 |
|  | 9319 | 9279 | 919 |
| ${ }_{14}^{14} 1007$ | 1003 | 999 | 9 |
| 15107910 | 107410 | 07010 | 5 |
| ${ }^{16} 6115111111$ | 114611 | 141 | 131113116 |
| 1771223118111 | 121811 | 12131 | 202120217 |
| ${ }^{18} 12951250 \mid 1$ | 12891256 | 28412 | 273127318 |
| 13671320 | 36113 | 355 |  |
| ${ }^{2} \mathbf{2} 1439138911$ | 1433139 | 12614 | 14 |
| ${ }^{21} 1{ }^{1511} 1459$ | 1504 146 | 4981 |  |
|  |  |  |  |
| 2316541598 | 16471605 | 164016 | 62616 |
| 2417261667 | 171916 | 析 | 6971 |
| 2517981737 | 1791174 | 788 |  |
| 2618701806 | 186218 | 1854 |  |
| 271942187 | 193418 | 192618 | 190919 |
| 28.20141945 | 200619 | 9719 |  |
|  |  | 268 |  |
| 30215820842 | 2149209 | 1402103 | 2121212130 |
| 3122302153 | 22212163 | 221121 | 2192 |
|  |  |  |  |
| 22 | 2364230 | 235423 | 2333 |
|  |  | 2523 |  |
|  | 072442 | 49624 | 475 |
| 25902501.2 | 25892512 |  | 546 |
| 3726622570,2 | 25025 | 263925 | 2616261637 |
| 3326 | 272225 |  |  |
|  | 27 |  |  |
| 1002877277928 | 28652791 |  | 82840 |
| Dep. Lat. ${ }^{\text {a }}$ | Dep. Lat. | $\overline{\text { Dep. }}$ Lat. |  |
| 46 Deg. | 453 | $\frac{451.2 \text { ner. }}{}$ |  |

Bo Sow the therquet it be chinetitrisk forin tuentwa-2 The argyle of eneven
 En a skijhtinntill ine Courd
 Whatur. the ilsilurvee a, Stheis altabies manll-be the theig her
 yfainaiy<t Live 4 H rox
 Nopzees ry allilude thes saj br line rif. 2 ithe. herg


## DO NOT REMOVE OR MUTILATE CARD




[^0]:    * When Degrees and Minutes are expressed, the Degrees are marked vith a small cipher, and the Minutes with a dash, as above.

[^1]:    * This nearest Mistance should be taken nearly at Right Anglesfrom the Random Line to the Bound. The Angle, howerver, should be as much less than a Right Angle, as half the quantity of the Angle at the first Station, made by the Random and true Iine; which would malse the Angles at $B$, and f, of the Triangle AfB, equal. (See Remark 1. Sec. 3. Part 1.)
    t " 57,3 is the Radius of a Circle (nearly) in such parts, as the Circamfortence contains $\mathbf{3 6 0 . 0}$
    $\ddagger$ The true Course, as found by this method, will seldom differ two minutes of a Degree, from a statement to find the Course by Trigononsetry.

[^2]:    *The practitioner is suppessd to stand at the Angular Point, and take the Bearings of the Lines ; by which it is easy to perceive that the last Course plotted, as taken in the Survey, must be reversed.

[^3]:    *For the manner of taking this Distance, yee the first note under Example 2; Sec. 4; Part 2.

