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1845. [illegible]855

TREATISE

ON THE

ART OF MEASURING;

CONTAINING ALL THAT IS USEFUL IN

BONNYCASTLE, HUTTON, HAWNEY, INGRAM,

AND SEVERAL OTHER

MODERN WORKS ON MENSURATION:

TO WHICH ARE ADDED

TRIGONOMETRY,

WITH

ITS APPLICATION TO HEIGHTS AND DISTANCES:

SURVEYING; GAUGING:

AND ALSO THE

MOST IMPORTANT PROBLEMS IN MECHANICS.

BY JAMES RYAN,

Author of a Treatise on Algebra, the New American Grammar of Astronomy,
The Differential and Integral Calculus, &c.

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PREFACE.

The art of Measuring, like all other useful inventions, appears to have been the offspring of want and necessity; and to have had its origin in those remote ages of antiquity, which are far beyond the reach of credible and authentic history. Egypt, the fruitful mother of almost all the liberal sciences, is imagined to have given birth, among the rest, to Geometry or Mensuration; it being to the inundations of the Nile that we are said to be indebted for this interesting and important branch of human knowledge.

After the overflowings of the river had deluged the country, and all artificial boundaries and land-marks were destroyed, there could have been no other method of ascertaining individual property, than by a previous knowledge of its figures and dimensions. From this circumstance, it appears highly probable, that Geometry was first known and cultivated by the Egyptians; as being the only science which could administer to their wants, and furnish them with the assistance they required. The name itself properly signifies the *art of measuring the earth*; which serves still further to confirm this opinion; especially, as it is well known that many of the ancient mathematicians applied their geometrical knowledge only to that purpose; and that even the Elements of Euclid, as they now stand, are only the theory from which we obtain the rules and precepts of our present more mechanical process.

But to trace the sciences to their first rude beginnings, is a matter only of learned curiosity, which could afford but little gratification to readers in general. It is of much more consequence to the rising generation to be informed that, in their present improved state, they are of the greatest utility and importance. And in this respect, the art I have undertaken to elucidate is inferior to none, Arithmetic only excepted. Its use in most of the different branches of the Mathematics is so general and extensive that it may justly be considered as the mother and mistress of all the rest; being the source from which their various properties and principles were at first chiefly derived.

As a testimony of this superior excellence, I need only mention a few of those who have studied and improved it; in which illustrious catalogue we have the names of Euclid, Archimedes, Thales, Anaxagoras, Pythagoras, Plato, Apollonius, and Ptolemy, amongst the ancients; and Huygens, Wallis, Gregory, Halley, the Bernoullies, Euler, Leibnitz, and Newton, among the moderns; all of whom applied themselves to particular parts of it, and greatly enlarged and improved the subject. To the latter especially, we are indebted for many valuable discoveries in the higher branches of the science; which have not only enhanced its dignity and importance, but rendered the practical application of it more general and extensive.

The degree of estimation in which the art was held by these, and other eminent characters, will, in general, it is apprehended, be thought a sufficient encomium on its merits. But, for the sake of young people, and those of a confined education, it may not be amiss to give a few more instances of its advantages, and show that its importance in trade and business is not inferior to its dignity as a science. Artificers of almost all denominations are indebted to this invention for the establishmemt of their several occupations, and the perfection and value of their workmanship. Without its assistance all the great and noble works of art would have been imperfect and useless. By this means the architect lays down his plan, and erects his edifice; bridges are built over large rivers; ships are constructed; and property of all kinds is accurately measured, and justly estimated. In short many of the elegancies and conveniencies of life owe their existence to this art, and will be multiplied in proportion as it is well understood, and properly practised.

From this view of the subject, it is hardly to be accounted for, that, in a commercial nation, like our own, an art of such general application should have been so greatly neglected. Mechanics of all kinds, it is well known, are but ill acquainted with its principles; and those who have been the best qualified to afford them any assistance, have thought it beneath their attention. Till within a few years past there could not be found a regular treatise upon this subject in the English language. Some particular branches, it is true, had been assiduously cultivated and improved; but these were only to be found in their miscellaneous state; interspersed through a number of large volumes, in the possession of but few, and in a form and language totally unintelligible to those for whom they were more immediately necessary.

In school books, and those designed for the use of learners, it has always appeared to me, that plain and concise rules, with proper exercises, are entirely sufficient for the purpose; it being obvious that example in science, as well as in morals, will ever enforce and illustrate precept. For this reason an operation, wrought out at length, will be found of more service to beginners, than all the tedious directions and observations that can possibly be given them. From constant experience I have been confirmed in this idea; and it is in pursuance of it I have formed the plan of the present publication. I have not been ambitious of adding much new matter to the subject: but chiefly to arrange and methodize it in a manner more easy and rational than had been done before.

N. B. The favourable reception this work has constantly met with, for a considerable number of years past, has induced me, in the present edition, to make a variety of alterations and improvements that have since occurred to me to be necessary, and which are such, as I hope, will render it still more acceptable to the public.

JOHN BONNYCASTLE.

Royal Academy, Woolwich,

ADVERTISEMENT.

THE favorable reception and great demand for BONNYCASTLE'S MENSURATION and PRACTICAL GEOMETRY, since its first publication in this country, induced me to publish the present edition, which contains not only the whole of that valuable work, (except a few Algebraical and Fluxional investigations,) but also, all that is useful in Hutton, Hawney, Ingram, and other modern works on the same subject.

To this edition, there is also added, TRIGONOMETRY, with its application to heights and distances, SURVEYING, GAUGING, and some of the most useful problems in MECHANICS and DYNAMICS.

The articles on Trigonometry, Surveying and Gauging, contain every thing, useful and necessary, which can be found in a variety of voluminous and expensive works on those subjects.

The article on *Mechanics* and *Dynamics* contains the principal problems in *Brunton's Mechanics* :—that is, *Falling Bodies ; the Pendulum ; the Lever, the Wheel and Axle, the Pulley, the Inclined Plane, the Wedge, and the Screw*, which are usually called *the six Mechanical Powers; Velocity of Wheels; Steam Engine; Water Wheels;* and *Pumps.*

So that, upon the whole, the present work on *The Art of Measuring*, will be found more useful than any similar performance which has yet been published in Europe, or in the United States of America.

JAMES RYAN.

New-York, Oct. 1*st*, 1831.

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ON

THE ART OF MEASURING.

INTRODUCTION.

DECIMALS.

If the numerator and denominator of a fraction be multiplied or divided by any number, its value will not be altered; thus, $\frac{1}{2} = \frac{5}{10}$, $\frac{1}{4} = \frac{25}{100}$, $\frac{1}{100} = \frac{10}{1000}$, and so on. Hence, it is evident, that we can reduce a fraction to another equivalent one, having a given denominator. It likewise follows, that a fraction may be reduced to another equivalent one, whose denominator shall be 10, or some number produced by the continued multiplication of 10, by annexing ciphers to the numerator and denominator, and dividing both, (with the ciphers annexed) by the original denominator.

Thus, the fraction $\frac{1}{5} = \frac{10}{50}$, and dividing both the numerator and denominator of the fraction $\frac{10}{50}$, by 5, the original denominator, we should have $\frac{10}{50} = \frac{2}{10}$.

Again, the fraction $\frac{1}{4} = \frac{100}{400}$; and dividing both the numerator and denominator by 4, we shall have $\frac{100}{400} = \frac{25}{100}$. Also, $\frac{3}{8} = \frac{3000}{8000}$; and dividing both the numerator and denominator of the fraction $\frac{3000}{8000}$, by 8, the original denominator, we shall have $\frac{3000}{8000} = \frac{375}{1000}$; hence $\frac{3}{8} = \frac{375}{1000}$; and so on.

Fractions whose denominators are 10, 100, 1000, &c. are called *decimal fractions;* and when these fractions are written without the denominator, they are usually called *decimals;* and to denote the value of a decimal, a point is prefixed to as many figures of the numerator as there are ciphers in the denominator.

Thus, the decimal fraction $\frac{2}{10}$ is written .2, the decimal fraction $\frac{25}{100}$ is written .25, the decimal fraction $\frac{375}{1000}$ is written .375, and so on. The expressions .2, .25, .375, &c. are called decimals. Hence, it is evident, that the figures next the decimal point indicates tenths, the next figure hundredths, the next thousandths, and so on.

The decimal .2 is read *two-tenths;* the decimal .25 is read *twenty-five hundredths;* the decimal .875 is read *eight hundred and seventy-five thousandths;* and so on.

Since, $\frac{2}{10} = \frac{20}{100} = \frac{200}{1000} = \frac{2000}{10000}$, and so on; $.2 = .20 = .200 = .2000$, &c. therefore the value of a decimal is not changed by annexing a cipher to the end of it, nor by taking one away.

If there be not as many figures in the numerator as there are ciphers in the denominator, ciphers must be put in the place of tenths, hundredths, &c. thus, $\frac{2}{100}$ is written .02; $\frac{25}{1000}$ is written .025; and $\frac{3}{100000}$ is written .00003; and so on.

Hence, *the value of figures in decimals are diminished in the same ratio from the decimal point towards the right, as whole numbers are increased from the right towards the left.*

When the fractional part of a mixed number is reduced to a decimal, the decimal part is separated from the whole number by a decimal point.

Thus, $3\frac{75}{100}$ is written 3.75; $4\frac{5}{1000}$ is written 4.005; and so on.

From what has been already observed, it is plain, that *any fraction may be reduced to a decimal by adding ciphers to the numerator and dividing by the denominator.*

Thus, the fraction $\frac{35}{700}$ is reduced to .05, by adding two ciphers to 35, and dividing the expression 35.00 by 700; as there are no tenths a cipher is put in the place of tenths; so that the decimal equivalent to the fraction $\frac{35}{700}$ is five hundredths.

Again, the fraction $\frac{3}{99}$, reduced to a decimal, is equivalent to .030303, an so on; here, there would still be a remainder, and it is also evident that the decimal would never terminate; in which case, it is only necessary in most calculations to use six or seven figures of the decimals.

A quantity of one denomination may be reduced to the decimal of another quantity of the same kind, but of a different denomination; by first expressing the ratio of the former to the latter by a common fraction, and then reducing the fraction thus formed to a decimal.

For example, 2 nails is the $\frac{2}{16}$ of a yard, or $\frac{1}{8}$ of a yard, which reduced to a decimal, is equivalent to .125; hence, 2 nails is the 125 *thousandths* of a yard.

The reduction of a fraction to a decimal, or of one quantity to the decimal of another, is usually called *reduction of decimals.*

Examples in Reduction of Decimals.

Example 1. What decimal of a foot is 9 inches?

Here, 9 inches is the $\frac{9}{12}$ or $\frac{3}{4}$ of a foot, which, reduced to a decimal, is equivalent to .75, or 75 hundredths.

Ex. 2. What decimal of a yard is 2 feet 6 inches?

Here, 2 feet 6 inches is the $\frac{30}{36}$ or $\frac{5}{6}$ of a yard, which, reduced to a decimal, is .833333, &c. This is a repeating or circulating decimal, never terminating.

Ex. 3. What decimal of an acre or 160 square poles, is 2 roods and 16 square poles?

Here, 2 roods and 16 square poles is the $\frac{96}{160}$ or $\frac{3}{5}$ of an acre, which, reduced to a decimal, is .6 or 6 tenths.

Ex. 4. What decimal of a cubic foot is 144 cubic inches?

Here, 144 cubic inches is the $\frac{144}{1728}$ or $\frac{1}{12}$ of a cubic foot; and $\frac{1}{12}$, reduced to a decimal, is equivalent to .083333, &c. being a repeating decimal.

Ex. 5. Reduce 8 feet 6 inches to the decimal of a mile.
Answer, .0016098.

Ex. 6. Reduce 2 feet 5 inches to the decimal of a yard.
Ans. .805555.

Ex. 7. Reduce $5\frac{1}{2}$ yards to the decimal of a mile or 1760 yards. *Ans.* .003125.

Ex. 8. Reduce $4\frac{1}{2}$ miles to the decimal of 40 miles.
Ans. .1125.

Ex. 9. Reduce 3 roods 11 poles to the decimal of an acre. *Ans.* .81875.

The decimal of one denomination may be reduced to whole numbers of lower denominations, as in the reduction of quantities of a higher denomination to a lower, observing, after each multiplication, to point off for decimals as many figures towards the right as there were figures in the given decimal. The figures on the left hand of the decimal points will be the whole numbers required.

For example, .3945 of a day is equal to the fraction $\frac{3945}{10000}$ of a day, which, expressed as the fraction of an hour, is $\frac{3945}{10000} \times 24 = \frac{94680}{10000}$, or 9 hours and $\frac{4680}{10000}$ of an hour; but $\frac{4680}{10000}$ of an hour is $\frac{4680}{10000} \times 60$ of a minute, or $\frac{280800}{10000}$, which is equal 28 minutes, $\frac{800}{10000}$ of a minute; again, this fraction of a minute is equal to $\frac{800}{10000} \times 60$ of a second, or $\frac{48000}{10000}$, which is equal to 4 seconds, and $\frac{8}{10}$ of a second; so that, the decimal .3945 of a day is equal to 9 hours, 28 minutes, $4\frac{8}{10}$ seconds. From this it appears that pointing off the decimals serves the same purpose as dividing by the denominator: thus,

```
 .3945 day
    24
 ------
 15780
 7890
 ------
 9.4680 hours
     60
 ------
28.0800 minutes.
```

Examples in finding the values of Decimals.

Ex. 1. Required the value of .375 of a yard.
Ans. 1 qr. 2 nails.

Ex. 2. Required the value of .625 of an acre.
Ans. 2 roods 20 poles.

Ex. 3. Required the value of .875 of a mile.
Ans. 7 furlongs.

Ex. 4. Required the value of .2385 of a degree.
Ans. 14′ 18″ 36 thirds.

ADDITION OF DECIMALS.

The addition of decimals is performed like that of whole numbers, observing however to arrange the numbers so that the separating points may be in the same column; that is, the tenths under tenths, the hundredths under hundredths, and so on.

For instance, the decimals .571, .672, .3, .003, and .0075, being arranged as follows:

```
 .571
 .672
 .3
 .003
 .0075
------
1.5535
```

their sum is found to be 1.5535; the reason of the arrangement is evident, since those figures are added together which are of the same local value.

Again, the sum of the numbers 3.5, 7.005, 4.325, .0003, and 1.000007, which contain whole units, is found in like manner, thus:

```
     3.5
     7.005
     4.325
      .0003
     1.000007
    ---------
Sum 15.830307
```

Examples in Addition of Decimals.

Ex. 1. Required the sum of 5.714, 3.456, .543, and 17.4957. *Ans.* 27.2087.

Ex. 2. Required the sum of 3.754, 47.5, .00857, and 37.5. *Ans.* 88.76257.

Ex. 3. Required the sum of 54.34, .375, 14.795, and 1.5. *Ans.* 71.01.

Ex. 4. Required the sum of 37.5, 43.75, 56.25, and 87.5 *Ans.* 225.

Ex. 5. Required the sum of .375, .625, .0625, .1875, .3125, .4375, .005, .9475, and .0075. *Ans.* 2.96.

The Substraction of Decimals is performed in the same manner as that of whole numbers ; observing to place each figure of the less below a figure of the same local value in the greater.

For instance, let the difference of .3765 and .1236 be required : the decimals being arranged thus ;

$$\begin{array}{r} .3765 \\ .1236 \\ \hline .2529 \end{array}$$

their difference will be .2529.

Again, let .7562 be taken from .82 ; by annexing ciphers to the greater and arranging the numbers thus ;

$$\begin{array}{r} .8200 \\ .7562 \\ \hline .0638 \end{array}$$

we shall find the difference to be .0638 : it must be observed, that the value of a decimal is not increased nor decreased by annexing ciphers to it ; for a fraction does not alter its value by annexing ciphers to its numerator and denominator, thus ; $\frac{82}{100} = \frac{820}{1000} = \frac{8200}{10000}$, and so on. This is also evident from the decimal notation, which is similar to that of whole numbers ; that is, the value of the decimal .82 is 8 tenths and 2 hundredths, the value of the decimal .820 is also the same, being 8 tenths 2 hundredths and 0 thousandths ; and so on.

Examples in Substraction of Decimals.

Ex. 1. Required the difference between 57.49 and 5.768. *Ans.* 51.722.

Ex. 2. Required the difference between .0076 and .00075. *Ans.* .00685.

Ex. 3. Required the difference between 3.468 and 1.2591. *Ans.* 2.2089.

Ex. 4. Required the difference between 3.1416 and .5236. *Ans.* 2.6180.

From the multiplication of fractions, (or even the decimal notation,) it appears evident that, *the multiplication of* *decimals is performed as in whole numbers, but if there be not as many decimals in the product as there are in both factors, ciphers must be prefixed to supply the deficiency.*

For instance, the product of $.06 \times .004$ is equal to .00024; since .06 is equal $\frac{6}{100}$, and .004 is equal $\frac{4}{1000}$; hence, $\frac{6}{100} \times \frac{4}{1000} = \frac{24}{100000}$, which, expressed according to the decimal notation, is equal to .00024.

Examples in Multiplication of Decimals.

Ex. 1. Multiply 3.125 by 2.75. *Ans.* 8.59375.

Ex. 2. Multiply 79.25 by .459. *Ans.* 36.37575.

Ex. 3. Multiply .135272 by .00425. *Ans.* .000574906.

Ex. 4. Multiply .004735 by .0375. *Ans.* .0001775625.

The Division of Decimals is performed in the same manner as that of whole numbers, but the dividend must contain as many decimal figures as the divisor, if not, ciphers must be annexed; and the decimals in the quotient, must be always equal to the excess of the decimal figures in the dividend above those in the divisor, if not, ciphers must be prefixed.

For instance, when the denominators of any two fractions are the same, their quotients are found by dividing their numerators: thus, $\frac{25}{1000} \div \frac{5}{1000}$ is equal to $25 \div 5$; that is 5: hence $.025 \div .005$ is equal to 5; that is, a whole number.

Again, $\frac{3}{10} \div \frac{4}{1000}$ is equal to $\frac{300}{1000} \div \frac{4}{1000}$, which is equal to $\frac{300}{4} = 75$; hence to the decimal .3, the dividend, two ciphers must be added, in order to have as many decimal places as the divisor .004, before the division can be performed.

It likewise follows, that $\frac{375}{1000} \div \frac{5}{10}$ is equal to $\frac{375}{1000} \div \frac{500}{1000}$, which is equal to $\frac{375}{500}$; this by reduction is equal to $\frac{75}{100}$, which may be written .75; hence, $.375 \div .5 = .75$; that is, the decimal figure in the quotient is equal to the excess of the decimal figures in the dividend above those in the divisor.

Examples in Division of Decimals.

Ex. 1. Divide .1342 by 67.1. *Ans.* .002.

Ex. 2. Divide 1.7144 by 1.5. *Ans.* 1.142955.

Ex. 3. Divide 24880 by 360. *Ans.* 69.111, &c. or $69\frac{1}{9}$.

Ex. 4. Divide 172.8 by .144. *Ans.* 1200.

Ex. 5. Divide .88 by 88. *Ans.* 100.

Ex. 6. When the diameter of a circle is 1 the circumference is 3.14159 nearly; what is the diameter of the earth, allowing its circumference to be 24880 miles?

Ans. 7919.53666 miles, *nearly.*

Extraction of the Square Root.

The square of the sum of two numbers is equal to the squares of the numbers with twice their product. Thus, the square of 24 is equal to the squares of 20 and 4 with twice the product of 20 and 4; that is, to $400 + 2 \times 20 \times 4 + 16 = 576$. Here in extracting the second root of 576, we separate it into two parts, 500 and 76. Thus, 500 contains 400, the square of 20, wtth the remainder 100; the first part of the root is therefore 20, and the remainder $100+76$, or 176.

Now, according to the principle above mentioned, this remainder must be twice the product of 20, and the part of the root still to be found, together with the square of that part. Now, dividing 176 by 40, the double of 20, we find for quotient 4; then this part being added to 40, the sum is 44, which being multiplied by 4, the product 176, is evidently twice the product of 20 and 4, together with the square of 4. The operation may, in every case, be illustrated in the same manner. Hence the following rule for extracting the square root of any number.

Commencing at the unit figure, cut off periods of two figures each, till all the figures are exhausted, the first figure of the square root will be the square root of the first period, or of the greatest square contained in it, if it be not a square itself. Subtract the square of this figure from the first period; to the remainder annex the next period for a dividend; and, for part of a divisor, double the part of the root already obtained. Try how often this part of the divisor is contained in the dividend wanting the last figure, and annex the figure thus found to the parts of the root and of the divisor already determined. Then multiply and subtract as in division; to the *remainder bring down the next period; and, adding to the divisor the figure of the root last found, proceed as before.*

For instance, the square root of 106929, is found thus:

```
      Square.  Root.
      106929 | 327
       9
      ----
   62)169
      124
      -----
  647)4529
      4529
      ----
```

If any thing remain, after continuing the process till all the figures in the given number have been used, proceed in the same manner to find decimals, adding, to find each figure two ciphers.

If the root of a fraction be required, let the fraction be reduced to a decimal, and then proceed as in the extraction of the roots of whole numbers.

Examples in extracting the Square Root.

Ex. 1. Required the square root of $2\frac{1}{4}$ or 2.25.
Answer, 1.5.

Ex. 2. Required the square root of 152399025.
Ans. 12345.

Ex. 3. Required the square root of 5499025.
Ans. 2345.

Ex 4. Required the square root of 36372961.
Ans. 6031.

Ex. 5. Required the square root of 10.4976.
Ans. 3.24.

Ex. 6. Required the square root of 9980.01.
Ans. 99.9.

Ex. 7. Required the square root of 2.
Ans. 1.414213, *nearly.*

Extraction of the Third or Cube Root.

The cube or third power of the sum of two numbers is equal to the cubes of the numbers increased by 300 times the square of the first number multiplied by the second, and also increased by 30 times the first multiplied by the square of the second, thus:

$$\left.\begin{array}{l}20+4\\20+4\end{array}\right\}\text{ Multiplied.}$$

$$\begin{array}{l}20\times20+4\times20\\ \qquad\qquad+4\times20+16\end{array}$$

$$\text{Multiplied}\left\{\begin{array}{l}20\times20+2\times4\times20+16=\text{2d power.}\\20\ +\ 4\end{array}\right.$$

$$\begin{array}{r}20\times20\times20+2\times4\times20\times20+20\times16\\4\times20\times20+2\times20\times16+64\end{array}$$

$$\text{Third power}=8000+3\times4\times20\times20+3\times20\times16+64,$$
$$\text{or } 8000+300\times4\times4+30\times2\times16+64.$$

Hence, this rule for extracting the third or cube root of any given number:—*Commencing at the unit figure, cut off periods of three figures each till all the figures of the given number are exhausted. Then find the greatest cube number contained in the first period and place the cube root of it in the quotient. Subtract its cube from the first period and bring down the next three figures; divide the number thus brought down by* 300 *times the square of the first figure of the root and it will give the second figure; add* 300 *times the square of the first figure,* 30 *times the product of the first and second figures, and the square of the second figure together, for a divisor; then multiply this divisor by the second figure, and subtract the result from the dividend, and then bring down the next period, and so proceed till all the periods are brought down.*

For instance, in finding the cube root of 48228544, the operation will stand thus:

```
      48'228'544(364 root.
      27
      ----
  3276)21228
       19656
       -----
393136)1572544
       1572544
       -------
```

Divided by $300\times3^2=2700$	Divided by $36^2\times300=288800$
$30\times3\times6=\ 540$	$30\times36\times4=\ 4320$
$6\times6=\ 36$	$4\times4=\ 16$
1st divisor 3276	2d divisor 393136

If any thing remains, add three ciphers, and proceed as before; but for every three ciphers that are added, one decimal figure must be cut off in the root. And if the cube root of a fraction or a mixed number be required, reduce tne fraction to a decimal, and proceed as in whole nnmbers: the decimal part however must consist of periods of three figures each, if not, ciphers must be added.

Examples in extracting the Cube Root.

Ex. 1. Required the cube root of 512000000.
Answer, 800.

Ex. 2. Required the cube root of 447697125.
Ans. 765.

Ex. 3. Required the cube root of 2.
Ans. 1.259921.

Ex. 4. Required the cube root of 44361864.
Ans. 354.

Ex. 5. Required the cube root of .0001357.
Ans. .05138, &c.

Ex. 6. Required the cube root of $\frac{5}{276}$ or .018115942.
Ans. .262 *nearly.*

Ex. 7. Required the cube root of $13\frac{2}{3}$.
Ans. 2.3908.

DUODECIMALS.

Fractions whose denominators are 12, 144, 1728, &c. are called *duodecimals;* and the division and subdivision of the integers are *understood* without being expressed as in *decimals.* The method of operating by this class of fractions, is principally in use among artificers, in computing the contents of work, of which the dimensions were taken in *feet, inches,* and *twelfths* of an inch.

Rule. Set down the two dimensions to be multiplled together, one under the other, so that feet shall stand under feet, inches under inches, &c. Multiply each term of the multiplicand beginning at the lowest, by the feet in the multiplier, and set the result of each immediately under its corresponding term, observing to carry 1 for every 12, from the inches to the feet. In like manner, multiply all the multiplicand by the inches of the multiplier, and then by the twelfth parts, setting the result of each term one place removed to the right hand when the multiplier is inches, and two places when the parts become the multiplier. The sum of these partial products will be the answer.

Or, instead of multiplying by the inches, &c. take such parts of the multiplicand as these are of a foot.

Or, reduce the inches and parts to the decimal of a foot, and proceed as in the multiplication of decimals.

For example, multiply 2 feet 6 inches by 2 feet 3 inches.

				or,		
2f.	6i.				2f.	6i.
2	3					2
5	0				5	0
	7	6		$3 = \frac{1}{4}$	0	$7\frac{1}{2}$
5	7	6			5	$7\frac{1}{2}$

Here, the 7, which stands in the second place, does not denote square inches, but rectangles of an inch broad and a foot long, which are to be added to the square inches in the third place; so that, $7 \times 12 + 6 = 90$ are the square inches, and the product is 5 square feet, 90 sq. inches. And this manner of estimating the inches must be observed in all cases where two dimensions in feet and inches are thus multiplied together.

Or, the product may be found by reducing the inches to the decimal of a foot: thus, 6 inches $=.5$ of a foot; hence, $2.5\times2.25=5.625$ square feet, but .625 of a square foot is equal to $.625\times144=90$ square inches, the same as before

Examples in Duodecimals.

Ex. 1. Multiply 35 feet $4\frac{1}{2}$ inches into 12 feet $3\frac{1}{3}$ inches.
Ans. 434 square feet 47 square inches.

Ex. 2. Multiply 7 feet 9 inches by 3 feet 6 inches.
Ans. 27 square feet 18 square inches.

Ex. 3. Multiply 7 feet 5 inches 9 parts by 3 feet 5 inches 3 parts. *Ans.* 25 square feet 102 $\frac{3}{6}$ square inches.

Ex. 4. Multiply 75 feet 9 inches by 17 feet 7 inches.
Ans. 1331 square feet 135 square inches.

Ex. 5. Multiply 97 feet 8 inches by 8 feet 9 inches.
Ans. 854 square feet 84 square inches.

Explanation of the characters used in the following part of the work.

The sign of equality is $=$; thus, 6 added to 4 *is equal to* 10, or 6 added to $4=10$.

The sign of addition is an erect cross; thus, $6+4$, which is read 6 *plus* 4, denotes that 4 is to be added to 6.

Subtraction is denoted by a single line; as $9-7$, read 9 *minus* 7, which expresses 9 diminished by 7, or $9-7=2$.

Multiplication is expressed by an oblique cross, or by a full point; thus, 7×9, or 7.9, denotes that 7 is to be multiplied by 9; that is, $7\times9=63$.

Division is denoted by placing the dividend before the sign $\div$, and the divisor after it; thus, $18\div2$, denotes 18 *divided by* 2, which is equal to 9: or, it may be expressed in the form of a fraction; thus, $\frac{18}{2}=9$.

The ratios of 6 is to 9 as 8 to 12, is usually expressed $6:9::8:12$.

The *second*, *third*, *fourth*, &c. powers of a number are written thus, 3^2, 3^3, 3^4, &c. that is, $3^2=9$, $3^3=27$, $3^4=81$, and so on.

The roots of quantities are expressed by means of the radical sign $\sqrt{}$, with the proper index annexed, or by fractional indices placed at the right hand of the number; thus, $\sqrt{9}$ or $9^{\frac{1}{2}}$ expresses the square root of 9, $\sqrt[3]{27}$ or $27^{\frac{1}{3}}$, the cube root of 27; and so on.

PRACTICAL GEOMETRY.

DEFINITIONS.

1. Geometry is the science which has for its object the measurement of extension.

Extension has three dimensions, length, breadth, and height.

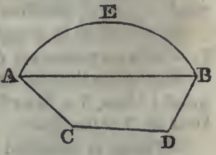
2. A *line* is length without breadth.

The extremities of a line are called *points:* a point, therefore, has no extension.

3. A *straight line* is the shortest distance from one point to another.

4. Every line, which is not straight, or composed of straight lines, is a *curve line.*

Thus, AB is a straight line; ACDB is a *broken* line, or one composed of straight lines; and AEB is a curve line.



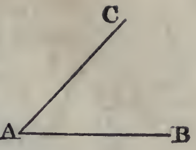
5. A *surface* is that which has length and breadth, without height or thickness.

6. A *plane* is a surface, in which, if two points be assumed at will, and connected by a straight line, that line will lie wholly in the surface.

7. Every surface, which is not plane, or composed of plane surfaces, is a *curved surface.*

8. A *solid* or *body* is that which combines all the three dimensions of extension.

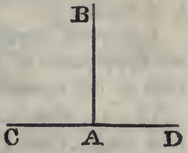
9. When two straight lines, AB, AC, meet together, the quantity, greater or less, by which they are separated from each other in regard to their position, is called an *angle;* the point of *intersection* A is the *vertex* of the angle; the lines AB, AC, are its *sides.*



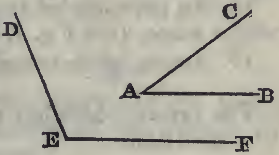
The angle is sometimes designated simply by the letter at the vertex A; sometimes by three letters BAC, or CAB, the letter at the vertex being always placed in the middle.

Angles, like all other quantities, are susceptible of addition, subtraction, multiplication, and division. Thus the angle BCE, is the sum of the two angles, DCB, DCE; and the angle DCB is the difference of the two angles BCE, DCE. (See Fig. Def. 22.)

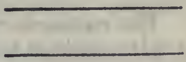
10. When a staight line AB meets another staight line CD, so as to make the adjacent angles BAC, BAD equal to each other, each of those angles is called a *right angle;* and the line AB is said to be *perpendicular* to CD.



11. Every angle BAC, less than a right angle, is an *acute angle;* every angle DEF, greater than a right angle, is an *obtuse angle.*



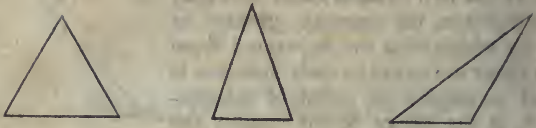
12. Two lines are said to be *parallel,* when, being situated in the same plane, they cannot meet, how far soever, either way, both of them be produced.



13. A *plane figure* is a plane terminated on all sides by lines.

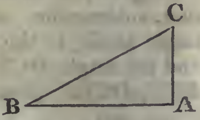
If the lines are straight, the space they enclose is called a *rectilineal figure,* or *polygon,* and the lines themselves taken together form the contour, or *perimeter* of the polygon.

14. The polygon of three sides, the simplest of all, is called a *triangle;* that of four sides, a *quadrilateral;* that of five, a *pentagon;* that of six, a *hexagon;* and so on.



15. An *equilateral* triangle is one which has its three sides equal; an *isosceles* triangle, one which has two of its sides equal; a *scalene* triangle, one which has its three sides unequal.

16. A *right-angled* triangle is one which has a right angle. The side opposite the right angle is called the *hypotenuse.* Thus, ABC is a triangle right-angled at A; the side BC is its hypotenuse.



It is very useful to observe that *the square described on the hypotenuse of a right-angled triangle is equivalent to the sum of the squares described on the two sides.*

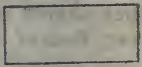
17. An *obtuse-angled* triangle is that which has one obtuse angle, (see Fig. 3, Art. 14.)

18. An *acute-angled* triangle is that which has all its angles acute, (see Fig. 1, 2, Art. 14.)

19. Among quadrilaterals, we distinguish:

The *square,* which has its sides equal, and its angles right.

The *rectangle,* which has its angles right angles, without having its sides equal.

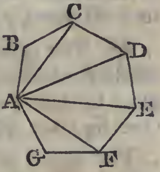


The *parallelogram,* or *rhomboid,* which has its opposite sides parallel.

The *lozenge,* or *rhombus,* which has its sides equal, without having its angles right angles.

And, lastly, the *trapezoid*, only two of whose sides are parallel.

20. A *diagonal* is a line which joins the vertices of two angles not adjacent to each other. Thus, AC, AD, AE, AF, are diagonals.

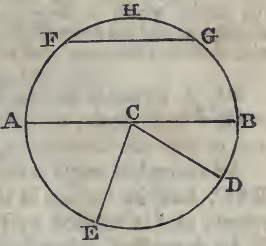


21. An *equilateral* polygon is one which has all its sides equal; an *equiangular* polygon, one which has all its angles equal.

A polygon, which is at once equilateral and equiangular, is called a *regular polygon*. Regular polygons may have any number of sides: the equilateral triangle is one of three sides; the square is one of four; and so on.

22. The *circumference of a circle* is a curve line, all the points of which are equally distant from a point within, called the *centre*.

The *circle* is the space terminated by this curved line.*



23. Every straight line, CA, CE, CD, drawn from the center to the circumference, is called a *radius* or *semidiameter*; every line which, like AB, passes through the centre, and is terminated on both sides by the circumference, is called a *diameter*.

From the definition of a circle, it follows that all the radii are equal; that all the diameters are equal also, and each double of the radius.

24. A portion of the circumference, such as FHG, is called an *arc*.

The *chord* or *subtense* of an arc is the straight line FG, which joins its two extremities.†

25. A *segment* is the surface, or portion of a circle, included between an arc and its chord.

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* *Note.* In common language, the circle is sometimes confounded with its circumference: but the correct expression may always be easily recurred to, if we bear in mind that the circle is a surface which has length and breadth, while the circumference is but a line.

† *Note.* In all cases, the same chord FG belongs to two arcs, FHG, FEG, and consequently also to two segments: but the smaller one is always meant, unless the contrary is expressed.

26. A *sector* is the part of the circle included between an arc DE, and the two radii CD, CE, drawn to the extremities of the arc.

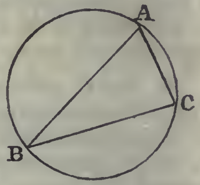
27. The circumference of every circle is supposed to be divided into 360 equal parts, called *degrees;* each degree into 60 equal parts, called *minutes;* and so on.

28. The *measure* of any right-lined angle is an arc of a circle contained between the two lines which form that angle; thus, the angle ECD is measured by the arc ED; (see fig. Art. 22.)

The angle is estimated by the number of degrees contained in the arc; whence a right angle is an angle of 90 degrees, or $\frac{1}{4}$ of the circumference.

From what is here said, it is plain that the angle may be of any magnitude less than 180 degrees; and that it is the same thing whether the circle, which is described from the angular point, be larger or smaller, as the arc contained between the two lines that form the angle, is always the same part of the whole circumference.

29. A *straight line* is said to be *inscribed in a circle,* when its extremities are in the circumference, as AB.

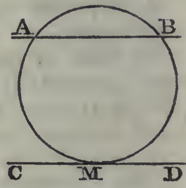


An *inscribed angle* is one which, like BAC, has its vertex in the circumference, and is formed by two chords.

An *inscribed triangle* is one which like BAC, has its three angular points in the circumference.

And, generally, an *inscribed figure* is one, of which all the angles have their vertices in the circumference. The circle is said to *circumscribe* such a figure.

30. A *secant* is a line which meets the circumference in two points. AB is a secant.

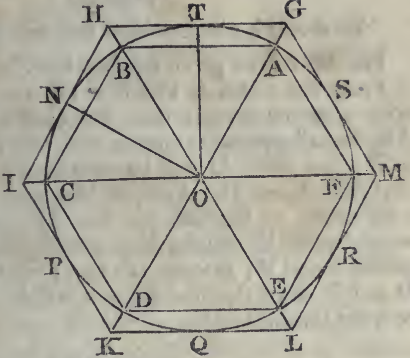


31. A *tangent* is a line which has but one point in common with the circumference CD is a tangent.

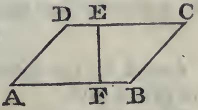
The point M is called the *point of contact.*

In like manner, two circumferences *touch* each other when they have but one point in common.

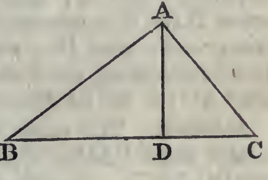
32. A polygon is *circumscribed about a circle,* when all its sides are tangents to the circumference : in the same case, the circle is said to be *inscribed* in the polygon.



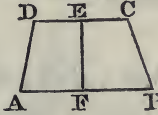
33. The *altitude* of a parallelogram is the perpendicular which measures the distance of two opposite sides, taken as bases. Thus, EF is the altitude of the parallelogram DB.



34. The *altitude* of a triangle is the perpendicular let fall from the vertex of an angle, on the opposite side taken as a base. Thus, AD is the altitude of the triangle BAC.



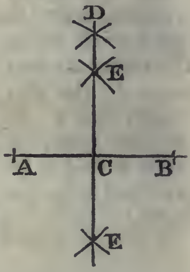
35. The *altitude* of a trapezoid is the perpendicular drawn between its two parallel sides. Thus, EF is the altitude of the trapezoid DB.



36. *Practical Geometry* is a mechanical method of describing mathematical figures by means of a ruler and compasses (usually called dividers) or other instruments proper for the purpose. It is founded upon the principles and properties of certain magnitudes, which are treated of in Legendre's Geometry; and other similar performances on Euclid's Elements.

PROBLEM I.

To divide a given straight line into two equal parts.

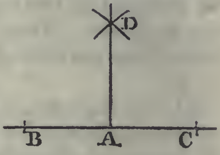


Let AB be the given straight line.

From the points A and B as centres, with a radius greater than the half of AB, describe two arcs cutting each other in D; the point D will be equally distant from A and B. Find, in like manner, above or beneath the line AB, a second point E, equally distant from the points A and B; through the two points D and E, draw the line DE; it will bisect the line AB in C.

PROBLEM II.

At a given point, in a given straight line, to erect a perpendicular to this line.



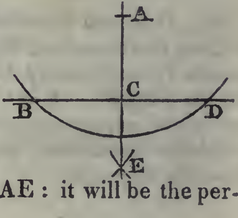
Let A be the given point, and BC the given line.

Take the points B and C at equal distances from A; then from the points B and C as centres, with a radius greater than BA, describe two arcs intersecting each other in D; draw AD: it will be the perpendicular required.

Scholium. The same construction serves for making a right angle BAD, at a given point A, on a given straight line BC.

PROBLEM III.

From a given point, without a straight line, to let fall a perpendicular on this line.



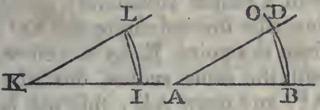
Let A be the point, and BD the straight line.

From the point A as a centre, and with a radius sufficiently great, describe an arc cutting the line BD in the two points B and D; then mark a point E, equally distant from the points B and D, and draw AE: it will be the perpendicular required.

PROBLEM IV.

At a point in a given line, to make an angle equal to a given angle.

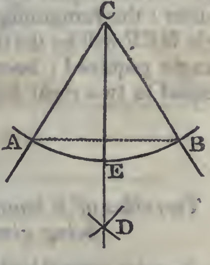
Let A be the given point, AB the given line, and IKL the given angle.



From the vertex K as a centre, with any radius, describe the arc IL, terminating in the two sides of the angle; from the point A as a centre, with a distance AB equal to KI, describe the indefinite arc BO; then take a radius equal to the chord LI, with which, from the point B as a centre, describe an arc cutting the indefinite one BO, in D; draw AD; and the angle DAB will be equal to the given angle K.

PROBLEM V.

To divide a given arc, or a given angle, into two equal parts.



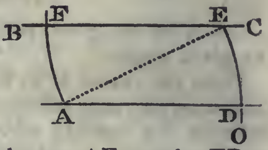
First. Let it be required to divide the arc AEB into two equal parts. From the points A and B, as centres, with the same radius, describe two arcs cutting each other in D; through the point D and the centre C, draw CD: it will bisect the arc AB in the point E.

Secondly. Let it be required to divide the angle ACB into two equal parts. We begin by describing, from the vertex C as a centre, the arc AB; which is then bisected as above. It is plain that the line CD will divide the angle ACB into two equal parts.

Scholium. By the same construction, each of the halves AE, EB, may be divided into two equal parts; and thus, by successive subdivisions, a given angle, or a given arc may be divided into four equal parts, into eight, into sixteen, and so on.

PROBLEM VI.

Through a given point, to draw a parallel to a given straight line.

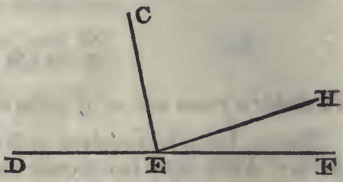


Let A be the given point, and BC the given line.

From the point A as a centre, with a radius sufficiently great, describe the indefinite arc EO; from the point E as a centre, with the same radius, describe the arc AF; make ED = AF, and draw AD: this will be the parallel required.

PROBLEM VII.

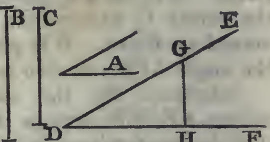
Two angles of a triangle being given, to find the third.



Draw the indefinite line DEF; at any point as E, make the angle DEC equal to one of the given angles, and the angle CEH equal to the other: the remaining angle HEF will be the third angle required; because those three angles are together equal to two right angles.

PROBLEM VIII.

Two sides of a triangle, and the angle which they contain, being given, to construct the triangle.

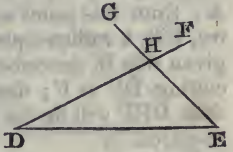


Let the lines B and C be equal to the given sides, and A the given angle.

Having drawn the indefinite line DE; at the point D, make the angle EDF equal to the given angle A; then take DG = B, DH = C, and draw GH; DGH will be the triangle required.

PROBLEM IX.

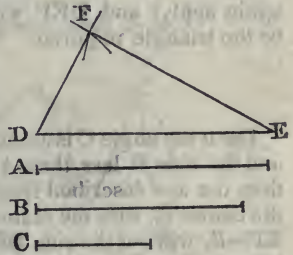
A side and two angles of a triangle being given, to construct the triangle.



The two angles will either be both adjacent to the given side, or the one adjacent and the other opposite: in the latter case, find the third angle (Prob. vii.); and the two adjacent angles will thus be known; draw the straight line DE equal to the given side: at the point D, make an angle EDF equal to one of the adjacent angles, and at E, an angle DEG equal to the other; the two lines DF, EG, will cut each other in H; and DEH will be the triangle required.

PROBLEM X.

The three sides of a triangle being given, to describe the triangle.



Let A, B, and C, be the sides. Draw DE equal to the side A; from the point E as a centre with a radius equal to the second side B, describe an arc; from D as a centre with a radius equal to the third side C, describe another arc intersecting the former in F; draw DF, EF; and DEF will be the triangle required.

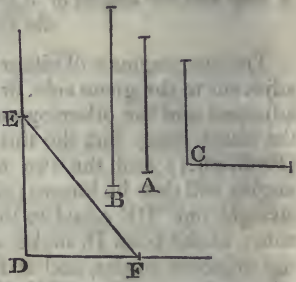
Scholium. If one of the sides were greater than the sum of the other two, the arcs would not intersect each other: but the solution will always be possible, when the sum of two sides, any how taken, is greater than the third.

PROBLEM XI.

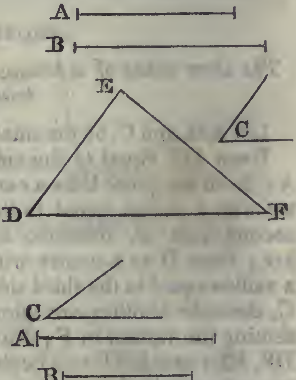
Two sides of a triangle, and the angle opposite one of them, being given, to describe the triangle.

Let A and B be the given sides, and C the given angle. There are two cases.

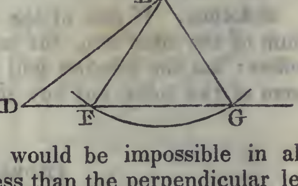
First. When the angle C is right or obtuse, make the angle EDF=C; take DE=A; from this point as a centre, with a radius equal to the given side B, describe an arc cutting DF in F; draw EF: then DEF will be the triangle required.



In this first case, the side B must be greater than A; for the angle C being right or obtuse, is the greatest angle of the triangle, and the side opposite to it must, therefore, also be the greatest.



Secondly. If the angle C is acute, and B greater than A, the same construction will again apply, and DEF will be the triangle required.



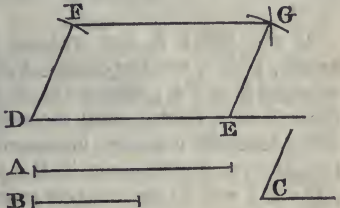
But if the angle C is acute, and the side B less than A, then the arc described from the centre E, with the radius EF=B, will cut the side DF in two points F and G, lying on the same side of D: hence there will be two triangles DEF, DEG, either of which will satisfy the conditions of the problem.

Scholium. The problem would be impossible in all cases, if the side B were less than the perpendicular let fall from E on the line DF.

PROBLEM XII.

The adjacent sides of a parallelogram, with the angle which they contain, being given, to describe the parallelogram.

Let A and B be the given sides, and C the given angle.

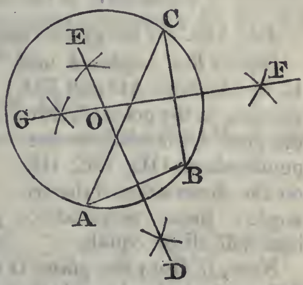


Draw the line DE=A; at the point D, make the angle FDE=C; take DF =B; describe two arcs, the one from F as a centre with a radius FG= DE, the other from E as a centre with a radius EG=DF; to the point G, where these arcs intersect each other, draw FG, EG; DEGF will be the parallelogram required.

Cor. If the given angle is right, the figure will be a rectangle; if, in addition to this, the sides are equal, it will be a square.

PROBLEM XIII.

To find the centre of a given circle or arc.

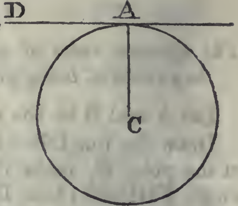


Take three points, A,B,C, any where in the circumference, or the arc; join AB, BC, or suppose them to be joined; bisect those two lines by the perpendiculars DE, FG: the point O, where these perpendiculars meet, will be the centre sought.

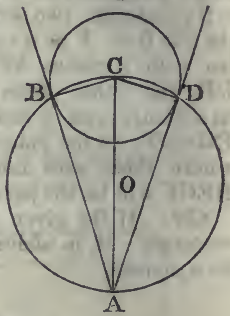
Scholium. The same construction serves for making a circumference pass through three given points A, B, C; and also for describing a circumference, in which a given triangle ABC shall be inscribed.

PROBLEM XIV.

Through a given point, to draw a tangent to a given circle.



If the given point A lies in the circumference, draw the radius CA, and erect AD perpendicular to it: AD will be the tangent required.



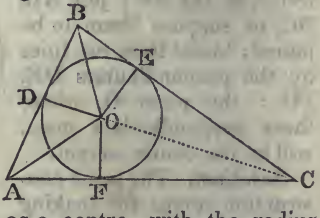
If the point A lies without the circle, join A and the centre, by the straight line CA; bisect CA in O; from O as a centre, with the radius OC, describe a circle intersecting the given circumference in B; join AB: this will be the tangent required.

Scholium. When the point A lies without the circle, there will evidently be always two equal tangents AB, AD, passing through the point A.

PROBLEM XV.

To inscribe a circle in a given triangle.

Let ABC be the given triangle.



Bisect the angles A and B, by the lines AO and BO, meeting in the point O; from the point O, let fall the perpendiculars OD, OE, OF, on the three sides of the triangle: these perpendiculars will all be equal.

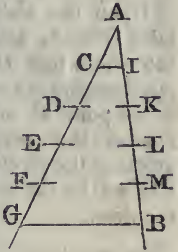
Now, if from the point O as a centre, with the radius OD, a circle be described, this circle will evidently be inscribed in the triangle ABC; for the side AB, being perpendicular to the radius at its extremity, is a tangent; and the same thing is true of the sides BC, AC.

Scholium. The three lines which bisect the angles of a triangle meet in the same point.

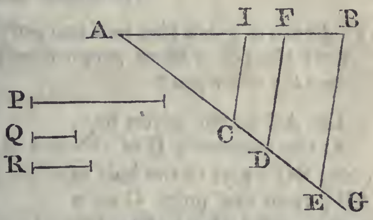
PROBLEM XVI.

To divide a given straight line into any number of equal parts, or into parts proportional to given lines.

First. Let it be proposed to divide the line AB into five equal parts. Through the extremity A, draw the indefinite straight line AG; and taking AC of any magnitude, apply it five times upon AG; join the last point of division G and the extremity B, by the straight line GB; then draw CI parallel to GB: AI will be the fifth part of the line AB; and thus by applying AI five times upon AB, the line AB will be divided into five equal parts.



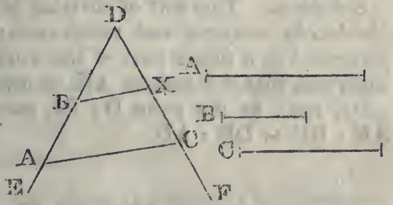
Secondly. Let it be proposed to divide the line AB into parts proportional to the given lines P, Q, R. Through A, draw the indefinite line AG; make AC=P, CD=Q, DE=R; join the extremities E and B; and through the points C, D draw CI, DF parallel to EB; the line AB will be divided into parts AI, IF, FB proportional to the lines P, Q, R.



PROBLEM XVII.

To find a fourth proportional to three given lines A, B, C.

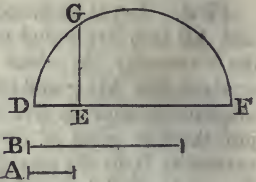
Draw the two indefinite lines DE, DF, forming any angle with each other. Upon DE take DA=A, and and DB=B; upon DF take DC=C; join AC; and through the point B, draw BX parallel to AC: DX will be the fourth proportional required.



Cor. A third proportional to two given lines A, B, may be found in the same manner, for it will be the same as a fourth proportional to the three lines A, B, C.

PROBLEM XVIII.

To find a mean proportional between two given lines A *and* B.



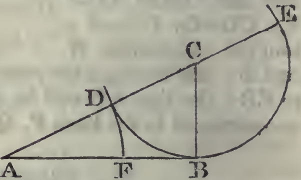
Upon the indefinite line DF, take DE=A, and EF=B; upon the whole line DF, as a diameter, describe the semicircle DGF; at the point E, erect upon the diameter the perpendicular EG meeting the circumference in G: EG will be the mean proportional required.

PROBLEM XIX.

To divide a given line into two parts, such that the greater part shall be a mean proportional between the whole line and the other part.

Let AB be the given line.

At the extremity B of the line AB, erect the perpendicular BC equal to the half of AB; from the point C as a centre, with the radius CB describe a semicircle; draw AC cutting the circumference in D; and take AF=AD: the line AB will be divided at the point F in the manner required; that is, we shall have AB : AF : : AF : FB.

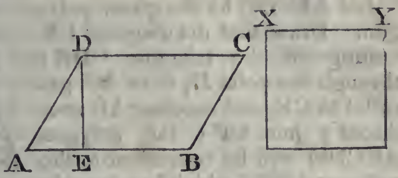


Scholium. This sort of division of the line AB is called division in extreme and mean ratio: the use of it will be perceived in a future part of the work. It may further be observed, that the secant AE is divided in extreme and mean ratio at the point D; for, since AB=DE, we have AE : DE : : DE : AD.

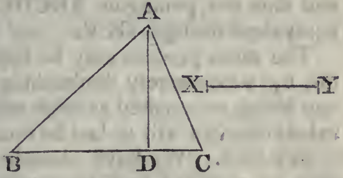
PROBLEM XX.

To describe a square that shall be equivalent to a given parallelogram, or to a given triangle.

First. Let ABCD be the given parallelogram, AB its base, DE its altitude: between AB and DE find a mean proportional XY: then will the square constructed upon XY be equivalent to the parallelogram ABCD.



Secondly. Let ABC be the given triangle; BC its base, AD its altitude: find a mean proportional between BC and the half of AD, and let XY be that mean; the square constructed upon XY will be equivalent to the triangle ABC.

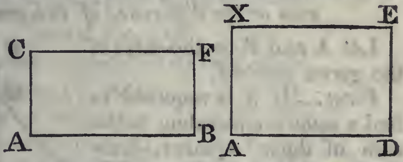


PROBLEM XXI.

Upon a given line, to describe a rectangle that shall be equivalent to a given rectangle.

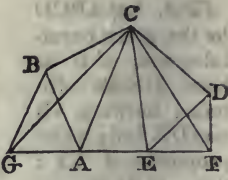
Let AD be the line, and ABFC the given rectangle.

Find a fourth proportional to the three lines AD, AB, AC, and let AX be that fourth proportional; a rectangle constructed with the lines AD and AX will be equivalent to the rectangle ABFC.



PROBLEM XXII.

To find a triangle that shall be equivalent to a given polygon.



Let ABCDE be the given polygon. Draw first the diagonal CE cutting off the triangle CDE; through the point D, draw DF parallel to CE, and meeting AE produced; join CF: the polygon ABCDE will be equivalent to the polygon ABCF, which has one side less than the original polygon.

The angle B may in like manner be cut off, by substituting for the triangle ABC the equivalent triangle AGC, and thus the pentagon ABCDE will be changed into an equivalent triangle GCF.

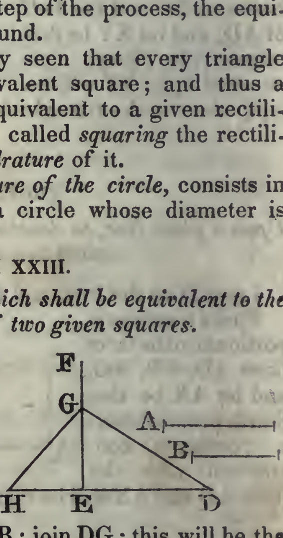
The same process may be applied to every other figure; for, by successively diminishing the number of its sides, one being retrenched at each step of the process, the equivalent triangle will at last be found.

Scholium. We have already seen that every triangle may be changed into an equivalent square; and thus a square may always be found equivalent to a given rectilineal figure, which operation is called *squaring* the rectilineal figure, or finding the *quadrature* of it.

The problem of *the quadrature of the circle,* consists in finding a square equivalent to a circle whose diameter is given.

PROBLEM XXIII.

To find the side of a square which shall be equivalent to the sum or the difference of two given squares.



Let A and B be the sides of the given squares.

First. If it is required to find a square equivalent to the sum of these squares, draw the two indefinite lines ED, EF at right angles to each other; take ED=A, and EG=B; join DG: this will be the side of the square required.

Secondly. If it is required to find a square equal to the difference of the given squares, form in the same manner the right angle FEH; take GE equal to the shorter of the sides A and B; from the point G as a centre, with a radius GH, equal to the other side, describe an arc cutting EH in

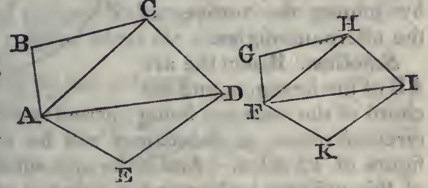
H ; the square described upon EH will be equal to the difference of the squares described upon the lines A and B.

Scholium. A square may thus be found equal to the sum of any number of squares; for a similar construction which reduces two of them to one, will reduce three of them to two, and these two to one, and so of others. It would be the same, if any of the squares were to be subtracted from the sum of the others.

PROBLEM XXIV.

Upon a given line to describe a polygon similar to a given polygon.

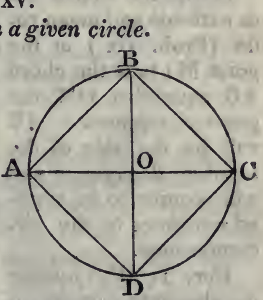
Let FG be the given line, and AEDCB the given polygon.



In the given polygon, draw the diagonals AC, AD; at the point F make the angle GFH=BAC, and at the point G the angle FGH = ABC ; the lines FH, GH will cut each other in H, and FGH will be a triangle similar to ABC. In the same manner upon FH, homologous to AC, construct the triangle FIH similar to ADC ; and upon FI, homologous to AD, construct the triangle FIK similar to ADE. The polygon FGHIK will be similar to ABCDE, as required.

PROBLEM XXV.

To inscribe a square in a given circle.

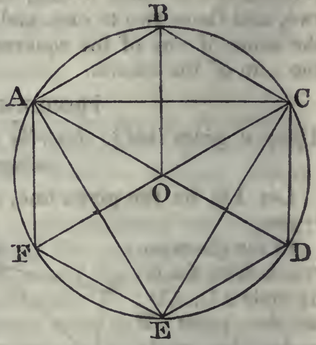


Draw two diameters AC, BD, cutting each other at right angles; join their extremities A, B, C, D : the figure ABCD will be a square.

Scholium. If the arc AB be bisected, the chord of one of those semi-arcs will form a regular octagon ; and if the arcs subtended by the sides of the octagon be severally bisected, the chords of those semi-arcs will form a new regular polygon of double the number of sides, that is, 16 sides ; and so on.

PROBLEM XXVI.

In a given circle, to inscribe a regular hexagon and an equilateral triangle.

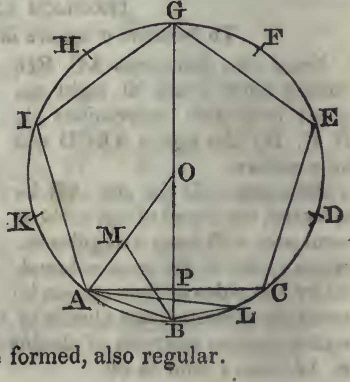


To inscribe a regular hexagon in a given circle, the radius must be applied six times to the circumference; which will bring us round to the point we set out from. And the hexagon ABCDEF being inscribed, the equilateral triangle ACE may be formed by joining the vertices of the alternate angles.

Scholium. Bisect the arc AB of the hexagon, and the chord of the semi-arc being carried twelve times round the circumference, a dodecagon will be formed; that is, a figure of 12 sides. And if the arcs subtended by the sides of this polygon be bisected, a polygon will be formed consisting of 24 sides; and so on.

PROBLEM XXVII.

In a given circle, to inscribe a regular decagon; then a pentagon, and also a regular polygon of fifteen sides.



Divide the radius AO in extreme and mean ratio (Prob. xix.) at the point M; take the chord AB equal to OM the greater segment; AB will be the side of the regular decagon, and will require to be applied ten times to the circumference.

Cor. 1. By joining the alternate corners of the regular decagon, the pentagon ACEGI will be formed, also regular.

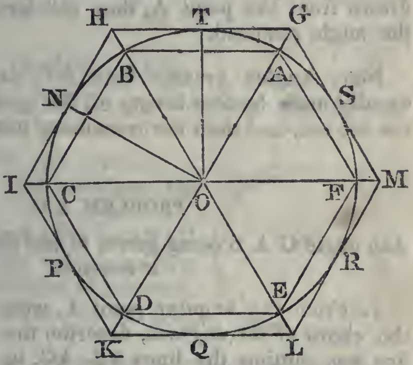
Cor. 2. AB being still the side of the decagon, let AL be the side of the hexagon; the arc BL will then, with reference to the whole circumference, be $\frac{1}{6}-\frac{1}{10}$, or $\frac{1}{15}$; hence the chord BL will be the side of the regular polygon of fifteen sides, or pentedecagon. It is evident, also, that the arc CL is the third of CB.

Scholium. By means of the decagon, regular polygons of 20, 40, 80, &c. sides may be inscribed; and by means of the pentedecagon, regular polygons of 30, 60, 120, &c. sides may be inscribed.

PROBLEM XXVIII.

A *regular inscribed polygon being given, to circumscribe a similar polygon about the same circle.*

Let ABCDE, &c. be the polygon.



At T, the middle point of the arc AB, apply the tangent GH, which will be parallel to AB; do the same at the middle point of each of the arcs BC, CD, &c.; those tangents, by their intersections, will form the regular circumscribed polygon GHIK, &c. similar to the inscribed one.

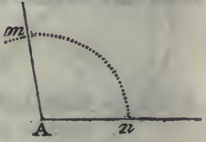
Cor. 1. Reciprocally, if the circumscribed polygon GHIK, &c. were given, and the inscribed one ABC, &c. were required to be deduced from it, it would only be necessary to draw from the angles G, H, I, &c. of the given polygon, straight lines OG, OH, &c. meeting the circumference in the points A, B, C, &c.; then to join those points by the chords AB, BC, &c.; which would form the inscribed polygon. An easier solution of this problem would be simply to join the points of contact T, N, P, &c.

by the chords TN, NP, &c. which likewise would form an inscribed polygon similar to the circumscribed one.

Cor. 2. Hence we may circumscribe about a circle any regular polygon, which can be inscribed within it; and conversely.

PROBLEM XXIX.

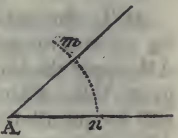
To make an angle of any proposed number of degrees.



1. Take the first 60 degrees from the scale of chords, and from the point A, with this radius, describe the arc *nm*.

2. Take the chord of the proposed number of degrees from the same scale, and apply it from *n* to *m*.

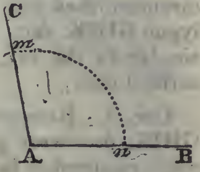
3. Then if the lines A*n* and A*m* be drawn from the point A, they will form the angle required.



Note. Angles greater than 90°, are usually made by first laying off 90° upon the arc *nm*, and then the remaining part.*

PROBLEM XXX.

Any angle C A B *being given, to find the number of degrees it contains.*



1. From the angular point A, with the chord of 60 degrees, describe the arc *nm*, cutting the lines AB, AC, in the points *n* and *m*.

2. Then take the distance *nm*, and apply it to the scale of chords, and it will show the degrees required.

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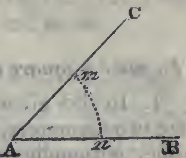
* The line of chords made use of in this and the following problems, is commonly put upon the plain scale, and is adapted to 90°, or the fourth part of a circle.

For a description of this and other instruments made use of in Practical Geometry, see *Robertson's Treatise on such mathematical instruments as are usually put into a portable case.*

And if the distance *nm* be greater than 90°, it must be taken at twice, and each part applied separately to the scale.*

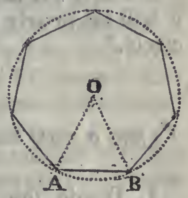


PROBLEM XXXI.

In a given circle to inscribe a polygon of any proposed number of sides.

1. Divide 360° by the number of sides of the figure, and make an angle AOB, at the centre, whose measure shall be equal to the degrees in the quotient.

2. Then if the points AB, be joined, the chord AB, applied to the circumference the given number of times, it will form the polygon required.

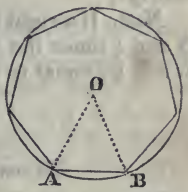


PROBLEM XXXII.

On a given line AB *to form a regular polygon of any proposed number of sides.*

1. Divide 360° by the number of sides of the figure, and substract the quotient from 180°.

2. Make the angles ABO and BAO each equal to half the difference last found; and from the point of intersection O, with the distance OA or OB, describe a circle.



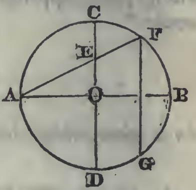
3. Then if the chord AB be applied to the circumference the proposed number of times, it will form the polygon required.†

* Both this and the last problem may be performed by means of a *protractor,* which is a graduated arc designed for the purpose of laying down arcs, or angles; and is usually given in the case of instruments before mentioned; or it may be had separately of any size that may be thought more convenient.

† By this method the circumference of a circle may also be divided into any number of equal parts; for if 360° be divided by the number of parts, and the angle AOB be made equal to the degrees in the quotient, the arc AB will be one of the equal parts required.

PROBLEM XXXIII.

To make a square that shall be nearly equal to a given circle.



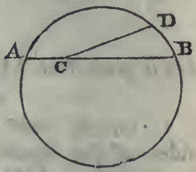
1. In the given circle ACBD, draw the two diameters, AB, CD, cutting each other perpendicularly in the centre O.

2. Bisect the radius OC in E, and through the points A, E, draw the chord AEF, which will be the side of a square that is nearly equal in area to the circle.

Note. If FG be drawn parallel to CD, it will be nearly equal to $\frac{1}{4}$ of the circumference of the circle.*

PROBLEM XXXIV.

To find a right line that shall be nearly equal to any given arc ADB *of a circle.*



1. Divide the chord AB into four equal parts, and set one of the parts AC, on the arc from B to D.

2. Draw CD, and the double of this line will be equal to the arc ADB *nearly.*†

Note. If a right line be made equal to 3 and $\frac{1}{7}$ times the diameter of a circle, it will be equal to the circumference nearly.

PROBLEM XXXV.

To construct a plain diagonal scale.

Draw eleven lines parallel to, and equidistant from each other; cut them at right angles by the equidistant lines BC, EF; 1, 9; 2, 7; &c.: then will BC, &c., be divided into ten equal parts; divide the lines EB, and FC, each into ten equal parts, and from the points of division on the line EB, draw diagonals to the points of division on the line

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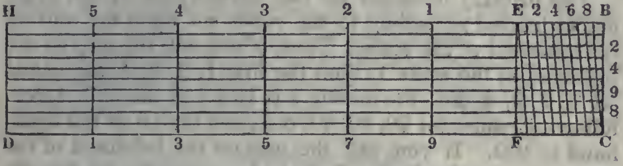
* The side AF of the square, in the above figure, is about the one-hundredth part of a unit too great, and the line FG about one-seventieth part too little.

† This construction furnishes a method of finding the length of a circular arc sufficiently near the truth for most mechanical purposes.

FC: thus join E and the first division on FC; the first division on EB, and the second on FC; and so on.



Diagonal scales serve to take off dimensions or numbers of three figures. If the first large divisions be units, the second set of divisions, along EB, will be tenth parts; and the divisions in the altitude along BC, will be 100th parts. If HE be tens, EB will be units, and BC will be 10th parts. If HE be hundreds, BE will be tens, and BC units. And so on, each set of divisions being tenth-parts of the former ones.

For example, suppose it were required to take off 244 from the scale. Extend the dividers from E towards H, and with one leg fixed in the point 2, extend the other till it reaches 4 in the line EB; move one leg of the dividers along the line 2, 7, and the other along the line 4, till they come to the line marked 4, in the line BC, and that will give the extent required.

Explanation of the line of numbers on Gunter's Scale.

The line of numbers on the two feet Gunter's Scale, marked *Number*, is numbered from the left hand of the Scale towards the right with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, which stands, exactly in the middle of the Scale; the numbers then go on 2, 3, 4, 5, 6, 7, 8, 9, 10, which stands at the right-hand end of the Scale.

These two *equal* parts of the Scale are also *equally divided*, the distance between the first, or left-hand 1, and the first 2, 3,4, &c. is exactly equal to the distance between the middle 1, and the numbers 2, 3, 4, &c. which follow it.

The subdivisions of these two equal parts of the scale are likewise similar, *viz.* they are each one tenth of the primary divisions, and are distinguished by lines of about half the length of the primary divisions. These subdivisions are again divided into ten parts, where room will admit, and where that is not the case, the units must be estimated, or guessed at by the eye, which is easily done by a little practice.

The primary divisions on the second part of the scale are estimated according to the value set upon the unit on the left-hand of the scale: Thus, if you call the unit on the left-hand of the scale 1, then the first 1, 2, 3, 4, &c. stand for 1, 2, 2, 4, &c., the middle 1 is 10; and the 2, 3, 4, &c. following, stand for 20, 30, 40, &c., and the 10 at the right-hand is 100. If you call the unit on the left-hand of the scale 10, then the first 1, 2, 3, 4, &c. stand for 10, 20, 30, 40, &c., the middle one will be 100; and the 2, 3, 4, &c. following will be 200, 300, 400, &c., and the 10 at the right-hand will be 1000. If you call the unit on the left-hand of the scale 100, then the first 1, 2, 3, 4, &c. will stand for 100, 200, 300, 400, &c., the middle one will be 1000; and the 2, 3, 4, &c. following 2000, 3000, 4000, &c., and the 10 at the right-hand will be 10000. Lastly, if you consider the unit on the left-hand of a scale as one-tenth of a unit, then the first 1, 2, 3, 4, &c. will be $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, &c., the middle one will stand for a unit, and the 2, 3, 4, &c., following it, will be 2, 3, 4, &c., and the 10 at the right-hand end of the scale will stand for 10.

From the above description it will be easy to find the divisions representing any given number. Suppose 12 was required; take the division at the figure 1, in the middle of the scale, for the first figure of 12; then for the second figure, count two of the longer strokes to the right-hand, and this last is the point representing 12, where there is a brass pin. If 34 were required; call the figure 3, on the right-hand half of the scale, 30, and count forward four of the longer divisions towards the right-hand; if 340 were required, it must be found in the same manner. If the point representing 345 were required, find 340 as above, then the middle distance between the point 340, and the point representing 350, will be the point representing 345.

By the line of numbers and a pair of compasses almost all the problems in mensuration may be readily done, for they in general depend upon proportion. And, as in natural numbers, the quotient of the first term, of any abstract proportion, by the second, is equal to the quotient of the third term by the fourth; so in logarithms (for the line of numbers is a logarithmical line) the difference between the first and second term, is equal to the difference between the third and fourth; consequently, on the line of numbers, the distance between the first and second terms will be equal to the distance between the third and fourth. And for a similar reason, because four proportional quantities are alternately proportional, the distance between the first and third term will be equal to the distance between the second and fourth. Hence the following

GENERAL RULE.

The extent of the compasses from the first term to the second, will reach, in the same direction, from the third to the fourth: Or, the extent of the compasses from the first term to the third, will reach, in the same direction, from the second to the fourth.

By the same direction must be understood, that if the second term lie on the right-hand of the first, the fourth term will lie on the right-hand of the third, and the contrary. Hence,

I. *To find the product of two numbers.*

As a unit is to the multiplier, so is the multiplicand to the product.

II. *To divide one number by another.*

As the divisor is to the dividend, so is a unit to the quotient.

III. *To find a mean proportional between two numbers.*

Because the distance between the first and second term is equal to the distance between the third and fourth; therefore, if you divide the space between the point representing the first term, and that representing the fourth, into two equal parts, the middle point must necessarily give the mean proportional sought.

IV. *To extract the square root.*

The square root of a quantity is nothing more than a mean proportional between a unit and the given number to be extracted; the unit being the first term, and the number to be extracted the fourth; therefore it may be done by the preceding direction.

MENSURATION OF SURFACES.

The area of any figure is the measure of its surface, or the space contained within the bounds of that surface, without any regard to the thickness.

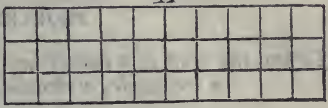
A square whose side is one inch, one foot, or one yard, &c. is called the *measuring unit;* and the area, or content, of any figure, is estimated by the number of squares of this kind that are contained in it.

Scholium. Hence the product of the base by the altitude may be assumed as the measure of a rectangle, provided we understand by this product, the product of two numbers, one of which is the number of linear units contained in the base, the other the number of linear units contained in the altitude.

Still this measure is not absolute but relative : it supposes that the area of any other rectangle is computed in a similar manner, by measuring its sides with the same linear unit : a second product is thus obtained, and the ratio of the two products is the same as that of the rectangles.

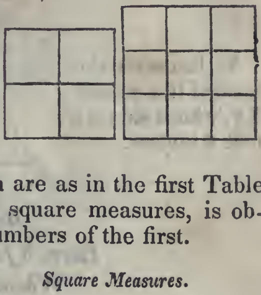
For example, if the base of the rectangle A contains three units, and its altitude ten, that rectangle will be represented by the number 3×10, or 30, a number which signifies nothing while thus isolated ; but if there is a second rectangle B, the base of which contains twelve units, and the altitude seven, this second rectangle will be represented by the number $12\times7=84$; and we shall hence be entitled to conclude that the two rectangles are to each other as 30 is to 84; and therefore, if the rectangle A were to be assumed as the unit of the measurement in surfaces, the rectangle B would then have $\frac{84}{30}$ for its absolute measure, in other words, it would be equal to $\frac{84}{30}$ of a superficial unit.

A



It is more common and more simple, to assume the square as the unit of surface; and to select that square, whose side is the unit of length. In this case, the measurement which we have regarded merely as relative, becomes absolute: the number 30, for instance, by which the rectangle A was measured, now represents 30 superficial units, or 30 of those squares, which have each of their sides equal to unity, as the diagram exhibits.

In geometry the product of two lines frequently means the same thing as their *rectangle*, and this expression has passed into arithmetic, where it serves to designate the product of two unequal numbers, the expression *square* being employed to designate the product of a number multiplied by itself.



The arithmetical squares of 1, 2, 3, &c. are 1, 4, 9, &c. So likewise the geometrical square constructed on a double line is evidently four times as great as on a single one; on a triple line, is nine times as great, &c.

The usual measures of length are as in the first Table below; and the annexed table of square measures, is obtained by squaring the several numbers of the first.

Lineal Measures.		*Square Measures.*	
12 inches	1 foot	144 inches	1 foot
3 feet	1 yard	9 feet	1 yard
6 feet	1 fathom	36 feet	1 fathom
$16\frac{1}{2}$ feet, or $5\frac{1}{2}$ yards	1 pole or rod	$272\frac{1}{4}$ feet, or $30\frac{1}{4}$ yards	1 pole or rod
40 poles	1 furlong	1600 poles	1 furlong
8 furlongs	1 mile	64 furlongs	1 mile

To this we may likewise add, that the chain made use of in measuring land, commonly called *Gunter's Chain*, is 4 poles, or 22 yards in length, and consists of 100 equal links, each of which are $\frac{22}{100}$ of a yard, or 7.92 inches long.

Note also, that in Land Measure

40 perches, or square poles, ... make 1 rood,
4 roods, or 160 square poles, or 10 square chains, ... make 1 acre.

PROBLEM I.

To find the area of a parallelogram; whether it be a square, a rectangle, a rhombus, or a rhomboid.

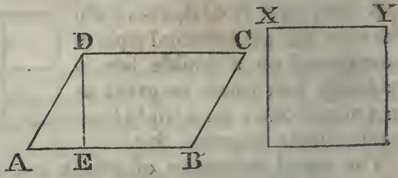
RULE.

Multiply the length by the perpendicular height, and the product will be the area; observing that since the length of a square is equal to its height, the area, in this case, will be obtained by multiplying the side by itself.

Or the area of either of these figures may be found by multiplying any two of its adjacent sides together, and that product again by the natural sine of their included angle.

EXAMPLES.

1. Required the area of the square XY, whose side is 5 feet 9 inches.



By decimals.

Here, 5 *fe.* 9 *in.* = 5.75 *fe.*

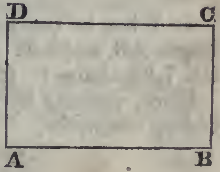
Whence 5.75 = XY.

```
      5.75
      5.75
   -------
      2875
     4025
    2875
   -------
   33.0625
        12
   -------
    0.7500
        12
   -------
    9.0000
```

Ans. 33 *fe.* 0 *in.* 9 *pts.*

By duodecimals.

fe.	*in.*	
5	9	
5	9	
28	9	
4	3	9
33	0	9 *as before.*



2. Required the area of the rectangle ABCD, whose length AB is 13.75 chains, and breadth BC 9.5 chains.

By decimals.

13.75 = AB.
9.5 = BC.

6875
12375

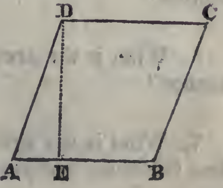
10)130.625

13.0625
4

0.2500
40

10.0000

Ans. 13 *ac.* 0 *ro.* 10 *po.*



3. Required the area of the rhombus ABCD, whose length AB, is 12 *fe.* 6 *in.* and its perpendicular height DE, 9 *fe.* 3 *in.*

By duodecimals.

fe.	*in.*	
12	6 = AB.	
9	3 = DE.	
112	6	
3	1	6
115	7	6

Ans. 115 *fe.* 7 *in.* 6 *pts.*

4. What is the area of the rhomboid ABCD, whose length AB is 10.52 chains, and its perpendicular height DE 7.63 chains? (See fig. Prob. I.)

Here, working by decimals, as in the first of the preceding examples,

We have 10.52 = AB.
7.63 = DE.

3156
6312
7364

10)80.2676

8.02676
4

0.10704
40

4.28160

Ans. 8 *ac.* 0 *ro.* 4 *po.*

5. What is the area of a square whose side is 35.25 chains?

	ac.	*ro.*	*po.*
Ans.	124	1	1

6. What is the area of a square whose side is 8 feet 4 inches?

	fe.	*in.*	*pa.*
Ans.	69	5	4.

7. What is the area of a rectangle whose length is 14 feet 6 inches, and breadth 4 feet 9 inches?

Ans. 68 *fe.* 10 *in.* 6 *pa.*

8. Required the area of a rhombus, the length of whose side is 12.24 feet, and height 9.16 feet.

Ans. 112 *fe.* 1 *in.* 5 *pa.*

9. Required the area of a rhomboid, whose length is 10.51 chains, and breadth 4.28 chains.

Ans. 4 *ac.* 1 *ro.* 39 *po.*

10. What is the area of a rhomboid, whose length is 7 feet 9 inches, and height 3 feet 6 inches?

Ans. 27 *fe.* 1 *in.* 6 *pa.*

11. Required the area of a rectangular board, whose length is $12\frac{1}{2}$ feet, and breadth 9 inches.

Ans. $9\frac{3}{8}$ *feet*

12. Required the area of a rhomboid, two of whose adjacent sides are 25.35 and 10.4 chains, and their included angle 30°.

	ac.	*ro.*	*po.*
Ans.	13	0	29

PROBLEM II.

To determine the area of a triangle.

RULE.

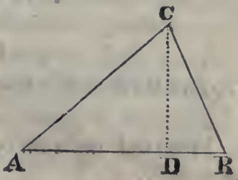
Multiply one of its sides, considered as a base, by the perpendicular falling upon it from the opposite angle, and half the product will be the area.

Or, multiply the product of any two of its adjacent sides by the natural sine of their included angle, and half this result will be the area, as before.

Thus, $\frac{AB \times CD}{2}$, or $\frac{AB \times AC \times \text{nat. sine} \angle A}{2}$ = area of the triangle ABC.

EXAMPLES.

1. Required the area of the triangle ABC, whose base AB is 10 feet 9 inches, and its perpendicular height, DC, 7 feet 3 inches.



Here, by decimals.

10 *fe.* 9 *in.* = 10.75, and 7 *fe.* 3 *in.* = 7.25.

Whence, 10.75 = AB.
7.25 = CD.

```
       5375
      2150
     7525
   ---------
   2)77.9375
   ---------
   38.96875
         12
   ---------
   11.62500
         12
    7.50000
```

Ans. 38 *fe.* 11 *in.* $7\frac{1}{2}$ *pts.*

2. What is the area of a triangle whose base is 20 feet and height $10\frac{1}{4}$ feet? *Ans.* $102\frac{1}{2}$ *feet.*

3. What is the area of a triangle whose base is 18 feet 4 inches, and height 11 feet 10 inches? *fe.* *in.* *pa.*
Ans. 108 5 8

4. What is the area of a triangle whose base is 16.75 feet, and height 6.24 feet? *fe.* *in* *pa.*
Ans. 52 3 1

5. Required the area of a triangle whose base is 12.25 chains, and height 8.5 chains. *ac.* *ro.* *po.*
Ans. 5 0 33

6. If the base of a triangle be $17\frac{1}{5}$ yards, and its perpendicular height $11\frac{1}{7}$ yards, what is its area in square feet?
Ans. $862\frac{16}{35}$ *fe.*

7. What is the area of a triangle, two of whose sides are 30 and 40, and their included angle $28° 57' \frac{3}{10}$?
Ans. 290.47356.

PROBLEM III.

To find the area of a triangle, when three sides only are given.

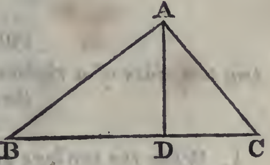
RULE.

From half the sum of the three sides substract each side severally.

Then multiply the half sum and the three remainders continually together, and the square root of the product will be the area.

EXAMPLES.

1. Required the area of the triangle ABC, whose three sides BC, CA, AB, are 15‘ 13, and 14 feet, respectively?



Here $\frac{13 + 14 + 15}{2} = \frac{42}{2} = 21 = \frac{1}{2}$ *sum of the sides.*

And	21	21	21
	15	13	14
	6	8	7

Whence 21
6
———
126
8
———
1008
7
———
7056(84
64
———
164) 656
656
———

Ans. 84 *square feet.*

2. Required the area of a triangle whose three sides are 20, 30, and 40 feet. *Ans.* 290.4737 *square feet.*

3. Required the area of an equilateral triangle, each of whose equal sides is 25 chains. *Ans.* 270.6329 *acres.*

4. Required the area of an isosceles triangle, whose base is 20, and each of its equal sides 15. *Ans.* 111.803.

5. Required the area of a right-angled triangle, whose hypotenuse, is 50, and the other two sides 30 and 40.

Ans. 600.

6. How many acres are there in the triangle, whose three sides are 380, 420, and 765 yards?

Ans. 9 *ac.* 0 *ro.* 38 *po.*

PROBLEM IV.

Any two sides of a right-angled triangle being given to find the third side.

RULE.

1. *When the two legs are given to find the hypotenuse.*

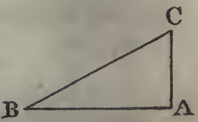
Add the square of one of the legs to the square of the other, and the square root of the sum will be equal to the hypotenuse.

2. *When the hypotenuse and one of the legs are given, to find the other leg.*

From the square of the hypotenuse take the square of the given leg, and the square root of the remainder will be equal to the other leg.

EXAMPLES.

1. In the right-angled triangle ABC, the base AB is 56, and the perpendicular AC 33; what is the hypotenuse?



Here 56 = BC, and 33 = AC.

56	33
56	33
336	99
280	99
3136	1089

Whence 3136
1089

4225(65
36

125) 625
625

Ans. hyp. BC 65.

2. If the hypotenuse BC be 53, and the base AB 45, what is the perpendicular AC?

Here 53 = BC, and 45 = AB.

```
 53        45
----      ----
 159       225
265       180
----      ----
2809      2025
   Whence 2809
          2025
          ----
          784(28
          4
          ----
      48) 384
          384
          ----
```

Ans. perpen. AC 28.

3. The base of a right-angled triangle is 77, and the perpendicular 36: what is the hypotenuse? *Ans.* 85.

4. The hypotenuse of a right-angled triangle is 109, and the perpendicular 60: what is the base? *Ans.* 91.

5. What must be the length of a scaling ladder that will reach the top of a wall, whose height is 28 feet, and the breadth of the ditch before it 45 feet. *Ans.* 53 *feet.*

6. It is required to find the length of a shore, which projecting 12 feet from the upright of a building, will support a jamb 20 feet from the ground? *Ans.* 23.32380 *feet.*

7. The height of a precipice, standing close by the side of a river, is 103 feet, and a line of 320 feet will reach from the top of it to the opposite bank: required the breadth of the river. *Ans.* 302.9703 *feet.*

8. A ladder 50 feet long being placed in a street, reached a window 28 feet from the ground, on one side; and by turning it over, without removing the foot, it reached another window, 36 feet high, on the other side; required the width of the street. *Ans.* 76.1233335 *feet.*

PROBLEM V.

To find the area of a trapezium.

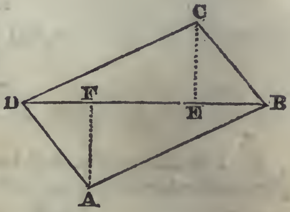
RULE.

Divide it into two triangles, by drawing a diagonal from either of its angles to the opposite one; then find the areas of these triangles separately, and add them together for the area of the whole figure.

Or, multiply the diagonal by the sum of the two perpendiculars falling upon it from the opposite angles, and half the product will be the area.

EXAMPLES.

1. Required the area of a trapezium ABCD, whose diagonal BD is 84, the perpendicular AF 21, and CE 28.



Here 84 = BD.
14 = ½ CE.

336
84

1176 = *area* △ BCD.

And 84 = BD.
21 = AF.

84
168

2)1764

882 = *area* △ BAD.

Hence 1176 + 882 = 2058 *area of the trapezium* BCDA.

Or, $\frac{(28+21)\times 84}{2} = 49 \times 42 = 2058 =$ *area as before.*

2. Required the area of a trapezium whose diagonal is $80\frac{1}{2}$, and the two perpendiculars $24\frac{1}{2}$ and $30\frac{1}{10}$.

Ans. 2197.65.

3. What is the area of a trapezium whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches? *Ans.* 6347 *fe.* 3 *in.*

4. How may square yards of paving are there in a trapezium, whose diagonal is 65 feet, and the two perpendiculars, let fall on it, from its opposite angles, 28 and $33\frac{1}{2}$ feet, respectively? *Ans.* $222\frac{1}{12}$ *yards.*

5. How many acres are there in the trapezium whose diagonal is 4.75 chains, and the two perpendiculars, falling on it, from its opposite angles, 2.25 and 3.6 chains, respectively? *Ans.* 13 *ac.* 2 *ro.* 25 *po.*

6. What is the area of a trapezium whose diagonal is 18 yards, and the perpendiculars 6 and 4, respectively?

Ans. 90 *square yards.*

PROBLEM VI.

To find the area of a trapezoid, or a quadrangle, which has two of its opposite sides parallel to each other.

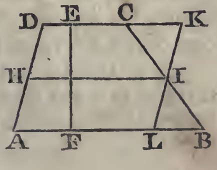
RULE.

Divide it into two triangles, by drawing a diagonal as in the trapezium, and add their areas together for the content.

Or, multiply the sum of the parallel sides by the perpendicular distance between them, and half the product will be the area.

EXAMPLES.

1. Required the area of the trapezoid ABCD, whose parallel sides AB, DC, are 321.51 and 214.24 respectively, and perpendicular EF 171.16.



Here 321.51 = AB.
214.24 = DC.

535.75 = *sum*.
171.16 = DE.

```
        321450
        53575
       53575
     375025
     53575
   ----------
  2)91698.9700
   ----------
    45849.485 the answer.
```

2. Required the area of a trapezoid whose two parallel sides are $20\frac{1}{2}$ and $12\frac{1}{4}$, and the perpendicular distance between them $10\frac{3}{4}$. *Ans.* 176.03125.

3. Required the area of a trapezoid whose two parallel sides are 25 feet 6 inches, and 18 feet 9 inches, and the perpendicular distance between them 10 feet 5 inches. *Ans.* 230 *fe.* 5 *in.* 7 *pa.*

4. The two parallel sides of a trapezoid are 12.41 and 8.22 chains, and the perpendicular distance between them 5.15 chains: required the area. *Ans.* 5 *ac.* 1 *ro.* 9 *po.*

5. Required the area of a trapezoid, whose two parallel sides are 750 and 1225 links, and the perpendicular distance between them 1540 links. *Ans.* 15 *ac.* 33 *po.*

PROBLEM VII.

To find the area of a regular polygon.

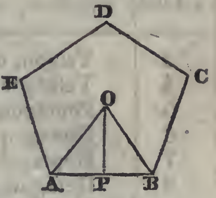
RULE.

Multiply the perimeter, or sum of all the sides of the figure, by the perpendicular falling from its centre upon one of the sides, and half the product will be the area.

This is, in effect, only resolving the polygon into as many equal triangles as it has sides, by drawing lines from the centre to all the angles, and then finding their areas, and adding them together, for the content.

EXAMPLES.

1. Required the area of the regular pentagon ABCDE, one of whose equal sides AB, or BC, &c. is 25 feet, and the perpendicular OP from its centre, 17.2 feet.



Here 25 = AB.
5

125
17.2 = OP,

250
875
125

2)2150.0

1075 *the Answer.*

2. Required the area of a hexagon, one of whose equal sides is 14.6 feet, and the perpendicular from the centre 12.64 feet? *Ans.* 553.632 *square feet.*

3. Required the area of a heptagon, one of whose equal sides is 19.38, and the perpendicular from the centre 20. *Ans.* 1356.6.

4. Required the area of an octagon, one of whose equal sides is 9.941, and the perpendicular from the centre 12. *Ans.* 477.168.

PROBLEM VIII.

To find the area of a regular polygon, when one of its equal sides only is given.

RULE.

Multiply the square of the side of the polygon, by the number standing opposite to its name in the following table, and the product will be the area.*

* The multipliers in the table are the areas of the polygons to which they belong when the side is unity or 1. And because

No. of sides.	Names.	Areas, or Multipliers.
3	Trigon, or equil. △	0.4330127
4	Tetragon, or square	1.0000000
5	Pentagon	1,7204774
6	Hexagon	2.5980762
7	Heptagon	3.6339124
8	Octagon	4.8284271
9	Nonagon	6.1818242
19	Decagon	7.6942088
11	Undecagon	9.3656399
12	Dodecagon	11.1961524

---

all regular polygons of the same number of sides, are similar figures, which are to each other as the squares of their like sides, (*Euc.* vi. 20) we shall have 1^2 : multiplier in the table : : square of the side of any polygon : area of that polygon ; or, which is the same thing, the square of the side of any polygon × by its tabular number = area of the polygon, *agreeably to the rule.*

The table is formed thus: divide 360° by the number of sides of the polygon, and the quotient will give the angle AOB at the centre, the half of which is the angle AOP ; and if this be taken from 90° it will give the angle OAP. Whence, by Trigonometry, rad. 1 : tan. ∠ OAP : : AP ($\frac{1}{2}$) : OP = $\frac{1}{2}$ tan. OAP ; and consequently $\frac{1}{2}$ AB × OP = $\frac{1}{4}$ tang. ∠ OAP = area of the triangle OAB ; and $\frac{1}{4}$ tang. ∠ OAP × number of sides = tabular area of the polygon.

The angle OAP, together with its tangent, for any polygon of not more than 12 sides, is shown in the following table.

No. of sides.	Names.	Angle OAP	Tangents.
3	Trigon or equi △.	30°	.5773503 = $\frac{1}{3}\sqrt{3}$
4	Tetragon, or square	45°	1.0000000 = 1
5	Pentagon	50°	1.3763819 = $\sqrt{(1 + \frac{2}{5}\sqrt{5})}$
6	Hexagon	60°	1.7320508 = $\sqrt{3}$
7	Heptagon	$64°\frac{2}{7}$	2.0765213
8	Octagon	$67°\frac{1}{2}$	2.4142136 = $1 + \sqrt{2}$
9	Nonagon	70°	2.7474774
10	Decagon	72°	3.0776835 = $\sqrt{(5 + 2\sqrt{5})}$
11	Undecagon	$73°\frac{7}{11}$	3.4056872
12	Duodecagon	75°	3.7320508 = $2 + \sqrt{3}$

EXAMPLES.

1. Required the area of a regular pentagon of which either of the equal sides is 15.

Here 15
15
—
75
15
——
225 = *square of the side.*
1.720477 = *area, when the side is* 1.
225
————
8602385
3440954
3440954
————
387.107325 = *area required.*

2. Required the area of a regular hexagon, either of the equal sides of which is 5 feet 4 inches.

Ans. 73.9 *feet.*

3. Required the area of a regular octagon, either of the equal sides of which is 16. *Ans.* 1236.0773.

4. Required the area of a regular decagon, either of the equal sides of which is $20\frac{1}{2}$. *Ans.* 3233.4913.

PROBLEM IX.

To find the area of an irregular right-lined figure of any number of sides.

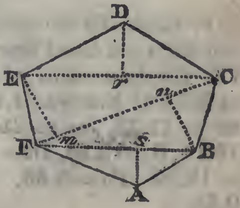
RULE I.

Divide the figure into triangles and trapeziums, and find the areas of each of them separately, by either of the rules before given for that purpose.

Then add these areas together, and their sum will give the area of the whole figure.

EXAMPLES.

1. Required the area of the irregular right-lined figure ABCD EF, the dimensions of which are as follows: FB = 20 75, FC = 27.48, EC = 18.5, Bn = 14.25, Em = 9.35, Dr = 12.8, and As = 8.6.



Here 20.75 FB.	*And* 18.5 EC.
8.6 A*s*.	12.8 D*r*.
12450	1480
16600	2220
2)178.450	2)236.80
89.225 *area* △ ABF.	118.40 *area* △ DEC.

Also 14.25 B*n*.
9 35 E*m*.

2)23.60

11.80
27.48 = FC.

9440
4720
8260
2360

324.2640 *area trap*. FBCE.

Whence 324.264 + 89.225 + 118.4 = 531.889 *area of the whole figure.*

2. Required the area of an irregular hexagon, like that in the last example, supposing the dimensions of the different lines to be the halves of those before given.

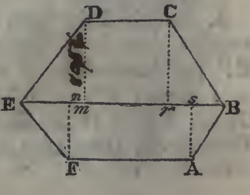
Ans. 132.97225.

RULE II.

The area of any irregular right-lined figure may also be determined, by drawing perpendiculars from all its angles, to one of its diagonals, considered as a base; and then adding the areas of all the triangles and trapezoids together for the content.

EXAMPLES.

1. Required the area of the irregular right-lined figure ABCDEF, the dimensions of which are as follows: E*m* = 4.54, E*n* = 3.26, *mr* = 11.75, *ns* = 21.68, B*r* = 10.14, B*s* = 3.93, D*m* = 10.56, C*r* = 12.24, F*n* = 8.56, and A*s* = 9.26.



Here 10.56 D*m*.	*And* 8.56 F*n*.
4.54 E*m*.	3.26 E*n*.
4224	5136
5280	1712
4224	2568
2)47.9424	2)27.9056
23.9712 *area* △ ED*m*.	13.9528 *area* △ EF*n*.
Also 12.24 C*r*.	*And* 9 26 A*s*.
10.14 B*r*.	9.93 B*s*.
4896	2778
1224	8334
12240	2778
2)124.1136	2)36.3918
62.0568 *area* △ CB*r*.	18.1959 *area* △ AB*s*.
Then 10.56 D*m*.	*And* 8.56 F*n*.
12.24 C*r*.	9.26 A*s*.
22.80	17.82
11.75 *mr*.	21.68 *ns*.
11400	14256
15960	10692
2280	1782
2280	3564
2)267.9000 *area trap*. C*m*.	2)193.1688 *area trap*. F*s*.

Hence 23.9712 + 13.9528 + 62.0568 + 18.1959 + 133.95 + 193.1688 = 445.2955 *the area required.*

2. Required the area of an irregular figure, like that in the last example, supposing the dimensions of the diagonal, and the several perpendiculars there given, to be doubled.

Ans. 1781.182.

PROBLEM X.

To find the circumference of a circle, when the diameter is given, or the diameter when the circumference is given.

RULE.

Multiply the diameter by 3.1416, and the product will be the circumference; or divide the circumference by 3.1416, and the quotient will be the diameter.

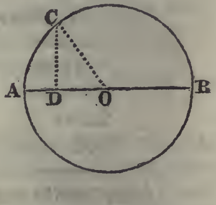
Where it may be observed that the number 3.1416 here used, is the approximate value of the circumference of a circle, of which the diameter is 1.

It may also be farther remarked, that the diameter of any circle is, to its circumference, nearly, as 7 is to 22; or, more nearly, as 113 is to 355.

The ratio, as 7 : 22 was first discovered by the celebrated Archimedes, more than 2000 years ago; the ratio, as 113 : 355 was given by Metius, and is much more correct than the former.

EXAMPLES.

1. If the diameter AB of the circle ACB, be 26, what is the circumference?



Here 3.1416
26
188496
62832
81.6816 *the circumference.*

2. If the circumference of a circle be 75, what is the diameter?

```
Here 3.1416)75.0000000(23.873
             62832
             ------
             121680
              94248
             -------
              274320
              219912
              -------
               229920
               251328
               -------
                100080
                 94248
                -------
                  5832
```

Ans. 23.873 *the diameter required.*

3. What is the circumference of a circle, the diameter of which is 7? *Ans.* 21.9912.

4. What is the diameter of a circle, the circumference of which is 50? *Ans.* 15.9156.

5. If the diameter of a circle be 17, what is the circumference? *Ans.* 53.4072.

6. If the circumference of a circle be 354, what is the diameter? *Ans.* 112.681.

7. What is the circumference of the globe of the earth, supposing its diameter to be 7920 miles, which it is very nearly? *Ans.* 24881.4720 *miles.*

8. Supposing the circumference of the earth to be 25000 miles, in round numbers, which it is very nearly, what is its diameter? *Ans.* 7920 *miles, nearly.*

PROBLEM XI.

To find the length of any arc of a circle.

RULE.

From eight times the chord of half the arc substract the chord of the whole arc, and one third of the remainder will be the length of the arc *nearly*: that is

$$\frac{8\,AC - AB}{3}, \text{ or } \frac{8\sqrt{\frac{1}{4}AB^2 + DC^2} - AB}{3} =$$

length of the arc ACB; using the latter formula when the chord AB of the whole arc, and its height, or versed sine, CD, are given.

And since the diameter CE of the circle, and the above-mentioned parts are readily found from each other, either by the rule for right-angled triangles, Prob. iv., or from the following properties of the circle;

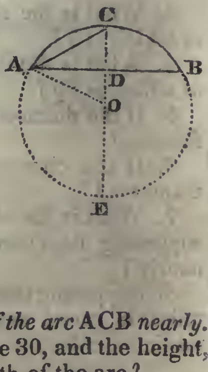
$EC \times CD = CA^2$, and $CD \times DE$ or $CD \times (CE - CD) = AD^2$;

the rule is equally applicable in all the cases of the problem that can occur.

It may here likewise be observed, as another rule for the same purpose, that if the number of degrees and decimal parts in the arc, be multiplied by the radius, and that product again by .01745, (which is the length of an arc of 1 degree,) the result will give the length of the arc *nearly*.

EXAMPLES.

1. The chord AB of the whole arc ACB, is 48.74, and the chord AC, of half the arc 30.25: what is the length of the arc?



$$\begin{array}{r} 30.25 = AC. \\ 8 \\ \hline 242.00 \\ 48.74 = AB. \\ \hline 3)193.26 \\ \hline 64.42 \end{array}$$

64.42 = *length of the arc* ACB *nearly*.

2. If the chord AB of the arc ACB be 30, and the height, or versed sine CD, 8, what is the length of the arc?

Here $15^2 + 8^2 = 225 + 64 = 289 = AC^2$, *and consequently* $AC = \sqrt{289} = 17$.

$$\begin{array}{r} \textit{Whence } 17 \\ 8 \\ \hline 136 \\ 30 \\ \hline 3)106 \\ \hline 35\frac{1}{3} \end{array}$$

$35\frac{1}{3}$ = *length of the arc* ABC *nearly*.

3. Required the length of an arc of 60 degrees, the radius of the circle being 7 feet.

Here 60
7
———
420
.01745
———
34900
6980
———
7.32900 *feet, the answer.*

4. The chord of the whole arc is $50\frac{4}{5}$, and the chord of half the arc is $30\frac{3}{5}$: required the length of the arc.

Ans. 64.6.

5. The length of the chord of the whole arc is $36\frac{3}{4}$, and the length of the chord of half the arc $23\frac{1}{5}$: what is the length of the arc? *Ans.* 49.783.

6. The chord of the whole arc is $48\frac{1}{2}$, and its height, or the versed sine of half the arc, $18\frac{1}{4}$: what is the length of the arc? *Ans.* 64·7677.

7. If the chord of the whole arc be 16, and the radius of the circle 10, what is the length of the arc?

Ans. 18.517.

8. If the height, or versed sine of half the arc, be 4, and the diameter of the circle 30, what is the length of the arc?

Ans. 22.412.

9. Required the length of an arc of 57° 17′ $44\frac{1}{2}$″, the diameter of the circle being 25 feet.

Ans. $12\frac{1}{2}$ *feet, which is = the radius.*

9. Reo̤uired the length of a degree, a minute, and a second, of a great circle of the earth, supposing its circumference to be 24,880 miles. *Ans.* 1° = $69\frac{1}{2}$ *miles.*

PROBLEM XII.

To find the area of a circle.

RULE.

Multiply half the circumference by half the diameter, and the product will be the area; or multiply the whole circumference by the whole diameter, and divide the product by 4.

Or, when the diameter, or circumference of the circle, only is given,

Multiply the square of the diameter by .7854; or the square of the circumference by .07958, and the product, in either case, will be the area.

Where it is to be observed, that .7854 is the area of a a circle whose diameter is 1; and .07958 the area when the circumference is 1.*

EXAMPLES.

1. What is the area of a circle whose diameter is 10 feet 6 inches, and its circumference 32 feet 11 inches?

---

* A circle may be considered as a regular polygon of an infinite number of sides, the circumference being equal to the perimeter, and the radius to the perpendicular, drawn from its centre to one of the sides. Hence the area of a regular polygon being equal to half the perimeter multiplied by the perpendicular, the area of a circle must also be equal to half the circumference multiplied by the radius, or half the diameter; agreeably to the first rule.

Again, since $3.1416 =$ circumference of a circle whose diameter is 1, and $.31831 =$ diameter of a circle whose circumference is 1, it follows that $\frac{3.1416 \times 1}{4} = .7854 =$ area of the circle whose diameter is 1, and $\frac{.31831 \times 1}{4} = .07958 =$ area of the circle whose circumference is 1.

Hence, because circles are to each other, as the squares of their diameters, or of their circumferences, (Leg. Art. 291.,) we shall

$$\text{have } 1^2 : d^2 :: .7854 : \frac{.7854 \times d^2}{1^2} = .7854 \times d^2.$$

$$\text{and } 1^2 : c^2 :: .07958 : \frac{.07948 \times c^2}{1^2} = .07958 \times c^2.$$

which is the same as the last of the two rules.

The following formulæ will likewise show most of the useful problems relating to the circle and its equal or inscribed square.

$$1.\ \text{Side of an equal square} = \begin{cases} \text{diameter} \times .8862 \\ \text{or} \\ \text{circumfe.} \times .2821 \end{cases}$$

$$2.\ \text{Side of the inscribed square} = \begin{cases} \text{diameter} \times .7071 \\ \text{or} \\ \text{circumfe.} \times .2251 \end{cases}$$

Here 32 *ft.* 11 *in.* = 32.9166
10 *ft.* 6 *in.* = 10.5

1645830
3291660

4)345.62430

86.40607
12

4.87284

Ans. 86 *feet* 4⅘ *inches.*

2. How many square yards are there in a circle whose diameter is 5½ feet?

.7854
30.25 = *square of* 5½

39270
15708
235620

9)23.758350

2.639816 = *square yards the ans.*

3. How many square feet are there in a circle whose circumference is 10¾ yards?

115.5625 = *square of the circumference.*
.07958

9245000
5778125
10400625
8089375

9.196463750
9

82.768173750 *square feet, the answer.*

4. What is the area of a circle, whose diameter is 7, and the circumference 22 *nearly*? *Ans.* 38½.

5. required the area of a circle, the diameter of which is 7⅔? *Ans.* 46.164.

6. Required the area of a circle, the circumference of which is $9\frac{1}{5}$? *Ans.* 6.73565.

7. How many square yards are there in a circle, whose radius is $15\frac{1}{4}$ feet? *Ans.* 81.1798.

8. How many square feet are there in a circle, whose circumference is $20\frac{1}{10}$ yards? *Ans.* 289.36.

9. It is required to find the radius of the circle, whose area is an acre. *Ans.* $39\frac{1}{4}$ *yards.*

PROBLEM XIII.

To find the area of a circular ring, or the space included between two concentric circles.

RULE.

Find the areas of each of the two circles separately, as in the last Problem. Then the difference of these areas will be the area of the ring.

Or, multiply the sum of the diameters by their difference, and this product again by .7854, and it will give the area required.*

EXAMPLES.

1. The diameters AB and CD being 20 and 15, it is required to find the area of the space included between the circumferences of those circles.

---

* *Demon.* The area of the outer circle AEBA $= AB^2 \times .7854$, and the area of the inner circle CD $= CD^2 \times .7854$; whence the area of the ring $= AB^2 \times .7854 - CD^2 \times .7854 = (AB^2 - CD^2) \times .7854 = (AB + CD) \times (AB - CD) \times .7854$; which is the rule.

Coroll. If CE be a perpendicular at the point C, the area of the ring will be equal to that of a circle whose radius is CE.

It may here likewise be observed, that if half the sum of the circumferences be multiplied by half the difference of the diameters, the product will be the area of the ring; which rule will also serve for any part of the ring, using half the sums of the intercepted arcs for half the sum of the circumferences.

Here .7854
400 = *square of* AB.

314.1600 = *area of the outer circle.*
And .7854
225 = *square* CD.

39270
15708
15708

176.7150 = *area of the inner circle.*

Hence 314.16 — 176.715 = 137.445 = *area of the ring.*

Or, 35 = AB + CD.
5 = AB — CD.

175
.7854

39270
54978
7854

137.4450 = *area, as before.*

2. Required the area of the ring, the diameters of whose bounding circles are 6 and 4. *Ans.* 15.708.

3. The diameters of two concentric circles are 16 and 10: what is the area of the ring formed by those circles? *Ans.* 122.5224.

4. The diameters of two circles are 21.75 and 9.5: required the area of the ring. *Ans.* 300.6609.

PROBLEM XIV.

To find the area of a sector of a circle.

RULE.

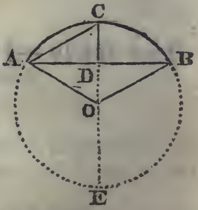
Multiply the radius, or half the diameter of the circle, by half the length of the arc of the sector, as found by Prob. X, and the product will be the area. Or multiply the whole diameter by the whole length of the arc, and divide the product by 4.

Or, if much accuracy be not required, the following approximating rule may be used for the same purpose; viz.

$\frac{1}{2} CE^2 \sqrt{\frac{3CD}{3CE - CD}}$ = area of the sector AOBCA *nearly*.

To this it may also be added, that as 360 degrees is to the number of degrees in the arc of the sector, so is the area of the circle to the area of the sector.

EXAMPLES.



1. The chord AB of the whole arc ACB is 24, and the chord AC of half the arc is 13: what is the area of the sector OBCAO.

Here $CD^2 = 13^2 - 12^2 = 169 - 144 = 25$
And consequently we shall have $CD = \sqrt{25} = 5$
Also, $CE \times CD = AC^2$, *or* $CE \times 5 = 13^2 = 169$
Whence, the diameter CE *of the circle* $= \frac{169}{5} = 33.8$

Then 13
8
——
104
24
——
3)80
——
26.666 = *length of the arc* ACB.
33.8 = *diameter* CE.
——
213328
79998
79998
——
4)901.3108
——
225.3277 = *area of the sector.*

2. Required the area of the sector, the arc of which is 30°, and the diameter 3 feet.

Here $.7854 \times 9 = 7.0686 =$ *area of the circle.*

Hence $360°: 30° :: 7.0686 : \frac{7.0686 \times 30}{360} = \frac{7.0686}{12} =$ $.58905 =$ *area of the sector.*

3. The chord of the whole arc is 16, and its height, or versed sine, is 6: what is the area of the sector?

Ans. 355.541.

4. What is the area of the sector whose chord is $18\frac{3}{5}$, and the radius of the circle 10? *Ans.* 115.33.

5. Required the area of the sector, whose height, or versed sine, is 4, and the diameter of the circle 16.

Ans. 33.5103.

6. Required the area of the sector, whose arc is 17° 15′, and the radius of the circle $9\frac{1}{2}$ feet. *Ans.* 13.5858.

PROBLEM XV.

To find the area of the segment of a circle.

RULE I.

Find the area of the sector, having the same arc as the segment, by the last problem.

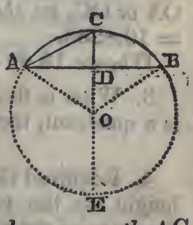
Also find the area of the triangle formed by the chord of the segment, and the two radii of the sector.

Then the sum, or difference, of these areas, according as the segment is greater or less than a semicircle, will be the area of the segment required.

Note. To this we may likewise add, as a useful approximating rule, for the same purpose, that

$\frac{4}{3} CD \sqrt{(\frac{1}{4} AB^2 + \frac{2}{5} CD^2)} =$ area of the segment ABCA, *nearly.*

EXAMPLES.



1. The chord AB is 24, and the height, or versed sine CD of half the arc ACB is 5: what is the area of the segment ABCA?

Here $12^2 + 5^2 = 144 + 25 = 169$, *and consequently* AC $= \sqrt{169} = 13$.

Also CE × CD = AC^2, or 5 CE = 169; *and therefore* CE = $\frac{169}{5}$ = 33.8.

$$\begin{array}{r} \textit{Then}\ 13 \\ 8 \\ \hline 104 \\ 24 \\ \hline 3)80 \\ \hline 26.666 \\ 33.8 \\ \hline 213328 \\ 79998 \\ 79998 \\ \hline 4)901.3108 \\ \hline 225.3277 \end{array}$$

26.666 = *length of the arc* ACB.

225.3277 = *area of the sector* OBCAO.

But CO — CD = 16.9 — 5 = 11.9 = OD.

Whence 11.9 × 12 = 142.8 = *area* △ OAB.

And conseq. 225.3277 — 142.8 = 82.5277 = *area of the segment* ABCA.

2. What is the area of a segment of a circle, whose arc is 60°, and the diameter of the circle 10 feet?

Here .7854 × 100 = 78.54 = *area of the whole circle.*

Then 360° : 60 : : 78.54 : 13.09 = *area of the sector* OA CBO.

And since the chord AB (*to an arc of* 60°) = 5 *the rad.* OA *or* OC, *we shall have by Prob. III. area of the* △ OAB = 10.8253.

Whence 13.09 — 10.8253 = 2.2647 = *segment required.*

3. What is the area of a segment of a circle whose arc is a quadrant, the diameter being 18 feet?

Ans. 23.1174.

4. Required the area of the segment of a circle, whose height, or the versed sine of half the arc, is 5, and the diameter of the circle 20. *Ans.* 61.4184.

5. Required the area of the segment of a circle, whose chord is 16, and the diameter of the circle $16\frac{2}{3}$.

Ans. 70.7083.

6. Required the area of a segment of a circle, whose arc contains 280 degrees, the diameter being 50.

Ans. 1834.9191.

RULE II.

Divide the height, or versed sine of half the arc of the segment, by the diameter of the circle, and find the quotient in the table of versed sines at the end of the volume.

Then multiply the tabular area on the right hand of the versed sine so found, (which is the tabular segment,) by the square of the diameter, and the product will be the area.

The table to which the rule refers, is formed of the areas of the segments of a circle whose diameter is 1; which is supposed to be divided by perpendicular chords into 10000 equal parts.

Note. When the quotient, arising from dividing the versed sine by the diameter, has a remainder, or fraction, after the third place of decimals, substract the tabular area, answering to the first 3 figures, from the next following area; then if the remainder be multiplied by the fractional part, and the result be added to the first area, it will give the tabular area for the whole quotient; which must be multiplied by the square of the diameter, as before.

EXAMPLES.

1. If the height, or versed sine, be 10, and the diameter of the circle 50, what is the area of the segment?

Here 50)10

.2 *the tabular versed sine.*

And .1118238 = *tabular segment.*

2500 = *square of* 50.

559119000

2236476

279.5595000 = *area required.*

2. What is the area of the segment, the versed sine of which is 2, and the diameter of the circle 52?

Here 52)2.000
.038$\frac{6}{13}$ *tab. vers. sine.*
And .009763 *tab. seg.* to 038.
.010148 *next tab. seg.*

.000385 *difference.*
6

13).002310

.000178
.009763 *tab. seg.* to .038.

.009941
2704 = 52^2

39764
695870
19882

26.880464 = *the area of the segment, as required.*

3. What is the area of a segment of a circle, whose height, or versed sine, is 5, and the diameter of the circle 25? *Ans.* 69.889375.

4. What is the area of the segment of a circle, whose height, or versed sine, is 3, and the diameter of the circle 8? *Ans.* 17.216832.

5. What is the area of the segment of a circle, whose height, or versed sine, is 6, and the diameter 21? *Ans.* 81.601758.

6. What is the area of the segment, whose height, or versed sine, is 7, and the diameter of the circle 38? *Ans.* 143.510496.

PROBLEM XVI.

To find the area of a circular zone, or the space included between two parallel chords and their intercepted arcs.

RULE.

Find the area of that part of the zone ABCD, which forms a trapezoid, by Prob. vi. and the area of the small segment B*n*CB by either of the rules in Prob. xv.

Then add the area of the trapezoid to twice the area of the segment, and it will give the area of the zone.

Note. If d be put $=$ the diameter EF; C, $c =$ the two parallel chords AB, DC; $v =$ versed sine nm, of half the arc CnB, and $b =$ GH, the breadth of the zone, we shall have

$$EF = \sqrt{\left\{ b^2 + \frac{C^2 + c^2}{2} + \left(\frac{C^2 - c^2}{4b}\right)^2 \right\}}$$

and

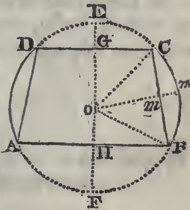
$$nm = \tfrac{1}{2} d - \tfrac{1}{2} \sqrt{\left\{ \left(\frac{C + c}{2}\right)^2 + \left(\frac{C^2 - c^2}{4b}\right)^2 \right\}}$$

Which expressions will be found of considerable use in facilitating the computation of the segment BnCB.

The formulæ, given in the rule, for facilitating the computation of the area of the segment BnCB, may be readily derived from the properties of right-angled triangles, &c.

Another method of resolving this problem is by finding the areas of each of the segments ADECB and DECD, and then taking their difference, which will be the area of the zone; but this rule, when the segments are large, and any of the approximating rules are used, is not so accurate as the former.

EXAMPLES.



1. The greater chord AB is 8, the less DC 6, and their perpendicular distance GH is 7; required the area of the zone.

Here AB + DC $= 8 + 6 = 14$, *and* GH $= 7$;

Whence $\frac{14 \times 7}{2} = 49 =$ *area of trapezoid* ABCD.

Also EF $= \sqrt{\left\{ 7^2 + \frac{8^2 + 6^2}{2} + \left(\frac{8^2 - 6^2}{4 \times 7}\right)^2 \right\}} = \sqrt{(49 + 50 + 1)} = \sqrt{100} = 10$, *the diameter.*

And $mn = \frac{10}{2} - \frac{1}{2} \sqrt{\left\{ \left(\frac{8 + 6}{2}\right)^2 + \left(\frac{8^2 - 6^2}{4 \times 7}\right)^2 \right\}} = 5 - \frac{1}{2} \sqrt{50} = 5 - \frac{5}{2} \sqrt{2} = 1.4644 =$ *versed sine.*

Therefore $\frac{1.4644}{10} = .01464 =$ *tabular versed sine.*

Answering to which is .071313 *the tabular area.*

Hence .071313
100 = *square of the diameter.*

7.131300 = *area of the segment* BnCB.
2

14.2626 = *twice ditto.*
49.0000 = *area of the trapezoid* ABCD.

63.2626 = *area of the zone.*

2. One of the parallel chords of a circular zone is 48, and the other 30, and its breadth is 13: what is the area of the zone? *Ans.* 534.1877.

3. The greater chord of a circular zone is 16, the less chord 12, and their perpendicular distance 2: what is the area of the zone? *Ans.* 28.376.

4. Supposing the greater chord of a circular zone to be 26, the less chord 15, and their diameter $17\frac{1}{2}$: what is the area of the zone? *Ans.* 395.4369.

6. Required the area of a circular zone, each of whose parallel chords are 50, and their perpendicular distance 30.

6. Required the area of a circular zone, the greater chord of which, being equal to the diameter of the circle, is 40, the less 20, and their perpendicular distance 10.

PROBLEM XXVII.

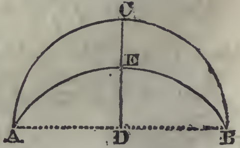
To find the area of a lune, or the space included between the intersecting arcs of two eccentric circles.

RULE.

Find the areas of the two segments, from which the lune is formed, by one of the rules in Prob. xv., and their difference will be the area of the lune.*

EXAMPLES.

1. The length of the chord AB is 40, the height DC 10, and DE 4: required the area of the lune ACBEA.



* For an account of the various properties of the lunes, and the theorems arising from them, see *Whiston's Commentaries on Tacquet's Euclid,* where the subject is very ingeniously handled.

Here, putting D, *d for the diameters of the two circles, we shall have, by Prob. X.* $CD \times (D - CD) = AD^2 = \frac{1}{4} AB^2$, *or*

$D = CD + \frac{AB^2}{4\,CD} = 10 + \frac{1600}{40} = 50$, *the diameter of the circle* ACB.

And $d = DE + \frac{AB^2}{4\,ED} = 4 + \frac{1600}{16} = 104$, *the diameter of the circle* AEB.

Whence $\frac{10}{50} = .2$, *the first tabular versed sine.*

Answering to which is .111823, *the first tabular segment.*

Therefore .111823
2500 = 50^2.

55911500
223646

279.557500 = *area of the segment* ABCA.

Also $\frac{4}{104} = \frac{1}{26} = .0384$, *the second tabular versed sine.*

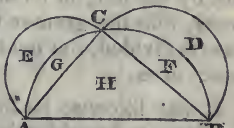
Answering to which is .009955, *the second tabular segment.*

Consequently .009955
10816 = 104^2

59730
9955
79640
99550

107.673280 = *area of the segment* AEBA.

One of the most curious of these properties, is the following: If ABC, or H, be a right-angled triangle, and semicircles be described on the three sides as diameters, then will the triangle H be equal to the two lunes D and E taken together.



For, if from the greater semicircle ABC there be taken the two segments F, G, there will remain the triangle H; and if the same segments be taken from the other two semicircles, there will remain the lunes D, E: hence, since the greater semicircle is equal to the sum of the other two, (Leg. Art. 186, and Euc. xii. 2,) the triangle H must also be equal to the sum of the lunes D, E.

Whence $279.5575 - 107.67328 = 171.88422$ = area of the lune ABCEA, as required.

2. The chord is 20, and the heights of the segments 10 and 2: required the area of the lune. *Ans.* 128.555.

3. Supposing the length of the chord to be 48, and the heights of the segments to be 18 and 7, what is the area of the lune? *Ans.* 405.8676.

PROBLEM XXVIII.

To find the area of a mixtilineal figure, or one formed by right lines and curves.

RULE I.

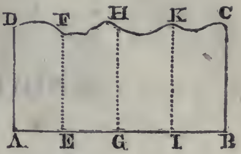
Take the perpendicular breadths of the figure in several places, at equal distances from each other, and divide their sum by the number of them, for the mean breadth; then this result being multiplied by the length, will give the area of the figure not far from the truth.

Or, if greater accuracy be required, take half the sum of the two extreme breadths, for one of the said breadths, and add it to the others, as before; then divide this sum by the number of parts in the base, (instead of by the number of breadths,) and the result being multiplied by the length will give the area, nearly.

Note. When the curved or mixtilineal boundary meets the base, as is frequently the case in surveying, the area is found by dividing the sum of all the breadths by the number of parts in the base, and then multiplying the result by the length, as before: observing, in each of these cases, that the greater the number of parts into which the base is divided, the nearer will the approximation be to the exact area.

It may likewise be further remarked, that if the perpendiculars, or breadths, be not at equal distances from each other, the parts should be computed separately, as so many trapezoids, and then added together, for the area.

EXAMPLES.



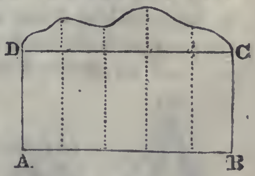
1. The perpendicular breadths of the irregular mixtilineal figure ABCD, at five equidistant places A, E, G, I, B, being 9.2, 10.5, 8.3, 9.4, and 10.7, and its length AB 20, what is its area ?

Here 9.2
10.5
8.3
9.4
10.7
———
5)48.1
———
9.62
20
———
192.40 = *area by the first rule.*

Or 9.2
10.7
———
2)19.9
———
9.95
10.5
8.3
9.4
———
4)38.15
———
9.5375
20
———
190.7500 = *area by the second rule.*

Here $\frac{1}{2}$ AD + $\frac{1}{2}$ BC is the arithmetical mean between the two extremes, or half the sum of them, and 4 is the number of parts. And the same rule will evidently hold for any number of parts whatever.

It may also be observed, that the latter result, in this case, differs from the former, by $1\frac{13}{20}$.



2. Required the area of the figure ABCD, of which the part AC is a rectangle, whose sides are $20\frac{1}{2}$ and $10\frac{1}{2}$, respectively; and the perpendicular breadths of the curvilineal space, reckoning from DC, at 4 equidistant places, are 10.2; 8.7; 10.9, and 8.5, respectively.

$$
\begin{array}{r l}
\textit{Here } 10.2 & \\
8.7 & \\
10.9 & \\
8.5 & \\
\hline
5)38.3 & \\
\hline
7.66 & \\
20.5 & \\
\hline
3830 & \\
15320 & \\
\hline
157.030 & = \textit{area of the curved space.} \\
20.5 & \\
10.5 & \\
\hline
1025 & \\
2050 & \\
\hline
215.25 & = \textit{area of the rectangle.} \\
157.03 & \textit{ditto curved space.} \\
\hline
372.28 & = \textit{area of the whole figure.}
\end{array}
$$

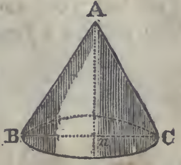
3. The length of an irregular mixtilineal figure is 47, and its breadths, at 6 equidistant places, beginning at the left hand extremity of the base, 5.7; 4.8; 7.5; 5.1; 8.4, and 6.5: what is its area? *Ans.* 299.86.

4. The length of an irregular mixtilineal figure, of which the curvilinear boundary meets the base, is $37\frac{1}{2}$, and its breadth, at 7 equidistant places, is 4.9; 5.6: 4.5: 8.2; 7.3; 5.9, and 8.5: what is its area? *Ans.* 210.46875.

CONIC SECTIONS.

DEFINITIONS.

1. The *conic sections* are certain plain figures, of great use in some of the higher branches of mathematics, which are formed by the cutting of a cone.



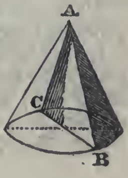
2. A *cone* is a solid described by the revolution of a right-angled triangle about one of its legs, which remains fixed, as ABC.

This is Euclid's definition, and is that which is generally best understood by learners; but the following one is more general:

Conceive the right line AB to move upon the fixed point A as a centre, so as continually to touch the circumference of the circle BC, placed in any position, except in that of a plane which passes through the said point; then, that part of the line which is intercepted between the fixed point and the periphery of the circle, will generate the convex superficies of a cone.

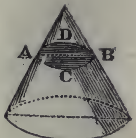
3. The *axis of a cone* is the right line about which the triangle revolves.

4. The *base of a cone* is the circle which is described by the revolving leg of the triangle; and its altitude is a perpendicular drawn from the vertex, to the base, as A*n*.

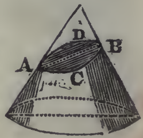


5. If a cone be cut through the vertex, by a plane perpendicular to that of the base, the section will be a *triangle*, as ABC.

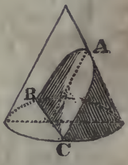
6. If a cone be cut into two parts, by a plane parallel to the base, the section will be a *circle*, as ACBD.



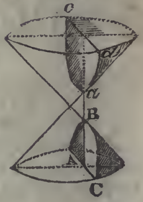
7. If a cone be cut by a plane which passes obliquely through its two slant sides, the section will be an *ellipse*, as ACBD.



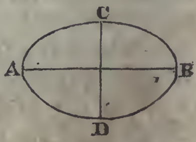
8. If a cone be cut by a plane, which is parallel to either of its slant sides, the section will be a *parabola*, as ABCA.



9. If a cone be cut into two parts, by a plane, which being continued, would meet the opposite cone, the section is called an *hyperbola*, as ABC.*

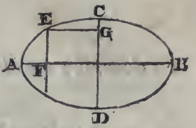


10. If two lines be drawn through the centre of an ellipse perpendicular to each other, and terminated on each side by the circumference, the longer of them AB is called the *transverse diameter*, or *axis*, and the shorter CD the *conjugate*.



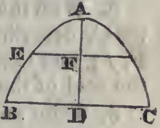
* The two opposite cones, in this definition, are supposed to be generated together, by the revolution of the same line.

It may here, also, be observed, that all the figures which can possibly be formed by the cutting of a cone, are mentioned in these definitions; being the five following: *viz. a triangle*, *a circle*, an *ellipsis*, *a parabola*, and an *hyperbola ;* but the last three only are usually called the *conic sections.*



11. *An ordinate of an ellipse,* is a right line EF, or EG, drawn from any point E in the curve, perpendicular to either of the diameters.

12. *An abscissa* is that part AF, or CG, of the diameter, which is contained between either of the extremities of that diameter and the ordinate.*



13. *The axis of a parabola* BAC, is a right line AD drawn from the vertex, so as to divide the figure into two equal parts.

14. *An ordinate of a parabola,* is a right line EF, drawn from any point in the curve perpendicular to the axis.

15. *An abscissa* is that part of the axis AF which is contained between the vertex of the curve and the ordinate.

16. *The transverse diameter of an hyperbola* is that part of the axis which is intercepted between the two opposite cones, as *a*B, in a former figure ; *def.* 9.

17. *The conjugate diameter* is a line drawn through the centre perpendicular to the transverse.

18. *An ordinate of an hyperbolu,* is a line drawn from any point in the curve perpendicular to either of the diameters : and an *abscissa* is that part of the diameter which is contained between either of the extremities of that diameter and the ordinate.

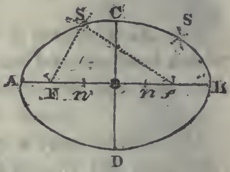
Hence, in the ellipse and hyperbola, every ordinate has two abscissæ, but in the parabola only one ; the other vertex of the diameter being at an infinite distance.

---

* An abscissa may be more generally considered as any part of the diameter or axis of a curve, comprised between any fixed point, from which all the abscissæ are supposed to take their origin, and another line called the ordinate, drawn so as to make a given angle with the former, and terminated in the curve; but when not otherwise specified, they are commonly taken as above. The abscissa and its corresponding ordinate, when considered together, are also frequently called co-ordinates.

PROBLEM I.

To describe an ellipse, the transverse and conjugate diameters being given.



Construction.–1. Draw the transverse and conjugate diameters, AB, CD, bisecting each other perpendicularly in the centre O.

2. With the radius AO, and centre C, describe arcs, cutting AB in F, *f*; and these two points will be the foci of the ellipse.

3. Take any number of points, *n*, *n*, &c., in the transverse diameter AB, and with the radii A*n*, *n*B, and centres F, *f*, describe arcs intersecting each other in *s*, *s*, &c.

4. Through the points *s*, *s*, &c. draw the curve A*s*CBD, and it will be the circumference of the ellipse required.

It is a known property of the ellipse, that the sum of two lines drawn from the foci, to meet in any point in the curve, is equal to the transverse diameter; and from this the truth of the construction is evident.

From the same principle is also derived the following method of describing an ellipse, by means of a string and two pins.

Having found the foci, F*f*, as above, take a thread of the length of the transverse diameter, and put round two pins fixed at the points F, *f*; then stretch the thread F*sf* to its greatest extent, and it will reach to the point *s* in the curve; and by moving a pencil round within the thread, keeping it always stretched, it will trace out the curve required.

PROBLEM II.

In an ellipse, any three of the four following terms being given, viz. the transverse and conjugate diameters, an ordinate and its abscissa, to find the fourth.

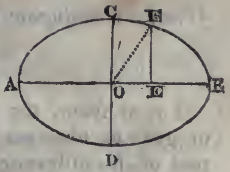
CASE I.

When the transverse, conjugate, and abscissa are given, to find the ordinate.

RULE.

As the transverse diameter is to the conjugate, so is the square root of the rectangle of the two abscissæ to the ordinate which divides them.

EXAMPLES.



1. In the ellipsis ADBC, the transverse diameter AB is 120, the conjugate diameter CD is 40, and the abscissa BE 24: what is the length of the ordinate EF?

Here AB $= 120$, CD $= 40$, *and* BE $= 24$.

Whence $120 : 40 :: \sqrt{24\,(120 - 24)}$: EF.

Or EF $= \dfrac{40}{120}\sqrt{24 \times 96} = \frac{1}{3}\sqrt{2304} = \dfrac{48}{3} = 16$.

Therefore 16 *is the ordinate required.*

2. If the transverse diameter be 35, the conjugate 25, and the abscissa 28, what is the ordinate? *Ans.* 10.

CASE II.

When the transverse, conjugate, and ordinate are given, to find the abscissa.

RULE I.

As the conjugate diameter is to the transverse, so is the square root of the difference of the squares of the ordinate and semi-conjugate, to the distance between the ordinate and centre.

And this distance being added to and subtracted from the semi-transverse, will give the two abscissæ required.

EXAMPLES.

1. The transverse diameter AB is 120, the conjugate diameter CD is 40, and the ordinate FE is 16: what is the abscissa BE?

Here AB $= 120$, CD $= 40$, *and* FE $= 16$.

Whence $40 : 120 :: \sqrt{20^2 - 16^2}$: OE.

Or OE $= \dfrac{120}{40}\sqrt{20^2 - 16^2} = 3\sqrt{144} = 36$.

Therefore $60 - 36 = 24 =$ BE, *and* $60 + 36 = 96 =$ AE.

2. What are the two abscissas to the ordinate 10, the diameters being 35 and 25? *Ans.* 7 *and* 28.

CASE III.

When the conjugate, ordinate, and abscissa are given, to find the transverse.

RULE.

To, or from, the semi-conjugate, according as the less or greater abscissa is used, add or substract the square root of the difference of the squares of the ordinate and semi-conjugate.

Then, as the square of the ordinate is to the rectangle of the conjugate and abscissa, so is the sum or difference above found, to the transverse diameter required.

EXAMPLES.

1. The conjugate diameter CD is 40, the ordinate EF is 16, and the abscissa BE 24: required the transverse AB.

Here OB $= 20$, EF $= 16$, and BE $= 24$.

Whence $20 + \sqrt{20^2 - 16^2} = 20 + \sqrt{144} = 20 + 12 = 32$.

And $16^2 : 40 \times 24 :: 32 : \frac{32 \times 40 \times 24}{16^2} = \frac{2 \times 24 \times 40}{16}$ $= \frac{2 \times 3 \times 40}{2} = 3 \times 40 = 120$ *the transverse diameter required.*

2. If an ordinate and its less abscissa be 10 and 7, and the conjugate 25, what is the transverse? *Ans.* 35.

CASE IV.

The transverse, ordinate, and abscissa being given, to find the conjugate.

RULE.

As the square root of the product of the two abscissæ is to the ordinate, so is the transverse diameter to the conjugate.

EXAMPLES.

1. The transverse AB is 120, the ordinate EF 16, and the abscissa EB 24: required the conjugate.

$5^2 \times \frac{210}{3} = 175$ ale gallons per minute.

$5^2 \times \frac{252}{3} = 2100$ libs water in a pipe.

$\frac{2100 \times 210}{44000} = 10$ horse power required to pump that quantity of water.

The following Table gives the contents of a pipe one inch in diameter, in weight and measure; which serves as a standard for pipes of other diameters, their contents being found by the following Rule.

Multiply the numbers in the following Table against any height, by the square of the diameter of the pipe, and the product will be the number of cubic inches avoirdupois ounces, and Wine gallons of water, that the given pipe will contain.

EXAMPLE.

How many wine gallons of water is contained in a pipe 6 inches diameter, and 60 feet long?

$2.4480 \times 36 = 88.1280$ wine gallons.

In a Wine gallon there are 231 cubic inches.

TABLE.

ONE INCH DIAMETER.			
Feet High.	Quantity in Cubic Inches.	Weight in Avoir, Oz	Gallons Wine Measure.
1	9.42	5.46	.0407
2	18.85	10.92	.0816
3	28.27	16.38	.1224
4	37.70	21.85	.1632
5	47.12	27.31	.2040
6	56.55	32.77	.2448
7	65.97	38.23	.2423
8	75.40	43.69	.3264
9	84.82	49.16	.3671
10	94.25	54 62	.4080
20	188.49	109.24	.8160
30	282 74	163.86	1.2240
40	376.99	218.47	1.6300
50	471.24	273.09	2.0400
60	565.49	327.71	2.4480
70	659.73	382.33	2.8560
80	753.98	436.95	3.2640
90	848.23	491.57	3.6700
100	942.48	546.19	4.0800
200	1884.96	1092.38	8.1600

The resistance arising from the friction of water flowing through pipes, &c. is directly as the velocity of the water, and inversely as the circumference of the pipe.

The data given is a medium, and which is 1-5th of the whole resistance; this is the standard generally adopted, being considered as most correct.

EXAMPLE I.

What is the power requisite to overcome the resistance and friction of a column of water 4 inches diameter, 100 feet high, and flowing at the velocity of 300 feet per minute?

$$\frac{546.19 \times 4^2}{16} = 546.19, \text{ say } 546.2$$

$$\frac{546.2 \times 300}{44000} = 3.7, \tfrac{1}{5}\text{th of which is } .7, \text{ therefore the power}$$

required to overcome the resistance occasioned by the weight and friction of the water will be $3.7 + .7 = 4.4$ H. P., say 4.5 horse power.

EXAMPLE II.

There is a cistern 20 feet square, and 10 feet deep, placed on the top of a tower 60 feet high, what power is requisite to fill this cistern in 30 minutes, and what will be the diameter of the pump, when the length of stroke is 2 feet, and making 40 per minute?

$20 \times 20 \times 10 = 4000$ cubic contents of cistern.

$$\frac{4000}{30} = 133.3 \text{ cubic feet of water per minute.}$$

$$\frac{133.3 \times 1000}{16} = 8331.25 \text{ libs avoirdupois per minute.}$$

$$\frac{8331.25 \times 60}{44000} = 11.36 \text{ horse power, 1-5th of which}$$

is $= 2.27 + 11.11 = 13.63$ horse power required.

$$2 \times 40 = 80 \quad \frac{133.3}{80} = 1.7 \times 144 = \frac{244.80}{.7854} = 311.7, \text{ now}$$

$\sqrt{311.7} = 17.6$ inches diameter of pump required.

Founders generally prove the pipes they cast to stand a certain pressure, which is calculated by the weight of a perpendicular column of water, the area being equal to the area of the pipe, and the height equal to any given height.

To ascertain the exact pressure of water to which a pipe is subjected, a safety valve is used, generally of 1 inch diameter, and loaded with a weight equal to the pressure required: for example, a pipe requires to stand a pressure of 300 feet, what weight will be required to load the safety-valve one inch diameter?

Feet. Inches. Ounces.

$$300 \times 12 = 3600 \times .7854 = \frac{2827.4400 \times 1000}{1728} = \frac{1636\frac{1}{4}}{16} = 102$$

libs $4\frac{1}{2}$ oz. weight required.

Each of the weights for the safety-valves of these hydrostatic proving-machines are generally made equal to a pressure of a column of water 50 feet high, the area being the area of the valve.

50 feet of pressure on a valve			1 inch diam.		= 17.06 libs
50 do.	do.	do.	$1\frac{1}{4}$	do.	= 26.65 do.
50 do.	do.	do.	$1\frac{1}{2}$	do.	= 38.38 do.
50 do.	do.	do.	2	do.	= 68.24 do.

In pumping, there is always a deficiency owing to the escape of water through the valves; to account for this loss, there is an allowance of 3 inches for each stroke of piston rod: for example, a three feet stroke may be calculated at 2 feet 9 inches.

There is a town, the inhabitants of which amount to 12000, and it is proposed to supply it with water, from a river running through the low grounds 250 perpendicular feet below the best situation from the reservoir.

It is required to know the power of an engine capable of lifting a sufficient quantity of water, the daily supply being calculated at 10 ale gallons to each individual: also what size of pump and pipes are requisite for such?

$12000 \times 10 = 120000$ gallons per day.

Engine is to work 12 hours, $\frac{120000}{12} = 10000$ gallons per hour.

$\frac{10000}{60} = 166.6$ gallons per minute.

The pump to have an effective stroke of $3\frac{3}{4}$ feet, and making 30 strokes per minute.

$\frac{166.6}{30} = 5.5533$ gallons each stroke.

$282 \times 5.6 = 1579.2$ cubic inches of water each stroke.

3 feet 9 inches $= \frac{1579.2}{45 \text{ in.}} = 35.1$ inches, area of pump

$\frac{35.1}{.7854} = 44.7$, therefore $\sqrt{44.7} = 6.7$ diam. of pump.

The pipes will require to be at least the diameter of the pump; if they are a little more, the water will not require to flow so quickly through them, and thereby cause less friction.

The power of the engine will be

166.6 gall. × $10\frac{1}{4}$ lb. × 250 feet = 426925 momentum.

$\frac{426925}{44000}$ = 9.7, add 1-5th = 11.64 horse power.

$\frac{426925}{32000}$ = 13.3, —— = 15.96 do. *Watt.*

$\frac{426925}{27500}$ = 15.5, —— = 18.6 do. *Desaguliers.*

$\frac{426925}{22916}$ = 18.6, —— = 22.32 do. *Smeaton.*

MISCELLANEOUS QUESTIONS.

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1. What difference is there between a floor 48 feet long, and 30 feet broad, and two others each of half the dimensions? *Ans.* 720 *feet.*

2. From a mahogany plank 26 inches broad, a yard and a half is to be sawn off; at what distance from the end must the line be strück? *Ans.* 6.23 *feet.*

3. The sides of 3 squares being 4, 5 and 6 feet, respectively, it is required to find the side of a square that shall be equal in area to all the 3? *Ans.* 8.7749 *feet.*

4. What quantity of canvass will be necessary for forming a conical tent, whose height is 8 feet, and the diameter at bottom 13 feet? *Ans.* $210\frac{1}{2}$ *square yds.*

5. How many square feet of board are required to make a rectangular box, whose length is to be $3\frac{1}{2}$ feet, breadth 2 feet, and depth 20 inches. *Ans.* $32\frac{1}{3}$ *feet.*

6. A joist is $8\frac{1}{2}$ inches deep, and 3' broad; what will be the dimensions of a scantling just as big again as the joist, that is $4\frac{3}{4}$ inches broad? *Ans* 12.52 *inches deep.*

7. A roof which is 24 feet 8 inches by 14 feet 6 inches, is to be covered with lead at 8lbs. to the foot; what will it come to at 18*s.* per cwt? *Ans.* 22*l.* 19*s.* $10\frac{1}{4}$*d.*

8. If the side of an equilateral triangle be 10 chains,

what will be the side of another equilateral triangle whose area is one fourth of the former? *Ans.* 5 *chains.*

9. What is the side of that equilateral triangle, whose area cost as much paving at 8*d.* per foot, as the pallisading the three sides did at a guinea a yard?

Ans. 72.746 *feet.*

10. What would a circular reservoir, whose diameter at top is 40 yards, at bottom $38\frac{2}{3}$ yards, and its side, or slant depth 11 feet, cost lining with brick work, at 3*s.* 10*d.* the square yard? *Ans.* 311*l.* 18*s.* 2*d.*

11. The four sides of a field, whose diagonals are equal to each other, are 25, 35, 31, and 19 poles, respectively; what is the area? *Ans.* 4 *ac.* 1 *ro.* 38 *po.*

12. What is the length of a cord which cuts off $\frac{1}{3}$ of the area from a circle whose diameter is 289?

Ans. 278.6716.

13. A cable which is 3 feet long, and 9 inches in compass, weighs 22lbs. what will a fathom of that cable weigh whose diameter is 9 inches? *Ans.* 434.26 *lbs.*

14. A circular fish-pond is to be dug in a garden, that shall take up just half an acre; what must the length of the cord be that strikes the circle? *Ans.* 27.75 *yards.*

15. A carpenter is to put an oaken curb to a round well, at 8*d.* per square foot; the breadth of the curb is to be $7\frac{1}{4}$ inches, and the diameter within $3\frac{1}{2}$ feet; what will be the expense? *Ans.* 5*s.* $2\frac{1}{4}$*d.*

16. Suppose the expense of paving a semicircular plot, at 2*s.* 4*d.* per foot, amounted to 10*l.* what is the diameter of it? *Ans.* 14.7737.

17. Seven men bought a grinding-stone of 60 inches in diameter, each paying $\frac{1}{7}$ part of the expense; what part of the diameter must each grind down for his share? *Ans.* 1*st*, 4.4508, 2*d.* 4.8400, 3*d.* 5.3535, 4*th.* 6.0765, 5*th.* 7.2079, 6*th.* 9.3935, *and the* 7*th.* 22.6778.

18. A garden 100 feet long, and 80 feet broad, is to have a gravel walk of an equal width half round it; what must the width of the walk be so as to take up just half the ground? *Ans.* 25.968 *feet.*

19. How many gallons, wine measure, will a cistern hold, supposing its length and breadth at top to be 5 and 4 feet respectively, and at bottom 4 and 3 feet; the perpendicular depth being $3\frac{1}{2}$ feet? *Ans.* $414\frac{6}{11}$ *gallons.*

20. A malster has a kiln that is 16 feet 6 inches square,

which he wants to pull down, and to build a new one that will dry three times as much at a time as the old one; what must be the length of its side? *Ans.* 28 *feet* 7 *inches.*

21. If a round cistern be 26.3 inches in diameter, and 52.5 inches deep; how many inches diameter must a cistern be to hold twice the quantity, the depth being the same? *Ans.* 37.19 *inches.*

22. A maypole, whose top was broken off by the wind, struck the ground at 15 feet distance from the bottom of the pole; what was the height of the whole maypole, supposing the length of the broken piece to be 39 feet?

Ans. 75 *feet.*

23. What will the diameter of a globe be, when its solidity and superficial content are equal to each other, or rather, when they are both expressed by the same number?

Ans. 6.

24. How many 3 inch cubes can be cut out of a 12 inch cube? *Ans.* 64.

25. A farmer borrowed part of a hay-rick of his neighbor, which measured 6 feet every way, and paid him back again by two equal cubical pieces, each of whose sides were 3 feet: Query, whether the lender was fully paid?

Ans. He was paid $\frac{1}{4}$ *part only.*

26. What will the painting of a conical church spire come to at 8*d.* per yard; supposing the circumference of the base to be 64 feet, and the altitude 118 feet?

Ans. 14*l.* 0*s.* 8$\frac{3}{4}$*d.*

27. What will the marble frustum of a cone come to at 12*s.* per solid foot; the diameter of the greater end being 4 feet, that of the less end 1$\frac{1}{2}$ feet, and the length of the slant side 8 feet? *Ans.* 30*l.* 1*s.* 10*d.*

28. The diameter of a legal Winchester bushel is 18$\frac{1}{2}$ inches, and its depth 8 inches; what must the diameter of that bushel be whose depth is 7$\frac{1}{2}$ inches? *Ans.* 19.10671.

29. Three men bought a tapering piece of timber, which was the frustum of a square pyramid; one side of the greater end was 3 feet, one side of the less 1 foot, and the length 18 feet; what must be the length of each man's piece, supposing they paid equally, and are to have equal shares? *Ans.* 1*st.* 3.269, 2*d.* 4.559, *and the* 3*d.* 10.172, *reckoning from the greater end to the less.*

30. Suppose the ball at the top of St. Paul's Church is 6 feet in diameter; what would the gilding of it come to at 3$\frac{1}{2}$*d.* per square inch? *Ans.* 237*l.* 10*s.* 1*d.*

31. A person wants a cylindric vessel of 3 feet deep, that shall hold twice as much as another of 28 inches deep, and 46 inches in diameter; what must be the diameter of the required vessel? *Ans.* 57.37 *inches.*

32. Two porters agreed to drink off a quart of strong beer between them, at two pulls, or a draught each; now, the first having given it a black eye, as it is called, or drank till the surface of the liquor touched the opposite edge of the bottom, gave the remaining part of it to the other; what was the difference of their shares, supposing the pot was the frustum of a cone, the depth being 5.7 inches, the diameter at the top 3.7 inches, and that of the bottom 4.23 inches? *Ans.* 7.07 *cubic inches.*

33. Three persons having bought a sugar loaf, want to divide it equally among them by sections parallel to the base; it is required to find the altitude of each person's share, supposing the loaf to be a cone, whose height is 20 inches? *Ans.* 13.867 *the upper part,* 3.604 *the middle part, and* 2.528 *the lower part.*

34. How high above the surface of the earth must a person be raised to see $\frac{1}{3}$ of its surface?

Ans. to the height of the earth's diameter.

35. A cubical foot of brass is to be drawn into a wire of $\frac{1}{40}$ of an inch in diameter; what will be the length of the wire, allowing no loss in the metal?

Ans. 97784.797 *yards, or near* 56 *miles.*

36. A bowling green, 300 feet long, and 200 feet broad, is to raised 1 foot higher, by means of the earth to be dug out of a ditch that goes round it; to what depth must the ditch be dug, supposing its breadth to be every where 8 feet? *Ans.* $7\frac{23}{86}$ *feet.*

37. Of what diameter must the bore of a piece of ordnance be, which is cast for a ball of 24lbs. weight, so that the diameter of the bore may be $\frac{1}{10}$ of an inch more than that of the ball? *Ans.* 5.757 *inches.*

38. If a sphere of copper, of one foot in diameter, was to be beat out into a circular plate of $\frac{1}{20}$ of an inch thick, what would be its diameter? *Ans.* 12.64 *feet.*

39. The perambulator, or surveying wheel, is so contrived as to turn just twice round, in the length of a pole, or $16\frac{1}{2}$ feet; what is its diameter? *Ans.* 2.626 *feet.*

40. The ellipse in Grosvenor square measures 840 links across the longest way, and 612 the shortest, within the

rails: now the walls being 14 inches thick, it is required to find what ground they inclose, and what they stand upon? *Ans. They inclose* 4*ac.* 0*ro.* 6*po. and the content* $1760\frac{1}{2}$ *square feet.*

41. If a heavy sphere whose diameter is 4 inches, be put into a conical glass, full of water, whose diameter is 5, and altitude 6 inches; it is required to know how much water will run over? *Ans.* $\frac{35}{47}$ *of a pint nearly.*

42. Suppose it to be found, by measurement, that a man of war, with its ordnance, rigging and appointments, displaces 50,000 cubic feet of water; what is the weight of the vessel? *Ans.* $1395\frac{5}{56}$ *tons.*

43. Supposing it were required to make a vessel of a foot deep, in the form of a frustum of a cone, that shall hold 13 ale gallons, and have its top and bottom diameters in proportion to each other, as 5 is to 3; what must be its dimensions? *Ans. the bottom diameter is* 14.64017, *and the top diameter* 24.40028.

Here AB = 120, EF = 16, EB = 24, *and* AE = 120 — 24 = 96.

$$\textit{Whence } \sqrt{24 \times 96} : 16 :: 120 : \frac{16 \times 120}{\sqrt{24 \times 96}} \frac{16 + 120}{\sqrt{2304}}$$

$$= \frac{16 \times 120}{48} = \frac{120}{3} = 40 \textit{ the conjugate diameter required.}$$

2. The transverse diameter is 35, the ordinate 10, and its abscissa 7: what is the conjugate? *Ans.* 25.

PROBLEM III.

To find the circumference of an ellipse, the transverse and conjugate diameters being given.

RULE.

Multiply the square root of half the sum of the squares of the two diameters by 3.1416, and the product will be the circumference *nearly*.

EXAMPLES.

1. The transverse diameter is 24, and the conjugate 20; required the circumference of the ellipse. (See Fig. Problem II.)

$$\textit{Here } \sqrt{\frac{24^2 + 20^2}{2}} = \sqrt{\frac{576 + 400}{2}} = \sqrt{288 + 200} =$$

$\sqrt{488} = 22.09.$

$$\begin{array}{r} \textit{Whence } 3.1416 \\ 22.09 \\ \hline 282744 \\ 628320 \\ 62832 \\ \hline 69.397944 \end{array} = \textit{circumference.}$$

2. The two axes are 24 and 18: what is the circumference? *Ans.* 66.6433.

3. The two axes are 50 and 40 yards: what is the circumference? *Ans.* 142.24.

4. The two axes are 70 and 50: what is the circumference? *Ans.* 191.096.

PROBLEM IV.

To find the area of an ellipse, the transverse and conjugate diameters being given.

RULE.

Multiply the transverse diameter by the conjugate, and this product again by .7854, and the result will be the area; or multiply .7854 by one axis and the product again by the other.

EXAMPLES.

1. Required the area of an ellipse whose transverse diameter AB is 24, and the conjugate CD 18.

Here $24 \times 18 \times .7854 = 339.2928 =$ *area required.*

Or, .7854
24 = *transverse.*

31416
15708

18.8496
18 = *conjugate.*

1507968
188496

339.2928 = *area as before.*

2. If the axes of an ellipse be 35 and 25, what is the area? *Ans.* 687.225.

3. Required the area of an ellipse whose two axes are 70 and 50. *Ans.* 2748.9.

4. What is the area of an ellipse whose two axes are 7 and 5? *Ans.* 27.489.

PROBLEM V.

To find the area of an elliptic segment, whose base is parallel to either of the axes of the ellipse.

RULE.

Divide the height of the segment by that axis of the ellipse of which it is a part, and find, in the table at the end of the work, a circular segment whose versed sine is equal to the quotient.

Then, multiply the segment thus found and the two axes of the ellipse continually together, and the product will give the area required.

The ellipse is equal to a circle whose diameter is a mean proportional between the two axes.

The area of an elliptic segment may also be determined by the following rule:

Find the corresponding segment of the circle described upon the same axis to which the base of the segment is perpendicular.

Then as this axis is to the other axis, so is the circular segment to the elliptic segment.

EXAMPLES.

1. Required the area of the elliptic segment, whose height is 10, and the axes of the ellipse 35 and 25 respectively.

Here $\frac{10.0000}{35} = \frac{2.0000}{7} = .2857 =$ *tabular versed sine.*

And the tabular segment belonging to this is .185135.

Whence .185135
35
925675
555405
6.479725
25
32398625
12959450
161.993125 *answer.*

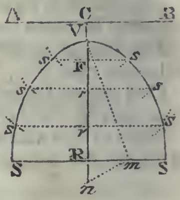
2. What is the area of an elliptic segment, cut off by a double ordinate parallel to the conjugate axis, at the distance of 36 from the centre, the axes being 120 and 40?
Ans. 536.7504.

3. What is the area of an elliptic segment, cut off by an ordinate parallel to the transverse diameter, whose height is 5, the axes being 35 and 25? *Ans.* 97.845725.

4. What is the area of an elliptic segment, its height being 2, vertical axis 20, and parallel axis 25?
Ans. 4.088.

PROBLEM VI.

To describe a parabola, any ordinate to the axis and its abscissa being given.



Construction.—1. Let VR and RS be the given abscissa and ordinate, and bisect the latter in *m*; join V*m*, and draw *mn* perpendicular to it, meeting the axis in *n*.

2. Make VC and VF each equal to R*n*, and F will be the focus of the curve.

3. Take any number of points, *r*, *r*, &c. in the axis, through which draw the double ordinates s*r*s, s*r*s, &c. of an indefinite length.

3. With the radii CF, C*r*, &c. and centre F, describe arcs cutting the corresponding ordinates in the points *s*, *s*, &c. and the curve svs, drawn through all the points of intersection, will be the parabola required.

Note. The line *s*F*s* passing through the focus F is called the parameter.

Besides the method above, given for determining the focus, it may also be found arithmetically thus:

Divide the square of the ordinate by four times the abscissa, and the quotient will be the focal distance VF.

Several other methods of doing this, as well as of describing the curve itself, may be found in Emerson's Conic Sections, and other similar performances.

PROBLEM VII.

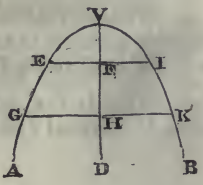
In a parabola, any three of the four following things being given, viz. *any two ordinates and their abscissæ, to find the fourth.*

RULE.

As any abscissa is to the square of its ordinate, so is any other abscissa to the square of its ordinate.

Or, as the square root of any abscissa is to its ordinate, so is the square root of any other abscissa to its ordinate; and conversely.

EXAMPLES.



1. The abscissa VF is 9, and its ordinate EF 6: required the ordinate GH, the abscissa of which VH is 16.

Here VF $= 9$, EF $= 6$, *and* VH $= 16$.

Whence $\sqrt{9} : 6 :: \sqrt{16} : \frac{6\sqrt{16}}{\sqrt{9}} = \frac{6 \times 4}{3} = 2 \times 4 = 8 =$ *ordinate* GH, *as required.*

2. The two abscissæ are 9 and 16, and their corresponding ordinates 6 and 8: from any three of these to find the fourth.

PROBLEM VIII.

To find the length of any arc of a parabola, cut off by a double ordinate.

RULE.

To the square of the ordinate add $\frac{4}{3}$ of the square of the abscissa, and twice the square root of the sum will be the length of the arc nearly.

EXAMPLES.

1. The abscissa VH is 2, and its ordinate GH 6: what is the length of the arc GVK? (See Fig. Prob. VII.)

Here $6^2 + \frac{4}{3} \times 2^2 = 36 + \frac{4}{3} \times 4 = 36 + \frac{16}{3} = 36 + 5.333333 = 41.333333$.

$$\begin{array}{rl} \textit{And } 41.333333(6.429 & \\ 36 & \quad 2 \\ \hline 124)533 & \quad 12.858 = \textit{arc } GVK. \\ 496 & \\ \hline 1282)3733 & \\ 2564 & \\ \hline 12849)116933 & \\ 115641 & \\ \hline 1292 & \end{array}$$

PROBLEM IX.

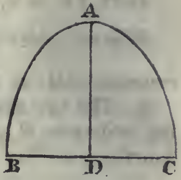
To find the area of a parabola, its base and height being given.

RULE.

Multiply the base by the height, and $\frac{2}{3}$ of the product will be the area required.

It may be observed, that every parabola is $\frac{2}{3}$ of its circumscribing parallelogram, as was first shown by ARCHIMEDES.

EXAMPLES.



1. What is the area of the parabola BAC, whose height AD is 12, and the base or double ordinate BC 16?

Here 16 = BC.
12 = AD.

192
2

3)384

128 = *area required.*

2. The abscissa is 12, and the double ordinate or base 38: what is the area? *Ans.* 304.

3. What is the area of a parabola, whose abscissa is 10, and its ordinate 8? *Ans.* $106\frac{2}{3}$.

4. What is the area of a parabola, its base being 6, and its height 9? *Ans.* 36.

PROBLEM X.

To find the area of the frustum of a parabola.

RULE.

Divide the difference of the cubes of the two ends of the frustum by the difference of their squares, and this quotient multiplied by $\frac{2}{3}$ of the altitude will give the area required.*

---

* *Demon.* Let D = EG, d = BD, and a = CF.

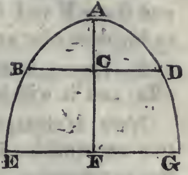
Then by the nature of the curve $D^2 - d^2 : a :: D^2 : \frac{a\,D^2}{D^2 - d^2} =$ AF, and $D^2 - d^2 : a :: d^2 : \frac{a\,d^2}{D^2 - d^2} =$ AC.

Whence $\frac{2}{3} \times \frac{a\,D^2}{D^2 - d^2} - \frac{2}{3} \times \frac{a\,d^2}{D^2 - d^2} = \frac{2}{3}\,a \times \frac{D^3 - d^3}{D^2 - d^2} =$ area of the frustum.

Note. Any parabolic frustum is equal to a parabola, of the same altitude, whose base is equal to one end of the frustum, increased by a third proportional to the sum of their two ends, and the other end.

The demonstration of which readily follows from the above expressions.

EXAMPLES.



1. In the parabolic frustum BEGD, the two parallel ends BD and EG are 6 and 10, and the altitude, or part of the abscissa CF, is 5: what is the area?

Here $10^3 - 6^3 = 1000 - 216 = 784$, *and* $10^2 - 6^2 = 100 - 36 = 64$.

Whence $\frac{10^3 - 6^3}{10^2 - 6^2} = \frac{784}{64} = \frac{98}{8} = \frac{49}{4} = 12.25$.

And 12.25
5
61.25
2
3)122.50
40.8333 *answer.*

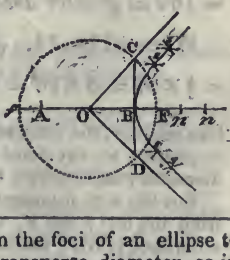
2. The greater end of the frustum is 24, the less end 20, and their distance $5\frac{1}{2}$: what is the area? *Ans.* 121.3.

3. Required the area of the parabolic frustum, the greater end of which is 10, the less 6, and the height 4. *Ans.* $32\frac{2}{3}$.

PROBLEM XI.

To construct an hyperbola, the transverse and conjugate diameters being given.

**Construction.*—1. Make AB the transverse diameter, and CD perpendicular to it, the conjugate.



2. Bisect AB in O, and from O with the radius OC, or OD, describe the circle DfCF, cutting AB produced in F and f, which points will be the two foci.

* As the sum of two lines drawn from the foci of an ellipse to any point in the curve, is equal to its transverse diameter, so in

3. In AB produced take any number of points, n, n, &c. and from F and f, as centres, with the distances Bn, An, as radii, describe arcs cutting each other in s, s, &c.

4. Through the several points s, s, &c. draw the curve sBs, and it will be the hyperbola required.

Note. If straight lines be drawn from the point O, through the extremities C, D, of the conjugate axis, they will be the asymptotes of the hyberbola, whose property it is to approach continually to the curve, without ever meeting it.

PROBLEM XII.

In an hyperbola, any three of the four following things being given, viz. *the transverse, diameter, an ordinate, and its abscissa, to find the fourth.*

CASE I.

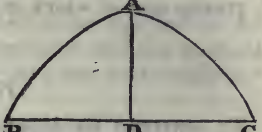
The transverse and conjugate diameters, and the two abscissa being given, to find the ordinate.

RULE.

As the transverse diameter is to the conjugate, so is the square root of the product of the two abscissæ to the ordinate required.

EXAMPLES.

1. In the hyperbola BAC, the transverse diameter is 120, the conjugate 72, and the less abscissa AD 40: required the ordinate DC.



Here trans. = 120, *conj.* 72, AD = 40, *and* 40 + 120 = 160 = *greater abscissa.*

Whence $120:72::\sqrt{160\times40}:\frac{72}{120}\sqrt{160\times40}=\frac{3}{5}\sqrt{6400}$ $=\frac{3}{5}\times80=3\times16=48=$ *ordinate required.*

---

like manner, the difference of two lines drawn from the fcci of an hyperbola to any point in the curve, is equal to its transverse diameter; as is shown by the writers on Conics.

But the arcs intersecting each other in s, s, &c. are described from the foci f and F, and with the distances An and Bn, whose difference is AB; therefore the points s, s, &c. are in the curve of an hyperbola.

2. The transverse diameter is 24, the conjugate 21, and the less abscissa 8 : what is its ordinate? *Ans.* 14.

3. The transverse diameter of an hyperbola is 50, the conjugate 30, and the greater abscissa 12 : required the ordinate. *Ans.*

CASE II.

The transverse and conjugate diameters, and an ordinate being given, to find the two abscissæ.

RULE.

As the conjugate diameter is to the transverse, so is the square root of the sum of the squares of the ordinate and semi-conjugate, to the distance between the ordinate and the centre, or half the sum of the abscissa.

Then the sum of this distance and the semi-transverse will give the greater abscissa, and their difference will give the less.

EXAMPLES.

1. The transverse diameter is 120, the conjugate 72 and the ordinate BD 48 : what are the two abscissæ.

1296 = *square of the semi conjugate.*
2304 = *square of the ordinate.*

3600(60 = *square root.*
36

00

As 72 : 120 : : 60
120

72)7200(100 = ½ *sum of the abscissæ.*
72   60 = *semi-transverse.*

160 = *greater abscissa.*

40 = *lesser abscissa.*

2. The transverse and conjugate diameters are 24 and 21 : required the two abscissæ to the ordinate 14.

Ans. 32 *and* 8.

3. The transverse being 60, and the conjugate 36, required the two abscissæ to the ordinate 24.

Ans. 80 *and* 20.

CASE III.

The transverse diameter, the two abscissæ, and the ordinate being given, to find the conjugate.

RULE.

As the square root of the product of the two abscissæ is to the ordinate, so is the transverse diameter to the conjugate.

EXAMPLES.

1. The transverse diameter is 120, the ordinate is 48, and the two abscissæ are 160 and 40: required the conjugate.

Here, $\sqrt{(160 \times 40)} = \sqrt{6400} = 80$.

Whence 80 : 48 : : 120 *the transverse axis.*

$$\begin{array}{r} 48 \\ \hline 960 \\ 480 \\ \hline 8.0)576.0 \\ \hline 72 \end{array}$$

72 = *conjugate required.*

2. The transverse diameter is 24, the ordinate 14, and the abscissæ 8 and 32: required the conjugate.

Ans. 21.

CASE. IV.

The conjugate diameter, the ordinate, and two abscissæ being given, to find the transverse.

RULE.

1. Add the square of the ordinate to the square of the semi-conjugate, and find the square root of their sum.

2. Take the sum or difference of the semi-conjugate and this root, according as the less or greater abscissæ is used, and then say,

As the square of the ordinate is to the product of the abscissa and conjugate, so is the sum, or difference, above found, to the transverse required.

EXAMPLES.

1. The conjugate diameter is 72, the ordinate is 48, and the less abscissa AD 40 : what is the transverse ?

Here $(48^2 + 36^2)^{\frac{1}{2}} = (2304 + 1296)^{\frac{1}{2}} = \sqrt{3600} = 60$, *and* $60 + 36 = 96$.

Also $72 \times 40 = 2880 =$ *product of the abscissa and conjugate. Whence,*

As 2304 : 2880 : : 96

```
              96
           -----
           17280
          25920
          ------
2304)276480(120 = transverse req.
     2304
     ----
      4608
      4608
      ----
```

2. The conjugate diameter is 21, the less abscissa 8, and its ordinate 14 ; required the transverse. *Ans.* 24.

3. Required the transverse diameter of the hyberbola, whose conjugate is 36, the less abscissa being 20, and the ordinate 24. *Ans.* 60.

PROBLEM XIII.

To find the length of any arc of an hyperbola, beginning at the vertex.

RULE.

1. To 19 times the square of the transverse, add 21 times the square of the conjugate; also to 9 times the square of the transverse add, as before, 21 times the square of the conjugate ; and multiply each of these sums by the abscissa.

2. To each of the two products, thus found, add 15 times the product of the transverse and the square of the conjugate.

3. Then as the less of these results is to the greater, so is the ordinate to the length of the curve, nearly.

EXAMPLES.

1. In the hyperbola BAC, the transverse diameter is 80, the conjugate 60, the ordinate BD 10, and the abscissa AD 2: required the length of the arc AC. (See Fig. Prob. XII.)

Here $2(19\times80^2+21\times60^2)=2(121600+75600)=2\times197200=314400$.

And $2(9\times80^2+21\times60^2)=2(57600+75600)=2\times133200=266400$.

Whence $15\times80\times60^2+314400=4320000+314400=4634400$.

And $15\times80\times60^2+266400=4320000+266400=4586400$.

Then $4586400:4634400::10:\frac{46344000}{4586400}=\frac{463440}{45864}=$

$=10.1046=$ *length of the arc* AC nearly.

The double of which will give the whole arc BAC.

2. The transverse diameter of an hyperbola is 120, the conjugate 72, the ordinate 48, and the abscissa 40: required the length of the curve. *Ans.* 62.6496.

3. Required the whole length of the curve of an hyperbola, to the ordinate 20, the transverse and conjugate axes being 70 and 80.

PROBLEM XIV.

To find the area of an hyberbola, the transverse, conjugate, and abscissa being given.

RULE.

1. To the product of the transverse and abscissa, add $\frac{5}{7}$ of the square of the abscissa, and multiply the square root of the sum by 21.

2. Add 4 times the square root of the product of the transverse and abscissa, to the product last found, and divide the sum by 75.

3. Then if 4 times the product of the conjugate and abscissa be divided by the transverse, this last quotient multiplied by the former will give the area required, *nearly.**

EXAMPLES.

1. In the hyperbola BAC, (See Fig. Prob. XII.) the transverse axis is 30, the conjugate 18, and the abscissa, or height is 10: what is the area?

Here $21\sqrt{(30\times10+\frac{5}{7}\times10^2)}=21\sqrt{(300+71.42857)}$ $=21\sqrt{(371.42857)}=21\times19.272=404.712.$

And $\frac{4\sqrt{(30\times10)}+404{\cdot}712}{75}=\frac{4\times17.3205+404.712}{75}$

$=\frac{69.282+404.712}{75}=\frac{473.994}{75}=\frac{157.998}{25}=6.3199.$

Whence $\frac{18\times10\times4}{30}\times6.3199=\frac{18\times4}{3}\times6.3199=6\times4$ $\times6.3199=24\times6.3199=151.6876=$ *area required.*

2. The transverse diameter is 100, the conjugate 60, and the less abscissa 50; what is the area of the hyperbola?

Ans. 3220.363472.

3. Required the area of the hyperbola to the abscissa 25, the two axes being 50 and 30. *Ans.* 805.0909.

4. Required the area of the hyperbola to the abscissa 30, the two axes being 60 and 40.

5. Required the area of the hyperbola to the abscissa 10, the two axes being 35 and 45.

---

* Or, from the product of the ordinate and its distance from the centre, subtract the product arising from the multiplication of the product of the semi-axis, by the hyperbolic logarithm of the quotient resulting from the division of the sum of the products of the semi-transverse multiplied by the ordinate, and the semi-conjugate multiplied by the said distance of the ordinate from the centre, by the product of the semi-axis; and the remainder will be the area cut off by the double ordinate.

MENSURATION OF SOLIDS.

DEFINITIONS.

1. All bodies are necessarily extended, and therefore are found existing under figure or shape, which is the boundary of extension. A *solid*, or body, is that which unites the three dimensions of extension, length, breadth, and thickness; of which the direction of each is perpendicular to those of the other two.

2. A *prism* is a solid, or body, whose ends are any plane figures, which are parallel, equal, and similar; and its sides are parallelograms.

It is called a triangular prism, when its ends are triangles; a square prism, when its ends are squares; a pentagonal prism, when its ends are pentagons; and so on.

3. A *cube* is a square prism, having six sides, which are all squares. It is like a die, having its sides perpendicular to one another.

4. A *parallelopipedon* is a solid, having its base and all its faces parallelograms. It is called a *rectangular* parallelopipedon, when all its faces are rectangles.

5. When the base, or end, is a circle, the prism is called a *cylinder*.

6. A *pyramid* is a solid formed by several triangular planes proceeding from the same point, called the *vertex*, and terminating in the different sides of the same polygonal plane, which is called its *base*. And the whole of the triangles form its *convex* or *lateral surface*.

A pyramid is *triangular*, *quadrangular*, &c. according as its base is a triangle, a quadrilateral, &c. Hence, the *cone* is a round pyramid having a circular base.

7. A *sphere* is a solid terminated by a curve surface, all the points of which are equally distant from a point within, called the *centre*.

The sphere may be conceived to be generated by the revolution of a semcircle about its diameter, which remains fixed: for the surface described in this movement, by the curve, will have all its points equally distant from its centre.

8. The *axis* of a solid is a line drawn from the middle of one end to the middle of the opposite end: as between the opposite ends of a prism.

Hence, the axis of a pyramid is the line from the vertex to the middle of the base, or the end on which it is supposed to stand. And the axis of a sphere is the same as a diameter, or a line passing through the centre, and terminated by the surface on both sides.

9. When the axis is perpendicular to the base, it is called a *right* prism or pyramid, otherwise it is an *oblique*.

10. The *height* or *altitude* of a solid, is the perpendicular let fall from its vertex, or top, upon the plane of the base, produced if necessary.

The altitude, in a right prism or pyramid, is equal to the axis; but in an oblique one, the altitude is the perpendicular side of a right-angled triangle, whose hypotenuse is the axis.

11. A pyramid is *regular* when its base is a regular polygon, and when, at the same time, the perpendicular let fall from the vertex to the plane of the base, passes through the centre of this base.

That perpendicular, as has been already observed, is then called the *axis* of the pyramid.

12. The segment of a pyramid, sphere, or any other solid, is a part cut off the top by a plane parallel to the base of that figure.

The section of a pyramid or sphere made by the plane, is a plane similar to the base of the figure; and every section of a sphere is a circle. If the section be made through the centre, it is a great circle of the sphere, having the same diameter with the sphere; if not, it is called a small circle. All the sections of a prism, formed by parallel planes, are equal polygons.

13. A *frustum*, or *trunk* of any solid, is the part remaining at the bottom, after the segment is cut off.

14. A *zone of a sphere*, is that part which is intercepted between two parallel planes; and it is the difference between two segments. When the ends, or planes, are equally distant from the centre on both sides, it is called the middle zone of the sphere.

15. The *sector of a sphere* is composed of a segment of a sphere less than a hemisphere or half-sphere, of a cone having the same base with the segment, and its vertex in the centre of the sphere.

16. A *circular* spine is a solid generated by the rotation of a segment of a circle about its chord, which remains fixed.

17. A *spheroid* or *ellipsoid*, is a solid generated by the revolution of a semi-ellipse about one of its axes, which remains fixed.

The spheroid is called *prolate* when the revolution is made about the transverse axis, and *oblate* when it is made about the conjugate axis.

18. *Paraboloids* and *hyperboloids* are solids formed by the revolution of a *semi-parabola* or hyperbola about its axis, which is considered as quiescent; and the same for any other solid of this kind.

19. A solid having a rectangular base, and two of the opposite faces ending in a *side* or *edge*, is called a *wedge*.

20. A *prismoid* is a solid having for its two ends any dissimilar parallel plane figures of the same number of sides, and all the upright faces of the solid trapezoids. If the ends of the prismoid be bounded by dissimilar curves, it is sometimes called a *cylindroid*.

21. An *ungula*, or *hoof*, is a part cut off a solid by a plane oblique to the base.

22. The *measure* of a solid is its solidity, capacity, or content.

23. By the mensuration of solids, then, are determined the spaces included by contiguous surfaces, and the sum of the measures of these including surfaces is the surface or superficies of the body.

24. A *cube* whose side is one inch, one foot, or one yard, &c. is called the *measuring unit*; and the content or solidity of any solid is estimated by the number of cubes of

this kind which are contained in it, or another of equal magnitude.

The least solid measure is the cubic inch, other cubes being taken from it, according to the proportion in the following table:

Cubic or solid measure.

1728	cubic, or solid inches, make	1	cubic foot.
27	feet	1	yard.
$166\frac{3}{8}$	yards	1	pole.
64000	poles	1	furlong.
512	furlongs	1	mile.

PROBLEM I.

To find the surface of a prism.

It is evident, that, if the area of each side and end be calculated separately, the sum of those areas will be the whole surface of any prism, whether right or oblique; or, indeed, of any other body whatever.

But for a right prism, observe the following *particular rule:*

Multiply the perimeter of the end by the height, and the product will be the sum of the sides or upright surface.

If the ends of the prism be regular plane figures, multiply the perimeter of the end by the sum of the height of the prism and the radius of the circle inscribed in the end, and the product will be the whole surface.

The surfaces of similar prisms, and in fact of any other bodies, are as the square of their lineal dimensions. This follows from their being composed of similar plane figures, alike placed.

EXAMPLES.

1. What is the upright surface of a triangular prism, whose length is 20 feet, and the ends of its base each 18 inches?

Here $18\times3=54$ *inches* $=4\frac{1}{2}$ *feet = the perimeter of the base. Therefore,* $4\frac{1}{2}\times20=90$ *square feet is the upright surface.*

Again, by rule, Prob. VIII. Mens. of Surfaces, we have $2\times\frac{3}{2}\times\frac{3}{2}\times.433013=1.9485585$ *= the area of the two ends.*

Therefore, 91.9485585 *is the area of the whole surface,*

2. What is the surface of a cube, the length of each of whose sides is 20 feet? *Ans.* 2400 *square feet.*

3. What must be paid for lining a rectangular cistern with lead at 3 cents a pound, the lead being 7 pound to the square foot, supposing the length, within side, to be 3 feet 2 inches, the breadth 2 feet 8 inches, and height 2 feet 6 inches? *Ans.* $7.90 *nearly.*

4. What is the convex surface of a round prism, or cylinder, whose length is 20 feet, and the diameter of its base 2 feet? *Ans.* 125.664 *square feet.*

5. What is the whole surface of a cylinder, whose length is 10 feet, and the circumference 3 feet?

Ans. 31.43239 *square feet.*

6. What is the convex surface of a right cylinder, the diameter of whose base is 30 inches, and length 60 inches? *Ans.* 5654.862 *inches.*

7. Required the convex surface of a right cylinder, whose circumference is 8 feet 4 inches, and its length 14 feet. *Ans.* 116.6666 *feet.*

PROBLEM II.

To find the solidity of a prism.

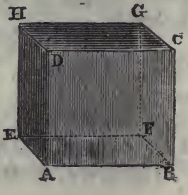
RULE.

Multiply the area of the base into the perpendicular height of the prism, and the product will be the solidity. The rule is also true for oblique prisms.

It is proper to observe that cubes, parallelopipedons, prisms, and cylinders of equal bases and altitudes, are equal to each other: this is shown in Legendre's Geometry, and other similar works on Euclid's Elements.

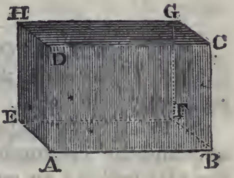
EXAMPLES.

1. The side AB, or BC, of the cube ABCGHE, is 25.5 inches: what is the solidity?



Here, 25.5
25.5
———
1275
1275
510
———
6502.5
25.5
———
325125
325125
130050
———
16581.375 *answer.*

2. Required the solidity of the parallelopiped ABCGHE, whose length AB is 8 feet, its breadth AE $4\frac{1}{2}$ feet, and its depth or altitude AD $6\frac{3}{4}$ feet.



Here, $6\frac{3}{4}$
8
———
54
$4\frac{1}{2}$
———
216
27
———
243 *solid feet, the answer.*

3. What is the solidity of the triangular prism ABCDEF, whose length AB is 10 feet, and either of the equal sides BC, CF, or FB, of one of its equilateral ends BCF, $2\frac{1}{2}$ feet?

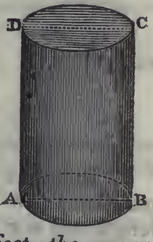
Here 2.5
2.5
2.5
———
2)7.5
———
3.75 = $\frac{1}{2}$ *sum of the 3 sides.*

And	3.75	3.75	3.75
	2.5	2.5	2.5
	1.25	1.25	1.25 *the* 3 *differences.*

Whence $\sqrt{(3.75 \times 1.25 \times 1.25 \times 1.25)} = \sqrt{7.32421875}$ $= 2.7063 =$ *area of the base* BCF.

And consequently $2.7063 \times 10 = 27.063 =$ *the solidity required.*

4. What is the solidity of the cylinder ABCD, the diameter of whose base AB is 30 inches, and the height BC 50 inches?



Here .7854
900 = *square of* 30.

706.8600
50

35343.0000 *cubic inches.*

And $35343 \div 1728 = 20.4531$ *solid feet, the answer required.*

5. What is the capacity of the oblique cylinder, whose axis and the circumference of its base are each 20 feet; the axis making an angle of 75 degrees with the base?

First $\frac{20}{3.1416} =$ *the diameter, and* $10 \times \frac{10}{3.1416} = \frac{100}{3.1416}$ $= \frac{25}{.7854} = \frac{125000}{3927} =$ *the area of the end.*

But, as radius : sine 75° : : 20 : 19.318516, *the height of the cylinder.*

Therefore $\frac{125000}{3927} \times 19.318516 = 614.92602$, *the capacity required.*

6. The side of a cube is 15 inches: what is the solidity?
Ans. 1 *ft.* 11 *in.* 5 *pa.*

7. What is the solidity of a cube whose side is 5 feet 3 inches?
Ans $144\frac{7}{10}$ *feet.*

8. The length of a parallelopiped is 15 feet, and each side of its square base 21 inches: what is the solidity?
Ans. 45.9 *feet.*

9. What is the solidity of a block of marble, whose length is 10 feet, breadth $5\frac{3}{4}$ feet, and depth $3\frac{1}{2}$ feet?
Ans. 201.25 *feet.*

10. What is the solidity of a triangular prism, whose length is 18 feet, and one side of the equilateral end $1\frac{1}{2}$ feet? *Ans.* 17.5370265.

11. Required the solidity of a prism whose base is a hexagon, supposing each of the equal sides to be 1 foot 4 inches, and the length of the prism 15 feet.

Ans. 69.282 *feet.*

12. What is the solidity of a cylinder, whose height is 5 feet, and the diameter of its base 2 feet?

Ans. 15.708 *feet.*

13. What is the solidity of a cylinder, whose height is 20 feet, and the circumference of its base 20 feet also?

Ans. 636.64 *feet.*

14. How many ale gallons of water will a cistern hold, whose length, breadth, and height, are 3 feet 2 inches, 2 feet 8 inches, and 2 feet 6 inches; supposing the ale gallon to contain 282 cubic inches? *Ans.* $129\frac{17}{47}$

15. Suppose the right cylinder, whose length is 20 feet, and diameter 3 feet, be cut by a plane parallel to, and at the distance of 1 foot from, its axis: required the solidities of the two prisms into which the cylinder is cut.

Ans. 15.4874 = *the solidity of the less prism, and* 125.88434 = *the solidity of the greater.*

PROBLEM III.

To find the surface of a right cone, or pyramid.

RULE.

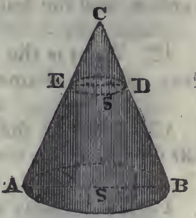
Multiply the circumference, or perimeter of the base, by the slant height, or length of the side of the cone or pyramid, and half the product will be the surface required. And if this be added to the base, it will give the whole surface.

The truth of the rule may be shown, by conceiving any upright hollow cone to be cut through one of its sides, and then spread out; in which case it will become the sector of a circle, having its bounding arc equal to the circumference of the base of the cone, and the radius equal to the slant height: the rule for finding the area of which, is the same as that given above.

Or, as the radius of the base is to the side, so is the base to the curve surface of the cone. And the same is true of any other pyramid whose base is a regular figure; that is, as the radius of the circle inscribed in the base is to the slant height, so is the base to the surface.

This is evident, since, in a right cone, the circumference of the base into half the side, is equal to the curve surface, and into half the radius, is equal to the base.

EXAMPLES.



1. The diameter of the base AB of a right cone CAB, is 3 feet, and the slant height AC or BC 15 feet: required the convex surface of the cone.

Here 3.1416
3
—————
9.4248 = *circumf. of the base.*
15
—————
471240
94248
—————
2)141.3720
—————
70.686 *square feet, the convex surface required.*

2. The diameter of the base of a right cone is 4.5 feet, and the slant height 20 feet: required the convex surface.
Ans. 141.372 *square feet.*

3. The circumference of the base of an upright cone is 10.75 feet, and the slant height 18.25 feet: what is the convex surface? *Ans.* 98.09375 *square feet.*

4. Required the outward surface of a triangular pyramid, each side of its base being $5\frac{1}{2}$ feet, and its slant height $17\frac{1}{4}$ feet. *Ans.* 284.625 *square feet.*

5. What is the surface of a triangular pyramid, the slant height being 20, and each side of the base 3?
Ans. 90.

6. What is the surface of a square pyramid, whose slant height is 20, and each side of the base 3? *Ans.* 120.

7. Required the whole surface of a cone, whose slant side is 20, and the circumference of the base 9.
Ans. 96.446, *nearly.*

PROBLEM IV.

To find the surface of the frustum of a right cone, or pyramid.

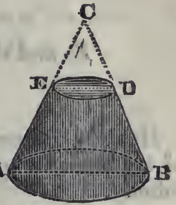
RULE.

Multiply the sum of the perimeters of the two ends, by the slant height of the frustum, and half the product will be the surface required.

For the surface is composed of a number of equal trapezoids, whose common height is equal to the slant height of the frustum, and the sums of whose parallel sides make up the perimeters of the ends of the frustum.

EXAMPLES.

1. In the frustum ABDE, the circumferences of the two ends AB and ED are 22.5 and 15.75 respectively, and the slant height BD is 26: what is the convex surface?



Here 22.5
15.75
———
38.25
26
———
22950
7650
———
2)994.50
———
497.25 *square feet, for the convex surface required.*

2. What is the convex surface of the frustum of a right cone, the circumference of the greater end being 30 feet, that of the less end 10 feet, and the length of the slant side 20 feet? *Ans.* 400 *square feet.*

3. What is the convex surface of the frustum of a right cone, the diameters of the ends being 8 and 4 feet, and the length of the slant side 20 feet?

Ans. 376.992 *square feet*

4. required the surface of the frustum of a square pyramid, one of the sides of the base being $12\frac{1}{2}$ feet, and of the upper end $5\frac{3}{4}$ feet, and its slant height $40\frac{1}{4}$ feet.

Ans. $2938\frac{1}{4}$ *square feet.*

5. If a segment of 6 feet slant height be cut off a cone whose slant height is 30 feet, and circumference of its base 10 feet, what is the surface of the frustum?

Ans. 144 *square feet.*

6. How many square feet are in the surface of the frustum of a square pyramid, whose slant height is 10 feet, each side of the greater base being 3 feet 4 inches, and each side of the less 2 feet 2 inches.

Ans. 110 *square feet.*

PROBLEM V.

To find the solidity of a cone or pyramid.

RULE.

Multiply the area of the base by the perpendicular height of the cone, or pyramid, and $\frac{1}{3}$ of the product will be the solidity.

EXAMPLES.

1. Required the solidity of the cone CAB, (See Fig. Prob. III) whose diameter AB is 20, and its perpendicular height CS 24.

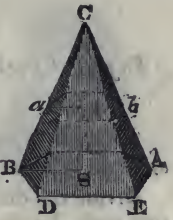
Here .7854
400 = *square of* AB.

314.1600 = *area of the base.*
24

12566400
6283200

3)7539.8400

2513.28 = *solidity required.*



2. Required the solidity of the hexagonal pyramid, CABC, each of the equal sides of its base being 40, and the perpendicular height CS 60.

Here, by the table, before given for polygons,

We have 2.598076 = *mult. when side is* 1.
1600 = *square of* 40.

1558845600
2598076

4156.921600 = *area of the base.*
60

3)24941529.6000

83138.432 = *solidity required.*

3. Required the solidity of a triangular pyramid, whose height is 30, and each side of the base 3.

Ans. 38.97117.

4. Required the solidity of a square pyramid, each side of whose base is 30, and the perpendicular height 20.

Ans. 6000.

5. What is the solidity of a cone, the diameter of whose base is 18 inches, and its altitude 15 feet?

Ans. 8.83575 *feet.*

6. If the circumference of the base of a cone be 40 feet, and its height 50 feet, what is its solidity?

Ans. 2122 *feet.*

7. What is the content of a pentagonal pyramid, its height being 12 feet, and each side of its base 2 feet?

Ans. 27.527.

PROBLEM VI.

To find the solidity of the frustum of a cone or pyramid.

RULE.

1. *For the frustum of a cone, the diameters of the two ends and the height being given.*

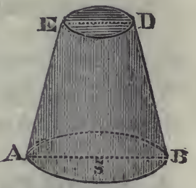
Divide the difference of the cubes of the diameters of the two ends, by the difference of the diameters, and this quotient being multiplied by .7854, and again by $\frac{1}{3}$ of the height, will give the solidity.

2. *For the frustum of a pyramid, the sides and height being given.*

To the areas of the two ends of the frustum add the square root of their product, and this sum being multiplied by $\frac{1}{3}$ of the height, will give the solidity.

EXAMPLES.

1. What is the solidity of the frustum of the cone EABD, the diameter of whose greater end AB is 5 feet, that of the less end ED 3 feet, and the perpendicular height 9 feet?



Here $5^3 = 125$, *and* $3^3 = 27$

Whence 125
27

$5 - 3 = 2)98$

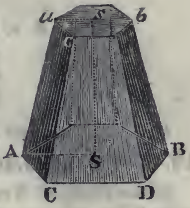
49
.7854

70686
31416

38.4846
3

115.4538 *content of the frustum.*

2. What is the solidity of the frustum *a*AB*b* of an hexagonal pyramid, the side AC of whose greater end is 4 feet, that of *ac*, the less end, 3 feet, and the height, S*s*, 9 feet?



Here 2.598076 = *tab. multiplier.*
9 = *square of* 3.

23.382684 = *area of ab.*
And 2.598076 = *tab. multiplier.*
16 = *square of* 4.

15588456
2598076

41.569216 = *area of* AB.

Whence $\sqrt{}$ (41.569216 × 23.382684) = $\sqrt{}$ 971.999841 = 31.176911.

And 41.569216
23.382684
31.176911

96.128811
3

288.386433 = *solidity required.*

3. If a cask, which is two equal conic frustums joined together at their bases, have its bung diameter 28 inches, its head diameter 20 inches, and length 40 inches; how many gallons of wine will it hold?

Here $20^2 \times .7854 = 314.16$ is the area at the end, and $28^2 \times .7854 = 615.7536$ = area of the bung circle, and $20 \times 28 \times .7854 = 439.824$, their proportion; their sum = 1369.7376, and its third part is 456.5792, which, multiplied by 40, the length of both frustums together, produces 18263.168 solid inches, which, divided by 231, the inches in a wine gallon; thus,

$$231 = 3 \times 7 \times 11 \left\{ \begin{array}{r} 3)18263.163 \\ \hline 7)6087.7226 \\ \hline 11)869.6746 \\ \hline \end{array} \right.$$

gives 79.0613 wine gallons.

Or, $(20^2 + 28^2 + 20 \times 28) \times 40 \times .2618 = 1744 \times 10.472 = 18263.168$, the solidity, as before.

In the State of New-York the gallon for measuring liquid contains 8 lbs. of pure water at its maximum of density, or 221.18 cubic inches. The barrel of ale or beer contains 32 gallons.

The gallon for dry measure contains 276.48 cubic inches, and the bushel therefore measures 2211.84 cubic inches.

4. What is the solidity of the frustum of a cone, the diameter of the greater end being 4 feet, that of the less end 2 feet, and the altitude 9 feet? *Ans.* 65.9736.

5. What is the solidity of the frustum of a cone, the circumference of the greater end being 40, that of the less end 20, and the length or height 50? *Ans.* 3713,64.

6. What is the solidity of the frustum of a square pyramid, one side of the greater end being 18 inches, that of the less end 15 inches, and the height 60 inches?

Ans. 16380 *inches*.

7. What is the solidity of the frustum of an hexagonal pyramid, the side of whose greater end is 3 feet, that of the less end 2 ft. and the length 12 ft.? *Ans.* 197.453776.

PROBLEM X.

To find the solidity of a Cuneus, or Wedge.

Add twice the length of the base to the length of the edge; then multiply this sum by the height of the wedge, and again by the breadth of the base, and $\frac{1}{6}$ of the last product will be the solidity.

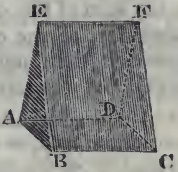
When the length of the base is equal to that of the edge, the wedge is evidently equal to half a prism of the same base and altitude.

And according as the edge is shorter or longer than the base, the wedge is greater or less than half a prism by a pyramid of the same height and breadth at the base as the wedge, and the length of whose base is equal to the difference of the lengths of the edge and base of the wedge.

It may be also observed, that the rule will apply, whether the two ends or the two sides of the wedge be equally or unequally inclined to the base.

EXAMPLES.

1. How many solid feet are there in a wedge whose base is 5 feet 4 inches long, and 9 inches broad, the length of the edge being 3 feet 6 inches, and the perpendicular height 2 feet 4 inches ?



Here 5 feet 4 inches = 64 inches, 3 feet 6 inches = 24 inches, and 2 feet 4 inches = 28 inches.

Whence $(2 \times 64 \times 42) \times 28 \times 9 = 170 \times 28 + 9 =$ $42840 =$ to 6 times the solidity; hence, $\frac{42840}{6} = 7140$ solid inches: or, $\frac{7140}{1728} = 4.1319$ solid feet, the answer required.

2. The length and breadth of the base of a wedge are 35 and 15 inches, and the length of the edge is 55 inches; what is the solidity, supposing the perpendicular height to be 17.14511 inches ? *Ans.* 3.1006 *solid feet.*

3. How many solid feet are in a wedge whose base is 3 feet 4 inches long, and 10 inches broad; the length of the edge being 2 feet 6 inches, and the height 13.737387 inches ? *Ans.* 1.45747 *solid feet.*

4. If the length and breadth of the base of a wedge be 35 and 15 inches, and the length of the edge 55; what is the solidity, supposing its perpendicular height to be 17.14508 inches ? *Ans.* 3.1006 *solid feet.*

PROBLEM XI.

To find the solidity of a Prismoid.

To the sum of the areas of the two ends add four times the area of a section parallel to and equally distant from both ends, and this last sum multiplied by $\frac{1}{6}$ of the height, will give the solidity.

Note.—The length of the middle rectangle is equal to half the sum of the lengths of the rectangles of the two ends, and its breadth equal to half the sum of the breadths of those rectangles.

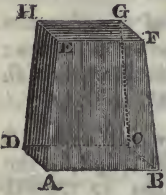
Particular Rule.—Or, if the bases be dissimilar rectangles, take two corresponding dimensions, and multiply each by the sum of the other dimension of the same end, and the dimension of the other end corresponding to this last dimension; then multiply the sum of the products by the height, and one-sixth of the last product will be the solidity.

Corresponding dimensions are those which are connected by a side of the solid, as is evident by the figure.

Demon.—The rectangular prismoid is evidently composed of two wedges, whose heights are equal to the height of the prismoid, and their bases its two ends.

The solidities of the two parts, commonly called the ungulas, or hoofs, into which the frustum of a rectangular pyramid or prismoid is divided, may be found by the last two rules, as they are only composed of wedges and prismoids.

EXAMPLES.



1. What is the solidity of a rectangular prismoid, the length and breadth of one end being 14 and 12 inches, and the corresponding sides of the other 6 and 4 inches; and the perpendicular $30\frac{1}{2}$ feet?

Here $14 \times 12 + 6 \times 4 = 168 + 24 = 192 =$ *sum of the areas of the two ends.*

Also $\frac{14+6}{2} = \frac{20}{2} = 10 =$ *length of the middle rectangle.*

And $\frac{12+4}{2} = \frac{16}{2} = 8 =$ *breadth of the middle rectangle.*

Whence $10 \times 8 \times 4 = 80 \times 4 = 320 = 4$ *times the area of the middle rectangle; also* $30\frac{1}{2}$ *feet* $= 366$ *inches;*

And consequently $(320 + 192) \times \frac{366}{6} = 512 \times 61 =$ 31232 *solid inches.*

Or $31232 \div 1728 = 18.074$ *solid feet the contents.*

2. If the frustum of a square pyramid be cut by a plane, passing through one side of the less end, and through the middle of the greater end, what are the contents of the two

parts, supposing each side of the greater end to be 15 inches, each side of the less 6, the slant height 24 feet?

The perpendicular height may be readily found, by Prob. IV. Mensuration of Surfaces; Since, $7\frac{1}{2} - 3 = 4\frac{1}{2}$ the base of a right angled triangle, and 24 feet is the hypotenuse, the perpendicular $= \sqrt{\{(24 \times 12)^2 - (4\frac{1}{2})^2\}} =$ 287.9649 inches.

First $\frac{2 \times 15 + 6}{6} \times 7\frac{1}{2} \times 287.9949 = 45 \times 287.9649$ $= 12958.4205$ inches $= 7.499086$ feet, is the content of the wedge.

And, $\frac{1}{6}\{(30 + 6) \times 7\frac{1}{2} + (12 + 15) \times 6\} \times 287.9649$ $= \frac{1}{6}(18 \times 15 + 27 \times 6) \times 287.9649 = 72 \times 287.9649 =$ 20733.4728 inches $= 11.998536$ feet, is the prismoid.

3. What is the solid content of a prismoid, whose greater end measures 12 inches by 8, the less end 8 inches by 6, and the length, or height, 60 inches? *Ans.* 2.453 *feet.*

4. What is the capacity of a coal-waggon, whose inside dimensions are as follow: at the top, the length is $81\frac{1}{2}$, and breadth 55 inches; at the bottom, the length is 41, and the breadth $29\frac{1}{2}$ inches; and the perpendicular depth is $47\frac{1}{4}$ inches?

Ans. 126340.59375 *cubic inches; which is nearly equal to a chaldron of coals.*

5. If the frustum of a square pyramid, the dimensions being the same as in Example 2, be divided by a plane into two wedges or hoofs, whose two bases are the ends or bases of the solid; what is the content of the two wedges into which it is cut?

Ans. 14.99817 feet, the greater wedge, and 4.49945 feet, the less wedge.

PROBLEM XII.

To find the convex surface of a Sphere.

RULE.

Multiply the diameter of the sphere by its circumference, or 3.1416 by the square of the diameter, and the product will be the convex superficies required.

The curve surface of any zone or segment will also be found by multiplying its height by the whole circumference of the sphere.

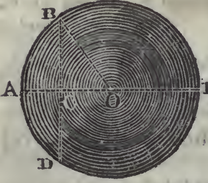
Note.—The lunar surface included between two great circles of the sphere may be found as follows.

RULE.—Multiply the diameter into the breadth of the surface in the middle, and the product will be the superficies required.

Or, as one right angle, or 90°, is to the area of a great circle of the sphere, so is the angle made by the two great circles, to the lunar surface included by them.

Also the area of a spherical triangle, or the surface included by the intersecting arcs of three great circles of the sphere, may be found thus:

RULE.—As two right angles, or 180°, is to the area of a great circle of the sphere, so is the excess of the three angles of the triangle above two right angles, to the area of the triangle.



EXAMPLES.

1. What is the convex superficies of a globe ADEB, whose diameter AE is 17 inches?

Here 3.1416
17 = AE.

219912
31416

53.4072
17 = AE.

3738504
534072

907.9224 *the surface in square inches.*

$$And\ \frac{907.9224}{144} = 6.305 = \text{the surface in square feet.}$$

2. What is the convex superficies of a sphere whose diameter is $1\frac{1}{3}$ feet, and the circumference 4.1888 feet?

Ans. 5.58506 *feet.*

3. If the diameter or axis of the earth be $7957\frac{3}{4}$ miles, what is the whole surface, supposing it to be a perfect sphere? *Ans.* 198943750 *square miles.*

4. The diameter of a sphere is 21 inches; what is the convex superficies of that segment of it whose height is $4\frac{1}{2}$ inches? *Ans.* 296.8812 *inches.*

5. What is the convex surface of a spherical zone, whose breadth is 4 inches, and the diameter of the sphere, from which it was cut, 25 inches? *Ans.* 314.16 *inches.*

PROBLEM XIII.

To find the solidity of a Sphere or Globe.

Multiply the cube of the diameter by .5236, (which is $\frac{1}{6}$ of 3.1416), and the product will be the solidity.

Or, multiply the surface by $\frac{1}{3}$ of the radius, or by $\frac{1}{6}$ of the diameter; and the product will be the solidity.

EXAMPLES.

1. What is the solidity of the sphere whose diameter is 17 inches?

Here $4913 = 17^3$.

$$\begin{array}{r} 4913 \\ .5236 \\ \hline 29478 \\ 14739 \\ 9826 \\ 24565 \\ \hline 2572.4468 \end{array}$$

2572.4468 *solid inches.*

$$Or,\ \frac{2572.4468}{1728} = 1.48868 \text{ solid feet, the answer.}$$

2. What is the solidity of a sphere, whose diameter is $1\frac{1}{3}$ feet? *Ans.* 1.2411 *feet.*

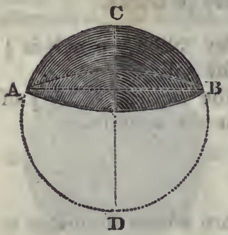
3. What is the solidity of the earth, supposing it to be perfectly spherical, and its diameter to be $7957\frac{3}{4}$ miles? *Ans.* 263858149120 *cubic miles.*

PROBLEM XIV.

To find the solidity of the Segment of a Sphere.

To three times the square of the radius of its base add the square of its height; and this sum multiplied by the height, and the product again by .5236, will give the solidity.

Or—From three times the diameter of the sphere, subtract twice the height of the segment, multiply the difference by the square of the height, and the product by .5236, for the solidity.



EXAMPLES.

1. The radius A*n* of the base of the segment CAB is 7 inches, and the height C*n* 4 inches; what is its solidity?

Here 7 = A*n*.

```
         7
        ——
        49
         3
      ————
       147
        16 = nC².
      ————
       163
         4 = nC.
      ————
       652
     .5236
    ——————
     10472
    26180
   31416
   ———————
   341.3872 = solid inches, the answer.
```

2. What is the solidity of the segment of a sphere, the diameter of whose base is 20, and its height 9?

Ans. 1795.4244.

3. What is the content of the spherical segment whose height is 4 inches, and the radius of its base 8?

Ans. 435.6352 *inches.*

4. What is the solidity of a spherical segment, the diameter of its base being 17.23368, and its height 4.5?

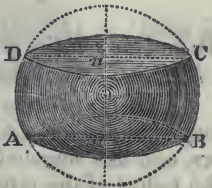
Ans. 572.5566.

PROBLEM XV.

To find the solidity of a Frustum or Zone of a Sphere.

To the sum of the squares of the radii of the two ends, add $\frac{1}{3}$ of the square of their distance, or the breadth of the zone, and this sum multiplied by the said breadth, and the product again by 1.5708 will give the solidity.

EXAMPLES.



1. What is the solid content of the zone ABCD, whose greater diameter AB is 20 inches, the less diameter DC 15 inches, and the distance nm of the two ends 10 inches?

Here $100 = 10^2 = mA^2$

$56\frac{1}{4} = 7.5^2 = n\,D^2$

$33\frac{1}{3} = \frac{1}{3}(10)^2 = \frac{1}{3}\,mn^2$

$189\frac{7}{12} = 189.5833$

Whence 189.5833

$10 = mn.$

1895.8330

1.5708

151666640

1327083100

94791650

18958330

2977.97447640 *solid inches, the answer.*

And if the solidity of the middle zone of a sphere be required—Multiply either the sum of the square of the diameter of the end and $\frac{2}{3}$ of the square of the height, or the difference between the square of the diameter of the sphere and $\frac{1}{3}$ of the square of the height, by the height, and the product by .7854, for the content.

2. What is the solid content of a zone whose greater diameter is 24 inches, the less diameter 20 inches, and the distance of the ends 4 inches? *Ans.* 1566.6112 *inches.*

3. Required the solidity of the middle zone of a sphere whose top and bottom diameters are each 3 feet, and the breadth of the zone 4 feet. *Ans.* 61.7848 *feet.*

4. What is the solidity of the torrid zone of the earth, supposing the diameter of the earth to be7957 $\frac{3}{4}$ miles, and, of course, the height of the zone must be 3173.14565052? *Ans.* 149455081137.

PROBLEM XVI.

To find the solidity of a circular Spindle, its length and middle diameter being given.

1. To the square of half the length of the spindle, or longest diameter, add the square of half the middle diameter, and this sum divided by the middle diameter, will give the radius of the circle.

2. Take half the middle diameter from the radius thus found, and it will give the central distance; or that part of the radius that lies between the centre of the circle and that of the spindle.

3. Find the area of the generating circular segment, by the Note to Rule 1, Prob. xv., or, if great accuracy be required, by Rule II. or III.

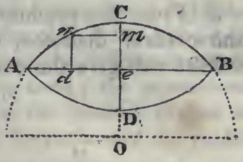
4. From $\frac{1}{3}$ of the cube of half the length of the spindle, subtract the product of the central distance and half the area last mentioned, and the remainder, multiplied by 12.5664, will give the content of the spindle.

The surface of a Circular Spindle, or of any segment or frustum of it, may be found thus:—

From the product of the height of the solid and radius of the revolving arc, subtract the product of said arc and central distance; multiply the remainder by 3.1416; and double the product will be the surface described by that arc.

EXAMPLES.

1. The longest diameter AB of the circular spindle ADBC is 48, and the middle diameter CD 36; what is the solidity of the spindle?



Here $\frac{24^2+18^2}{36}=\frac{576+324}{36}=\frac{900}{36}=25$ *the radius* OC.

And $25-18=7$ *the central distance* O *e.*

Also (by Prob. XV.) $\frac{4\times18}{3}\sqrt{(\frac{48^2}{4}+\frac{2\times18^2}{5})}=24$ $\sqrt{576+129.6}=24\sqrt{705.6}=24\times26.563=637.512$ *the area of the segment* ACBA.

Whence, $(\frac{24^3}{3}-\frac{637.512}{2}\times7)\times12.5664=(4608-2231.292)\times12.5664=2376.708\times12.5664=29866.6634$ *the solidity required.*

2. If the length of a circular spindle be 40, and its middle diameter 30, what is its solidity? *Ans.* 17312.858.

3. Required the surface of a circular spindle whose greatest diameter is 30, and its length 40 inches.

Ans. 3270.5335.

PROBLEM XVII.

To find the solidity of the middle frustum of a Circular Spindle, its length, the middle diameter, and that of either of the ends being given.

1. Divide the square of half the length of the frustum by half the difference of the middle diameter, and that of either of the two ends; and half this quotient added to $\frac{1}{4}$ of the said difference, will give the radius of the circle.

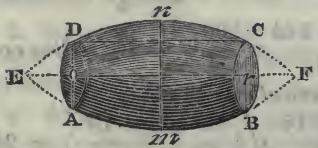
2. Find the central distance and the revolving area, as in the last problem.

3. From the square of the radius take the square of the central distance, and the square root of the remainder will give half the length of the spindle.

4. From the square of half the length of the spindle take $\frac{1}{3}$ of the square of half the length of the frustum, and multiply the remainder by the said half length.

5. From this product take that of the generating area and central distance, and the remainder multiplied by 6.2832 will give the content of the frustum.

EXAMPLES.



1. What is the solidity of the middle frustum ABCD, of a circular spindle, whose middle diameter nm is 36, the diameter DA or CB of the end 16, and its length or 40?

Here $\frac{1}{2}\{20^2 \div \frac{1}{2}(36-16)\} + \frac{1}{4}(36-16) = \frac{1}{2}(400 \div 10) + 5 = 25 =$ *radius of the circle.*

And $25 - \frac{1}{2}nm = 25 - 18 = 7 =$ *central distance; and* $\frac{1}{2}nm - \frac{1}{2}\text{DA} = 18 - 8 = 10 =$ *versed sine.*

Therefore we have $\frac{10}{50} = \frac{1}{5} = .2 =$ *tab. versed sine; and by the common table* $.111823 =$ *tab. segment.*

Whence $.111823 \times 50^2 = .111823 \times 2500 = 279.5575$ $=$ *area of the segment* DnC.

And $279.5575 + 8 \times 40 = 279.5575 + 320 = 599.5575$ $=$ *generating area* oDCr.

Again $\sqrt{(25^2 - 7^2)} = \sqrt{(625 - 49)} = \sqrt{576} = 24$ $= \frac{1}{2}$ *length of the spindle.*

And $\{(24^2 - \frac{1}{3} \times 20^2) \times 20 - 599.5575 \times 7\} \times 6.2832 = (8853.334 - 4196.9025) \times 6.2832 = 4656.4315 \times 6.2832 = 29257.2904$ *solidity required.*

2. The middle diameter of the frustum of a circular spindle is 32, the diameter at the end 24, and the length 40; what is the solidity? *Ans.* 27286.5478 *cubic inches.*

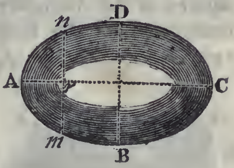
PROBLEM XVIII.

To find the solidity of a Spheriod, its two axes being given.

Multiply the square of the revolving axis by the fixed axis, and this product again by .5236, (or $\frac{1}{6}$ of 3.1416); and it will give the solidity required.

The surface of a spheroid may be found thus :—To 9 times the square of the fixed axis add 3 times the difference of the squares of the fixed and revolving axes; multiply the square root of this sum by $\frac{1}{3}$ of the revolving axis and 3.1416, and it gives the surface of an oblate spheroid. From 9 times the square of the fixed axis take 3 times the difference of the squares of the fixed and revolving axes; the square root of the remainder multiplied by $\frac{1}{3}$ of the product of the revolving axis and 3.1416, will give the surface of the prolate spheroid.

EXAMPLES.



1. In the prolate spheroid ABCD, the transverse or fixed axis AC is 90, and the conjugate or revolving axis DB is 70; what is the solidity?

```
Here 70
     70
   ----
   4900
     90
  ------
 441000
  .5236
 --------
   5236000
  20944
 20944
 ---------
 230907.6000 the solidity required.
```

2. What is the solidity of a prolate spheroid, whose fixed axis is 100, and its revolving axis 60? *Ans.* 188496.

3. What is the solidity of an oblate spheroid, whose fixed axis is 60, and its revolving axis is 100? *Ans.* 314160.

4. the axes of a spheroid being 50 and 30, required the surface of each. *Ans.* 5868.49918 = *the surface of the oblate spheroid*, and 4144.8263 = *the surface of the prolate or oblong spheroid.*

PROBLEM XIX.

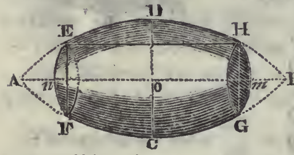
To find the solidity of the middle frustum of a spheroid, its length, the middle diameter, and that of either of the ends being given.

CASE I.

When the ends are circular, or perpendicular to the transverse axis.

To twice the square of the middle diameter add the square of the diameter of either of the ends, and this sum multiplied by the length of the frustum, and the product again by .2618, (or, $\frac{1}{12}$ of 3.1416), will give the solidity.

EXAMPLES.



1. In the middle frustum of a prolate spheroid EFGH, the diameter DC is 50 inches, that of either of the ends EF or GH is 40 inches, and its length *n m* 18 inches; what is its solidity?

Here $(50^2 \times 2 + 40^2) \times 18 = (2500 \times 2 + 1600) \times 18 = (5000 + 1600) \times 18 = 118800.$

$$\begin{array}{r} \textit{And } 118800 \\ .2618 \\ \hline 950400 \\ 118800 \\ 712800 \\ 237600 \\ \hline 31101.8400 \end{array}$$ *the solidity required.*

2. What is the solidity of the middle frustum of a prolate spheroid, the middle diameter being 60, that of either of the two ends 36, and the distance of the ends 80?

Ans. 177940.224.

3. What is the solidity of the middle frustum of an oblate spheroid, the middle diameter being 100, that of either of the ends 80, and the distance of the ends 36?

Ans. 248814.72.

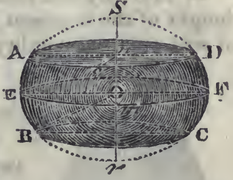
CASE II.

When the ends are elliptical or perpendicular to the conjugate axis.

1. Multiply twice the transverse diameter of the middle section by its conjugate diameter, and to this product add the product of the transverse and conjugate diameters of either of the ends.

2. Multiply the sum thus found, by the distance of the ends, or the height of the frustum, and this product again by .2618, and it will give the solidity required.

EXAMPLES.



1. In the middle frustum ABCD of an oblate spheroid, the diameters of the middle section are 50 and 30; those of the end 40 and 24: and its height 18; what is the solidity?

Here $(50 \times 2 \times 30 + 40 \times 24) \times 18 = (3000 + 960) \times 18 = 3960 \times 18 = 71280.$

And 71280
.2618
———
570240
71280
427680
142560
———
18661.1040 *the solidity required.*

2. In the middle frustum of a prolate spheroid, the diameters of the middle section are 100 and 60; those of the end 80 and 48; and the length 36: what is the solidity? *Ans.* 149288.832.

3. In the middle frustum of an oblate spheroid, the diameters of the middle section are 100 and 60; those of the end 60 and 36; and the length 80: what is the solidity of the frustum? *Ans.* 296567.04.

PROBLEM XX.

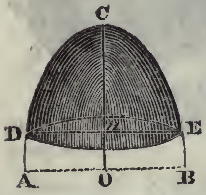
To find the solidity of the Segment of a Spheroid.

CASE I.

When the base is perpendicular to the transverse axis.

1. Divide the square of the conjugate axis by the square of the transverse axis, and multiply the quotient by the difference between three times the transverse axis and twice the height of the segment.

2. Multiply the product thus found by the square of the height of the segment, and this product again by .5236, and it will give the solidity required.



EXAMPLES.

1. In the prolate spheroid ACB, the transverse axis 2CO is 100, the conjugate AB 60, and the height C n, of the segment DCE 10 : what is the solidity ?

Here $\left\{\frac{60^2}{100^2} \times (300 - 20)\right\} \times 10^2 = .36 \times 280 \times 10^2$ $= 100.8 \times 100 = 10080.$

$$\begin{array}{r} \textit{And}\ 10080 \\ .5236 \\ \hline 60480 \\ 30240 \\ 20160 \\ 50400 \\ \hline 5277.8880 \end{array}$$ *the solidity required.*

2. The axes of a prolate spheroid are 50 and 30 : what is the solidity of that segment whose height is 5, and its base perpendicular to the transverse axis ?

Ans. 659.734.

3. The diameters of an oblate spheroid are 100 and 60 :

what is the solidity of that segment whose height is 12, and its base perpendicular to the transverse axis?

Ans. 32672.64.

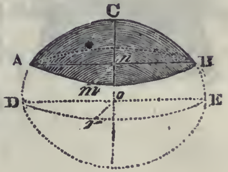
CASE. II.

When the base is perpendicular to the conjugate axis.

1. Divide the fixed axis by the revolving axis, and multiply the quotient by the difference between three times the revolving axis and twice the height of the segment.

2. Multiply the product thus found by the square of the height of the segment, and this product again by .5236, and it will give the solidity required.

EXAMPLES.



1. In the oblate spheroid DCE, the transverse axis DE is 100, the conjugate 2C o, 60, and the height C n of the segment ACB, 12: what is the solidity?

Here $\frac{100}{60}(3 \times 60 - 2 \times 12) \times 12^2 = \frac{5}{3}(180 - 24) \times 144 = \frac{5}{3} \times 156 \times 144 = 5 \times 52 \times 144 = 37440.$

And	37440
	.5236
	224640
	112320
	74880
	187200
	19603.5840 *the solidity required.*

2. Required the content of the segment of a prolate spheroid, its height being 6, and the axes 50 and 30.

Ans. 2450.448.

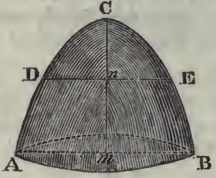
PROBLEM XXI.

To find the solidity of a Paraboloid.

1. Multiply the area of the base by the altitude of the figure, and half the product will be the solidity.

To find the curve surface of a Paraboloid.

To the square of the ordinate, or semi-diameter of the base, add four times that of the axis, and the square root of the sum will be the tangent of the curve at the base, and intercepted by the axis produced. Divide the difference between the cube of the tangent and the cube of the ordinate, by 4 times the square of the axis; multiply the quotient by $\frac{2}{3}$ of the ordinate, and the product again by 3.1416 for the surface.



EXAMPLES.

1. What is the solidity of the paraboloid ACBA, whose height, C *m* is 84, and the diameter AB of its circular base 48?

Here .7854
2304 = 48^2.

31416
235620
15708

1809.5616 = *area of the base.*
42 = $\frac{1}{2}$ *the altitude.*

36191232
72382464

76001.5872 *solidity required.*

2. What is the solidity of a paraboloid, whose height is 60, and the diameter of its circular base 100?

Ans. 235620.

3. Required the solidity of a paraboloid, whose height is 30, and the diameter of its base 40? *Ans.* 18849.6.

4. Required the solidity of a paraboloid, whose height is 50, and the diameter of its base 100? *Ans.* 196350.

5. Required the curve surface of a paraboloid, whose axis is 20 and the diameter of its base 60?

Ans. 3848.451.

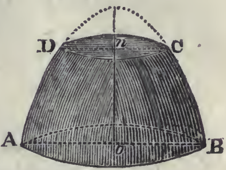
PROBLEM XXII.

To find the solidity of the frustum of a Paraboloid, when its ends are perpendicular to the axis of the solid.

Multiply the sum of the squares of the diameters of the two ends by the height of the frustum, and the product again by .3927 (or $\frac{1}{2}$ of .7854), and it will give the solidity.

EXAMPLES.

1. Required the solidity of the parabolic frustum ABCD, the diameter AB of the greater end being 58, that of the less end DC 30, and the height *n o* 18.



Here $58^2 + 30^2 = 3364 + 900 = 4264$.

```
And 4264
      18
  ------
   34112
   3264
  ------
   76752
   .3927
  ------
  537264
 153504
690768
230256
----------
30140.5104 solidity required.
```

2. What is the solidity of the frustum of a parabolic conoid, the diameter of the greater end being 60, that of the less end 48, and the distance of the ends 18?

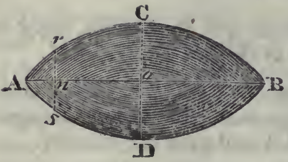
Ans. 41733.0144.

PROBLEM XXIII.

To find the solidity of a Parabolic Spindle.

Multiply the square of the middle diameter by the length of the spindle, and the product again by .418879, (or $\frac{8}{15}$ of .7854), and it will give the solidity.

EXAMPLES.



1. The length of the parabolic spindle ACBD is 60, and the middle diameter DC 34; what is the solidity?

Here $34^2 \times 60 = 1156 \times 60 = 69360$.

And .418879
79360

25132740
1256637
3769911
2513274

29053.447440 *the solidity required.*

2. The length of a parabolic spindle is 9 feet, and the middle diameter 3 feet; what is the solidity?

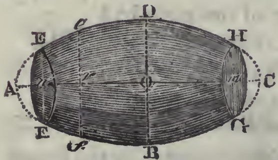
Ans. 33.929199.

PROBLEM XXIV.

To find the solidity of the middle frustum of a Parabolic Spindle.

Add 8 times the square of the middle diameter, 3 times the square of the less, and 4 times the product of these diameters together; then this sum being multiplied by the length, and the product again by .05236, (or $\frac{1}{60}$ of 3.1416), will give the solidity.

EXAMPLES.



1. In the middle frustum EFGH, of the parabolic spindle ABCD, the middle diameter DB is 36, the diameter of the end EF is 20, and the length nm 36; what is the solidity?

Here $36^2 \times 8 + 20^3 \times 3 + 4 \times 36 \times 20 = 10368 + 1200 + 2880 = 14448$.

And	14448
	36
	86688
	43344
	520128
	.05236
	3120768
	1560384
	1040256
	2600640
	27233.90208 *the solidity required.*

2. Required the solidity of the middle frustum of a parabolic spindle, the middle diameter being 32, the diameter at the end 24, and the length 40? *Ans.* 27210.4448.

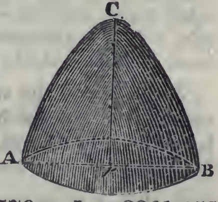
PROBLEM XXV.

To find the solidity of an Hyperboloid.

As the sum of the transverse axis of the generating hyperbola, and the height of the solid, is to the sum of the said transverse, and $\frac{2}{3}$ of the height, so is half the area of the base multiplied by the altitude, to the solidity of the hyperbola.

EXAMPLES.

1. In the hyperboloid ACB, the altitude Cr is 10, the radius Ar of the base 12, and the transverse diameter 30; what is the solidity?



Here $.7854 \times 24^2 \times 5 = .7854 \times 576 \times 5 = 2261,952$ $=$ *area base* $\times \frac{1}{2}$ *the altitude.*

Whence $30 + 10\ (40) : 30 + \frac{2}{3} \times 10 \left(\frac{110}{3}\right) :: 248814.72$ $: 2073.456$ *the solidity required.*

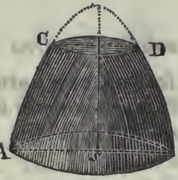
2. In an hyperboloid the altitude is 30, the radius of the base 40, and the transverse diameter 50; what is the solidity? *Ans.* 48473.6.

PROBLEM XXVI.

To find the solidity of the frustum of an Hyperboloid.

Add together the squares of the greater and less semidiameters, and the square of the whole diameter in the middle; then this sum being multiplied by the altitude, and the product again by .5236 will give the solidity.

EXAMPLES.



1. In the hyperbolic frustum ACDB, the length rs is 20, the diameter AB of the greater end 32, that DC of the less end 24, and the middle diameter 28.1708; required the solidity?

Here $16^2 + 12^2 + (28.1708)^2 = 1193.5939.$

And 1193.5939
20
———
23871.8780
.5236
———
1432212680
716156340
477437560
1193593900
———
12499.0766203 *the solidity.*

2. Required the solidity of the frustum of an hyperbolic conoid, the height being 12, the greater diameter 10, the less diameter 6, and the middle diameter $8\frac{1}{2}$? *Ans.* 667.57725.

3. What is the content of the middle frustum of an hyperbolic spindle, the length being 20, the middle or greater diameter 16, the diameter at each end 12, and the diameter at $\frac{1}{4}$ of the length $14\frac{1}{2}$? *Ans.* 3248.87595.

MENSURATION OF REGULAR BODIES.

DEFINITIONS.

1. The name *solid polyedron*, or simply *polyedron*, is given to every solid terminated by planes or plane faces; which planes, it is evident, will themselves be terminated by straight lines.

The plane figures, under which the polyedron is contained, are called the *faces* of the polyedron; and the common intersection of two adjacent faces of a polyedron, is called the *side*, or *edge*.

2. Polyedrons having equal, regular polygons for their faces, and whose solid angles are all equal to each other, are called *regular solids*, or *bodies*.

These conditions cannot be fulfilled except in a small number of cases.

3. If the faces are equilateral triangles, polyedrons may be formed of them, having solid angles contained by three of those triangles, by four, or by five: hence arises three regular bodies; the *tetraedron*, or triangular pyramid, having four triangular faces; the *octaedron*, having eight triangular faces; and the *icosaedron*, having twenty triangular faces.

No other can be formed with equilateral triangles; for six angles of such a triangle are equal to four right angles, and cannot form a solid angle. The tetraedron is the simplest of all polyedrons; because at least three planes are required to form a solid angle, and these three planes leave a void, which cannot be closed without at least one other plane.

4. If the faces are squares, their angles may be arranged by threes: hence results the *hexaedron*, or *cube*. Four angles of a square are equal to four right angles, and cannot form a solid angle.

5. In fine, if the faces are regular pentagons, their angles may be likewise arranged in threes. The regular *dodecaedron* will result, having twelve pentagonal faces.

We can proceed no farther: three angles of a regular hexagon are equal to four right angles; three of a regular heptagon are greater. Hence there are only five regular polyedrons; three formed with equilateral triangles, one with squares, and one with pentagons.

The solids above described, on account of their singularity, and the mysterious nature usually ascribed to them, were formerly known by the name of the *five Platonic bodies;* and though now in a great measure neglected, they were so far regarded by the ancients, that Euclid is said to have composed his celebrated work on the Elements of Geometry, for the purpose of being able to display some of their most remarkable properties.

From the properties of these figures we may likewise easily deduce the following consequences:

1. Every regular polyedron consists of as many regular pyramids as the polyedron has faces; the common vertex of each of which pyramids is the centre of the polyedron, as well as that of the inscribed and circumscribed sphere.

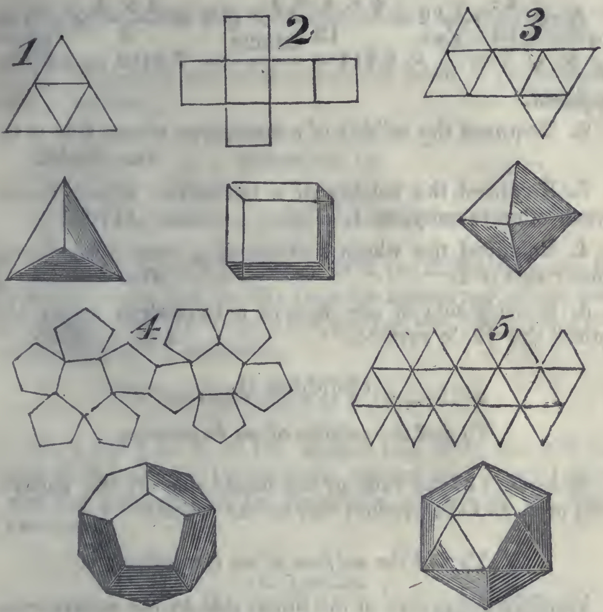
2. The solidity of any polyedron is equal to its surface multiplied by a third of the radius of the inscribed sphere.

3. Any two regular polyedrons of the same name are similar solids, having their like dimensions proportional; and the radii of the inscribed and circumscribed spheres are as the sides of the polyedrons.

4. If a regular polyedron be inscribed in a sphere, the planes, drawn from the centre, through the lengths of the different sides, will divide the surface of the sphere into as many equal and similar spherical polygons as the polyedron has faces.

To construct or form the regular Solids or Bodies.

Having described the following figures on pasteboard, or some other pliable matter, cut them by the extreme sides; and cut the other lines half through; then fold them at the sides so cut, till the sides meet; which being fastened together with glue, or otherwise, will form the regular bodies: namely, figure 1 will form the tetraedron; figure 2, the hexaedron, or cube; figure 3, the octaedron; figure 4, dodecaedron; figure 5, icosaedron.



PROBLEM I.

To find the solidity of a Tetraedron.

Multiply the cube of the linear side, or edge of the figure by the square root of 2, (or 1.4142), and $\frac{1}{12}$ of the product will be the solidity.

To find the surface of a Tetraedron.

Multiply the square of the linear edge by the square root of 3, and the product will be the sum of the four faces.

The rule for finding the surface and solidity of the hexaedron, or cube, has been given before.

EXAMPLES.

1. The linear side of a tetraedron is 4: what is the solidity?

Here, $\frac{4^3}{12} \times \sqrt{2} = \frac{4 \times 4 \times 4}{12} \times \sqrt{2} = \frac{4 \times 4}{3} \times \sqrt{2} = \frac{16}{3} \times \sqrt{2} = \frac{16}{3} \times 1.414 = \frac{22.624}{3} = 7.5413 =$ *solidity required.*

2. Required the solidity of a tetraedron whose side is 6.
Ans. 25.452.

3. Required the solidity of a tetraedron whose linear linear edge is unity, or 1. *Ans.* .11785113.

4. Required the whole surface of a tetraedron whose linear side is 2. *Ans.* 7.9282.

5. If each side of one face of a tetraedron be 1; required the whole surface. *Ans.* 1.73205.

PROBLEM II.

To find the solidity of an Octaedron.

Multiply $\frac{1}{3}$ of the cube of the linear side by the square root of 2, and the product will be the solidity.

To find the surface of an Octaedron.

Multiply the square of the linear side by the square root of 3, and double the product will be the surface.

EXAMPLES.

1. What is the solidity of the octaedron whose linear side is 4?

$$
\begin{array}{r}
\textit{Here } \sqrt{2} = 1.4142 \\
64 \\
\hline
56568 \\
84852 \\
\hline
3)90.5088 \\
\hline
30.1696
\end{array}
$$
solidity required.

2. Required the solidity of an octaedron whose side is 8?
Ans. 241.3568.

3. If the linear side of an octaedron be 1, what is its solidity? *Ans.* .47140452.

4. The linear side of an octaedron being 1, required the whole surface. *Ans.* 3.4641.

PROBLEM III.

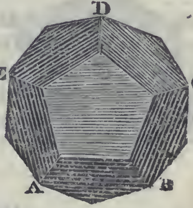
To find the solidity of a Dodecaedron.

To 21 times the square root of 5, (or 2.236068), add 47, and divide the sum by 40; then the square root of the quotient multiplied by 5 times the cube of the linear side, will give the solidity required.

To find the surface of a Dodecaedron.

To 1 add $\frac{2}{5}$ or $\frac{4}{10}$ of the square root of 5; multiply the root of the sum by 15 times the square of the linear side; and the product will be the whole surface of the dodecaedron.

EXAMPLES.



1. The linear side of the dodecaedron ABCDE is 3; what is the solidity?

Here 2.236068 = $\sqrt{5}$.

```
       21
   -------
   2236068
  4472136
  --------
 46.957428
 47
 ---------
40)93.957428
 ---------
 2.3489357
```

And, $5 \times 3^3 \times \sqrt{2.3489357} = 5 \times 27 \times \sqrt{2.3489357}$ $= 206.901$, *the solidity.*

2. The linear side of a dodecaedron is 1; what is the solidity? *Ans.* 7.6631.

3. If the linear side of a dodecaedron be 1, what is the whole surface? *Ans.* 20.6457788.

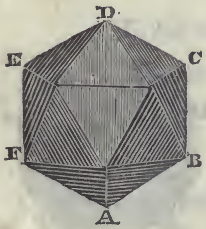
PROBLEM IV.

To find the solidity of an Icosaedron.

To three times the square root of 5, (or 2.236068,) add 7, and divide the sum by 2; then the square root of this quotient being multiplied by $\frac{5}{6}$ of the cube of the linear side, will give the solidity required.

To find the surface of an Icosaedron.

Multiply 5 times the square of the linear side by the square root of 3, and the product will be the whole surface.



EXAMPLES.

1. The linear side of the icosaedron ABCDEF is 3; what is the solidity?

$$\begin{array}{r} \textit{Here } 2.236068 = \sqrt{5}. \\ 3 \\ \hline 6.708204 \\ 7 \\ \hline 2)13.708204 \\ \hline 6.854102 \end{array}$$

And $\frac{5 \times 3^3}{6} \times \sqrt{6.854102} = \frac{45}{2} \times 2.618 = 58.905$ *the solidity.*

2. Required the solidity of an icosaedron whose linear side is 1. *Ans.* 2.181695.

3. The linear side of an icosaedron being 1; required the whole surface. *Ans.* 8.660254.

PROBLEM V.

To find the surface or solidity of any of the five Regular Bodies.

1. Multiply the tabular area, (taken from the following table,) by the square of the linear edge, and the product will be the surface or area of the solid.

2. Multiply the tabular solidity by the cube of the linear edge, and the product will be the solid content of the body.

The numbers in the table denote the surface and solidity of each body, when its edge is 1; and because, in similar bodies, the surfaces are as the squares of the linear edges, and the solidities as the cubes of the same; therefore the truth of the rules is manifest.

SURFACES AND SOLIDITIES OF THE REGULAR BODIES.

No. of Sides.	*Name.*	*Surface.*	*Solidity.*
4	Tetraedron.	1.7320508	0.1178513
6	Hexaedron.	6.0000000	1.0000000
8	Octaedron.	3.4641016	0.4714045
12	Dodecaedron.	20.6457788	7.6631189
20	Icosaedron.	8.6602540	2.1816950

EXAMPLES.

1. The linear side of a tetraedron being 3, required its surface and solidity.

Here, $1.7320508 \times 3^2 = 15.5884572 =$ *its surface.*
And $0.1178513 \times 3^3 = 3.1819851$, *its solidity.*

2. What is the surface and solidity of a Hexaedron whose linear edge is 2?

Ans. 24 = *its surface*, and 8 = *its solidity*,

3. What is the surface and solidity of an octaedron whose linear edge is 2?

Ans. 13.8564064 = *its surface*, and 3.7712360 = *its solidity.*

4. What is the surface and solidity of a dodecaedron whose linear edge is 2?

Ans. 82.5831152 = *its surface,* and 61.3049512 = *its solidity.*

5. What is the surface and solidity of an icosaedron whose linear edge is 2?

Ans. 34.641016 = *its surface, and* 17.45356 = *its solidity.*

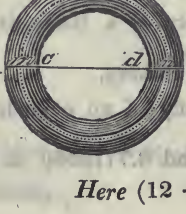
CYLINDRIC OR SOLID RINGS.

Definition.—By a ring, in general, is meant a solid returning into itself; of which every section perpendicular to the axis, or line passing through the middle of the solid, is everywhere the same figure, and of the same magnitude.

PROBLEM I.

To find the convex superficies of a Cylindric Ring.

To the thickness of the ring add the inner diameter, and this sum being multiplied by the thickness, and the product again by 9.8698, (or the square of 3.14159,) will give the superficies required.

A  B

EXAMPLES.

1. The thickness AC of a cylindric ring is 3 inches, and the inner diameter *cd* 12 inches; what is the convex superficies?

Here $(12 + 3) \times 3 = 15 \times 3 = 45$

And 9.8698
45

493490
394792

444.1410 *square inches, the answer.*

2. The thickness of a cylindric ring is 4 inches, and the inner diameter 18; what is the convex superficies?
Ans. 868.52 *square inches.*

3. The thickness of a cylindric ring is 2 inches, and the inner diameter 18; what is the convex superficies?
Ans. 394.734 *square inches.*

PROBLEM II.

To find the solidity of a Cylindric Ring.

To the thickness of the ring add the inner diameter, and this sum being multiplied by the square of half the thickness, and the product again by 9.8698, will give the solidity.

This figure being only a cylinder bent round into a ring, its surface and solidity may also be found as in the cylinder, by multiplying the axis, or length of the cylinder, by the circumference of the ring for the surface, and by the area of any perpendicular section of it for the solidity.

Thus, if $c =$ circumference of the ring, or section, $a =$ area of that section, and $l =$ length of the axis; then will $cl =$ surface of the ring, and $al =$ to its solidity; which rules are the same as for the cylinder, and may be easily converted into those given in the text.

So that a ring being equal to a prism, whose altitude and end are respectively equal to the axis and section of the ring, both in surface and solidity, the rules for them both must be the same; those above given being only preferred from their being more commodious in practice.

EXAMPLES.

1. What is the solidity of an anchor-ring, whose inner diameter is 8 inches, and thickness in metal 3 inches?

Here $(8 + 3) \times (1\frac{1}{2})^2 = 11 \times \frac{9}{4} = \frac{99}{4} = 24.75.$

And	9.8698
	24.75
	493490
	690886
	394784
	197392
	244.272750 *solidity required.*

2. The inner diameter of a cylindric ring is 18 inches; and its thickness 4 inches; what is the solidity?

Ans. 868.5248.

3. Required the solidity of a cylindric ring, whose thickness is 2 inches, and its inner diameter 12.

Ans. 138.1744.

4. What is the solidity of a cylindric ring, whose thickness is 4 inches, and inner diameter 16?

Ans. 789.568.

OF ARTIFICERS' WORK.

Artificers estimate, or compute, the value of their works by different measures, viz.*

1. *Glazing* and *Masons'* flat work, &c. by the square foot.

2. *Painting*, *Plastering*, *Paving*, &c. by the square yard.

3. *Flooring*, *Partitioning*, *Roofing*, *Tiling*, &c. by the square of 100 square feet.

* The best method of taking the dimensions of all sorts of artificers' work is by tenths and hundredths, or by feet and decimal parts; because the computations may then be performed by common multiplication, or by the sliding rule; the latter of which is the method generally used by workmen.

4. *Brick-work*, &c. by the rod of $16\frac{1}{2}$ feet, whose square is $272\frac{1}{4}$.

In all which cases, the measures made use of are contained in the following

TABLE.

12	inches	make	1 lineal foot
144	square inches	make	1 square foot
9	square feet	make	1 square yard
100	square feet	make	a square
$272\frac{1}{4}$	square feet, or	make	1 rod, perch, or
$30\frac{1}{4}$	square yards	make	square pole.

All works, whether superficial or solid, are computed by the rules proper to the figure of them, whether it be a triangle, or rectangle, or parallelopiped, or any other figure.

For taking measures, the most common instrument is what is called the carpenter's rule, of which it will be necessary here to give a description.

CARPENTER'S RULE.

This instrument, which is commonly called the sliding rule, consists of two pieces, of a foot in length each, which are connected together by means of a folding joint.

The whole length of the rule, on one side, is divided into inches and half quarters, for the purpose of taking dimensions ; and on this face there are also several plain scales, divided by diagonal lines into twelve parts, which are designed for planning such dimensions as are taken in feet and inches.

On one part of the other face there is a slider, and four lines marked A, B, C, D ; the two middle ones, B and C, being upon the slider.

Three of these lines, A, B, C, are also double ones, because they proceed from one to ten, twice over : and the fourth line, D, is a single one, proceeding from four to forty, and is called the girt *line*.

The use of the double lines, A and B, is for working proportions, and finding the areas of plane figures. And the use of the girt line, D, and the other double line, C, is for measuring solids.

When 1 at the beginning of any line is counted 1, the 1 in the middle will be 10, and the 10 at the end 100. And when 1 at the beginning is counted 10, the 1 in the middle is 100, and the 10 at the end 1000, &c. and all the small divisions are to be altered in value accordingly.

Upon the other part of this face there is a table of the value of a load of timber, at all prices, from 6*d.* to 2*s.* a foot.

Some rules have likewise a line of inches, or a foot divided decimally into 10th parts; as well as tables of board-measure, timber-measure, &c. but these will be best understood from a sight of the instrument.

THE USE OF THE SLIDING RULE.

PROBLEM I.

To find the product of two numbers, as 7 *and* 26.

Set 1 upon A, to one of the numbers (26) upon B; then against the other number (7) on A, will be found the product (182) upon B.

Note.—If the third term runs beyond the end of the line, seek it on the other radius, or part of the line, and increase the product 10 times.

PROBLEM II.

To divide one number by another, as 510 *by* 12.

Set the divisor (12) on A, to 1 on B; then against the dividend (510) on A, is the quotient ($42\frac{1}{2}$) on B.

Note.—If the dividend runs beyond the end of the line, diminish it 10 or 100 times, to make it fall on A, and increase the quotient accordingly.

PROBLEM III.

To find the square of any number, as 27.

Set 1 upon D to 1 upon C; then against the number (27) upon D, will be found the square (729) upon C.

And if you would square 270, reckon the 1 on D as 100; and then the 1 on C will be 1000, and the product will be 72900.

PROBLEM IV.

To extract the square root of any number, as 4268.

Set 1 upon C, to 1 upon D; then against (4268) the number on C, is (65.3) the root on D.

But to value this right, you must suppose the 1 on C to be some of the squares 1, 100, 1000, &c. which is the nearest to the given number, and then the root corresponding to it will be the value of the 1 upon D.

PROBLEM V.

To find a mean proportional between any two numbers, as 27 *and* 450.

Set one of the numbers (27) on C, to the same on D; then against the other number (450) on C, will be the mean (110.2) on D.

Note.—If one of the numbers overruns the line, take the 100th part of it, and augment the answer 10 times.

PROBLEM VI.

Three numbers being given, as 12, 28, *and* 57, *to find a fourth proportional to them.*

Set the first number (12) upon A, to the second (28) upon B; then against the third number (57) on A, is the fourth (133) on B.

Note.—If one of the middle numbers runs off the line, take the 10th part of it only, and augment the answer 10 times.

The finding a third proportional is also the same; the second number being twice repeated.

Thus, if it were required to find a third proportional to 21 and 32,

Set the first (21) on B, to the second (32) on A; then against the second (32) on B, is 48.8 on A, which is the third proportional sought.*

---

* The use of the rule in board and timber measure may be shown as follows:

When the breadth of a board is given; to find how much in length will make a square foot.

Rule.—If the board be narrow, it will be found in the table of

BRICKLAYERS' WORK.*

Bricklayers compute or value their work at the rate of a brick and a half thick; and if a wall be more or less than this standard, it must be reduced to it, as follows:

Multiply the superficial content of the wall in feet, by the number of half-bricks in the thickness, and $\frac{1}{3}$ of that product will be the content required.

Note.—In practice, it is usual to divide the square feet by 272 only, for rods, omitting the fractional part, $\frac{1}{4}$. But as this is not exact, it will be both better and easier to multiply the feet by 4, and then divide successively by 9, 11, and 11. Also, to divide square yards by $30\frac{1}{4}$, first multiply them by 4, and then divide twice by 11.

In the United States, bricklayers' work is usually reckoned by the thousand bricks: and, in fact, artificers' work, in general, is performed according to a certain contract made between the parties concerned; for instance, a master builder agrees to build a house according to a plan laid down, and the manner in which it shall be finished, for a specific sum of money: the quality of the materials are, however, taken into consideration.

---

board-measure on the rule; but if not, shut the rule, and seek the breadth in the line of board-measure, running along the rule, from that table; then over against it, on the opposite side, is the length, in inches, required.

The side of the square of a piece of timber being given; to find how much in length will make a solid foot.

Rule.—If the timber be small, it will be found in the table of timber-measure on the rule, but if not, look for the side of the square, in the line of timber-measure, running along the rule, from that table, and against it, in the line of inches, is the length required.

* The method of taking the dimensions of a building, in this kind of work, is to measure half round its middle on the outside, and half round it on the inside; and to consider the sum as the true compass, in which the thickness of the wall is supposed to be included.

When the height of the building is unequal, take several different altitudes, and their sum, being divided by the number of them, may be taken as the mean height.

EXAMPLES.

1. How many rods of brick-work are there in a wall $52\frac{1}{2}$ feet long, $12\frac{3}{4}$ feet high, and $2\frac{1}{2}$ bricks thick?

Here $52\frac{1}{2} \times 12\frac{3}{4} = 52.5 \times 12.75 = 669.375 =$ *content in square feet.*

And $669.375 \div 272$, *or* $\frac{669.375}{272} = 2.4609$ *square rods.*

Whence $\frac{2.4609 \times 5}{3} = 4.1015 = 4$ *ro.* 27 *fe.* 7 *in. the answer.*

2. How many yards and rods of standard brick-work are in a wall, whose length or compass is 57 feet 3 inches, and height 24 feet 6 inches; the walls being $2\frac{1}{2}$ bricks, or 5 half-bricks thick?

Here, $57.25 \times 24.5 = 1402.625 =$ *content in square feet.*

To measure a chimney standing by itself, without any party-wall adjoining, take the girt of it for the length, and reckon the height of the story for the breadth; but if it stand against a wall, measure it round to the wall for the girt, and take the height as before.

If the chimney be wrought upright from the mantle-tree to the ceiling the thickness must be considered the same as that of the jambs; and nothing is ever deducted for the vacancy between the floor and the mantle-tree, on account of the gathering of the breast and wings to make room for the hearth in the next story.

For the measure of chimney-shafts, or that part which appears above the roof, girt them with a line about the least place for the length, and take the height for the breadth; and if they be four inches thick, set down the thickness at one brickwork; but if they be nine inches thick, reckon it a brick and a half, in consideration of the plastering and scaffolding.

All windows, doors, &c. are to be deducted out of the contents of the walls in which they are placed. But this deduction is only to be made with regard to materials; as the value of their workmanship is to be added to the bill at the stated rate agreed on.

There are also other allowances to be made to the workman, for returns or angles formed by two adjoining walls, or double measure for feathered gable ends, &c.

All ornamented work is generally valued by the foot square, such as arches, doors, architraves, frizes, cornices, &c. But carved mouldings, &c. are often agreed for by the running foot, or lineal measure.

Multiplying 1402.625 by 5, and dividing the result by 3, we shall have $2337.708\frac{1}{3}$ = the content in square feet, reduced to the standard. And by multiplying $2337.708\frac{1}{3}$ by 4, and dividing the product by 9, 11, and 11, successively, we shall find 8.5866 rods, the answer required.

B A B A
By the Sliding Rule—As, $1 : 24\frac{1}{2} :: 57\frac{1}{4} : 1403$.

3. How many rods of brick-work are there in a wall $62\frac{1}{2}$ feet long, 14 feet 8 inches high, and $2\frac{1}{2}$ bricks thick? *Ans.* 5 *ro.* 167 *fe.* 9 *in.* 4 *p.*

4. If each side-wall of a building be 45 feet long on the outside, each end-wall 15 feet broad on the inside, the height of the building 20 feet, and the gable at each end of the wall 6 feet high, the whole being 2 bricks thick; what is the true content in standard rods? *Ans.* 12.134 *rods.*

5. A triangular gable is raised $17\frac{1}{2}$ feet high, on an end-wall whose length is 24 feet 9 inches, the thickness being 2 bricks: required the reduced content.

Ans. $32.08\frac{1}{3}$ *yards.*

MASONS' WORK.

To Masonry belongs all sorts of stone-work, and the measure made use of is a foot, either superficial or solid.

Walls, blocks of marble or stone, columns, &c. are measured by the solid foot; and pavements, slabs, chimney-pieces, &c. by the superficial foot: the content, in the first of these cases, being found by multiplying the length, breadth, and thickness together; and in the latter by multiplying the length by the breadth.*

---

* In works of this kind, solid measure is principally used for materials, and superficial measure for workmanship: in the former of which cases, the length, breadth and thickness must be

EXAMPLES.

1. Required the solid content of a stone wall whose length is $48\frac{1}{2}$ feet, its height $10\frac{3}{4}$ feet, and thickness 2 feet.

Here 48.5 *length.*			
10.75 *height.*			
2425			
3395			
4850			
521.375			
2 *thickness.*			
1042.750 *the answer.*			
Or 48 *ft.*	6 *in.*		
10 *ft.*	9 *in.*		
485	0		
86	4	6	
521	4	6	
		2	
1042	9	0	*as before.*

2. Required the solid content of a wall whose length is 53 feet 6 inches, its height 12 feet 3 inches, and its thickness 2 feet. *Ans.* 1310 *feet* 9 *in.*

3. What is a marble slab worth, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at 6*s.* per foot. *Ans.* 3*l.* 1*s.* 5*d.*

4. What is the solid content of a wall whose length is 60 feet 9 inches, its height 10 feet 3 inches, and its thickness $2\frac{1}{2}$ feet? *Ans.* 1556.71875 *feet.*

---

taken simply; but in the latter, the length and breadth of every part of the projection must be taken, as it appears without the general upright face of the building.

Deductions and allowances, for trouble and additional workmanship, &c. are also made in the same way as is mentioned in bricklayers' work, in the preceding article.

5. In a chimney-piece, suppose the

	fe.	*in.*
Length of the mantle and slab, each . . .	4	6
Breadth of both together, ...	3	2
Length of each jamb, ...	4	4
Breadth of both together, ...	1	9

What will be the content of the chimney-piece?

Ans. 21 *feet* 10 *inches.*

CARPENTERS' AND JOINERS' WORK.

Carpenters' and Joiners' work is that of flooring, partitioning, roofing, &c. and is commonly measured by the foot, yard, or square of 100 feet; which is done by multiplying the length of the work by its breadth; and, in the atter case, dividing the result by 100.

Enriched mouldings, however, in works of this kind, and some other articles, are often estimated by running or lineal measure, and some things are rated by the piece.*

---

* In measuring of joists it is to be observed, that only one of their dimensions is the same with that of the floor, and the other will exceed the length of the room by the thickness of the wall, and $\frac{1}{3}$ of that thickness, because each end is let into the wall about $\frac{2}{3}$ of its thickness.

No deductions are made for hearths, on account of the additional trouble and waste of materials.

Partitions are measured from wall to wall, for one dimension, and from floor to floor, as far as they extend, for the other.

No deduction is made for door-ways, on account of the trouble of framing them.

In measuring of joiners' work, the string is made to ply close to every part of the work over which it passes.

The measuring of centering for cellars is done by making a string pass over the surface of the arch for the breadth, and taking the length of the cellar for the length; but in groin-centering, it is usual to allow double measure, on account of the additional trouble.

In roofing, the length of the house in the inside, together with $\frac{2}{3}$

EXAMPLES.

1. If a floor be 57 feet 3 inches long, and 28 feet 6 inches broad, how many squares will it contain?

Here 57.25 *length.*
28.5 *breadth.*

28625
25800
11450

100)1631.625 *square feet.*

$16\frac{1}{3}$ *squares nearly.*

Or 75 *ft.*	3 *in.*	
28 *ft.*	6 *in.*	
1603	0	
28	7	6
100)1631	7	6

$16\frac{1}{3}$ *squares, nearly, as before.*

---

of the thickness of one gable, is to be considered as the length, and the breadth is equal to double the length of a string which is stretched from the ridge down to the rafter, along the eaves-board, till it meets with the top of the wall.

For stair-cases, take the breadth of the steps, and having made a line ply close over them, from the top to the bottom, multiply the length of this line by the length of a step for the whole area. By the length of a step is meant the length of the front and the returns at the two ends, and by the breadth is to be understood the girt of its two upper surfaces, or the tread and riser.

For the ballustrade, take the whole length of the upper part of the hand-rail, and girt over its end till it meet the top of the newel-post, for the length; and take twice the length of the baluster upon the landing, with the girt of the hand-rail, for the breadth.

For wainscoting, take the compass of the room for the length, making the string ply close into all the mouldings; and for the breadth, take the height from the floor to the ceiling. Out of this must be made deductions for windows, doors, chimneys, &c. but workmanship is counted for the whole, on account of the trouble they occasion.

For doors, it is usual to allow for their thickness, by adding it

2. A floor is 53 feet 6 inches long, and 47 feet 9 inches broad; how many squares will it contain?

Ans. 25 *sq. and* 54 *feet.*

3. A partition is 91 feet 9 inches long, and 11 feet 3 inches broad; how many squares will it contain?

Ans. 10 *sq. and* 32 *feet.*

4. If a house, within the walls, be 44 feet 6 inches long, and 18 feet 3 inches broad, how many squares of roofing will cover it? *Ans.* 12 *sq. and* 18 *feet.*

5. If a house measure, within the walls, 52 feet 8 inches in length, and 30 feet 6 inches in breadth, and the roof be of a true pitch, what will it cost roofing at 10*s.* 6*d.* per square? *Ans.* 12*l.* 12*s.* 11$\frac{3}{4}$*d.*

SLATERS' AND TILERS' WORK.

In works of this kind, the content of a roof is found by multiplying the length of the ridge by the girt from eave to eave; and, in slating, allowance must be made for the double row at the bottom.

In taking the girt, the line must be made to ply over the lowest row of slates, and returned upon the under side, till

---

into both the dimensions of length and breadth, and then multiplying them together for the area. If the door be paneled on both sides, take double its measure for the workmanship; but if one side only be paneled, take the area and its half for the workmanship.

For the surrounding architrave, gird it about the outermost part of its length; and measure over it, as far as it can be seen when the door is open, for its breadth.

Window-shutters, bases, &c. are also measured in the same manner.

In the measuring of roofing for workmanship alone, all holes for chimney-shafts and sky-lights are generally deducted.

But in measuring for work and materials, they commonly measure in all sky-lights, luthern-lights, and holes for the chimney-shafts, on account of the additional trouble and the waste of materials.

it meet with the wall or eaves-board; but in tiling, the line is stretched down only to the lowest part, without returning it up again.

Double measure is generally allowed for hips, vallies, gutters, &c. but no deductions are made for the openings of chimneys, &c.*

EXAMPLES.

1. The length of a slated roof is 45 feet 9 inches, and its girt 34 feet 3 inches; what is the content?

```
   Here 45.75 length.
        34.25
        -----
        22875
        9150
       18300
      13725
      ---------
   9)1566.9375
      ---------
       174.104 yards, the answer.
```

* In angles formed in a roof, running from the ridge to the eaves, that angle of the roof which bends inwards is called a *valley*, and the angle bending outwards is called a *hip*. And in tiling and slating, it is common to add the length of the vallies to the content in feet; and sometimes also the hips are added.

In slating it is usual to reckon the breadth of the roof 2 or 3 inches broader than what it measures, because the first row is almost covered by the second; and this is done sometimes when a roof is tiled.

Sky-lights and chimney-shafts are generally deducted; but they seldom deduct luthern lights, or garret windows on the roof; as the covering them is reckoned equal to the hole in the roof.

In all works of this nature the content is computed either in yards of 9 square feet or in squares of 100 feet, and the same allowance of hips and vallies is to be made as in roofing.

It is customary to reckon the flat and half-flat of any building within the walls, for the measure of the roof of the building, when the roof is of a *true pitch*, or such that the rafters are $\frac{3}{4}$ of the breadth of the building.

Or 45 *ft.*	9 *in.*	
34 *ft.*	3 *in.*	
1555	6	
11	5	3
9)1566	11	3
174	1	3

Ans. $174\frac{1}{10}$ *yards nearly, as before.*

2. What will the tiling of a barn cost at 25*s.* 6*d.* per square, the length being 43 feet 10 inches, and the breadth 27 feet 5 inches, on the flat, the eaves-boards projecting 16 inches on each side? *Ans.* 24*l.* 9*s.* $5\frac{1}{2}$*d.*

PLASTERERS' WORK.

Plasterers' work is of two kinds, viz. plastering upon laths, called ceiling; and plastering upon walls, called rendering; each of which kinds must be measured separately, by multiplying their lengths by their breadths, and then adding their contents together.

Note.—Proper deductions must be made for doors, chimneys, &c. And in measuring between quarters, there is commonly $\frac{1}{5}$ part of the whole area allowed; but when rendering between quarters is whitened or colored, there is $\frac{1}{5}$ part to be added to the whole, for the sides of the quarters and braces.*

* Plasterers' plain work is measured by the square foot, or yard of 9 square feet; and enriched mouldings, &c. by running or lineal measure.

Windows are seldom deducted, as the plastered returns at the top and sides are allowed to compensate for the window openings; the trouble and expense being nearly as great as if the whole was plastered.

EXAMPLES.

1. If a ceiling be 59 feet 9 inches long, and 24 feet 6 inches broad; how many square yards does it contain?

Here 59.75 *length.*
24.5 *breadth.*

```
    29875
   23900
  11950
 -----------
9)1463.875
 -----------
```

Answer, 162.652 *square yards.*

2. If the partitions between rooms be 141 feet 6 inches about, and 11 feet 3 inches high; how many yards do they contain? *Ans.* 176.87.

3. How much will the ceiling of a room amount to at 10*d.* a yard, the length being 21 feet 8 inches, and the breadth 14 feet 10 inches? *Ans.* 1*l.* 9*s.* $8\frac{3}{4}$*d.*

4. There is a quantity of partitioning that measures 234 feet 8 inches about, and 14 feet 6 inches high, and is rendered between quarters; the lathing and plastering of which is 8*d.* per yard, and the whiting 2*d.* per yard; what will the whole come to? *Ans.* 13*l.* 17*s.* $2\frac{3}{4}$*d.*

5. The length of a room is 14 feet 5 inches, its breadth 13 feet 2 inches, and height 9 feet 3 inches to the under side of the cornice, whose girt is $8\frac{1}{2}$ inches, and its projection 5 inches from the wall, on the upper part next the ceiling; what will be the quantity of plastering, supposing there are no deductions but for one door, which measures 7 feet by 4 feet?

Ans. 53 *yards* 5 *feet* 3 *inches of rendering,* 18 *yards* 5 *feet* 6 *inches of ceiling, and* 39 *feet* $0\frac{11}{12}$ *inches of cornice.*

PAINTERS' WORK.

Painters' work is measured in the same manner as that of Joiners' and other similar work, by multiplying the length by the breadth; and in taking the dimensions, the line must be forced into all the mouldings and corners.

Windows are done at so much a-piece, and in carved mouldings, &c. it is customary to allow double the usual measure.*

EXAMPLES.

1. If a room is to be painted whose height is 16 feet 6 inches, and its compass 97 feet 9 inches, how many yards does it contain?

Here 97.75 *compass.*
16.5 *height.*

```
   48875
  58650
 9775
--------
9)1612.875
--------
 179.208
```

179.208 *square yards, the answer.*

2. The height of a room is 14 feet 10 inches, and the circumference 21 feet 8 inches: how many square yards does it contain? *Ans.* 35.7.

---

* Balustrades, and most other works of this kind, that are to be painted, are measured as in Joiners' work, excepting for doors, window-shutters, &c. where the Joiner is only allowed the area and half-area; but the Painter has always double the area of one side, because every part that is painted must be measured.

Painters take their dimensions with a string, and measure from the top of the cornice to the floor, girting the string over all the mouldings and swellings.

All work of this kind is done by the square yard, and every part where the color lies must be measured, and estimated in the general account of the work.

Deductions are to be made for chimneys, casements, &c. and the price is generally proportioned to the number of times they lay on their color.

3. How many yards of painting are there in a room which is 65 feet 6 inches in compass, and 12 feet 4 inches high? *Ans.* 89.788.

4. What will the painting of a room cost at 6*d* a yard; its length being 24 feet 6 inches, its breadth 16 feet 3 inches, and height 12 feet 9 inches; also the door is 7 feet by 3 feet 6 inches, and two window-shutters each 7 feet 9 inches by 3 feet 6 inches, the breaks being 8 feet 6 inches high, and 1 foot 3 inches deep; deducting the fire-place of 5 feet by 5 feet 6 inches? *Ans.* 3*l.* 3*s.* 10½*d.*

5. Suppose a room, that is to be painted at 8*d.* per yard, measures as follows: the height 11 feet 7 inches, the girt or compass 74 feet 10 inches, the door 7 feet 6 inches by 3 feet 9 inches; 5 window-shutters, each 6 feet 8 inches by 3 feet 4 inches, the breaks in the windows 14 inches 14 inches deep and 8 feet high, the chimney 6 feet 9 inches by 5 feet, a closet the height of the room, 3½ feet deep and 4¾ feet in front, with shelving, 22 feet 6 inches by 10 inches; the shutters, doors and shelves being all colored on both sides; what will the whole come to, at the price above mentioned? *Ans.* 4*l.* 18*s.* 9*d.*

GLAZIERS' WORK.

Glaziers take their dimensions in feet, inches, and parts, and estimate their work by the square foot.

In taking the length and breadth of a window, the cross-bars between the panes are always included.

Windows of every form are measured as if they were squares, and the greatest lengths and breadths are constantly to be taken, on account of the waste attending the cutting of the glass.*

---

* In some cases, windows are measured by taking the dimensions of one pane, and multiplying it by the number of panes; and no allowance is ever made for round or oval windows, as the trouble of cutting them to those shapes is more than the value of the glass omitted.

EXAMPLES.

1. If a window be 2 feet 8 inches and $\frac{3}{4}$ long, and 1 foot 4 inches and $\frac{1}{4}$ broad, how many feet does it contain?

Here the decimal $8\frac{3}{4}$ *inches is* .729 *feet.*
And that of $4\frac{1}{4}$ *inches is* .354
Whence 2.729

```
     2.729
     1.354
  --------
     10916
    13645
   8187
  2729
  --------
  3.695066
        12
  --------
  8.340792
```

Ans. 3 *feet* $8\frac{1}{4}$ *inches nearly.*

2. What will the glazing of a triangular sky-light come to at 20*d.* per foot, supposing the base to be 12 feet 6 inches and the perpendicular height 6 feet 9 inches?

Ans. 4*l.* 7*s.* $2\frac{3}{4}$*d.*

Here	12 *ft.*	6 *in.*	
	6 *ft.*	9 *in.*	
	75	0	
	9	4	6
	84	4	6 = $84\frac{2}{5}$ *ft. nearly.*

Hence	$84\frac{2}{5}$		
6*d.* *is* $\frac{1}{2}$	42		
2*d.* *is* $\frac{1}{3}$	14		
$\frac{2}{5}$ *of* 20*d.*	0	0	8
20)	140	0	8
	7*l.*	0*s.*	8*d.* *answer.*

3. There is a house with three tier of windows, 3 in a tier; the height of the first tier is 7 feet 10 inches; of the second 6 feet 8 inches; of the third 5 feet 4 inches, and the breadth of each 3 feet 11 inches; what will the glazing come to at 14*d* per foot? *Ans.* 13*l.* 11*s.* $10\frac{1}{2}$*d.*

4. Required the expense of glazing the windows of a house at 13*d* a foot; there being three stories, and three windows in each story:

	ft.	in.
The height of the lower tier is	7	9
of the middle tier	6	6
of the upper	5	$3\frac{1}{4}$
and of an oval window over the door .	1	$10\frac{1}{2}$

The common breadth of all the windows being 3 feet 9 inches. *Ans.* 12*l*. 5*s*. 6*d*.

PAVERS' WORK.

Pavers' work is done by the square yard, and the content is found by multiplying the length by the breadth.

Or if the dimensions be taken in feet, and the area be found in the same measure, the result being divided by 9, will give the number of square yards required.

EXAMPLES.

1. What will the paving a rectangular court-yard come to at 3*s*. 6*d*. per yard, supposing the length to be 27 feet 9 inches, and the breadth 14 feet 6 inches?

```
Here 27.75 length.
      14.5 breadth.
     13875
    11100
    2775
  9)402.375
    44.7083 square yards.
        3.5
    2235415
   1341249
20)156.47905
   7.823952
         20
  16.479040
         12
   5.748480               Ans. 7l. 16s. 5d.
```

Or	27 *ft.*	9 *in.*	
	14 *ft.*	6 *in.*	
	388	6	
	13	10	6
	9)402	4	6

$44\frac{7}{10}$ *yards nearly.*

Whence $44\frac{7}{10}$

3

$134\frac{1}{10}$

6*d* is $\frac{1}{2}$ - $22\frac{7}{20}$

$20)156\frac{9}{20}$

7*l.* 16*s.* 5*d.* *as before.*

2. What will the paving a foot-path cost, at 3*s.* 4*d.* a yard, the length being 35 feet 4 inches, and breadth 8 feet 3 inches? *Ans.* 5*l.* 7*s.* $11\frac{1}{2}$*d.*

3. What will the paving of a court-yard cost, at 3*s.* 2*d.* a yard, the length being 27 feet 10 inches, and the breadth 14 feet 9 inches? *Ans.* 7*l.* 4*s.* $4\frac{1}{2}$*d.*

4. A rectangular court-yard is 42 feet 9 inches long, and 68 feet 6 inches in depth, and a foot-way goes quite through it, of 5 feet 6 inches in breadth, which is laid with stone at 3*s.* 6*d.* per yard, and the rest with pebbles at 3*s.* per yard; what will the whole come to?

Ans. 49*l.* 17*s.* $0\frac{1}{4}$*d.*

5. A stable and a court-yard in front, are to be paved with Dutch clinkers, at 5*s.* 6*d.* a hundred; the stable is 20 feet long and 15 feet wide; and the court is of the same length, and 18 feet wide: what will the whole come to, at 144 clinkers to a square yard?

PLUMBERS' WORK.

Plumbers' work is measured like other flat work, and is usually done by the pound or hundred weight, of 112*lbs.*; the price being regulated according to the value of the lead at the time the contract was made, or when the work is performed.*

EXAMPLES.

1. Required the weight of a sheet of lead, which is 39 feet 6 inches long, and 3 feet 3 inches broad at $8\frac{1}{2}$ pounds to the square foot.

Here $39\frac{1}{2} \times 3\frac{1}{4} = 35.5 \times 3.75 = 128.375 =$ *superficial content.*

Whence 128.375
8.5

641875
1027000

1091.1875 *lbs. the answer.*

* Sheet-lead, used in roofing, guttering, &c. is generally between 7 and 12 pounds weight to the square foot; and a pipe of an inch bore is commonly about 13 or 14 pounds to the yard in length.

The following table will show the weight of a square foot to each of these thicknesses.

Thick.	*lbs. sq. foot.*	*Thick.*	*lbs. sq. foot.*	*Thick.*	*lbs. sq. foot.*
$\frac{1}{8}$	7.373	.15	8.848	.18	10.618
.13	7.668	.16	9.438	.19	11.207
.14	8.258	$\frac{1}{6}$	9.831	$\frac{1}{5}$	11.797
$\frac{1}{7}$	8.427	.17	10.028	.21	12.387

Or 39 *ft.*	6 *in.*	
3 *ft.*	3 *in.*	
118	6	
9	10	6
128	4	6

$$\begin{array}{r} Or\ 128\frac{3}{8} \\ 8\frac{1}{2} \\ \hline 1027 \\ 64\frac{3}{16} \\ \hline 1091\frac{3}{16}\ lbs. \end{array}$$

Which answer is the same as before.

2. How much lead, of 10 pounds to the square foot, will line a rectangular cistern 6 feet 6 inches long, 4 feet deep, and 3 feet 9 inches broad? *Ans.* 1076$\frac{3}{4}$ *lbs.*

3. What will the covering and guttering a roof with lead cost, at 18*s.* a *cwt.*; the length of the roof being 43 feet, and breadth, or girt over it, 32 feet; the guttering 57 feet long and 2 feet wide; supposing the former to be 9.831 *lbs.* and the latter 7.373 *lbs.* to the square foot? *Ans.* 115*l.* 9*s.* 1$\frac{1}{2}$*d.*

VAULTED AND ARCHED ROOFS.

Arched roofs are either *vaults*, *domes*, *saloons*, or *groins*.

Vaulted roofs are formed by arches springing from the opposite walls, and meeting in a line at the top.

Domes are made by arches springing from a circular or polygonal base, and meeting in a point at the top.

Saloons are formed by arches connecting the side-walls to a flat roof, or ceiling, in the middle.

Groins are formed by the intersection of vaults with each other.

Domes and saloons rarely occur in the practice of measuring, but vaults and groins cover the cellars of most houses.

Vaulted roofs are generally one of the three following sorts:

1. *Circular roofs*, or those of which the arch is some part of the circumference of a circle.

2. *Elliptical roofs*, or those of which the arch is some part of the circumference of an ellipse.

3. *Gothic roofs* are those which are formed by two circular arcs that meet in a point directly over the middle of the breadth, or span of the arch.

PROBLEM I.

To find the solid content of Circular, Elliptic, or Gothic Vaulted Roofs.

Multiply the length of the vault by the area of one end, or that of a vertical transverse section, and the product will be the solidity required.*

EXAMPLES.

1. What is the solid content of a semi-circular vault, whose span is 40 feet, and its length 120 feet?

Here .7854
1600 = *square of* 40.

4712400
7854

2)1256.6400

628.32 = *area of the end.*
120 = *length.*

75398.40 = *solidity required.*

2. What is the solidity of an elliptic vault whose span is 40 feet, height 12 feet, and length 80?

Ans. 30159.36 *feet.*

* The solidity of the materials, in either of these arches, may be found by the following rule:

From the solid content of the whole arch take the solid content of the void space, and the remainder will be the solidity of the arch.

3. What is the solid content of a gothic vault, whose span is 48, the chord of its arch 48, the distance of the arch from the middle of the chord 18, and the length of the vault 60? *Ans.* 136228.044.

PROBLEM II.

To find the concave, or convex surface, of Circular, Elliptic, or Gothic Vaulted Roofs.

Multiply the length of the arch by the length of the vault, and the product will be the superficies required.*

EXAMPLES.

1. What is the concave surface of a semi-circular vault, whose span is 40 feet, and its length 120?

Here 3.1416
40 = *span.*

2)125.6640

62.832 = *length of the arch.*
120

7539.840 = *concave surface req'd.*

PROBLEM III.

To find the solid content of a Circular or Polygonal Dome, its height and the dimensions of its base being known.

Multiply the area of the base by $\frac{2}{3}$ of the height, and the product will be the solidity.†

EXAMPLES.

1. what is the solid content of a spherical dome, the diameter of whose circular base is 60 feet?

---

* The length of the convex surface of a vault may be found by stretching a string over it; but for the concave surface this method is not applicable, and therefore its length must be found from proper dimensions.

† Domes of this kind are considered as double cones, and their contents determined accordingly.

Here .7854
3600 = *square of* 60.

4712400
23562

2827.4400 = *area of the base.*
20 = $\frac{2}{3}$ *of the height* (30).

56548.8000 = *solidity required.*

2. In an hexagonal spherical dome, one side of the base is 20 feet; what is the solidity? *Ans.* 12000 *feet.*

PROBLEM IV.

To find the superficial content of a Spherical Dome.

Multiply the area of the base by 2, and the product will be the superficial content required.*

EXAMPLES.

1. What will the painting of an hexagonal spherical dome come to at 1*s.* per yard; each side of the base being 20 feet?

Here 2.598076 = { *area of a hexagon whose side is* 1.
400 = *square of* 20.

1039.230400 = *area of the base.*
2

9)2078.460800 = *superfi. content required.*

2.0)230.940088

11.5470044 = 11*l.* 10*s.* 11*d. the expense of painting.*

* The practical rule for elliptical domes is as follows:
Add the height to half the diameter of the base, and this sum, multiplied by 1.5708, will give the superficial content *nearly*.

2. What will the painting of a pentagonal spherical dome come to at 1*s*. 6*d*. per yard; each side of the base being 10 feet?

PROBLEM V.

To find the solid content of a Saloon.

1. Multiply continually together the height of the arc, its projection, $\frac{1}{4}$ of the perimeter of the ceiling, and 3.1416, and call the product A.

2. From a side or diameter of the room take a like side or diameter of the ceiling, and multiply the square of the remainder by the proper factor, (page 62) and this product again by $\frac{2}{3}$ of the height, and call the last product B.

3. Multiply the area of the flat ceiling by the height of the arch, and this product, added to the sum of A and B, will give the content required.*

EXAMPLES.

1. What is the solid content of a saloon with a circular quadrantal arch of 2 feet radius, springing over a rectangular room of 20 feet long and 16 feet wide, the projection on each side being 2 feet?

Here, the flat part of the ceiling is 16 *feet by* 12, *and consequently its perimeter* = 56 *feet.*

Whence $2 \times 14 \times 2 \times 3.1416 = 175.9296 = $ A.

And $(20 - 16)^2 \times 1$ (*factor for a rectangle*) $\times \frac{4}{3} = 4^2 \times \frac{4}{3} = 16 \times \frac{4}{3} = 21.333 = $ B.

---

* The superficial content of a saloon is determined as follows:

1. Find the area of the flat part of the ceiling.

2. Find the convex surface of a cylinder, or cylindroid, whose length is equal to $\frac{1}{4}$ the perimeter of the ceiling, and its diameters twice the height and twice the projection of the arch.

3. Find the superficial content of a dome of the same figure as the arch, and whose base is either a square or a figure similar to that of the ceiling, the side being equal to the difference of a side of the room, and a side of the ceiling.

4. Add these three results together, and the sum will give the superficial content required.

Note.—In a rectangular, circular, or polygonal room, the base of the dome will be a square, a circle, or a polygon.

Therefore $16 \times 12 \times 2 + 175.9296 + 21.333 = 384 + 175.9296 + 21.333 = 581.2629$ *the solid content required.*

2. A circular building of 40 feet diameter, and 25 feet high to the ceiling, is covered with a saloon, whose circular arch is 5 feet radius; required the capacity of the room in cubic feet? *Ans.* 30779.45948 *feet.*

PROBLEM VI.

To find the solid content of the vacuity formed by a groin arch, either circular or elliptical.

Multiply the area of the base by the height, and this product again by .904, and it will give the solidity required.*

EXAMPLES.

1. What is the solid content of the vacuity formed by a circular groin, one side of its square base being 12 feet?

Here 12
12
———
144 = *area of the base.*
6 = *height.*
———
864
.904
———
3456
77760
———
781.056 = *solidity required.*

2. What is the solid content of the vacuity formed by an elliptical groin, one side of its square base being 20 feet, and the height 6 feet? *Ans.* 2169.6.

* For an account of this, and other rules relating to the present subject, see the following note.

PROBLEM VII.

To find the concave superficies of a Circular Groin.

Multiply the area of the base by 1.1416, and the product will be the superficies required.*

EXAMPLES.

1. What is the curve superficies of a circular groin arch, one side of its square being 12 feet?

---

* This rule may also serve for elliptical groins, the error being too small to be regarded in practice.

In measuring works where there are many groins in a range, the cylindrical pieces between the groins, and on their sides, must be computed separately.

And to find the solidity of the brick or stone work, which forms the groin arches, observe the following rule.

Multiply the area of the base by the height, including the work over the top of the groin, and this product lessened by the solid content, found as before, will give the solidity required.

The general rule for measuring all arches is this:

From the content of the whole, considered as solid, (measuring from the springing of the arch to the outside of it,) deduct the vacuity contained between the said springing and the under side of it, and the remainder will be the content of the solid part.

And because the upper sides of all arches, whether vaults or groins, are built up solid, above the haunces, to the same height with the crown, it is evident that the area of the base will be the whole content above mentioned, taking for its thickness the height from the springing to the top. And for the content of the vacuity to be deducted, take the area of its base, accounting its thickness to be $\frac{2}{3}$ of the greatest inside height.

But it is to be noted that the area used in the vacuity is not exactly the same with that used in the solid; for the diameter of the former is twice the thickness of the arch less than that of the latter; and although the deduction of the vacuity is mentioned as common to both the vault and the groin, it is reasonable to reckon them so only in the former, on account of the waste of materials and trouble to the workman, in cutting and fitting them for the angles and intersections.

Here 12
12

144 = *area of the base.*
1.1416

45664
45664
11416

164.3904 = *superficies required.*

2. What is the concave superficies of a circular groin arch, one side of its square being 9 feet?

Ans. 92.4696.

3. What is the concave superficies of a circular groin arch, one side of its square being 15 feet?

Ans. $257\frac{1}{7}$.

TIMBER MEASURE.

PROBLEM I.

To find the area, or superficial content, of a Board or Plank.

Multiply the length by the breadth, and the product will be the content required.

Note.—When the board is tapering, add the breadths of the two ends together, and take $\frac{1}{2}$ the sum for the mean breadth.

BY THE SLIDING RULE.

Set 12 on B to the breadth in inches on A, then against the length in feet on B, is the content on A, in feet and fractional parts.

EXAMPLES.

1. What is the value of a plank, whose length is 8 feet 6 inches, and breadth 1 foot 3 inches throughout, at $6\frac{1}{2}$ cents per foot?

BY DECIMALS.	BY DUODECIMALS.
1.25	8 6
8.5	1 3
625	8 6
900	2 1 6
9.625	10 7 6 *the content.*

Here, 9.625 or 10 square feet and 90 square inches is the content: this, at $6\frac{1}{2}$ cents, comes to 62.562 cents, or $62\frac{1}{2}$ cents *nearly.*

BY THE SLIDING RULE.

As 12 *on* B : 15 *on* A :: $8\frac{1}{2}$ *on* B : $10\frac{1}{2}$ *on* A.

2. What is the content of a board, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches?

Ans. 10 *fe.* 2 *in.* 10 *pa.*

3. At $1\frac{1}{2}d.$ per foot, what is the value of a plank, whose length is 12 feet 6 inches, and breadth 11 inches throughout? *Ans.* 1*s.* 5*d.*

4. Find the value of 5 oaken planks at 3*d.* per foot, each being $17\frac{1}{2}$ feet long, and their particular breadths as follows: viz. two of $13\frac{1}{2}$ inches in the middle, 1 of $14\frac{1}{2}$ inches in the middle and the two remaining ones, each 18 inches at the broader end, and $11\frac{1}{4}$ at the narrower.

Ans. 1*l.* 5*s.* $9\frac{1}{2}d.$

PROBLEM II.

To find the solidity of squared or four-sided Timber.

Multiply the mean breadth by the mean thickness, and this product again by the length, and it will give the solidity required.

Note 1.—If the tree be equally broad and thick throughout, the breadth and thickness, any where taken, will be the mean breadth and thickness; but if it tapers regularly from the one end to the other, the breadth and thickness, taken in the middle, will be the mean breadth and thickness.

2. If it does not taper regularly, but is thicker in some places

than in others, let several different dimensions be taken, and their sum divided by the number of them will give the mean dimensions.

It is to be observed, however, that this method of finding the mean dimensions, though generally used in practice, is, in many cases, exceedingly erroneous.

The quarter-girt, likewise, which is mentioned in the proportion by the sliding rule, is subject to error; since it is not the fourth part of the circumference, but the square root of the product arising from multiplying the mean breadth by the mean thickness.

Thus, in order to show the fallacy of taking $\frac{1}{4}$ of the girt for the side of a mean square, take the following example:

Suppose a piece of timber to be 24 feet long, and a foot square throughout, and let it be slit into two equal parts, from end to end.

Then the sum of the solidities of the two parts, by the quarter-girt method, will be 27 feet, but the true solidity is 24 feet; and if the two pieces were very unequal, the difference would be still greater; also, if the two equal parts be again bisected, and so on, the measure will be continually increased.

BY THE SLIDING RULE.

As the length in feet on C : 12 on D : : quarter-girt in inches on D : solidity on C.

EXAMPLES.

1. The length of a piece of timber is $20\frac{1}{2}$ feet, the breadth at the greater end is $1\frac{3}{4}$ feet, and the thickness $1\frac{1}{4}$ feet; also, at the less end the breadth is $1\frac{1}{2}$ feet, and the thickness 1 foot; what is the solidity?

Here $\frac{1.75 + 1.5}{2} = 1.625 =$ *the mean breadth.*

And $\frac{1.25 + 1}{2} = 1.125 =$ *the mean thickness.*

Whence $1.625 \times 1.125 = 1.828125$.

And $1.828125 \times 20.5 = 37.4765625$ *the solidity required.*

As 1 *upon* B : $19\frac{6}{12}$ *upon* A : $13\frac{6}{12}$ *upon* B : $263\frac{25}{100}$ *upon* A, *the mean square.*

As 16 *upon* C : 4 *upon* D : : 1.8 *upon* C : 16.2 *upon* D, *the side of the mean square.*

As $20\frac{1}{2}$ *upon* C : 12 *upon* D : : 16.2 *upon* D : $37\frac{5}{12}$ *upon* C, *the answer.*

2. The length of a piece of timber is 24.5 feet, and its ends are equal squares, whose sides are each 1.04 feet; what is the solidity? *Ans* 26 *feet* 6 *inches.*

3. The length of a piece of timber is 20.38 feet, and the ends are unequal squares, the side of the greater being $19\frac{1}{2}$ inches, and that of the less $9\frac{7}{8}$ inches; what is the solidity? *Ans.* 29.756 *feet.*

4. The length of a piece of timber is 27.36 feet; at the greater end the breadth is 1.78 feet, and the thickness 1.23 feet; and at the less end the breadth is 1.04 feet, and the thickness .91 feet; what is the solidity. *Ans.* 41.278 *feet.*

PROBLEM III.

To find the solidity of round or unsquared Timber.

RULE I.

Multiply the square of the quarter-girt (or $\frac{1}{4}$ of the circumference) by the length, and the product will be the content, *according to the common practice.**

Note 1.—When the tree is tapering, take the mean dimension, as in the former problems, either by girting it in the middle for the mean girt, or at the two ends, and take half the sum of the two. But when the tree is very irregular, divide it into several lengths, and find the content of each part separately.

2. This rule, which is commonly used, gives the answer about $\frac{1}{4}$ less than the true quantity in the tree, or nearly what the quantity would be after the tree is hewed square in the usual way: so that it seems intended to make an allowance for the squaring of the tree. When the true quantity is desired, use the subsequent rule.

---

* When trees have their bark on, an allowance is generally made, by deducting so much from the girt as is judged sufficient to reduce it to such a circumference as it would have without its bark.

In oak this allowance is about $\frac{1}{10}$ or $\frac{1}{12}$ part of the girt; but for elm, beech, ash, &c. whose bark is not so thick, the deduction ought to be less.

It may here also be observed, that that part of a tree, or the branches, which is less than two feet in circumference, is cut off; not being accounted timber.

BY THE SLIDING RULE.

As the length in feet upon C : 12 upon D : : $\frac{1}{4}$ girt in inches upon D : content upon C.

EXAMPLES.

1. A piece of timber is $9\frac{3}{4}$ feet long, and the quarter-girt is 39 inches; what is the solidity?

Here 3.25 = 39 *inches.*

```
     3.25
     ----
     1625
     650
    975
   ------
  10.5625
     9.75 = 9¾ feet.
  -------
   528125
  739375
 950625
---------
102.984375 = solidity required.
```

BY THE SLIDING RULE.

As $9\frac{3}{4}$ *upon* C : 12 *upon* D : : 39 *upon* D : 103 *upon* C, *the content.*

2. The length of a tree is 25 feet, and the girt throughout $2\frac{1}{2}$ feet; what is its solidity? *Ans.* 9 *feet* 9 *inches.*

3. The length of a tree is $14\frac{1}{2}$ feet, and its girt in the middle 3.15 feet; required the solidity! *Ans.* 9 *feet, nearly.*

4. An oak tree is 45 feet 7 inches long, and its quarter-girt 3 feet 8 inches; what is the solid content, allowing $\frac{1}{17}$ for the bark? *Ans.* 515 *feet, nearly.*

5. The girts of a tree in 4 different places are as follows: in the first place 5 feet 9 inches. in the second 4 feet 6 inches, in the third 4 feet 9 inches, and in the fourth 3 feet 9 inches; and the length of the whole tree is 15 feet; what is its solidity? *Ans.* 20 *feet* 7 *inches.*

RULE II.

Multiply the square of $\frac{1}{5}$ of the girt by twice the length, and the product will be the solidity, *nearly.**

BY THE SLIDING RULE.

As twice the length in feet upon C : 12 upon D :: $\frac{1}{5}$ of the girt in inches, upon D : content upon C.

---

* This rule is full as easy in practice as the false one, and therefore ought to be generally used, since the ease of the other method is the only argument which is alleged for employing it.

The following Problems will show the artifices that may be used in measuring timber according to the false method now practised, and the necessity there is for abolishing it.

I. *To find where a tree must be cut, so that the two parts, measured separately, shall produce a greater solidity than that of the whole tree, or any other two parts of it.*

Rule.—Cut it through exactly in the middle, or at $\frac{1}{2}$ of the length, and the two parts will measure the most possible.

Thus, supposing a tree to girt 14 feet at the greater end, 2 feet at the less, and 8 feet in the middle, and that its whole length is 32 feet.

Then, by the common method, the whole tree measures only 128 feet.

But when it is cut through the middle, the greater part measures 121 feet, and the less part 25 feet.

Whence, the sum of these two parts is 146 feet; which exceeds the whole by 18 feet, and is the most that it can be made measure by cutting it into two parts.

II. *To find where a tree must be cut, so that the part next the greater end may measure the most possible.*

Rule.—Cut it where the girt is $\frac{1}{3}$ of the greatest girt, and the greater end will measure the most possible.

Whence, taking here the same example as before, we shall have $12 : 8 :: \frac{32}{3} : 7\frac{1}{9} =$ length to be cut off; $24\frac{8}{9} =$ length of the remaining part; and $4\frac{2}{3} =$ girt at the section.

But the content of the whole tree is only 128 feet; and the content of the greater part is $135\frac{41}{81}$; which exceeds 128 by $7\frac{41}{81}$, and is the greatest possible.

Note.—If the greater girt does not exceed three times the less, the tree cannot be cut as is required by the problem; for when the less girt is exactly equal to $\frac{1}{3}$ of the greater, the tree already measures the most possible.

EXAMPLES.

1. A piece of timber is $9\frac{3}{4}$ feet long, and $\frac{1}{5}$ of the girt is 2.6 feet; what is the solidity?

Here 2.6 = $\frac{1}{5}$ *of the girt.*

```
     2.6
     2.6
    ----
     156
     52
    ----
    6.76 = square of do.
    9.75
    ----
    3380
   4732
  6084
  ------
 65.9100
       2
 -------
131.8200 = content.
```

III. *To cut a tree so that the part next the greater end may measure exactly the same as the whole tree.*

Rule 1.—Call the sum of the girts of the two ends s, and their difference d.

2. Multiply d by the sum of d and 4 s, and from the square root of the product take the difference between d and 2 s.

3. Then as 2 d is to this remainder, so is the whole length of the tree to the length to be cut off the smaller end.

Thus, taking the same example as in the last problems, we shall have $s = 16$, $d = 12$, and the length $L = 32$; whence $\frac{32}{24}(\sqrt{76 \times 12} - 20) = 13.599118$ = length to be cut off; and consequently the length of the remaining part is 18.400882.

Also, $\frac{1}{2}\sqrt{76 \times 12} - 8 = 7.099669$ = girt at the section. Whence the girt in the middle of the greater part is $\frac{1}{2}(13 + 7.099699) = 10.549834$, whose $\frac{1}{4}$ part is 2.637458; and consequently the content of this part is $(2.637458)^2 \times 18.400882 = 128$, the same as the content of the whole tree.

Note.—The principles of these last three problems are also applicable to the second rule in page 180, and, indeed, to any other approximate rule, or such as is not founded on the true rule for the frustum of a cone.

BY THE SLIDING RULE.

As 19.15 *upon* C : 12 *upon* D :: $31\frac{1}{5}$ *in. upon* D : 132, *the content upon* C.

2. If the length of a tree be 24 feet, and the girt throughout 8 feet, what is the content? *Ans.* 123 *feet, nearly.*

3. If the length of a tree be $16\frac{1}{2}$ feet, and the girt throughout $5\frac{1}{2}$ feet, what is the content?

Ans. 39.93 *feet, nearly.*

4. If a tree girt 14 feet at the thicker end, and 2 feet at the smaller end, required the solidity when the length is 24 feet. *Ans.* 123 *feet, nearly.*

5. A tree girts in five different places as follows: in the first place 9.43 feet, in the second 7 92 feet, in the third 6.15 feet, in the fourth 4.74 feet, and in the fifth 3.16 feet; and the whole length is $17\frac{1}{4}$ feet; what is the solidity?

Ans. 54.4249 *feet.*

THE WEIGHT AND DIMENSIONS

OF

BALLS AND SHELLS.

The weight and dimensions of any ball or shell being first found by experiment, the weight and dimensions of any other ball or shell, of the same kind, may be determined by the following rules.

PROBLEM I.

Having the diameter of an iron shot given, to find its weight.

Multiply the cube of the diameter by 9 and divide the product by 64, and the quotient will be the weight in pounds.

Or take $\frac{1}{8}$ of the cube of the diameter, and $\frac{1}{8}$ of that eighth, and the sum will be the weight, as before.*

EXAMPLES.

1. The diameter of an iron shot is 3.5 inches; what is its weight?

Here $(3.5)^3 = 3.5 \times 3.5 \times 3.5 = 42.875.$

$$\begin{array}{r} \textit{Whence } 42.875 \\ 9 \\ \hline 8)385.875 \\ \hline 8)48.234 \\ \hline 6.029 \\ \textit{Or } 8)42\ 875 \\ \hline 8)5.359 \\ .669 \\ \hline 6.028 \end{array}$$

6.028 *lbs. the answer.*

2. The diameter of an iron shot is 6.7 inches; what is its weight? *Ans.* 42.294 *lbs.*

PROBLEM II.

The diameter of a leaden ball being given, to find its weight.

Multiply the cube of the diameter by 2, and divide the product by 9, and the quotient will be the weight in pounds, *nearly.*

---

* The weight of an iron shot whose diameter is 4 inches, is known to be 9 pounds; and as spheres are to each other, as the cubes of their diameters, $4^3 : 9$ pounds :: D^3 (diameter being $=$ D) : $\frac{9}{64}D^3 =$ weight. But $\frac{9}{64} = \frac{8}{64} + \frac{1}{64} = \frac{1}{8} + \frac{1}{8}$ of $\frac{1}{8}$.

Whence, the weight required is either $\frac{9}{64}D^3$, or $\frac{1}{8}D^3 + \frac{1}{8}$ of $\frac{1}{8}D^3$, agreeing with the rule above given, as was to be shown.

Or take $\frac{1}{3}$ of the cube of the diameter, and from it subtract $\frac{1}{3}$ of this third, and the remainder will be the weight, as before.*

EXAMPLES.

1. What is the weight of a leaden ball, whose diameter is 3.3 inches?

Here $(3.3)^3 = 3.3 \times 3.3 \times 3.3 = 35.937.$

Whence 35.937
2
9)71.874
7.986 *lbs. the answer.*

Or 3)35.937
3)11.979
3.993
7.986 *lbs. as before.*

2. What is the weight of a leaden ball, whose diameter is 5.24 inches? *Ans.* 32 *lbs. nearly.*

PROBLEM III.

The weight of an iron ball being given, to find its diameter.

Multiply the weight by 7, and to the product add $\frac{1}{9}$ of the weight, and the cube root of the sum will be the diameter in inches.†

---

* The weight of a leaden ball, of $4\frac{1}{4}$ inches diameter, is known to be 17 lbs. Whence $(4\frac{1}{4})^3 : 17$ lbs. :: $D^3 : \frac{64}{289} D^3 = \frac{2}{9} D^3$ *nearly*, $= (\frac{3}{9} - \frac{1}{9}) \times D^3 = \frac{1}{3} D^3 - \frac{1}{3}$ of $\frac{1}{3} D^3$; which is the rule.

† By Problem I. $\frac{9}{64} D^3 =$ weight $= w$; whence $D^3 = \frac{64}{9} \times w = \frac{63}{9} w + \frac{1}{9} w = 7w + \frac{1}{9} w$; and consequently $D = \sqrt[3]{7w + \frac{1}{9}w}$; as was to be shown.

EXAMPLES.

1. The weight of an iron ball is 24 pounds; what is the diameter?

$$\begin{array}{r} \textit{Here } 24 \\ 64 \\ \hline 96 \\ 144 \\ \hline 9)1536 \\ \hline 170\frac{2}{3} \\ \textit{Or } 24 \\ 7 \\ \hline 168 \\ \tfrac{1}{9} \text{ of } 24 = 2.666 \\ \hline 170.666 \end{array}$$

And $\sqrt[3]{170.666} = 5.54$ *the diameter required.*

2. The weight of an iron ball is 12 pounds; what is the diameter? *Ans.* 4.403 *inches.*

PROBLEM IV.

Having the weight of a leaden ball given, to find its diameter.

Multiply the weight of the ball by 289, and $\frac{1}{4}$ of the cube root of the product will be the diameter.

Or, divide the weight of the ball by 6, and 3 times the cube root of the quotient will be the diameter, *nearly.**

---

* By Problem II. it appears that $\frac{64}{289} D^3 =$ weight $= w$; whence we have $D = \sqrt[3]{\frac{289\, w}{64}} = \frac{1}{4}\sqrt[3]{289\, w}$.

Or, since $\frac{289}{64} = \frac{9}{2}$ nearly, we shall also have $\frac{2}{9} D^3 = w$; or $D = \sqrt[3]{\frac{9}{2} w} = \sqrt[3]{\frac{27\, w}{6}} = 3\sqrt[3]{\frac{w}{6}}$; which agrees with the rule laid down in the text.

EXAMPLES.

1. The weight of a leaden ball is 8 pounds; what is the diameter?

$$\begin{array}{r} \textit{Here } 289 \\ 8 \\ \hline 2312 \end{array}$$

And $\sqrt[3]{2312} = 12.94$.

Whence $\frac{12.94}{4} = 3.23$ *the diameter.*

Or $\frac{8}{6} = 1.333$; *and* $\sqrt[3]{1.333} = 1.099$.

Whence $1.099 \times 3 = 3.27$ *as before, nearly.*

2. What is the diameter of a leaden ball whose weight is 12 pounds? *Ans.* 3.78 *inches.*

3. Required the diameter of a leaden ball whose weight is 18 pounds? *Ans.* 4.32 *inches.*

PROBLEM V.

Having the external and internal diameters of an iron Shell given, to find its weight.

Multiply the difference of the cubes of the two diameters by 9, and divide the quotient by 64, and the product will be the weight in pounds.

Or take $\frac{1}{8}$ of the difference of the cubes of the diameters in inches, and $\frac{1}{8}$ of that eighth, and their sum will be the weight, as before.*

EXAMPLES.

1. What is the weight of a 13 inch iron bomb-shell, the metal being 2 inches thick on a mean?

Here $13 \times 13 \times 13 = 2197$ *cube of external diameter.*

---

* The weight, by a former problem, is $\frac{9}{64} \times (D^3 - d^3) = (\frac{8}{64} + \frac{1}{64}) \times (D^3 - d^3) = \frac{1}{8} \times (D^3 - d^3) + \frac{1}{8}$ of $\frac{1}{8} \times (D^3 - d^3)$; which is the rule.

And $9 \times 9 \times 9 = 729$ *ditto of internal diameter.*

$$\begin{array}{r} 1468 \text{ difference.} \\ 9 \\ \hline 8)13212 \\ \hline 8)1651.5 \\ \hline 206.4 \text{ weight required.} \end{array}$$

$$\begin{array}{r} \text{Or } 8)1468 \\ \hline 8)183.5 \\ 22.9 \\ \hline 206.4 \text{ as before.} \end{array}$$

2. What is the weight of a 9 inch iron bomb-shell, the metal being $1\frac{1}{2}$ inches thick? *Ans.* 72.14 *lbs.*

PROBLEM VI.

To find the number of pounds of powder that a hollow shell will hold.

Divide the cube of the internal diameter in inches, by 59.32, and the quotient will be the weight, *nearly.**

EXAMPLES.

1. How many pounds of powder will a hollow shell hold, whose internal diameter is 9 inches?

Here $9^3 = 9 \times 9 \times 9 = 729 =$ *cube of the diameter.*

And 59.32)729.0(12.28 *lbs. the answer.*

$$\begin{array}{r} 5932 \\ \hline 13580 \\ 11864 \\ \hline 17160 \\ 11864 \\ \hline 52960 \end{array}$$

* The content of the shell is $= D^3 \times .5236$; whence, 31.06 (*in. in* 1*lb.*) : 1*lb.* :: $D^3 \times .5236 : \frac{.5236}{31.06} \times D^3 = \frac{D^3}{59.32}$; which was to be shown.

Or the same rule will hold by multiplying D^3 by .01668; which is the reciprocal of 59.32.

2. How many pounds of powder will a hollow bomb-shell hold, whose internal diameter is 13 inches?

Ans. 37 lbs. nearly.

PROBLEM VII.

To find the dimensions of a cubical box, that shall hold a given quantity of powder.

Multiply the weight in pounds by 31 06, and the cube root of the product will give the length of the side in inches.*

EXAMPLES.

1. What must be the length of the side of a cubical box that is to hold 15 pounds of powder?

```
Here 31.06
         15
     ------
      15530
      3106
     ------
     465.90
```

And $\sqrt[3]{465.9} = 7.75$ *inches, the answer.*

2. What is the side of a cubical box, that is to hold 12 pounds of powder? *Ans. 7.19 inches.*

PROBLEM VIII.

Given the length, breadth and depth of a rectangular box, to find how many pounds of powder will fill it.

Multiply the length, breadth and depth in inches together; and this product being again multiplied by .0322, will give the answer in pounds, *nearly.*†

---

* As 31.06 inches : 1 *lb.* :: s^3 (the cube of the side) : w (the weight); whence $s = \sqrt[3]{(31.06 \times w)}$; as was to be shown.

† Let L, B, D, = length, breadth and depth of the box, respectively.

Then $\frac{L \times B \times D}{31.06}$ = weight in pounds = L B D × .03219574, as was to be shown.

EXAMPLES.

1. The length of a rectangular box is 15 inches, the breadth 13 inches, and the depth 5 inches; how many pounds of powder will it hold?

$$\textit{Here } 15 \times 13 \times 5 = 975$$

$$\begin{array}{r} \textit{And } 975 \\ .0322 \\ \hline 1950 \\ 1950 \\ 2925 \\ \hline 31.3950 \end{array} = \textit{pounds required.}$$

2. The length of a rectangular box is 1 foot, the breadth 9 inches, and the depth 4 inches; how many pounds of powder will it hold? *Ans.* 13.9104 *pounds.*

PROBLEM IX.

Given the diameter and length of a hollow cylinder, to find how many pounds of gunpowder will fill it.

Multiply .02528 by the square of the diameter, in inches, and this again by the inches in the length, and the product will be the number of pounds required.

Or multiply the square of the diameter by the length, and $\frac{1}{40}$ of the product will be the weight, *nearly.**

EXAMPLES.

1. The diameter of a hollow cylinder is 4 inches, and the length 1 foot; how many pounds of powder will it hold?

---

* Here $\frac{D^2 \times .7854 \times L}{31.06}$ = quantity of powder in the cylinder.

But $\frac{.7854}{31.06} = .02528 = \frac{1}{40}$ nearly.

Whence $.02528 \times D^2 \times L = \frac{D^2 L}{40}$ nearly, for the quantity of powder required; as was to be shown.

Here .02528
16 = *square of* 4.

15168
2528

.40448
12

4.85376 *lbs. the answer.*

Or 16
12

40)192

4.8 *answer, nearly.*

2. The diameter of a hollow cylinder is 6 inches, and its length 10 inches; how many pounds of powder will it hold? *Ans.* 9.1 *lbs.*

PROBLEM X.

Given the diameter of a hollow cylinder, to find what length of it will be filled by a given quantity of powder.

Divide 40 times the weight in pounds by the square of the diameter in inches, and the quotient will be the length in inches, *nearly.**

EXAMPLES.

1. The diameter of a hollow cylinder is 1 foot; what length of it will be filled by 10 pounds of powder?

Here 10 = *weight, and* 144 = *square of the diameter.*

---

* By the last problem $\frac{D^2 \times .7854 \times L}{31.06} = .02528 \times D^2 \times L$ $= \frac{D^2 L}{40.} = w$ nearly; agreeably to what is there shown.

Whence $L = \frac{40\,w}{D^2}$ nearly; which is the same as the rule.

Whence 10
40

```
144)400(2.77  the length required.
    288
    ----
    1120
    1008
    ----
     1120
     1008
     ----
      112
```

2. The diameter of a hollow cylinder is 6 inches; what length of it will be filled by 12 pounds of powder?

Ans. 13.185.

3. The diameter of a hollow cylinder is 8 inches; what length of it will be filled by 20 pounds of powder;

Ans. $12\frac{1}{2}$.

4. The diameter of a hollow cylinder is 9 inches; what length of it will be filled by 15 pounds of powder?

Ans. $7\frac{2}{5}$.

THE PILING OF BALLS AND SHELLS.

Iron shot and shells are usually piled in horizontal courses, in a pyramidical or wedge-like form, the base being either an equilateral triangle, a square, or a rectangle; where it is to be observed, that the triangular and square pile finishes in a single ball; but in the rectangle, the top is a single row of balls; and the whole pile consists of a series of figures similar to that of the base, the side of each successive row diminishing by one, from the bottom upwards.

To this, it may also be added, that in triangular and square piles the number of horizontal rows or courses is always equal to the number of shot on one side, in the bottom row. But, in rectangular piles, the number of courses is equal to the number of shot in the breadth of the bottom row; and the number in the top row, less one, is the difference between the number in the length and breadth of the bottom row.

PROBLEM I.

To find the number of shot in a finished triangular pile.

Multiply the number in one side of the bottom row by that number plus one; and this product again by the same number plus two, and $\frac{1}{6}$ of the result will give the answer required.*

EXAMPLES.

1. Required the number of shot in a finished triangular pile, the number in one side of the base being 20?

Here 20 = *the number.*
And 21 = *number plus one.*

```
   20
  40
 ----
  420
   22 = number plus 2.
 ----
  840
 840
 ----
6)9240
 ----
 1540 the number of shot in the pile.
```

* In triangular piles, each horizontal course, beginning at the top, is a triangular number, produced by taking the successive sums of the numbers 1; 1, 2; 1, 2, 3; 1, 2, 3, 4; 1, 2, 3, 4, 5, &c. and the whole number of shot in such a pile is equal to the sum of all these triangular numbers, taken to as many terms, as are equal to the number in one side of the bottom course.

Thus, if 1, 2, 3, 4, 5, 6, 7, &c. be the natural numbers, then will 1, 3, 6, 10, 15, 21, 28, &c. be the triangular numbers; or the number of shot in each course from the top.

But the sum of the series $1 + 3 + 6 + 10 + 15 + 21 + 28$, &c. to n terms is $= \frac{n}{1} \times \frac{n+1}{2} + \frac{n+2}{3}$ (as is shown in the N. Y. ed. of Bonnycastle's Algebra,) $= (n + 2) \times (n + 1) \times \frac{n}{6}$; which is the same as the rule.

2. Required the number of shot in a finished triangular pile, the number in one side of the base being 40?

Ans. 11480.

3. Required the number of shot in a finished triangular pile, the number in one side of the base being 25?

Ans. 2925.

PROBLEM II.

To find the number of shot in a finished square pile.

Multiply the number in one side of the bottom course by that number plus 1, and this product again by double that number plus 1; and $\frac{1}{6}$ of the last product will give the answer.*

EXAMPLES.

1. Required the number of shot in a finished square pile, the number in one side of its base being 20?

```
Here  20 = number in side.
      21 = 20 plus 1.
     ---
     420
      41 = double of 20 plus 1.
     ---
     420
    1680
   -----
  6)17220
   -----
    2870 number of shot in the pile.
```

* In square piles, each horizontal course is a square number, produced by taking the square of the number in its side; and the whole number of shot in such a pile is equal to the sum of all those squares, beginning with one at the top, and proceeding downwards, as far as the number in the side of the bottom course.

Thus, if 1, 2, 3, 4, 5, 6, 7, &c. be the natural numbers, then will 1, 4, 9, 16, 25, 36, 49, &c. be the square numbers, or the number of shot in each course from the top.

But the sum of the series $1 + 4 + 9 + 16 + 25 + 36 + 49$, &c. to n terms is $= \frac{n \times (n + 1) \times (2n + 1)}{6}$ (see the N. Y. ed. of Bonnycastle's Algebra,) $= (n + 1) \times (2n + 1) \times \frac{n}{6}$; which is the rule given in the text.

2. Required the number of shot in a finished square pile, one side of the lower tier having 40 shot in it?

Ans. 22140.

PROBLEM III.

To find the number of shot in a finished rectangular pile.

From three times the number in the length of the bottom course, subtract one less than the number in the breadth of the same course; then multiply the remainder by the said breadth, and this product again by one more than the breadth, and $\frac{1}{6}$ of the last result will be the answer.*

EXAMPLES.

1. How many shot are there in a finished rectangular pile, the length and breadth of the bottom course being 46 and 15?

---

* Let the number of shot in the upper row be denoted by $r + 1$; then, by the nature of the pile, the *2d* course will contain $2(r+2)$ shot; the *3d*, $3(r+3)$; and soon, to $n(r+n)$, which is the number in the bottom course; n being the number in the breadth of that course.

Whence, the whole number of shot in all the courses, is $= (r+1) + 2(r+2) + 3(r+3)$, &c.... $+ n(r+n)$.

But $1 + 2 + 3$, &c...$n = \frac{1}{2}n(n+1)$; and $1^2 + 2^2 + 3^2$, &c....$n^2 = \frac{1}{6}n(n+1)(2n+1)$; therefore the whole number $= \frac{1}{2}n(n+1)r + \frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}n(n+1)(3r + 2n+1)$.

Hence, since the number in the breadth of the bottom course is n, and the number in the length $r+n$, if $m-n$ be put for r, we shall have $\frac{1}{6}n(n+1)(3m-n+1) =$ whole number of shot in the pile; which is the rule.

The several rules for finding the number of shot in the different kinds of piles, may be expressed algebraically thus:

$\frac{n}{6} \times (n+1) \times (n+2) =$ number in the triangular pile.

$\frac{n}{6} \times (n+1) \times (2n+1) =$ number in the square pile.

$\frac{n}{6} \times (n+1) \times (3n-m+1) =$ No. in the rectangular pile.

Where n denotes the number in one side of the bottom row,

```
Here 46 = length bottom course.
      3
     ---
    138
     14 = breadth of do. less 1.
     ---
    124
     15 = breadth of do.
     ---
    620
   124
   ----
   1860
     16 = breadth plus 1.
  -----
  11160
  1860
  -----
6)29760
  -----
   4960 = number required.
```

2. How many shot are there in a finished rectangular pile, the length of the bottom course being 59, and its breadth 20? *Ans.* 11060.

PROBLEM IV.

To find the number of shot in an incomplete pile.

From the number in the whole pile, considered as complete, subtract the number that is wanting in the upper

---

for the triangular and square piles; and in the rectangular pile n denotes the breadth of the bottom row, and m its length.

To this we may likewise add the following general rule; which is better adapted to the memory than those above given.

Rule.—In every pile there may be found three parallel rows of balls, the sum of which being multiplied by the number of balls in the triangular face of the pile, and then divided by 3, will give the whole number of balls in the pile. Observing, that in the rectangular pile, the three parallel rows are the two bottom rows in length and the upper ridge of the pile. In the square pile, any two opposite sides of the square base and the upper ball are the parallel rows; and in the triangular pile, they are the bottom row, the opposite extreme ball, and the upper ball.

part; computing them both by the rule for the proper form, and the remainder will be the number in the frustum, or incomplete pile.

EXAMPLES.

1. Required the number of shot in an incomplete triangular pile, one side of the bottom course being 40, and that of the upper course 20.

Here 19	*And* 40
20	41
380	1640
21	42
380	3280
760	6560
7980	68880

Whence 68880
7980

6)60900

10150

Therefore 10150 = *the number required.*

2. How many shot are there in an incomplete triangular pile, the side of the base being 24, and of the top 8?
Ans. 2516.

3. How many shot are there in an incomplete square pile, the side of the base being 24, and of the top 8?
Ans. 4760.

4. How many shot are there in an incomplete rectangular pile, of 12 courses, the length and breadth of the base being 40 and 26? *Ans.* 6146.

5. How many shot are there in an incomplete rectangular pile of 15 courses, the length and breadth of the base being 42 and 28 respectively?

TRIGONOMETRY.

SECTION 1. *Plane Trigonometry.*

1. *Plane Trigonometry* is that branch of mathematics by which we learn how to determine or compute three of the six parts of a plane, or rectilinear triangle, from the other three, when that is possible.

The determination of the mutual relation of the sines, tangents, secants, &c. of the sums, differences, multiples, &c. of arcs or angles; or the investigation of the connected formulæ, is, also, usually classed under plane trigonometry.

2. Let ACB be a rectilinear angle, if about C as a centre, with any radius CA, a circle be described, intersecting CA, CB, in A, B, the arc AB is called the *measure* of the angle ACB. (See the next figure.)

3. The circumference of a circle is supposed to be divided, or to be divisible into 360 equal parts, called *degrees;* each degree into 60 equal parts, called *minutes;* each of these into 60 equal parts, called *seconds;* and so on, to the minutest possible subdivisions. Of these, the first is indicated by a small circle, the second by a single accent, the third by a double accent, &c. Thus, $47° 18' 34'' 45'''$, denotes 47 degrees, 18 minutes, 34 seconds, and 45 thirds. So many degrees, minutes, seconds, &c. as are contained in any arc of so many degrees, minutes, seconds, &c. is the angle of which that arc is the measure said to be. Thus, since a quadrant, or quarter of a circle, contains 90 degrees, and a quadrantal arc is the measure of a right angle, a right angle is said to be one of 90 degrees.

4. The *complement of an arc* is its difference from a quadrant; and the *complement of an angle* is its difference from a right angle.

5. The *supplement of an arc* is its difference from a semi-circle, and the *supplement of an angle* is its difference from two right angles.

6. The *sine* of an arc is a perpendicular let fall from one extremity upon a diameter passing through the other.

7. The *versed sine* of an arc is that part of the diameter which is intercepted between the foot of the sine and the arc.

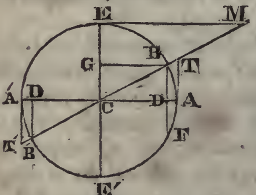
8. The *tangent* of an arc is a right line which touches it in one extremity, and is limited by a right line drawn from the centre of the circle through the other extremity.

9. The *secant* of an arc is the sloping line which thus limits the tangent.

10. These are also, by way of accommodation, said to be the sine, tangent, &c. of the angle measured by the aforesaid arc, to its determinate radius.

11. The *cosine* of an arc or angle, is the sine of the complement of that arc or angle: the *cotangent* of an arc or angle is the tangent of the complement of that arc or angle. The *co-versed sine* and *cosecant* are defined similarly.

To exemplify these definitions by the annexed diagram: let AB be an assumed arc of a circle described with the radius AC, and let AE be a quadrantal arc; let BD be demitted perpendicularly from the extremity B upon the diameter AA′; parallel to it let AT be drawn and limited by CT: let GB and EM be drawn parallel to AA′, the latter being limited by CT or CT produced. Then BE is the *complement* of BA, and angle BCE the *complement* of angle BCA; BEA′ is the *supplement* of BA, and angle BCA′ the *supplement* of BCA; BD is the *sine*, DA the *versed sine*, AT the *tangent*, CT the *secant*, GB the *cosine*, GE the *coversed sine*, EM the *cotangent*, and CM the *cosecant*, of the arc AB, or, by convention, of the angle ACB.



Note.—These terms are indicated by obvious contractions:

Thus, for sine of the arc AB we use sin AB,
tangent......ditto.......tan AB,
secant ......ditto.......sec AB,
versed sine...ditto.......versin AB,
cosine.......ditto.......cos AB,
cotangent....ditto.......cot AB,
cosecant.....ditto.......cosec AB,
coversed sine.ditto.......coversin AB.

Corolleries from the above Definitions.

12. (A.) Of any arc less than a quadrant, the arc is less than its corresponding tangent; and of any arc whatever, the chord is less than the arc, and the sine less than the chord.

(B.) The sine BD of an arc AB, is half the chord BF of the double arc BAF.

(C.) An arc and its supplement have the same sine, tangent, and secant. (The two latter, however, are affected by different signs, + or —, according as they appertain to arcs less or greater than a quadrant).

(D.) When the arc is evanescent, the sine, tangent, and versed sine, are evanescent also, and the secant becomes equal to the radius, being its minimum limit. As the arc increases from this state, the sines, tangents, secants, and versed sines increase; thus they continue till the arc becomes equal to a quadrant AE, and then the sine is in its maximum state, being equal to radius, thence called the *sine total;* the versed sine is also then equal to the radius; and the secant and tangent becoming incapable of mutually limiting each other, are regarded as infinite.

(E.) The versed sine of an arc, together with its cosine, are equal to the radius. Thus, $AD + BG = AD + DC = AC$. (This is not restricted to arcs less than a quadrant).

(F.) The radius, tangent, and secant, constitute a right angled triangle CAT. The cosine, sine, and radius, constitute another right angled triangle CDB, similar to the former. So again, the cotangent, radius, and cosecant, constitute a third right angled triangle, MEC, similar to both the preceding. Hence, when the sine and radius are known, the cosine is determined by the property of the right angled triangle.

The same may be said of the determination of the secant, from the tangent and radius, &c. &c. &c.

(G.) Further, since CD : DB :: CA : AT, we see that the tangent is a fourth proportional to the cosine, sine, and radius.

Also, CD : CB :: CA : CT; that is, the secant is a third proportional to the cosine and radius.

Again, CG : GB :: CE : EM; that is, the cotangent is a fourth proportional to the sine, cosine, and radius.

And, BD : BC :: CE : CM; that is, the cosecant is a third proportional to the sine and radius.

(H.) Thus, employing the usual abbreviations, we should have

1. $\cos = \sqrt{(\text{rad}^2 - \sin^2)}$. 2. $\tan = \sqrt{(\sec^2 - \text{rad}^2)}$.
3. $\sec = \sqrt{(\text{rad}^2 + \tan^2)}$. 4. $\text{cosec} = \sqrt{(\text{rad}^2 + \cot^2)}$.

5. $\tan = \frac{\text{rad} \times \sin}{\cos} = \frac{\text{rad}^2}{\cot}$. 6. $\cot = \frac{\text{rad} \times \cos}{\sin} = \frac{\text{rad}^2}{\tan}$.

7. $\sec = \frac{\text{rad}^2}{\cos}$. 8. $\text{cosec} = \frac{\text{rad}^2}{\sin}$.

These, when unity is regarded as the radius of the circle, become

1. $\cos = \sqrt{(1 - \sin^2)}$. 2. $\tan = \sqrt{(\sec^2 - 1)}$.
3. $\sec = \sqrt{(1 + \tan^2)}$. 4. $\text{cosec} = \sqrt{(1 + \cot^2)}$.

5. $\tan = \frac{\sin}{\cos} = \frac{1}{\cot}$. 6. $\cot = \frac{\cos}{\sin} = \frac{1}{\tan}$. 7. $\sec = \frac{1}{\cos}$. 8. $\text{cosec} = \frac{1}{\sin}$.

13. From these, and other properties and theorems, mathematicians have computed the lengths of the sines, tangents, secants, and versed sines, to an assumed radius, that correspond to arcs from 1 second of a degree, through all the gradations of magnitude, up to a quadrant, or 90°. The results of the computations are arranged in tables called *Trigonometrical Tables* for use. The arrangement is generally appropriated to two distinct kinds of these artificial numbers, classed in their regular order upon pages that face each other. On the left-hand pages are placed the sines, tangents, secants, &c. adapted at least to every degree and *minute* in the quadrant, computed to the radius 1, and expressed decimally. On the right-hand pages are placed, in succession, the corresponding *logarithms* of the numbers that denote the several sines, tangents, &c. on the respective opposite pages. Only, that the necessity of using negative indices in the logarithms may be precluded, they are supposed to be the logarithms of sines, tangents, secants, &c. computed to the radius 10000000000. The numbers thus computed and placed on the successive right-hand pages are called *logarithmic* sines, tangents, &c. The numbers of which these are the logarithms, and which are

arranged on the left-hand pages, are called *natural* sines, tangents, &c.

II. *General Properties and Mutual relations.*

1. The chord of any arc is a mean proportional between the versed sine of that arc and the diameter of the circle.
2. As radius, to the cosine of any arc; so is twice the sine of that arc, to the sine of double the arc.
3. The secant of any arc is equal to the sum of its tangent, and the tangent of half its complement.
4. The sum of the tangent and secant of any arc, is equal to the tangent of an arc exceeding that by half its complement. Or, the sum of the tangent and secant of an arc is equal to the tangent of 45° *plus* half the arc.
5. The chord of 60° is equal to the radius of the circle; the versed sine and cosine of 60° are each equal to half the radius, and the secant of 60° is equal to double the radius.
6. The tangent of 45° is equal to the radius.
7. The square of the sine of half any arc or angle is equal to a rectangle under half the radius and the versed sine of the whole; and the square of its cosine, equal to a rectangle under half the radius and the versed sine of the supplement of the whole arc or angle.
8. The rectangle under the radius and the sine of the sum or of the difference of two arcs, is equal to the sum or the difference of the rectangles under their alternate sines and cosines.
9. The rectangle under the radius and the cosine of the sum or the difference of two arcs, is equal to the difference or the sum of the rectangles under their respective cosines and sines.
10. As the difference or sum of the square of the radius and the rectangle under the tangents of two arcs, is to the square of the radius; so is the sum or difference of their tangents, to the tangent of the sum or difference of the arcs.
11. As the sum of the sines of two unequal arcs, is to their difference, so is the tangent of half the sum of those two arcs to the tangent of half their difference.
12. Of any three equidifferent arcs, it will be as radius, to the cosine of their common difference, so is the sine of the mean arc, to half the sum of the sines of the extremes; and, as radius to the sine of the common difference, so is

the cosine of the mean arc to half the difference of the sines of the two extremes.

(A.) If the sine of the mean of three equidifferent arcs (radius being unity) be multiplied into twice the cosine of the common difference, and the sine of either extreme be deducted from the product, the remainder will be the sine of the other extreme.

(B.) The sine of any arc above 60°, is equal to the sine of another arc as much below 60°, together with the sine of its excess above 60°.

Remark.—From this latter proposition, the sines below 60° being known, those of arcs above 60° are determinable by addition only.

13. In any right angled triangle, the hypothenuse is to one of the legs, as the radius to the sine of the angle opposite to that leg; and one of the legs is to the other, as the radius to the tangent of the angle opposite to the latter.

14. In any plane triangle, as one of the sides is to another, so is the sine of the angle opposite to the former to the sine of the angle opposite to the latter.

15. In any plane triangle it will be, as the sum of the sides about the vertical angle, is to their difference, so is the tangent of half the sum of the angles at the base, to the tangent of half their difference.

16. In any plane triangle it will be, as the cosine of the difference of the angles at the base, is to the cosine of half their sum, so is the sum of the sides about the vertical angles to the third side. Also, as the sine of half the difference of the angles at the base, is to the sine of half their sum, so is the difference of the sides about the vertical angle to the third side, or base.*

17. In any plane triangle it will be, as the base, to the sum of the two other sides, so is the difference of those sides, to the difference of the segments of the base made by a perpendicular let fall from the vertical angle.

18. In any plane triangle it will be, as twice the rectangle under any two sides, is to the difference of the sum of the squares of those two sides and the square of the base,

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* These propositions were first given by *Thacker*, in his *Mathematical Miscellany* published in 1744; their practical utility has been recently shown by *Professor Wallace*, in the *Edinburgh Philosophical Transactions.*

so is the radius to the cosine of the angle contained by the two sides.

Cor.—When unity is assumed as radius, then if AC, AB, BC, are the sides of a triangle, this prop. gives cos C $= \frac{AC^2 + BC^2 - AB^2}{2\,CB\,.\,CA}$: and similar expressions for the other angles.

19. As the sum of the tangents of any two unequal angles, is to their difference, so is the sine of the sum of those angles, to the sine of their difference.

20. As the sine of the difference of any two unequal angles, is to the difference of their sines, so is the sum of those sines, to the sine of the sum of the angles.

These and other propositions are the foundation of various formulæ, for which the reader who wishes to pursue the inquiry may consult the best treatises on Trigonometry.

III. *Solution of the Cases of Plane Triangles.*

There are usually three methods of resolving triangles, or the cases of trigonometry; namely, Geometrical Construction, Arithmetical Computation, and Instrumental Operation

In the first method. The triangle is constructed by making the parts of the given magnitudes, namely, the sides from a scale of equal parts, and the angles from a scale of chords, or by some other instrument. Then, measuring the unknown parts by the same scales or instruments, the solution will be obtained near the truth.

In the second method. Having stated the terms of the proportion according to the proper rule or theorem, resolve it like any other proportion, in which a fourth term is to be found from three given terms, by multiplying the second and third terms together, and dividing the product by the first, in working with the natural numbers: or, in working with the logarithms, add the logs of the second and third terms together, and from the sum take the log. of the first term; then the natural number answering to the remainder, is the fourth term sought.

In the third method, or instrumentally, as suppose by the log. lines on one side of the common two-foot scales. Extend the dividers, (usually called compasses), from the first term to the second or third, which happens to be of the same kind with it; then that extent will reach from the other term to the fourth term, as required, taking both extents towards the same end of the scale.

Note 1. In every triangle, or case in trigonometry, there must be given three parts, to find the other three. And of the three parts that are given, one of them at least must be a side; because the same angles are common to an infinite number of the triangles.

Note 2.—Although the three sides and three angles of a plane triangle, when combined three and three, constitute twenty varieties, yet they furnish only *three* distinct cases in which separate rules are required.

CASE I.

When a side and an angle are two of the given parts.

The solution may be effected by prop. 14 of the preceding section, wherein it is affirmed that the sides of plane triangles are respectively proportional to the sines of their opposite angles.

In practice, if a *side* be required, begin the proportion with a sine, and say,

As the sine of the given angle,
To its opposite side;
So is the sine of either of the other angles,
To its opposite side.

If an *angle* be required, begin the proportion with a side, and say,

As one of the given sides,
Is to the sine of its opposite angle;
So is the other given side,
To the sine of its opposite angle.

The third angle becomes known by taking the sum of the two former from 180°.

Note 1.—Since sines are *lines*, there can be no impropriety in comparing them with the sides of triangles; and the rule is better remembered by young mathematicians, than when the sines and sides are compared each to each.

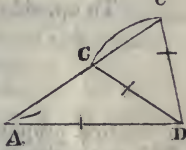
Note 2. An angle found by this rule is ambiguous, or uncertain whether it be acute or obtuse, unless it be a right angle, or unless its magnitude be such as to prevent the ambiguity; because the sine answers to two angles, which are supplements to each other: and accordingly the geometrical construction forms two triangles with the same parts that are given, as in the example below; and when there is no restriction or limitation included in the question, either of them may be taken. The degrees in the Table, answering to the sine, is the acute angle; but if the angle be obtuse, subtract those degrees from 180°, and the remainder will be the obtuse angle. When a given angle is obtuse or a right one, there can be no ambiguity; for then neither of the other angles can be obtuse, and the geometrical construction will form only one triangle.

EXAMPLE I.

In the plane triangle ABC,

Given { AB 345 yards,
BC 232 yards,
< 37° 20′,

Required the other parts.



1. *Geometrically.*

Draw an indefinite line, upon which set off AB = 345, from some convenient scale of equal parts; make the angle A = 37° 20′; with a radius of 232, taken from the same scale of equal parts, and centre B, describe the arc CC, cutting AC in C, C: lastly, join BC, BC, and the figure is constructed, which gives two triangles, and showing that the case is ambiguous. Then, the side AC, measured by the scale of equal parts, and the angles B and C measured by the line of chords, or other instrument, will be found to be nearly as below; namely,

AB 174,	∠ B 27°,	∠ C 115° 30′,
or $374\frac{1}{2}$,	or $78\frac{1}{4}$,	or 64 30.

2. *Arithmetically.*

First, to find the angle at C.

As side BC 232 . . . . log	2.3654880
To sine op. ∠ A 37° 20′	9.7827958
So is side AB 345	2.5378191
To sine op. ∠ C 115° 36′, or 64° 24′	9.9551269

Add ∠ A	37 20	37 20
The sum	152 56	101 44
Take from	180 00	180 00
	27 04	78 16

Then to find the side AC.

As sine < A 37° 20′	........log.	9.7827958
To op. side BC 232	....	2.3654880
So is sine < B { 27°. 04′	....	9.6580371
{ 78 16	....	9.9908291
To op. side AC 174.07	....	2.2407293
Or 374.56	....	2.5735213

3. *Instrumentally.*

In the first proportion. Extend the compasses from 232 to 345 upon the line of numbers: then that extent will reach, on the line of sines, from 37°$\frac{1}{3}$ to 64°$\frac{1}{2}$, the angle C nearly.

In the second proportion. Extend the compasses from 37°1 to 27° or 78°$\frac{1}{4}$, on the line of sines: then that extent will reach on the line of numbers, from 232 to 174, or 374, the two values, nearly, of the side AC.

EXAMPLE II.

In the plane triangle ABC,

Given { AB 365 poles, < A 57° 12′, < B 24 45. } *Ans.* { < C 98° 3′, AC 154.33, BC 309.86. }

Required the other parts.

EXAMPLE III.

In the plane triangle ABC.

Given { AC 120 feet, BC 112 feet, < A 57° 27′ } *Ans.* { < B 64° 34′ 21″ or 115 25 39, < C 57 58 39 or 7 7 21, AB 112.65 feet, or 16.47 feet. }

CASE II.

When two sides and the included angle are given,

The solution may be effected by means of props. 15 and 16 of the preceding section.

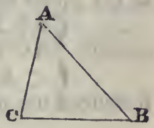
Thus: take the given angle from 180°, the remainder will be the sum of the other two angles.

Then say—As the sum of the given sides,
Is to their difference;
So is the tangent of half the sum of the remaining angles,
To the tangent of half their difference.

Then secondly say—As the cosine of half the said difference
Is to the cosine of half the sum of the angles;
So is the sum of the given sides,
To the third, or required side.

Or, As the sine of half the difference of the angles,
Is to the sine of half their sum;
So is the difference of the given sides,
To the third side.

Example 1.—In the triangle ABC are given AC = 450, and the included angle C = 80°; to find the third side, and the two remaining angles.



Here BC + AC = 990, BC — AC = 90, 180° — C = 100° = A + B.

Hence, BC + AC....990	..Log.	=	2·9956352
To BC — AC.. .. 90	..Log.	=	1·9542425
So is tan $\frac{1}{2}$ (A + B)... 50°	Log.	=	10·0761865
To tan $\frac{1}{2}$ (A — B).....6° 11′	Log.	=	9·0347938

Cos $\frac{1}{2}$ (A — B) ....6° 11′	Log.	=	9·9974660
Cos $\frac{1}{2}$ (A + B) ......50°	Log.	=	9·8080675
So is BC + AC..... 990	Log.	=	2·9956352
To AB............ 640.08	Log.	=	2·8062367

Also $\frac{1}{2}$ (A + B) + $\frac{1}{2}$ (A — B) = 56° 11′ = A; and $\frac{1}{2}$ (A + B) — $\frac{1}{2}$ (A — B) = 43° 49′ = B.

Here, much time will be saved in the work by taking $\cos \frac{1}{2}(A + B)$ from the tables, at the same time with $\tan \frac{1}{2}(A + B)$; and $\cos \frac{1}{2}(A - B)$ as soon as $\tan \frac{1}{2}(A - B)$ is found. Observe, also, that the log. of BC + AC is the same in the second operation as in the first. Thus the tables need only be opened in *five* places for both operations.

Another solution to Case II.

Supposing C to be the given angle, and CA, CB, the given sides; then the third side may be found by this theorem: viz.

$$AB = \sqrt{(AC^2 + BC^2 - 2\,AC\,.\,CB\,.\cos C)}.$$

Thus, taking AC = 450, BC = 540, C = 80°, its cos ·1736482

$$AB = \sqrt{(450^2 + 540^2 - 2\,.\,450\,.\,540 \times \cdot1736482)}$$
$$= \sqrt{[90^2 (5^2 + 6^2 - 2\,.\,5\,.\,6 \times \cdot1736482)]}$$
$$= 90\sqrt{50\cdot58118} = 90 \times 7\cdot112 = 640\cdot08, \text{ as bef.}$$

EXAMPLE II.

In the plane triangle ABC,

Given { AB 345 yards, AC 174·07 yards, < A 37° 20′ } *Ans.* { ∠ B 27° 4′, BC 232, < C 115° 36′.

Required the other parts.

EXAMPLE III.

In the plane triangle ABC,

Given { AB 365 poles, AC 154.33 poles, < A 57° 12′ } *Ans.* { BC 309 86, < B 24° 45′, < C 98° 3′.

Required the other parts.

EXAMPLE IV.

In the plane triangle ABC,

Given { AC 120 yards, BC 112 yards, < C 57° 58′ 39″ } *Ans.* { AB 112 65, < A 57° 27′, < B 64° 34′ 21″

Required the other parts.

CASE III.

10, When the 3 sides of a plane triangle are given, to find the angles.

1st *Method.* Assume the longest of the three sides as base, then say, conformably with prop. 16,

As the base,
To the sum of the two other sides;
So is the difference of those sides,
To the difference of the segments of the base.

Half the base added to the said difference, gives the greater segment, and made less by it gives the less; and thus, by means of the perpendicular from the vertical angle, divides the original triangle into two, each of which falls under the first case.

2d *Method.* Find any one of the angles by means of prop. 18, of the preceding section; and the remaining angles either by a repetition of the same rule, or by the relation of sides to the sines of their opposite angles.

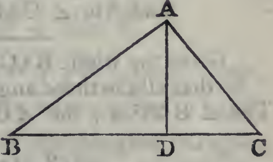
Thus, $\cos C = \frac{AC^2 + BC^2 - AB^2}{2\,AC\,.\,BC}$; $\cos B = \frac{AB^2 + BC^2 - AC^2}{2\,AB\,.\,BC}$

And $\cos A = \frac{BA^2 + AC^2 - BC^2}{2\,AB\,.\,AC}$.

EXAMPLE I.

In the plane triangle ABC,

Given the sides { BC 345 yards, AB 232, AC 174.07;
to find the angles.



1. *Geometrically.*

Draw the base BC = 345 by a scale of equal parts; with radius 232, and centre B, describe an arc; and with radius 174, and centre C, describe another arc, cutting the former in A: join AB, AC, and it is done.

Then by measuring the angles, they will be found to be nearly as follows: that is, < A 27°, < B 37°$\frac{1}{3}$, < C 115°$\frac{2}{3}$.

2. *Arithmetically.*

Having let fall the perpendicular AD, it will be,
As the base BC : AB + AC :: AB — AC : BD — CD; that is, 345 : 406.07 :: 57.93 : 68.18 = BD — CD,

Its half is 34.09
The half base is 172.50

The sum of these is 206.59 = BD
And their difference 138.41 = CD

Then in the triangle ABD, right-angled at D:

As the side AB........232.........log. 2.3654880
To sine op. ∠ D.....90°............10.0000000
So is side BD.........206.59...........2.3151093
To sine op. ∠ BAD...62° 56′...........9.9496213
Which, taken from...90° 00

Leaves the ∠ B 27° 04′

Again in the triangle CDA, right-angled at D;

As the side AC........174.07.....log. 2.2407239
To sine op. ∠ D......90°............10.0000000
So is side CD.........138.41...........2.1411675
To sine op. ∠ CAD....52° 40′..........9.9004436
Which, taken from.....90° 00

Leaves the ∠ C 37° 20′

Also, the ∠ BAD 62° 56′
Added to ∠ CAD 52 40

Gives the whole BAC 115 36

So that all the three angles are as follows:
The ∠ B 27° 4′; the ∠ C 37° 20′; and the ∠ A 115° 36′.

3. *Instrumentally.*

In the first proportion. Extend the compasses from 345 to 406, on the line of numbers; then that extent will reach, on the same line, from 58 to 68.2, nearly, which is the difference of the segments of the base.

In the second proportion. Extend from 232 to 206½, on the line of numbers; then that extent will reach, on the sines, from 90° to 63°.

In the third proportion. Extend from 174 to $138\frac{1}{2}$; then, that extent will reach from 90° to $52^{\circ}\frac{2}{3}$ on the line of sines.

EXAMPLE II.

In the plane triangle ABC,

Given the sides	AB 365 poles, AC 154.33 BC 309.86.	*Ans.*	∠ A 57° 12', ∠ B 24 45, ∠ C 98 3,

To find the angles.

EXAMPLE III.

In the plane triangle ABC,

Given the sides	AB 120, AC 112.6 BC 112.	*Ans.*	∠ A 57° 27′ 00″ ∠ B 57 58 39 ∠ C 64 34 21

To find the angles.

Right-angled Plane Triangles.

1. Right-angled triangles may, as well as others, be solved by means of the rule to the respective case under which any specified example falls: and it will then be found, since a right angle is always one of the data, that the rule usually becomes simplified in its application.

2. When two of the sides are given, the third may be found by means of the property in *Practical Geometry, Def.* 16.

$$\text{Hypoth.} = \sqrt{(\text{base}^2 + \text{perp.}^2)}$$

$$\text{Base} = \sqrt{(\text{hyp.}^2 - \text{perp.}^2)} = \sqrt{(\text{hyp.} + \text{per.}) . (\text{hyp.} - \text{per})}$$

$$\text{Per.} = \sqrt{\text{hyp.}^2 - \text{base}^2)} = \sqrt{(\text{hyp} + \text{base}) . . (\text{hyp.} - \text{base})}.$$

3. There is another method for right-angled triangles, known by the phrase *making any side radius*; which is this:

" *To find a side.* Call any one of the sides radius, and write upon it the word *radius*; observe whether the other sides become sines, tangents, or secants, and write those words upon them accordingly. Call the word written upon each side the *name* of each side: then say,

As the *name* of the given side,
Is to the given side;

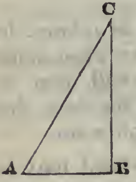
So is the *name* of the required side,
To the required side."

"*To find an angle.* Call either of the *given* sides radius, and write upon it the word *radius*; observe whether the other sides become sines, tangents, or secants, and write those words on them accordingly. Call the word written upon each side the *name* of that side: then say,

As the side made radius,
Is to radius;
So is the other given side,
To the *name* of that side,
which determines the opposite angle."

4. When the numbers which measure the sides of the triangle are either under 12, or resolvable into factors which are each less than 12, the solution may be obtained, conformably with this rule, easier without logarithms than with them. For,

Let ABC be a right-angled triangle, in which AB, the base, is assumed to be radius; BC is the tangent of A, and AC, its secant, to that radius; or, dividing each of these by the base, we shall have the tangent and secant of A, respectively, to radius 1. Tracing, in like manner, the consequences of assuming BC, and AC, each for radius, we shall readily obtain these expressions.



1. $\frac{\text{perp.}}{\text{base}}$ = tan angle at base.

2. $\frac{\text{base}}{\text{perp.}}$ = tan angle at vertex.

3. $\frac{\text{hyp.}}{\text{base}}$ = sec angle at base.

4. $\frac{\text{hyp.}}{\text{perp.}}$ = sec angle at vertex.

5. $\frac{\text{perp.}}{\text{hyp.}}$ = sin angle at base.

6. $\frac{\text{base}}{\text{hyp.}}$ = sin angle at vertex.

EXAMPLE I.

In the right angled triangle ABC,

Given { the leg AB 162 ; ∠ A 53° 7′ 48″ } to find AC and AB.

1. *Geometrically.*

Make AB = 162 equal parts, and the angle A = 53° 7′ 48″; then raise the perpendicular BC, meeting AC in C. So shall AC measure 270, and BC 216.

2. *Arithmetically.*

As radius......tang.	45°..........	log. 10.0000000
To leg AB..........	162..............	2.2095150
So tang. ∠ A........	53° 7′ 48″.........	10.1249371
To leg BC...........	216................	2 3344521
So secant ∠ A......	53° 7′ 48″.........	10.2218477
To hyp. AC.........	270................	2.4313627

3. *Instrumentally.*

Extend the compasses from 45° to $53\frac{1}{8}$°, on the tangents: then that extent will reach from 162 to 216 on the line of numbers.

EXAMPLE II.

In the right-angled triangle ABC,

Given { the leg AB 180 ; the < A 62°40′ } *Ans.* { AC 392.046. ; BC 348.246.

To find the other two sides.

EXAMPLE III.

In the right-angled triangle ABC,

Given { the < A 39° 10′ ; the perpen. BC = 384 } *Ans.* { AC 480. ; AB 288.

To find the other two sides.

SECTION 2. ***On the Heights and Distances of Objects.***

The instruments employed to measure angles are quadrants, sextants, theodolites, &c. the use of either of which may be sooner learned from an examination of the instruments themselves, than of any description independently of them. For military men and for civil engineers, a good pocket sextant, and an accurate micrometer, (such as Cavallo's,) attached to a telescope, are highly useful. For measuring small distances, as bases, 50 feet and 100 feet chains, and a portable box of graduated tape, will be necessary.

We shall here present a selection of such examples as are most likely to occur.

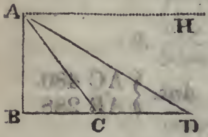
EXAMPLE I.

In order to find the distance between two trees, A and B, which could not be directly measured because of a pool which occupied much of the intermediate space, I measured the distance of each of them from a third object, C, viz. AC = 588, BC = 672, and then at the point C took the angle ACB between the two trees = 55° 40′. Required their distance.

This is an example to case 2 of plane triangles, in which two sides and the included angle are given. The work, therefore, may exercise the student: the answer is 593·8.

EXAMPLE II.

Wanting to know the distance between two inaccessible objects, which lie in a direct line from the bottom of a tower on whose top I stood, I took the angles of *depression* of the two objects, viz. of the most remote 25½°, of the nearest 57°. What is the distance between them, the height of the tower being 120 feet?



The figure being constructed, as in the margin, AB = 120 feet, the altitude of the tower, and AH, the horizontal line drawn through its top; there are given,

HAD = 25° 30′, hence BAD = BAH − HAD = 64° 30′.
HAC = 57° 0′, hence BAC = BAH − HAC = 33° 0′.

Hence the following calculation, by means of the *natural* tangents. For, if AB be regarded as radius, BD and BC will be the tangents of the respective angles BAD, BAC, and CD the difference of those tangents. It is, therefore, equal to the product of the difference of the natural tangents of those angles into the height AB.

Thus, nat. tan $64\frac{1}{2}°$ =	2·0965436
nat. tan 33° =	0·6494076
difference	1·4471360
multiplied by height ...	120
gives distance CD ..	173·6563200

EXAMPLE III.

Standing at a measurable distance, on a horizontal plane, from the bottom of a tower, I took the angle of elevation of the top; it is required from thence to determine the height of the tower.

In this case, there would be given AB and the angle A (see the figure in *Right-angled Triangles*,) to find BC = AB × tan A.

By logarithms, when the numbers are large, it will be, BC = log. AB + log. tan A.

Note. If angle A = 11° 19′ then BC = $\frac{1}{5}$ AB *very nearly.*
A = 16 42 ....BC = $\frac{3}{10}$ AB ....
A = 21 48 ....BC = $\frac{2}{5}$ AB ....
A = 26 34 ....BC = $\frac{1}{2}$ AB ....
A = 30 58 ....BC = $\frac{3}{5}$ AB ....
A = 35 0 ....BC = $\frac{7}{10}$ AB ....
A = 38 40 ....BC = $\frac{4}{5}$ AB ....
A = 45° ....BC = AB, *exactly.*

To save the time of computation, therefore, the observer may set the instrument to one of these angles, and advance or recede, till it accords with the angle of elevation of the object: its height above the horizontal level of the observer's eye, will at once be known, by taking the appropriate fraction of the distance AB.

EXAMPLE IV.

Wanting to know the height of a church-steeple, to the bottom of which I could not measure on account of a high wall between me and the church, I fixed upon two stations at the distance of 93 feet from each other, on a horizontal line from the bottom of the steeple, and at each of them took the angle of elevation of the top of the steeple, that is, at the nearest station 55° 54′, at the other 33° 20′. Required the height of the steeple.

Recurring to the figure of Example II, we have given the distance CD, and the angles of elevation at C and D. The quickest operation is by means of the natural tangents, and the theorem $AB = \frac{CD}{\cot D - \cot C}$.

Thus $\cot D = \cot 33° 20' = 1.5204261$

$\cot C = \cot 55\ 54 = .6770509$

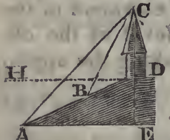
Their difference $= .8433752$

Hence, $AB = \frac{93}{.8433752} = 110.27$ feet.

EXAMPLE V.

Wishing to know the height of an obelisk standing at the top of a regularly sloping hill, I first measured from its bottom a distance of 36 feet, and there found the angle formed by the inclined plane and a line from the centre of the instrument to the top of the obelisk 41°; but after measuring on downward in the same sloping direction 54 feet farther, I found the angle formed in like manner to be only 23° 45′. What was the height of the obelisk, and what the angle made by the sloping ground with the horizon?

The figure being constructed, as in the margin, there are given in the triangle ACB, all the angles and the side AB, to find BC. It will be obtained by this proportion, as sin C (= 17° 15′ = B — A): AB (= 54) :: sin A (= 23° 45′) : BC = 73.3392. Then, in the triangle DBC are known BC as above, BD = 36, CBD = 41°; to find the other angles, and the side CD. Thus, first, as CB + BD : CB — BD :: tan ½ (D + C) = ½ (139°) : tan ½ (D — C) = 42° 24½′. Hence



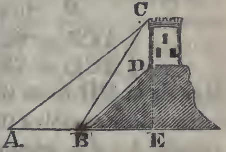
$69° 30' + 42° 24\frac{1}{2}' = 112° 54\frac{1}{2}° =$ CDB, and $69° 30' - 42° 24\frac{1}{2}' = 26° 5\frac{1}{2}' =$ BCD. Then, sine BCD : BD :: sin CBD : CD = 51.86, height of the obelisk.

The angle of inclination DAE = HDA = CDB — 90° = $22° 54\frac{1}{2}'$.

Remark.—If the line BD cannot be measured, then the angle DAE of the sloping ground must be taken, as well as the angles CAB, and CBD In that case DAE + 90° will be equal to CDB : so that after CB is found from the triangle ACB, CD may be found in the triangle CBD, by means of the relation between sides and the sines of their opposite angles.

EXAMPLE VI.

Being on a horizontal plane, and wanting to ascertain the height of a tower standing on the top of an inaccessible hill, I took the angle of elevation of the top of the hill 40°, and of the top of the tower 51°, then measuring in a direct line 180 feet farther from the hill, I took in the same vertical plane the angle of elevation of the top of the tower 33° 45′. Required from hence the height of the tower.



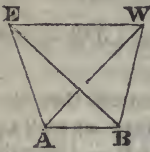
The figure being constructed as in the margin, there are given AB = 180, CAB = 33° 45′, ACB = CBE — CAE = 17° 15′, CBD = 11°, BDC = 180° — (90° — DBE) = 130°. And CD may be found from the expression CD rad^4 = AB sin A sin CBD cosec ACB sec DBE.

Or, using the logarithms, it will be log AB + log sin A + log sin B + log cosec ACB + log sec DBE — 40 (in the index) = log CD ; in the case proposed = log. of 83.9983 feet.

EXAMPLE VII.

In order to determine the distance between two inaccessible objects, E and W, on a horizontal plane, we measured a convenient base AB of 536 yards, and at the extremities A and B took the following angles, viz. BAW =

40° 16′, WAE = 57° 40′, ABE = 42° 22′, EBW = 71° 7′. Required the distance EW.

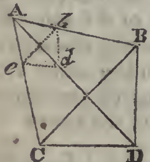


First, in the triangle ABE are given all the angles, and the side AB, to find BE. So, again, in the triangle ABW, are given all the angles and AB to find BW. Lastly, in the triangle BEW are given the two sides EB, BW, and the included angle EBW, to find EW = 939.52 yards.

Remark.—In like manner, the distances taken two and two, between any number of remote objects posited around a convenient station line, may be ascertained.

EXAMPLE VIII.

Suppose that in carrying on an extensive survey, the distance between two spires A and B has been found equal to 6594 yards, and that C and D are two eminences conveniently situated for extending the triangles, but not admitting of the determination of their distance by actual admeasurement: to ascertain it, therefore, we took at C and D the following angles, viz.



{ ACB = 85° 46′ { ADC = 31° 48′
{ BCD = 23° 56′ { ADB = 68° 2′

Required CD from these data.

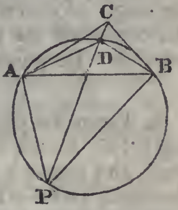
In order to solve this problem, construct a similar quadrilateral A *c d b*, assuming *c d* equal to 1, 10, or any other convenient number: compute A *b* from the given angles, according to the method of the preceding example. Then, since the quadrilaterals A *c d b*, ACDB, are similar, it will be, as A *b* : *c d* :: AB : CD ; and CD is found = 4694 yards.

EXAMPLE IX.

From a convenient station P, where could be seen three objects, A, B, and C, whose distances from each other were known (viz. AB = 800, AC = 600, BC = 400 yards), I took the horizontal angles APC = 33° 45′, BPC = 22° 30′. It is hence required to determine the respective distances of my station from each object.

Here it will be necessary, as preparatory to the computation, to describe the manner of

Construction —Draw the given triangle ABC from any convenient scale. From the point A draw a line AD, to make with AB an angle equal to 22° 30′, and from B a line BD, to make an angle DBA = 33° 45′. Let a circle be described to pass through their intersection D, and through the points A and B. Through C and D draw a right line to meet the circle again in P: so shall P be the point required. For, drawing PA, PB, the angle APD is evidently = ABD, since it stands on the same arc AD: and for a like reason BPD = BAD. So that P is the point where the angles have the assigned value.



The result of a careful construction of this kind, upon a good sized scale, will give the values of PA, PC, PB, true to within the 200dth part of each.

Manner of Computation.—In the triangle ABC where the sides are known, find the angles. In the triangle ABD, where all the angles are known, and the sides AB, find one of the other sides AD. Take BAD from BAC, the remainder, DAC is the angle included between two known sides AD, AC; from which the angles ADC and ACD may be found, by Trig. case 2. The angle CAP = 180° − (APC + ACD). Also, BCP = BCA − ACD; and PBC = ABC + PBA = ABC + sup. ADC. Hence, the three required distances are found by these proportions. As sin APC : AC :: sin PAC : PC, and :: sin PCA : PA; and lastly, as sin BPC : BC :: sin BCP : BP. The results of the computation are, PA = 709.33, PC = 1042.66, PB = 934 yards.

⁎ The computation of problems of this kind, however, may be a little shortened by means of an analytical investigation. Those who wish to pursue this department of trigonometry may consult the treatises by *Bonnycastle, Gregory, Woodhouse*, and *Legendre's Geo. and Trig.*

Note.—If C had been *below* AB, the general principles of construction and computation would be the same; and the modification in the process very obvious.

Practical Questions in Trigonometry.

1. Having measured 100 feet from the bottom of a tower, in a direct line from it, on a horizontal plane, I then took the angle of elevation of the top, and found it to be 47° 30′; the centre of the quadrant being fixed five feet above the ground. Required the height of the tower.

Ans. 114.13 feet.

2. From the edge of a ditch of 18 feet wide, surrounding a fort, I took the angle of elevation of the top of the wall, and found it to be 62° 40′. Required the height of the wall, and the length of the ladder necessary to reach from my station to the top of it.

Ans. 34.82 = *height, and* 39.2 = *the length of the ladder.*

3. From the top of a ship's mast, which was 80 feet above the water, the angle of depression of another ship's hull, at a distance on the water, is 20 feet: what is their distance? *Ans.* 219.79 *feet, nearly.*

4. What is the perpendicular height of a hill, whose angle of elevation, taken at the bottom of it, was 46°; and 100 yards farther off, on a level with the bottom of it, the angle was 31°? *Ans.* 143.14 *feet.*

5. An obelisk standing on the top of a declivity, I measured from its bottom a distance of 40 feet, and then took the angle formed by the plane and a line drawn to the top 41°; going on in the same direction 60 feet farther, the same angle was 23° 45′, the height of the instrument being 5 feet: what was the height of the obelisk?

Ans. 62.623 *feet.*

6. Wanting to know the height of an inaccessible object; at the least distance from it, on a horizontal plane, I took its angle of elevation equal to 58°, and going 100 yards directly farther from it, found the angle there to be only 32°: required its height, and my distance from it at the first station, the instrument being 5 feet above the ground at each observation.

Ans. 104.17 = *the height, and* 64.05 *yards* = *the distance.*

7 Being on a horizontal plane, and wanting to know the height of an object on the top of an inaccessible hill: I

took the angle of elevation of the top of the hill 40°, and of the top of the object equal 51°; measuring then in a direct line from it, to the distance of 100 yards farther, I found the angle of the top of the object to be 33° 45′: what is the object's height? *Ans.* 46.66574.

8. From a window near the bottom of a house, which seemed to be on a level with the bottom of a steeple, I took the angle of elevation of the top of the steeple, equal 40°, and from another window 18 feet directly above the former, the same angle was 37° 30′: what, then, is the height and distance of the steeple?

Ans. 210.44 *feet = height and* 250.79 = *distance.*

9. What is the perpendicular height of a cloud, whose angle of elevation are 35° and 64°, taken by two observers, at the same time, both on the same side of the cloud, and at the distance of 880 yards asunder, so placed that a vertical plane would pass through both their stations and the cloud; and what are its distances from the places of observation?

Ans. 937.757 = *height;* 1041.125 *and* 1631.442 = *the distances.*

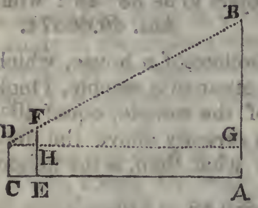
10 Wanting to know the breadth of a river, I measured 100 yards in a straight line close by one side of it; and at each end of this line I found the angles subtended by the other end and a tree close by the other side of the river, to be 35° and 79° 12′: what is the perpendicular breadth? *Ans.* 105.897 *yards.*

II. *Determination of Heights and Distances by approximate mechanical methods.*

1. *For Heights.*

1. *By shadows*, when the sun shines. Set up vertically a staff of known length, and measure the length of its shadow upon a horizontal or other plane; measure also the length of the shadow of the object whose height is required. Then it will be, as the length of the shadow of the staff is to the length of the staff itself; so is the length of the shadow of the object, to the object's height.

2. *By two rods or staves set up vertically.* Let two staves, one say of 6 feet, the other of 4 feet long, be placed upon horizontal, circular, or square feet, on which each may stand steadily. Let AB be the object, as a tower or steeple, whose altitude is required, and AC the horizontal plane passing through its base. Let CD and EF, the two rods, be placed with their bases in one and the same line CA, passing through A the foot of the object; and let them be moved nearer to, or farther from, each other, until the summit B of the object is seen in the same line as D and F, the tops of the rods. Then by the principle of similar triangles, it will be, as DH (= CE) : FH :: DG (= CA) : BG; to which add AG = CD, for the whole height AB.



3. *By Reflection.*—Place a vessel of water upon the ground, and recede from it, until you see the top of the object reflected from the smooth surface of the liquid. Then, since by a principle in optics, the angles of incidence and reflection are equal, it will be as your distance measured horizontally from the point at which the reflection is made, is to the height of your eye above the reflecting surface; so is the horizontal distance of the foot of the object from the vessel, to its altitude above the said surface.*

4. *By means of a portable barometer and thermometer.*—Observe the altitude, B, of the mercurial column, in inches, tenths, and hundredths, at the *bottom* of the hill, or other object whose altitude is required: observe, also, the altitude, b, of the mercurial column at the *top* of the object; observe the temperatures on Fahrenheit's thermometer, at the times of the two barometrical observations, and take the mean between them. Then $55000 \times \frac{B-b}{B+b}$ = height of the hill, in

* Leonard Digges, in his curious work the Pantometria, published in 1571, first proposed a method for the determination of altitudes by means of a geometrical square and plummet, which has been described by various later authors, as Ozanam, Donn, Hutton, &c. But, as it does not seem preferable to the methods above given, I have not repeated it here.

feet, for the temperature of 55° on Fahrenheit. Add $\frac{1}{440}$ of this result for every degree which the mean temperature exceeds 55°; subtract as much for every degree below 55°.

This will be a good approximation when the height of the hill is below 2000; and it is easily remembered, because 55°, the assumed temperature, agree with 55, the effective figures in the coefficient; while the effective figures in the denominator of the correcting fraction are two *fours*.

*** Where great accuracy is required, logarithmic rules become necessary, of which various are exhibited in treatises on Pneumatics.

5. *By an extension of the principle of pa* 214. Set the sextant, or other instrument, to the angle 45°, and find the point C (pa. 215) on the horizontal plane, where the object AB has that elevation: then set the instrument to 26° 34′, and recede from C, in direction BCD, till the object has that elevation. *The distance* CD *between the two stations will be* = AB.

So, again, if $C = 40°$, $D = 24° 31\frac{1}{2}'$, CD will be = AB.
or, if $C = 35°$, $D = 22° 33'$, CD = AB.
or, if $C = 30°$, $D = 20° 6'$, CD = AB.
or, generally, if $\cot D - \cot C = \text{rad}\ CD = AB$.

6. *For deviation from level.*—Let E represent the elevation of the tangent line to the earth above the true level, in feet and parts of a foot, D the distance in miles: then $E = \frac{2}{3} D^2$.

This gives 8 inches for a distance of one mile; and is a near approximation when the distance does not exceed two or three miles.

EXAMPLES.

1. What is the height of an object whose shadow is 40 feet; the shadow of a pole 6 feet in height being 4 feet?
Ans. 60 *feet.*

2. Two poles, the one 4 feet, and the other 7 feet, are set up vertically; their distance asunder being 8 feet, and the distance of the shorter pole from an object is equal to 100 feet; what is the height of the object?
Ans. 64 *feet.*

3. A reflecting surface is placed 84 feet from the bottom of an object; and a person at the distance of 7 feet, in

a direct line from the object, with his eye 5 feet above the ground, views the image of the object at the reflecting surface: what is the height of the object? *Ans.* 60 *feet.*

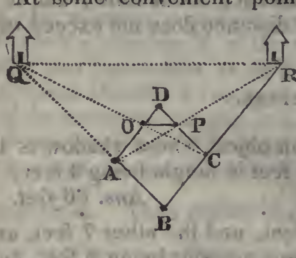
2. *For Distances.*

1. *By means of a rhombus set off upon a horizontal plane.*

Suppose O the object and OB the required distance. With a line or measuring tape, whose length is equal to the side of the intended rhombus, say 50 or 100 feet, lay down one side BA in the direction BO toward the object, and BC another side in any convenient direction (for whether B be a right angle or not, is of no consequence); and put up rods or arrows at A and C. Then fasten two ends of two such lines at A and C, and extend them until the two other ends just meet together at D; let them lie thus stretched upon the ground, and they will form the two other sides of the rhombus AD, CD. Fix a mark or arrow at R, directly between C and O, upon the line AD; and measure RD, RA upon the tape. Then it will be as RD : DC :: CB : BO, the required distance.

Otherwise. To find the length of the inaccessible line QR.

At some convenient point B, lay down the rhombus BADC, so that two of its sides BA, BC, are directed to the extremities of the line QR. Mark the intersections, O and P, of AR, CQ, with the sides of the rhombus (as in the former method): then the triangle ODP will be similar to the triangle RBQ; and the inaccessible distance RQ will be found $= \frac{OP \times BA^2}{OD \times DP}$.*



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* For PD : DA :: AB : BR $= \frac{AB^2}{PD}$;

and OD : OP :: BR : RQ $= \frac{AB^2 . OP}{OD . DP}$.

Thus, if BA = BC, &c. = 100 f. OD = 9 f. 5 in., DP = 11 f. 10 in. OP = 13 f. 7 in. then $QR = \frac{10000 + 13\frac{7}{12}}{9\frac{5}{12} + 11\frac{10}{12}}$ = 1219 feet.

2. *By means of a micrometer attached to a telescope.*

Portable instruments, for the purpose of measuring extremely small angles, have been invented by Martin, Cavallo, Dollond, Brewster, and others. In employing them for the determination of distances, all that is necessary in the practice is to measure the angle subtended by an object of known dimensions, placed either vertically or horizontally, at the remoter extremity of the line whose length we wish to ascertain. Thus, if there be a house, or other erection, built with bricks, of the usual size; then four courses in *height* are equal to *a foot*, and four in *length* equal to *a yard:* and distances measured by means of these will be tolerably accurate, if care be taken with regard to the angle subtended by the horizontal object, to stand directly in front of it. A man, a carriage-wheel, a window, a door, &c. at the remoter extremity of the distance we wish to ascertain, may serve for an approximation. But in all cases where it is possible, let a foot, a yard, or a six-feet measure, be placed vertically, at one end of the line to be measured, while the observer, with his micrometer, stands at the other. Then, if h be the height of the object,

either $\frac{1}{2} h \times \cot \frac{1}{2}$ angle subtended
or $h \times \cot$ angle

will give the distance, according as the eye of the observer is horizontally opposite to the *middle*, or to *one extremity* of the object whose angle is taken.

When a table of natural tangents is not at hand, a *very* near approximation for all angles less than *half a degree*, and a tolerably near one up to angles of *a degree*, will be furnished by the following rules.

1. If the distant object whose angle is taken be 1 foot in length, then

3437.73 ÷ the angle in minutes
or 206264· ÷ the angle in seconds } will give the distance in feet.

2. If the remote object be 3, 6, 9, &c. feet in length, multiply the former result by 3, 6, 9, &c. respectively.

Ex. 1. What is the distance of a man 6 feet high, when he subtends an angle of 30 seconds?

$206264 \times 6 \div 30 = 206264 \div 5 = 41252{\cdot}8$ feet $= 13750{\cdot}9$ yards, the distance required.

Ex. 2. In order to ascertain the length of a street, I put up a foot measure at one end of it, and standing at the other, found that measure to subtend an angle of 2 minutes: required the length of the street.

$$3437{\cdot}73 \div 2 = 1718{\cdot}86 \text{ feet} = 572{\cdot}95 \text{ yards.}$$

3. *By means of the velocity of sound.*

Let a gun be fired at the remoter extremity of the required distance, and observe, by means of a chronometer that measures tenths of seconds, the interval that elapses between the flash and the report: then estimate the distance for one second by the following rule, and multiply that distance by the observed interval of time; the product will give the whole distance required.

At the temperature of freezing, 33°, the velocity of sound is 1100 feet per second.

For lower temperatures deduct } half a foot.
For higher temperatures add }

From the 1100 } for every degree of difference from 33°
to the 1100 }

on Fahr. therm.; the result will show the velocity of sound, very nearly, at all such temperatures.

Thus, at the temperature of 50°, the velocity of sound is,

$$1100 + \tfrac{1}{2}(50 - 33) = 1108\tfrac{1}{2} \text{ feet.}$$

At temperature 60°, it is $1100 + \frac{1}{2}(60 - 33) = 1113\frac{1}{2}$ feet.

Note 1. The velocity of sound is usually reckoned at the rate of 1142 feet per second, without regarding the temperature of the atmosphere; therefore, any distance may be readily found, in feet, by multiplying 1142 by the time, in seconds, which the sound takes to arrive at the ear; or by $\frac{3}{14}$ of the time, for the distance in miles.

Note 2. The time taken for the passage of sound, in the interval between seeing a flash of lightning, or that of a gun, and hearing the report, may be determined by the beats of the pulse, counting, on an average, about 70 to a minute, for persons in moderate health, or $5\frac{1}{2}$ pulsations for a mile.

EXAMPLES.

1. After observing a flash of lightning, it was 12 seconds before I heard the thunder: required the distance of the cloud from whence it came.

Here, $12 \times \frac{3}{14} = \frac{36}{14} = 2\frac{4}{7}$ *miles, the answer.*

2. How long, after firing the Navy-Yard guns, may the report be heard at Harlem, supposing the distance to be 8 miles in a straight line?

Here, $8 \times \frac{14}{3} = \frac{112}{3} = 37\frac{1}{3}$ *seconds.*

3. After observing the firing of a piece of ordnance at a distance, it was 7 seconds before I heard the report: what was its distance? *Ans.* $1\frac{1}{2}$ *miles.*

4. Perceiving a man at a distance, hewing down a tree with an axe, I remarked that 6 of my pulsations passed between seeing him strike and hearing the report of the blow: what was the distance between us, allowing 70 pulsations to a minute? *Ans.* 1 *mile and* 198 *yards.*

5. How far off was the cloud from which thunder issued, whose report was 5 pulsations after the flash of lightning, counting 75 to a minute? *Ans.* 1523 *yards.*

6. Counting 17 seconds between the time of seeing the flash and hearing the report of a gun; what is the distance from it? *Ans.* $3\frac{2}{3}$ *miles.*

SURVEYING.

The art of measuring and dividing land, and of delineating its boundaries, is usually called *surveying.*

The instruments generally used in surveying, are the *chain* and the *circumferentor*, commonly called the *surveyor's compass.*

The length of a line in surveying, estimated horizontally, is called the *distance* of that line; and distances, or the boundaries of a survey, are measured either by a *four-pole chain* or by a *two-pole chain.*

The four-pole chain is 4 poles, or 66 feet in length, and is divided into 100 equal parts, called *links:* hence, the length of a link is 7·92 inches.

The two-pole chain is two poles, or 33 feet in length, and is divided into 50 links.

For the more ready reckoning the links of a four-pole chain, there is a large ring, or sometimes a round piece of brass, fixed at every 10 links; and at 50 links, or in the middle, there are two large rings. In such chains as have a brass piece at every ten links, there is the figure 1 on the first piece, 2 on the second, 3 on the third, &c. to 9. By leading, therefore, that end of the chain forward which has the least number next to it, he who carries the hinder end may easily determine any number of links. Thus, if he has the brass piece number 8 next to him, and six links more in a distance, that distance is 86 links. After the same manner 10 may be counted for every large ring of a chain that has not brass pieces on it; and the number of links is thus readily determined.

The two-pole chain has a large ring at every 10 links, and in its middle, or at 25 links, there are 2 large rings; so that any number of links may be the more readily counted off, as before.

The surveyor should be careful to have his chain measured before he proceeds on business; for the rings are apt to open by frequently using it, and its length is thereby increased; so that no one can be too circumspect in this point.

In measuring a stationary distance, there is an object fixed in the extreme point of the line to be measured; this is a direction for the hinder chainman to govern the foremost one by, in order that the distance may be measured in a right line; for if the hinder chainman causes the other to cover the object, it is plain the foremost is then in a right line towards it. For this reason it is necessary to have a person that can be relied on, at the hinder end of the chain, in order to keep the foremost man in a right line; and a surveyor who has no such person, should chain himself. The inaccuracies of most surveys arise from bad chaining, that is, from straying out of the right line, as well as from other omissions of the hinder chainman: no person, therefore, should be admitted at the hinder end of the chain, of whose abilities in this respect, the surveyor was not previously convinced; since the success of the survey, in a great measure, depends on his care and skill.

In setting out to measure any stationary distance, the fore man of the chain carries with him 10 iron pegs pointed, each about 10 inches long; and when he has stretched the chain to its full length, he, at the extremity thereof, sticks one of those pegs perpendicularly in the ground; and leaving it there, he draws on the chain till the hinder man checks him when he arrives at that peg: the chain being again stretched, the fore man sticks down another peg, and the hind man takes up the former; and thus they proceed at every chain's length contained in the line to be measured, counting the surplus links contained between the last peg and the object at the termination of the line, as before: so that the number of pegs taken up by the hinder chainman expresses the number of chains; to which, if the odd links be annexed, the distance line required in chains and links is obtained, which must be registered in the field-book, as will hereafter be shown.

If the distance exceeds 10, 20, 30, &c. chains, when the leader's pegs are all exhausted, the hinder chainman, at the extremity of the 10 chains, delivers him all the pegs; from whence they proceed to measure as before, till the leader's pegs are again exhausted, and the hinder chainman at the extremity of these 10 chains again delivers him the pegs, from whence they proceed to measure the whole distance line in the like manner; then it is plain, that the number of pegs the hinder chainman has, being added to 10, if he had delivered all the pegs once to the leader, or to 20 if twice, or to 30 if thrice, &c. will give the number of chains in that distance; to which, if the surplus links be added, the length of the stationary distance is known in chains and links.

It is customary, and, indeed, necessary, to have red, or other colored cloth, fixed to the top of each peg, that the hinder man at the chain may the more readily find them; otherwise, in chaining through corn, high grass, briers, rushes, &c. it would be extremely difficult to find the pegs which the leader puts down: by this means no time is lost, which otherwise must be, if no cloths are fixed to the pegs, as before.

It will be necessary here to observe, that all slant or inclined surfaces, as sides of hills, are measured horizontally, and not on the plane or surface of the hill, and is thus effected:

If the chain, instead of being carried parallel to the surface of the ground, be kept constantly parallel to the horizon, the line thus measured will be the base line required.

If the inclined side of the hill be the plane surface, the angle of the hill's inclination may be taken, and the slant height may be measured on the surface; and thence (by case 1. of right-angled trigonometry) the horizontal line answering to the top may be found; and if we have the angle of inclination given on the other side, with those already given, we can find the horizontal distance across the hill, by case 2. of oblique trigonometry.

All inclined surfaces are considered as horizontal ones; for all trees which grow upon any inclined surface, do not grow perpendicular thereto, but to the plane of the horizon: hence, the base will be capable to contain as many trees as are on the surface of the hill, which is manifest from the continuation of them thereto. And this is the reason that the area of the base of a hill is considered to be equal, in value, to the hill itself.

Besides, the irregularities of the surfaces of hills in general are such, that they would be found impossible to be determined by the most able mathematicians. Certain regular curve surfaces have been investigated with no small pains, by the most eminent; therefore an attempt to determine in general the infinity of irregular surfaces which offer themselves to our view, to any degree of certainty, would be idle and ridiculous; and for this reason also, the horizontal area is only attempted.

Again, if the circumjacent lands of a hill be planned or mapped, it is evident we shall have a plan of the hill's base in the middle: but were it possible to put the hill's surface in lieu thereof, it would extend itself into the circumjacent lands, and render the whole a heap of confusion: so that if the surfaces of hills could be determined, no more than the base could be mapped.

Since some surveys are taken by a four-pole, and others by a two-pole chain, and as ground for houses is measured by feet, we will show how to reduce one to the other, in the following problems.

PROBLEM I.

To reduce two-pole chains and links to four-pole ones.

If the number of chains be even, the half of them will be the four-pole ones, to which annex the given links, thus,

EXAMPLES.

1. In 16 chains 37 links of two-pole chains, how many four-pole ones? *Ans.* 8 chains 37 links.

But if the number of chains be odd, take the half of them for chains, and add 50 to the links, and they will be four-pole chains and links, thus,

2. In 17 chains 42 links of two-pole chains, how many four-pole ones? *Ans.* 8 ch. 92 links.

3. In 21 two-pole chains and 25 links, how many four-pole chains and links? *Ans.* 10 ch. 75 links, or 10·75 ch.

4. In 24 two-pole chains and 9 links, how many four-pole chains and links? *Ans.* 12·09 chains.

PROBLEM II.

To reduce four-pole chains and links to two-pole ones.

Double the chains, to which annex the links, if they be less than 50; but if they exceed 50, double the chains, add one to them, and take 50 from the links, and the remainder will be the links, thus,

EXAMPLES.

Ch. L.
1. In 8 37 of four-pole ch. how many two-pole ones?
2
———
16 37
———

Ch. L.
2. In 8 82 of four-pole ch. how many two-pole ones?
2 50
———
17 32 Answer.

3. How many four-pole chains and links are there in 37 four-pole chains and 57 links? *Ans.* 75 chains and 27 links.

4. In 31 four-pole chains and 25 links, how many two-pole chains and links? *Ans.* 62 ch. 25 links.

PROBLEM III.

To reduce four-pole chains and links to perches and decimals of a perch.

The links of a four-pole chain are decimal parts of it, each link being the hundredth part of a chain; therefore, if the chain and links be multiplied by 4, (for 4 perches are a chain) the product will be the perches and decimal parts of a perch. Thus,

EXAMPLES.

Ch. L.
1. How many perches in 13 64 of four-pole chains?
4
———
Answer, 54 56 perches.

2. Reduce 20 four-pole chains and 17 links to perches and hundredths. *Ans.* 80·68 perches.

3. Reduce 18 four-pole chains and 37 links to perches and hundredths. *Ans.* 73·58 perches.

4. Reduce 33 four.pole chains and 9 links to perches and hundredths. *Ans.* 152·36 perches.

PROBLEM IV.

To reduce two-pole chains and links to perches and decimals of a perch.

They may be reduced to four-pole ones, (by prob. 1), and thence to perches and decimals, (by the last), or,

If the links be multiplied by 4, carrying one to the chains, when the links are, or exceed 25; and the chains by two, adding one, if occasion be; the product will be the perches and decimals of a perch. Thus,

EXAMPLES.

	Ch.	*L.*	
1. In	17	21	of two-pole chains, how many perches?
	2	4	
Ans.	34	84	perches.

	Ch.	*L.*	
2. In	15	38	of two-pole chains, how many perches?
	2	4	
Ans.	31.	52	perches.

3. Reduce 25 two-pole chains and 25 links to perches and decimals of a perch. *Ans.* 51 perches.

4. Reduce 15 two-pole chains and 17 links to perches and decimals of a perch. *Ans.* 30·68 perches.

PROBLEM V.

To reduce perches and decimals of a perch to four-pole chains and links.

Divide by 4, so as to have two decimal places in the quotient, and that will be four-pole chains and links. Thus,

EXAMPLES.

1. In 31.52 perches, how many four-pole ch. and links?

```
      Ch.  L.
4)31.52(7  88 Answer.
  ----
   35
   --
   32
```

2. In 18·24 perches, how many four-pole chains and links? *Ans.* 4 ch. 56 links.

3. Reduce 37·12 perches to four-pole chains and links. *Ans.* 9 ch. 28 links.

4. Reduce 23 perches to four-pole chains and links. *Ans.* 5 ch. 75 links.

PROBLEM VI.

To reduce perches and decimals of a perch to two-pole chains and links.

The perches may be reduced to four-pole chains (by the last), and from thence to two-pole chains (by prob. 2), or,

Divide the whole number by 2, the quotient will be chains; to the remainder annex the given decimals, and divide by 4, the last quotient will be the links. Thus,

EXAMPLES.

1. In 31.52 perches, how many two-pole ch. and links?

```
      Ch.   L.
2)31.52(15  38 Answer.
  ----
  11
  ------
  4)152(38
  ------
    32
```

2. In 12·96 perches, how many two-pole chains and links? *Ans.* 6 ch. 24 links.

3. In 38·08 perches, how many two-pole chains and links? *Ans.* 19 ch. 2 links.

4. Reduce 49·04 perches to two-pole chains and links.
Ans. 24 ch. 26 links.

PROBLEM VII.

To reduce chains and links to feet and decimal parts of a foot.

If they be two-pole chains, reduce them to four-pole ones: (by prob. 1,) these being multiplied by the feet in a four-pole chain, will give the feet and decimals of a foot. Thus,

EXAMPLES.

Ch. L.
In 17 21 of two-pole chains, how many feet?

```
            Ch.  L.
             8   71 of four-pole chains.
                 66 feet = 1 chain.
            -------
              5226               Feet  Inches.
             5226        Answer  574   10¼.
            -------
      Feet  574.86
                12
            -------
    Inches  10.32
                 4
            -------
              1.28
```

2. In 12.37 four-pole chains, how many feet and inches?
Ans. 816 feet 5¼ inches, nearly.

3. Reduce 25 two-pole chains and 25 links to feet and inches. *Ans.* 841 feet 6 inches.

4. Reduce 20.50 four-pole chains to feet.
Ans. 1353 feet.

PROBLEM VIII.

To reduce feet and inches to chains and links.

Reduce the inches to the decimal of a foot, and annex that to the feet; that divided by the feet in a four-pole chain, will give the four-pole chains and links in the quotient; these may be reduced to two-pole chains and links, if required, by prob. 2. Thus,

EXAMPLES.

Feet Inches.
1. In 217 9 how many two-pole chains?
12)9.00.(75 the decimal of 9 inches.
60

66)217.75(3. 29 of four-pole chains, or
197
655 — *Ch.* 6 *L.* 29
61

2. Reduce 185 feet 9 inches to four-pole chains.
Ans. 2.81½, *nearly.*

3. Reduce 750 feet 6 inches to two-pole chains.
Ans. 22 chains 37 links.

4. In 1278 feet 5 inches, how many four-pole chains?
Ans. 19.37 chains, *nearly.*

PROBLEM IX.

To reduce square four-pole chains to acres.

Divide by 10, and the quotient will be acres and decimals of an acre; multiply the decimals, if any, by 4, for the roods, and the decimal of these by 40, for the poles.

EXAMPLES.

1. Reduce 31.375 square chains to acres.

```
10)31.375
   3.1375
        4
   0.5500
       40
  22.0000
```

Ans. 3 ac. 0 r. 22 p.

2. Reduce 255.875 square chains to acres.

Ans. 25 ac. 2 r. 14 poles.

3. Reduce 1875.625 square chains to acres.

Ans. 187 ac. 2 r. 10 poles.

PROBLEM X.

To reduce acres, roods, and perches to square chains.

Reduce the roods and perches to the decimal of an acre; then, the acres and decimal of an acre, multiplied by 10, will give the square chains.

EXAMPLES.

1. Reduce 12 acres, 2 roods, 14 poles, to square chains.

Here, 2 roods and 14 poles is equal to .5875 of an acre; hence,

$$\begin{array}{r} 12.5875 \\ 10 \\ \hline 125.8750 \end{array} \text{ acres.}$$

Ans. 125 acres, 87½ links.

2. Reduce 25 acres and 22 perches to square chains.

Ans. 251.375 acres.

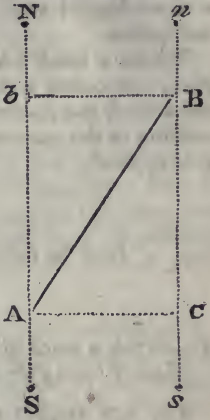
3. Reduce 19 acres, 2 roods, 10 perches to acres.

Ans. 195.625 acres.

A line due north and south, and passing through any point or station of the survey, is called a *meridian line:* and a line in direction of the magnetic needle, is called the *magnetic meridian.* The angle which those two meridians make with one another, is commonly called the *variation of the compass;* but it should be called the *declination* of the needle, reserving the term *variation* for the increase or decrease of the angle which the magnetic meridian makes with different meridians, and, at different epochs, with the same meridian line.

The angle which any line makes with the meridian, reckoning from the north or south points of the horizon, towards the east or west, is called the *bearing* or *course* of that line.

Thus, supposing the line NS represent the meridian passing through the point A, one of the extremities of the line AB, and that the angle NAB is found to be 30° ; then the bearing of the line AB from the point A, is north, 30°, east, which is usually written N. 30° E. The bearing of the line AB, taken from the point B, is called the *reverse bearing*. The bearing and reverse bearing of a line are equal in magnitude, but situated between directly opposite points: thus, the bearing of the line AB, from the point B, is S. 30° W.



This is not strictly true, since the meridians passing through the different points of a survey are not parallel; but as farms of moderate extent, or small portions of the earth's surface, may be considered as planes, there can be no sensible error in considering the meridians passing through any two points of the survey as parallel lines; and, consequently, the bearing of a line must be equal to the reverse bearing. This is of use, in order to ascertain whether the bearing has been taken correctly.

Surveyors sometimes call the angle which a line makes with the magnetic meridian, the bearing: although by this method we could find the area, still it is not very convenient, in delineating and marking a map of the survey.

The *circumferentor*, or *surveyor's compass*, used to find the bearing of lines, is composed of a brass circular box, about five or six inches in diameter, within which is a brass ring, divided on the top into 360°, and graduated from 0 to 360°; that is, numbered 10, 20, 30, &c. to 360°. The brass ring of its face, or in the box, is also divided into 360°, and sometimes into half degrees ; but the degrees are reckoned from the north and south points, to the east and west, reckoning from 0 to 90° each way.

A steel pin, finely pointed, is fixed in the centre of the box, on which is placed a magnetic needle, which points towards the north and south points of the horizon ; allowance, however, must be made for the declination of the needle.

The instrument is fixed on a ball and socket, placed on a three-legged staff, or on a staff having a pointed iron at the bottom.

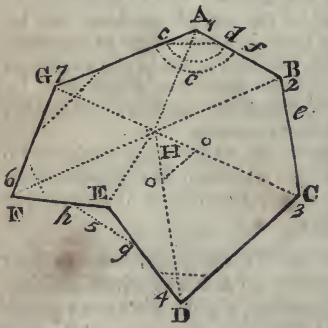
When a small telescope is added to the surveyor's compass, it is called a *theodolite;* and when other appendages are added, the instrument is called by different names, according to the particular construction and the uses to which it is applied.

How to take a survey by the chain only.

PROBLEM I.

To survey a piece of ground by going round it, and the method of taking the angles of the field, by the chain only.

Let ABCDEFG be a piece of ground to be surveyed: beginning at the point A, let one chain be laid in a direct line from A, towards G, where let a peg be left, as at *c;* and again, the like distance from A, in a direct line towards B, where another peg is also to be left, as at *d:* let the distance from *d* to *c* be measured, and placed in the field-book, in the second column under the denomination of angles, in a line with station No. 1; and in the same line, under the title of distances, in the third column, let the measure of the line AB in chains and links be inserted. Being now arrived at B, let one chain be laid in a direct line from B towards A, where let a peg be left, as at *f*, and again, the like distance from B, in a direct line towards C, where let another peg be left, as at *e;* the distance from *e* to *f* is to be inserted in the field-book, in the second column, under angles, in a line with station No. 2; and in the same line, under the title of distances in the third column, let the measure of the line CB, in chains and links, be inserted:



after the same manner we may proceed from C to D, and thence to E; but because the angle at E, viz. FED, is an external angle, after having laid one chain from E to *h*, and to *g*, the distance from *g* to *h* is measured, and inserted in the column of angles, in a line with station No. 5, and on the side of the field-book against that station, we make an asterisk, thus *, or any other mark, to signify that to be an external angle, or one measured out of the ground. We then proceed as before, from E to F, to G, and thence to A, measuring the angles and distances, and placing them as before, in the field-book, opposite to their respective stations; so will the field-book be completed in the manner following.

N. B. After this manner the angles for inaccessible distances may be taken, and the method of constructing or laying them down, as well as the construction of the map, from the following field-notes, must be obvious from the method of taking them.

The form of the Field-Book, with the Title.

A Field-Book of part of the land of Benjamin Townsend, of Oysterbay, Long-Island, State of New-York; surveyed by Gerardus B. Docherty.

TAKEN BY A FOUR-POLE CHAIN.

REMARKS.	NO. STA.	ANGLES. CH. L.	DISTANCE. CH. L.
Mr. J. D's part of the land.......	1	1.80	17.65
	2	1.79	18.50
Mr. L. P's part of Oysterbay strand	3	1.76	28.00
	4	1.41½	20.00
*	5	1.87½	14.83
	6	1.14	19.41
Widow J. G's part of the land....	7	1.89	24.53

Close at the first station.

EXPLANATION OF THE REMARKS.

Mr. J. D.'s part bounds, or is adjacent to the surveyed land from the first to the third station; Mr. L. P's part of Oysterbay strand bounds it from the third to the fourth station; the strand, then, is boundary from thence to the sixth; and from the sixth to the first station, the Widow J. G.'s part is the boundary.

It is absolutely necessary to insert the persons' names, and townlands, strands, rivers, woods, rivulets, &c. which bound or circumscribe the land which is surveyed, for these must be expressed in the map.

In a survey of a town-land, or estate, it is sufficient to mention only the circumjacent town-lands, without the occupiers' names: but when a part only of a town-land is surveyed, then it is necessary to insert the person or persons' names, who hold any particular parcel or parcels of such town-land as bound the parts surveyed.

When an angle is very obtuse, as most in our present figure are, viz. the angles at A, B, C, E, and G; it will be best to lay a chain from the angular point, as at A, on each of the containing sides to *c* and to *d;* and any where nearly in the middle of the angle as at *e:* measuring the distances *ce* and *ed;* and these may be placed for the angle in the field-book. Thus,

No.	Sta.	Angle.	
		Ch. L.	*Ch. L.*
		1.03 } 1.09 }	17.65

For when an angle is very obtuse, the chord line, as *cd*, will be nearly equal to the radii *Ac* and *Ad;* so if the arc *ced* be described, and the chord line *cd* be laid on it, it will be difficult to determine exactly that point in the arc where *cd* cuts it: but if the angle be taken in two parts, as *ce* and *ed*, the arc, and the angle thence, may be truly determined and constructed.

After the same manner, any piece of ground may be surveyed by a two-pole chain.

PROBLEM II.

To take a survey of a piece of ground from any point within it, from whence all the angles can be seen, by the chain only.

Let a mark be fixed at any point in the ground, as at H, (see the fig. to the last Prob.) from whence all the angles can be seen; let the measures of the lines HA, HB, HC, &c. be taken to every angle of the field from the point H; and let those be placed opposite to No. 1, 2, 3, 4, &c. in the second column of the radii: the measures of the respective lines of the mearing, viz. AB, BC, CD, DE, &c. being placed in the third column of distances, will complete the field-book: thus,

REMARKS.	NO.	RADII. CH. L.	DISTANCE. CH. L
	1	20.00	17.65
	2	21.72	18.50
	3	21.74	28.00
	4	25.34	20.00
	5	17.20	14.83
	6	29.62	19.41
	7	21.20	24.53

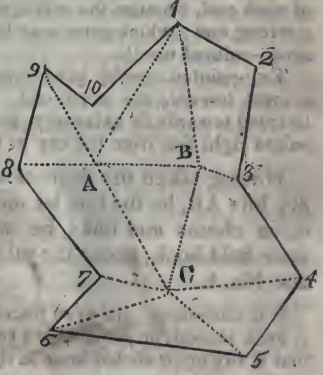
Close at the first station.

If any line of the field be inaccessible, as suppose CD to be, then by way of proof that the distance CD is true, let the measure of the angle CHD be taken by the line *oo*, with the chain: if this angle corresponds with its containing sides, the length of the line DC is truly obtained, and the whole work is truly taken.

Note.—That in setting off an angle it is necessary to use the largest scale of equal parts, viz. that of the inch, which is diagonally divided into 100 parts, in order that the angle should be accurately laid down; or if two inches were thus divided for angles, it would be the more exact; for it is by no means necessary that the angles should be laid from the said scale with the stationary distances.

PROBLEM III.

To take a survey by the chain only, when all the angles cannot be seen from one point within.



Let the ground to be surveyed be represented by 1, 2, 3, 4, &c. Since all the angles cannot be seen from one point, let us assume three points, as A, B, C, from whence they may be seen; at each of which let a mark be put, and the respective sides of the triangle be measured and set down in the field-book; let the distance from A to 1, and from B to 1, be measured, and these will determine the point 1; let the other lines which flow from A, B, C, as well as the circuit of the ground, be then measured as the figure directs; and thence the map may be easily constructed.

There are other methods which may be used, as dividing the ground into triangles, and measuring the three sides of each; or by measuring the base and perpendicular of each triangle. But this we shall speak of hereafter.

To take field-notes by the Circumferentor.

Let your instrument be fixed at any angle, as A, your first station; and let a person stand at the next angle, B, or cause a staff, with a white sheet, to be set there perpendicularly for an object to take your view to: then having placed your instrument horizontally, which is easily done by turning the box so that the ends of the needle may be equidistant from its bottom, and it traverses or plays freely,) turn the flower-de-luce, or north part of the box, to your eye, and looking through the small aperture, turn the index about, till you cut the person or object in the next angle, B, with the horse-hair, or thread, of the opposite sight; the degrees then cut by the south end of the needle, will give the number to be placed in the second column of your field-book, in a line with station No. 1, and expresses the number of degrees the stationary line is from the north, counting quite round with the sun.

Most needles are pointed at the south end, and have a small ring at the north: such needles are better than those which are pointed at each end, because the surveyor cannot mistake by counting to a wrong end; which error may be frequently committed, in using a two-pointed needle.

Two-pointed needles have sometimes a ring, but more usually a cross towards the north end: and the south end is generally bearded towards its extremity, and sometimes not, but its arm is a naked right line from the cap at the centre.

Having taken the degrees or bearing of the first stationary line AB, let the line be measured, and the length thereof, in chains and links, be inserted in the third column of your field-book, under the title of distances, opposite to station No. 1.

It is customary, and even necessary, to cause a sod to be dug up at each station, or place where you fix the instrument: to the end, that if any error should arise in the field-book, it may be the more readily adjusted and corrected, by trying over the former bearings and stationary distances.

Having done with your first station, set the instrument over the hole or spot where your object stood, as at B, for your second station, and send him forward to the next angle of the field, as at C; and having placed the instrument in a horizontal direction, with the sights directed to the object at C, and the north of the box next your eye, count your degrees to the south end of the needle, which register in your field-book, in the second column, opposite to station No. 2; then measure the stationary distance BC, which insert in the third column, and thus proceed from angle to angle, sending your object before you, till you return to the place where you began, and you will have the field-book complete; observing always to signify the parties' names who hold the contiguous lands, and the names of the town-lands, rivers, roads, swamps, lakes, &c. that bound the land you survey, as before; and this is the manner of taking field-notes by what is called fore-sights.

But the generality of mearsmen frequently set themselves in disadvantageous places, so as often to occasion two or more stations to be made, where one may do, which creates much trouble and loss of time; we will therefore show how this may be remedied, by taking back-sights, thus: let your object stand at the point where you begin your survey, as at A; leaving him there, proceed to your next angle, B, where fix your instrument so that you may have the longest view possible towards C. Having set the instrument in a horizontal position, turn the south part of the box next your eye, and having cut your object at A, reckon the degrees to the south point of the needle, which will be the same as if they were taken from the object to the instrument, the direction of the index being the same. Let the degree be inserted in the field-book, and the stationary distance be measured and annexed thereto, in its proper column; and thus proceed from station to station, leaving your object in the last point you left, till you return to the first station, A.

By this method your stations are laid out to the best advantage, and two men may do the business of three; for one of those who chain may be your object; but in fore-sights, you must have an object before you, besides two chainmen.

It was said before, that a surveyor should have a person with him to carry the hinder end of the chain, on whom he can depend: this person should be expert and ready at taking off-sets, as well as exact in giving a faithful return of the length of every stationary line. One who has such a person, and who uses back-sights, will be able to go over nearly double the ground he could at the same time, by taking fore-sights, because of overseeing the chaining; for should he take back-sights, he must be obliged, after taking his degree, to go back to the foregoing station, to oversee the chaining, and by this means to walk three times over every line, which is a labor not to be borne.

Or a back and a fore-sight may be taken at one station, thus: with the south of the box to your eye, observe from B the object A, and set down the degree in your field-book, cut by the south end of the needle. Again, from B observe an object at C, with the north of the box to your eye, and set down the degree cut by the south point of the needle; so you have the bearings of the lines AB and BC; you may then set up your instrument at D, from whence take a back-sight to C, and a fore-sight to E: thus the bearings may be taken quite round, and the stationary distances being anuexed to them, will complete the field-book.

But in this last method, care must be taken to see that the sights have not the least cast on either side; if they have, it will destroy all: and yet with the same sights you may take a survey by fore-sights, or by back-sights only, with as great truth as if the sights were ever so erect, provided the same cast continues without any alteration; but, upon the whole, back-sights only will be found the readiest method.

If your needle be pointed at each end, in taking fore-sights, you may turn the north part of the box to your eye, and count your degrees to the south part of the needle, as before; or you may turn the south of the box to your eye, and count your degrees to the north end of the needle.

But in back-sights you may turn the north of the box to your eye, and count your degrees to the north point of the needle; or you may turn the south of the box to your eye, and count your degrees to the south end of the needle.

The brass ring in the box is divided on the side into 360 degrees, thus; from the north to the east, into 90; from the north to the west, into 90; from the south to the east, into 90; and from the south to the west, into 90 degrees; so the degrees are numbered from the north to the east or west, and from the south to the east or west.

The manner of using this part of the instrument is this; having directed your sights to the object, whether fore or back, as before, observe the two cardinal points of your compass, the point of the needle lies between, (the north, south, east and west being called the four cardinal points, and are graved on the bottom of the box) putting down those points together, by their initial letters, and thereto annexing the number of degrees, counting from the north or south, as before, thus: if the point of your needle lies between the north and east, north and west, south and east, or south and west points in the bottom of the box, then put down NE, NW, SE, or SW, annexing thereto the number of degrees cut by the needle on the side of the ring, counting from the north or south, as before.

But if the needle point exactly to the north, south, east, or west, you are then to write down N, S, E, or W, without annexing any degree.

This is the manner of taking field-notes, whereby the content of ground may be universally determined by calculation; and they are said to be taken by the quartered compass, or by the four nineties.

To find the number of degrees contained in any given angle.

Set your instrument at the angular point, and thence direct the sights along each leg of the angle, and note down their respective bearings, as before; the difference of these bearings, if less than 180, will be the quantity of degrees contained in the given angle; but if more, take it from 360, and the remainder will be the degrees contained in the given angle.

Ex. Let the angle proposed be GAB (see the last fig.); place the instrument at A, with the flower-de-luce towards you; then direct the sights to B, and observe what degrees are cut by the south end of the needle, which let be 250°; then turning the instrument about on its stand, direct the sights to G; note again what degrees are cut by the south end of the needle, which suppose are 172°. Then $250° - 172° = 78° =$ the $\angle$ GAB; but if the degrees cut should be 298° and 105°, then $298° - 105° = 193°$, which, taken from 360°, leaves $167° =$ the $\angle$ GAB.

THE THEODOLITE.

This instrument is a circle, commonly of brass, of ten or twelve inches in diameter, whose limb is divided into 360 degrees, and those again are subdivided into smaller parts, as the magnitude of it will admit; sometimes by equal divisions, and sometimes by diagonals, drawn from one concentric circle of the limb to another.

In the middle is fixed a circumferentor, with a needle; but this is of little or no use, except in finding a meridian line, or the proper situation of the land.

Over the brass circle is a pair of sights, fixed to a moveable index, which turns on the centre of the instrument, and upon which the circumferentor-box is placed.

This instrument will either give the angles of the field, or the bearing of every stationary distance-line, from the meridian.

To take the angles of the field.

Lay the ends of your index to 360°, and 180° ; turn the whole about with the 360 from you ; direct the sights from A to G, (see the last fig.) and screw the instrument fast ; direct them from A, to cut the object at B ; the degrees then cut by that end of the index which is opposite you, will be the quantity of the angle GAB, to place in your field-book ; to which annex the measure of the line AB, in chains and links ; set up your instrument at B, unscrew it, and lay the ends of your index to 360 and 180; turn the whole about with the 360 from you, or 180 next you, till you cut the object at A ; screw the instrument fast, and direct your sights to the object at C, and the degree then cut by that end of the index which is opposite to you, will be the quantity of the angle ABC. Then proceed from station to station, still laying the index to 360, turning it from you, and observing the object at the foregoing station, screwing the instrument fast, and observing the object at the following station, and counting the degrees to the opposite end of the index, will give you the quantity of each respective angle.

LEMMA.

All the angles of any polygon are equal to twice as many right angles as there are sides less by four. Thus, all the angles, A, B, C, D, E, F, G, are equal to twice as many right angles as there are sides in the figure, less by four.

Let the polygon be disposed into triangles, by lines drawn from any assigned point H within it, (see fig. prob. 1,) as by the lines HA, HB, HC, &c. It is evident, then, that the three angles of each triangle are equal to two right ; and consequently, that the angles in all the triangles are twice as many right ones as there are sides : but all the angles about the point H, are equal to four right ; therefore the remaining angles are equal to twice as many right ones as there are sides in the figure, abating four.

SCHOLIUM.

Hence we may know if the angles of a survey be truly taken; for if their sum be equal to twice as many right angles as there are stations, abating four right angles, we may conclude that the angles were truly taken, otherwise not.

If you take the bearing of any line with the circumferentor, that bearing will be the number of degrees the line is from the north; consequently the north must be a like number of degrees from the line; and thus the north, and of course the south, as well as the east and west, or the situation of the land, is obtained.

To take the bearing of each respective line from the meridian; or to perform the office of the circumferentor, by the theodolite.

Set your instrument at the first station, and lay the index to 360° and 180°, with the flower-de-luce of the box next 360; unscrew the instrument, and turn the whole about, till the north and south points of the needle cut the north and south points in the box; then screw it fast, and the instrument is north and south, if there be no variation in the needle; but if there be, and its quantity known, it may be easily allowed.

The circumferentor-box may then be taken off.

Direct the sights to the object at the second station, and the degree cut by the opposite end of the index will be the bearing of that line from the north, and the same that the circumferentor would give.

After having measured the stationary distance, set up your instrument at the second station; unscrew it, and set either end of the index to the degree of the last line, and turning the whole about with that degree towards you, direct your sights to an object at the foregoing station, and screw the instrument fast; it will then be parallel to its former situation, and consequently north and south; direct then your sights to an object at the following station, and the degree cut by the opposite end of the index will be the bearing of that line.

In the like manner you may proceed through the whole.

If the brass circle be divided into four nineties, from 360 and 180, and the letters N, S, E, W, be applied to them, the bearings may be obtained by putting down the letters, between which the far or opposite end of the index lies, and annexing thereto the degrees from the N. or S., and this is the same as the quartered compass.

If you keep the compass-box on, to see the mutual agreement of the two instruments; after having fixed the theodolite north and south, as before; turn the index about the north end or flower-de-luce next your eye, and count the degree to the opposite, or south end of the index, and this will correspond with the degree cut by the south end of the needle.

At the second, or next station, unscrew the instrument, and set the south of the index to the degree of the last station; turn the whole about with the south of the index to you, and cut the object at the foregoing station; screw the instrument fast, and with the north of the index to you, cut the object at the next following station; the degree then cut by the south of the index, will correspond with the degree cut by the south end of the needle, and so through the whole.

Some theodolites have a standing pair of sights, fixed at 360 and 180, besides those on the moveable index; if you would use both, look through the standing sights, with the 180 next you, to an object at the foregoing station; screw the instrument fast, and direct the upper sights on the moveable index to the object at the following station, and the degree cut by the opposite end of the index, will give you the quantity of the angle of the field.

Two pair of sights will be of no use in finding the angles from the meridian; and inasmuch as one pair is sufficient to find the angles of the field, the second can be of no use; besides, they obstruct the free motion of the moveable index, and therefore are rather an incumbrance than of any real use. Some will have it that they are useful with the others, for setting off a right angle, in taking an off-set: and surely this is as easily performed by the one pair on the moveable index: thus, if you lay the index to 360 and 180, and cut the object either in the last or following station, screw the instrument fast, and turn the index to 90 and 270, and then it will be at right angles with the line. So that the small sights, at those of the circle, can be of no additional use to the instrument, and therefore should be laid aside as useless.

This instrument may be used in windy and rainy weather, as well as in mountainous and hilly grounds; for it does not require a horizontal position to find the bearing, or angle, as the needle does, and therefore is preferred to any instrument that is governed by the needle.

To measure Angles of Altitude by the Circumferentor, or Theodolite.

1. To take an angle of altitude by the circumferentor, let the glass lid be taken off, and let the instrument be turned on one side, with the stem of the ball into the notch of the socket, so that the circle may be perpendicular to the plane of the horizon; let the instrument be placed in this situation before the object, so that the top thereof may be seen through the sights; let a plummet be suspended from the centre-pin, and the object being then observed, the complement of the number of degrees, comprehended between the thread of the plummet and that part of the instrument which is next your eye, will give the angle of altitude required.

2. If an angle of altitude is to be taken by the theodolite, let a thread be run through a hole at the centre, and a plummet be suspended by it; turn the instrument on one side, by the help of the ball and notch in the socket for that purpose, so that the thread may cut 90, having 360 degrees next you; screw it fast in that position, and through the sights cut the top of the objects; and the degrees then cut by the end of the index next you, are the degrees of elevation required. An angle of depression is taken the contrary way.

THE PROTRACTOR.

The protractor is a semicircle annexed to a scale, and is made of brass, ivory, or horn; its diameter is generally about five or six inches.

The semicircle contains three concentric semicircles, at such distances from each other that the spaces between them may contain figures.

The outward circle is numbered from the right to the left hand, with 10, 20, 30, &c. to 180 degrees; the middlemost the same way, from 180 to 360 degrees; and the innermost from the upper edge of the scale both ways, from 10, 20, 30, &c. to 90 degrees.

It is easy to conceive that the protractor, though a semicircle, may be made to supply the place of a whole circle; for if a line be drawn, and the centre-hole of the protractor be laid on any point in that line, the upper edge of the scale corresponding with that line, the divisions on the edge of the semicircle will run from 0 to 180, from right to left: again, if it be turned the other way, or downwards, keeping the centre-hole thereof on the aforesaid point, in the line, then the divisions will run from 180 to 360, and so completes an entire circle with the former semicircle.

The use of the protractor is to lay off angles, and to delineate or draw a map or plan of any ground from the field-notes, and is performed in the following manner.

To protract a field-book, taken by the angles of the field.

Note.—We here suppose the land surveyed is kept on the right hand as you survey.

Draw a blank line with a ruler of a length greater than the diameter of the protractor; pitch upon any convenient point therein, to which apply the centre-hole of your protractor with your pin, turning the arc upwards if the angle be less than 180, and downwards if more; and observe to keep the upper edge of the scale, or 180 and 0 degrees upon the line: then prick off the number of degrees contained in the given angle, and draw a line from the first point through the point at the degrees; upon which lay the stationary distance. Let this line be lengthened forwards and backwards, keeping your first station to the right, and second to the left; and lay the centre of your protractor over the second station, with your pin turning the arc upwards, if the angle be less than 180, and downwards, if more; and keeping the 180 and 0 degrees on the line, prick off the number of degrees contained in the given angle, and through that point and the last station draw a line, on which lay the stationary distance: and in like manner proceed through the whole.

In all protractions, if the end of the last station falls exactly in the point you began at, the field-work and protraction are truly taken and performed; if not, an error must have been committed in one of them: in such case, make a second protraction; if this agrees with the former, and neither meets nor closes, the fault is in the field-work, and not in the protraction; and then a re-survey must be taken.

REMARKS.

The accuracy of geometrical and trigonometrical mensuration, depends in a great degree on the exactness and perfection of the instruments made use of; if these are defective in construction, or difficult in use, the surveyor will either be subject to error, or embarrassed with continual obstacles. If the adjustments, by which they are to be rendered fit for observation, be troublesome and inconvenient, they will be taken upon trust, and the instrument will be used without examination, and thus subject the surveyor to errors, that he can neither account for nor correct.

In the present state of science, it may be laid down as a maxim, that every instrument should be so contrived that the observer may easily examine and rectify the principal parts; for however careful the instrument-maker may be, however perfect the execution thereof, it is not possible that any instrument should long remain accurately fixed in the position in which it came out of the maker's hand, and therefore the principal parts should be moveable, to be rectified occasionally by the observer.

Method of determining the areas of right-lined figures universally, or by calculation.

DEFINITIONS.

1. Meridians are north and south lines, which are supposed to pass through every station of the survey.

2. The difference of latitude, or the northing or southing of any stationary line, is the distance that one end of the line is north or south from the other end; or it is the distance which is intercepted on the meridian, between the beginning of the stationary line and a perpendicular drawn from the other end to that meridian. Thus, if N. S. (see fig. page 237,) be a meridian line passing through the point A of the line AB, then is A*b* the difference of latitude or southing of that line.

3. The departure of any stationary line, is the nearest distance from one end of the line to a meridian passing through the other end. Thus B*b* is the departure or easting of the line AB: but if CB be a meridian, and the measure of the stationary distance be taken from B to A; then is BC the difference of latitude, or northing, and AC the departure or westing of the line BA.

4. That meridian which passes through the first station, is sometimes called the first meridian; and sometimes it is a meridian passing on the east or west side of the map, at the distance of the breadth thereof, from east to west, set off from the first station.

5. The meridian distance of any station is the distance

thereof from the first meridian, whether it be supposed to pass through the first station, or on the east or west side of the map.

THEOREM I.

In every survey that is truly taken, the sum of the northings will be equal to that of the southings; and the sum of the eastings equal to that of the westings.

Let 1, 2, 3, 4, &c. (see figure page 260,) represent a tract or plot of land: let 1 be the first station, 2 the second, and so on. Then, the surveyor going completely round it, and arriving at the place of beginning, it is evident that the whole northing which has been made, must be equal to the southing, and the easting to the westing.

Scholium.—This theorem is of use to prove whether the field-work be truly taken, or not; for if the sum of the northings be equal to that of the southings, and the sum of the eastings to that of the westings, the field-work is right, otherwise not.

Since the proof and certainty of a survey depend on this truth, it will be necessary to show how the difference of latitude and departure for any stationary line, whose course and distance are given, may be obtained by the table, usually called the traverse table.*

To find the difference of latitude and departure by the Traverse-Table.

This table is so contrived, that by finding therein the given course, and a distance not exceeding 120 miles, chains, perches, or feet, the difference of latitude and departure is had by inspection: the course is to be found at the top of the table when under 45 degrees; but at the bottom of the table when above 45 degrees. Each column signed with a course consists of two parts, one for the difference of latitude, marked Lat., the other for the departure, marked Dep., which names are both at the top and bottom of these columns. The distance is to be found in the column marked Dist., next the left-hand margin of the page.

* This table is calculated by the first case of right-angled plane trigonometry, where the hypothenuse and an acute angle are given to find the legs.

EXAMPLE.

In the use of this table, a few observations only are necessary.

1. If a station consist of any number of even chains or perches, (which are almost the only measures used in surveying), the latitude and departure are found at sight under the bearing or course, if less than 45 degrees; or over it if more, and in a line with the distance.

2. If a station consist of any number of chains and perches, and decimals of a chain or perch, under the distance 10, the lat. and dep. will be found as above, either over or under the bearing; the decimal point or separatrix being removed one figure to the left, which leaves a figure to the right to spare.

If the distance be any number of chains or perches, and the decimals of a chain or perch, the lat. and dep. must be taken out at two or more operations, by taking out the lat. and dep. for the chains or perches in the first place; and then for the decimal parts.

To save the repeated trouble of additions, a judicious surveyor will always limit his stations to whole chains, or perches and lengths, which can commonly be done at every station, save the last.

1. In order to illustrate the foregoing observations, let us suppose a course or bearing to be S. 35° 15′ E. and the distance 79 four-pole chains. Under 35° 15′, or 35½ degrees, and opposite 79, we find 64·51 for the latitude, and 45.59 the departure, which signify that the end of that station differs in latitude from the beginning 64.51 chains, and in departure 45.59 chains.

Note.—We are to understand the same things if the distance is given in perches or any other measures, the method of proceeding being exactly the same in every case.

Again, let the bearing be 54¾ degrees, and distance as before; then over said degrees we find the same numbers, only with this difference, that the lat. before found, will now be the dep., and the dep. the lat., because 54¾ is the complement of 35¼ degrees to 90, viz. lat. 45.59, dep. 64.51.

2. Suppose the same course, but the distance 7 chains 90 links, or as many perches. Here we find the same numbers, but the decimal point must be removed one figure to the left.

Thus, under $35\frac{1}{4}$, and in a line with 79 or 7.9, are

Lat. 6.45
Dep. 4.56

the 5 in the dep. being increased by 1, because the 9 is rejected; but over $54\frac{3}{4}$ we get

Lat. 4.56
Dep. 6.45

3. Let the course be as before, but the distance 7.79; then opposite

7.70	Lat. 6.29	Dep. 4.43
9	7	6
7.79	6.36	4.49

Or opposite

7.00	Lat. 5.72	Dep. 4.03
.79	.64	.46
7.79	6.36	4.49

THEOREM II.

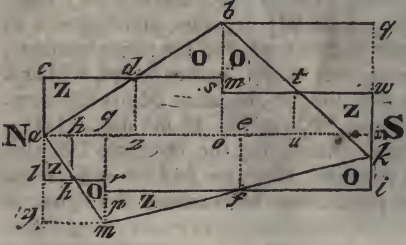
WHEN THE FIRST MERIDIAN PASSES THROUGH THE MAP.

If the east meridian distances in the middle of each line be multiplied into the particular southing, and the west meridian distances into the particular northing, the sum of these products will be the area of the map.

Let the figure *abkm* be a map, the lines *ab*, *bk* to the southward, and *km*, *ma* to the northward, N S, the first meridian line, passing through the first station, *a*.

$$\left.\begin{matrix}\text{The mer.}\\ \text{Dist. east}\end{matrix}\right\}\left\{\begin{matrix} zd\times ao \\ tu\times ox\ (bq)\end{matrix}\right\} = \text{Area}\left\{\begin{matrix} am \\ ow\end{matrix}\right.$$

$$\left.\begin{matrix}\text{The mer.}\\ \text{Dist. west}\end{matrix}\right\}\left\{\begin{matrix} ef\times gx \\ hh\times ga\ (my)\end{matrix}\right\} = \text{Area}\left\{\begin{matrix} xp \\ gl\end{matrix}\right.$$



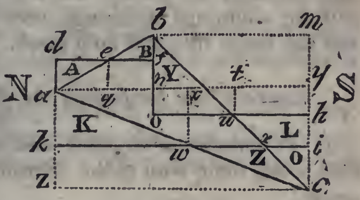
These four areas, $am + ow + xp + gl$ will be the area of the whole figure *cmswiprlc*, which is equal to the area of the map *abkm*. Complete the fig.

The parallelograms *am* and *ow* are made of the east meridian distances *dz* and *tu*, multiplied into the southings *ao* and *ox*. The parallelograms *xp* and *gl* are composed of the west meridian distances *ef* and *hh*, multiplied into the northings *xg* and *ga* (*my*); but these four parallelograms are equal to the area of the map; for if from them be taken the four triangles marked Z, and in the place of those be substituted the four triangles marked O, which are equal to the former; then it is plain the area of the map will be equal to the four parallelograms.

THEOREM III.

If the meridian distance, when east, be multiplied into the southings, and the meridian distance, when west, be multiplied into the northings, the sum of these less by the meridian distance when west, multiplied into the southings, is the area of the survey.

Let *a b c* be the map. The figure being completed, the rectangle *af* is made of the meridian distance *eq* when east, multiplied into the southing *an*; the rectangle *yk* is made of the meridian distance *xw*, multiplied into the northings *cz* or *ya*. These two rectangles, or parallelograms, *af* + *yk*, make the area of the figure *dfnyikd*, from which taking the rectangle *oy*, made of the meridian distance *tu* when west, into the southings *oh* or *bm*, the remainder is the area of the figure *dfohikd*, which is equal to the area of the map.



Let *bou* = Y, *urih* = L, *ric* = O, *wrc* = Z, *akw* = K, and *efb* = B, *ade* = A: then Y + Z + B = K + L + A.

Y = L + O, add Z to both, then Y + Z = L + O + Z; but Z + O = K, put K instead of Z + O; then Y + Z = L + K, add to both sides the equal triangles B and A, then Y + Z + B = L + K + A. If, therefore, B + Y + Z be taken from *abc*, and in lieu thereof we put L + K + A, we shall have the figure *dfohikd* = *abc*; but that figure is made up of the meridian distance when east, multiplied into the southing, and the meridian distance, when west, multiplied into the northing less by the meridian distance, when west, multiplied into the southing.

COROLLARY.

Since the meridian distance (when west) multiplied into the southing, is to be subtracted, by the same reasoning the meridian distance when east, multiplied into the northing, must be also subtracted.

SCHOLIUM.

From the two preceding theorems we learn how to find the area of the map, when the first meridian passes through it; that is, when one part of the map lies on the east and the other on the west side of that meridian. Thus,

RULE.

<table>
<tr><td>The merid.
Dist. when</td><td>{ east
{ west</td><td>multiplied
into the</td><td>{ southings
{ northings</td><td>their sum</td></tr>
</table>

is the area of the map.

But

<table>
<tr><td>The merid.
Dist. when</td><td>{ east
{ west</td><td>multiplied
into the</td><td>{ northings
{ southings</td><td>the sum of</td></tr>
</table>

these products taken from the former gives the area of the map.

These theorems are true, when the surveyor keeps the land he surveys on his right hand, which we suppose to be done through the whole; but if he goes the contrary way, call the southings northings, and the northings southings, and the same rule will hold good.

General rule for finding the Meridian Distances.

1. The meridian distance and departure, both east, or both west, their sum is the meridian distance of the same name.

2. The meridian distance and departure of different names; that is, one east and the other west, their difference is the meridian distance of the same name with the greater.

Thus, in the first method of finding the area, as in the following field-book.

The first departure is put opposite the northing or southing of the first station, and is the first meridian distance of the same name. Thus, if the first departure be east, the first meridian distance will be the same as the departure, and east also; and if west, it will be the same way.

The first meridian distance.......	6.61 E.
The next departure...............	6.61 E.
The second meridian distance....	13.22 E.
The next departure...............	1.80 E.
The third meridian distance.....	15.02 E.
At station 5, the meridian distance..	5.78 E.
The next departure...............	7.76 W.
The next meridian distance.......	1.98 W.
At station 11, the meridian distance	0.12 W.
The next departure...............	5.84 E.
The next meridian distance........	5.72 E.

In the 5th and 11th stations, the meridian distance being less than the departures, and of a contrary name, the map will cross the first meridian, and will pass as in the 5th line, from the east to the west line of the meridian; and in the 11th line it will again cross from the west to the east side, which will evidently appear, if the field-work be protracted, and the meridian line passing through the first station, be drawn through the map.

The field-book cast up by the first method, will be evident from the two foregoing theorems, and therefore requires no farther explanation; but *to find the area by the second method* take this

RULE.

When the meridian distances are east, put the products of north and south areas in their proper columns; but when west, in their contrary columns; that is, in the column of south area, when the difference of latitude is north; and in north when south; the reason of which is plain, from the last two theorems. The difference of these two columns will be the area of the map.

Field-Book, Method 1.

No. Sta.	Bearings.	C. L.	Lat. and half Dep.	Merid. Dist.	Area.	Deduct.
1	NE 75°	13.70	N 3.54 E 6.61	6.61 E 13.22 E		23.3994
2	NE 20½	10.30	N 9.67 E 1.80	15.02 E 16.82 E		144.9430
3	East	16.20	0.00 E 8.10	24.92 E 33.02 E		
4	SW 33½	35.30	S 29.44 W 9.74	23.28 E 13.54 E	685.3632	
5	SW 76	16.00	S 3.87 W 7.76	5.78 E 1.98 W	22.3686	
6	North	9.00	N 9.00 0.00	1.98 W 1.98 W	17.8200	
7	SW 84	11.60	S. 1.21 W 5.77	7.75 W 13.52 W		9.3775
8	NW 53¼	11.60	N 6.94 W 4.64	18.16 W 22.80 W	126.0304	
9	NE 36¾	19.20	N 15.38 E 5.74	17.06 W 11.32 W	262.3828	
10	NE 22½	14.00	N 12.93 E 2.68	8.64 W 5.96 W	111.7152	
11	SE 76¾	12.00	S 2.75 E 5.84	0.12 W 5.72 E		0.3300
12	SW 15	10.85	S 10.48 W 1.40	4.32 E 2.92 E	45.2736	
13	SW 16¾	10.12	S 9.69 W 1.46	1.46 E 0.00	14.1474	
					1285.1012 178.0499	178.0499
	Content in Chains				1107.0513	

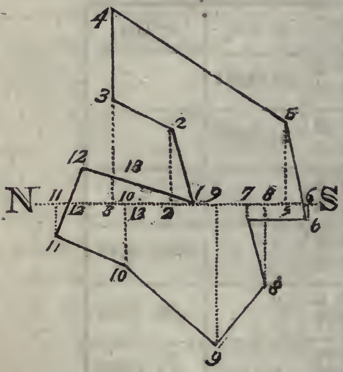
The foregoing Field-Book, Method 2.

It is needless here to insert the columns of bearing or distances in chains, they being the same as before.

No. St.	Lat. and half Dep.	Merid. Dist.	N. Area.	S. Area.
1	N 3.54 E 6.61	6.61 E 13.22 E	23.3994	
2	N 9.65 E 1.80	15.02 E 16.82 E	144.9430	
3	0.00 E 8.10	24.92 E 33.02 E		
4	S 29.44 W 9.74	23.28 E 13.54 E		685.3632
5	S 3.87 W 7.76	5.78 E 1.98 W		22.3686
6	N 9.00 0.00	1.98 W 1.98 W		17.8200
7	S 1.21 W 5.77	7.75 W 13.52 W	9.3775	
8	N 6.94 W 4.64	18.16 W 22.80 W		126.0303
9	N 15.38 E 5.74	17.06 W 11.32 W		262.3328
10	N 12.93 E 2.68	8.64 W 5.96 W		111.7152
11	S 2.75 E 5.84	0.12 W 5.72 E	0.3300	
12	S 10.48 W 1.40	4.32 E 2.92 E		45.2736
13	S 9.69 W 1.46	1.46 E 0.00		14.1474
			178.0499	1284.1012 178.0499
Area in chains, as before . . .				1107.0513

Construction of the Map from either the 1st or the 2d Table.

Draw the line N. S. for a north and south line, which call the first meridian ; in this line assume any point, as 1, for the first station. Set the northing of that stationary line, which is 3.54, from 1 to 2, on the said meridian line. Upon



the point 2 raise a perpendicular to the eastward, the meridian distance being easterly, and upon it set 13.22, the second number in the column of meridian distances from 2 to 2, and draw the line 1, 2, for the first distance line : from 2 upon the first meridian, set the northing of the second stationary line, that is, 9.65 to 3, and on the point 3 erect a perpendicular eastward, upon which let the meridian distance of the second station 16 82, from 3 to 3, and draw the line 2, 3, for the distance-line of the second station. And since the third station has neither northing nor southing, set the meridian distance of it 33.02, from 3 to 4, for the distance-line of the third station. To the fourth station there is 29.44, southing, which set from 3 to 5 ; upon the point 5, erect the perpendicular 5, 5 ; on which lay 13.54, and draw the line 4 to 5.

In the like manner proceed to set the northings and southings on the first meridian, and the meridian distances upon the perpendiculars raised to the east or west ; the extremities of which, connected by right lines, will complete the map.

The area of the survey may be readily found by protracting it, and dividing the plot into triangles and trapeziums ; then, measuring the several bases and perpendiculars, on the same scale that was used in the protraction, and the sum of the areas of the triangles and trapeziums, found by Prob. II and V, Mens. of surfaces, will be the whole area required.

The following method, which for its simplicity and ease in finding the meridian distances, is considered to be preferable to any other that has been published on the subject.*

Find, in the first place, by the Traverse-Table, the lat. and dep. for the several courses and distances, as already taught; and if the survey be truly taken, the sums of the northings and southings will be equal, and also those of the eastings and westings. Then, in the next place, find the meridian distances, by choosing such a place in the column of eastings or westings as will admit of a continual addition of one, and subtraction of the other; by which means we avoid the inconvenience of changing the denomination of either of the departures.

The learner must not expect that in real practice the columns of lat. and those of dep. will exactly balance when they are at first added up, for little inaccuracies will arise, both from the observations taken in the field, and in chaining; which to adjust, previous to finding the meridian distances, we may observe, That if, in small surveys, the difference amount to two-tenths of a perch for every station, there must have been some error committed in the field; and the best way in this case, will be to rectify it on the ground, by a re-survey, or at least as much as will discover the error. But when the differences are not within those limits, the columns of northing, southing, easting, and westings, may be corrected as follows.

Add all the distances into one sum, and say, as that sum is to each particular distance, so is the difference between the sums of the columns of northing and southing to the correction of northing or southing belonging to that distance: the corrections thus found are respectively additive, when they belong to the column of northing or southing, which is the less of the two, and subtractive when they belong to the greater; if the course be due east or west, the correction is always additive to the less of the two columns of northing or southing. The corrections of easting and westing are found exactly in the same manner.

The following example will sufficiently illustrate the manner of applying the rule.

---

* This method, with very little alteration and improvement, usually called the Pennsylvania method of calculation in America, was first invented by Thomas Burgh, Esq. for which he obtained a reward of twenty thousand pounds sterling from the Irish Parliament.

In this example the sum of the distances is 791, and the difference between the columns of northing and southing, is .4, also the first distance is 70 ; say, then,

$$791 : 70 :: .4 : .04$$

which fourth proportional .04 is the first correction belonging to the southing 53.6, from which the correction .04 should be subtracted.

In this manner the several corrections of the southings

$$\left.\begin{array}{r}53.6\\29.1\\135.7\end{array}\right\}\text{ are found to be }\left\{\begin{array}{l}.04\\.09\\.07\end{array}\right.\text{ respectively.}$$

But as only two of these corrections amount to half a tenth, we must use .1 for each of the corrections .09 and .07, and neglect the correction .04 ; thus the correct southings become

$$\left.\begin{array}{r}53.6\\29.0\\135.0\end{array}\right\}$$

In like manner, from the remaining distances we obtain to

$$\text{the northings }\left.\begin{array}{r}62.9\\101.1\\54.0\\00.0\end{array}\right|\text{ the additive corrections }\left\{\begin{array}{l}.04\\.06\\.03\\.07\end{array}\right.$$

And consequently, by neglecting .04, and .03, and using .1 for each of the two .06 and .07, the northings

$$\text{when corrected are }\left\{\begin{array}{r}62.9\\101.2\\54.0\\00.1\end{array}\right.$$

In obtaining these corrections, it is commonly unnecessary to use all the significant figures of the distances : thus, for the ratio of 791 to 70, we may say, as 80 to 7.

EXAMPLE OF CORRECTING A SURVEY.

FIELD-NOTES.			FROM THE TABLES.				CORRECTED.			
NO.	COURSES.	DIST. PER.	N.	S.	E.	W.	N.	S.	E.	W.
1	S. 40° W.	70		53.6		45.0		53.6		45.0
2	N. 45 W.	89	62.9			62.9	62.9			62.9
3	N. 36 E.	125	101.1		73.5		101.2		73.5	
4	North.	54	54.0				54.0			
5	S. 81 E.	186		29.1	183.7			29.0	183.6	
6	S. 8 W.	137		135.7		19.1		135.6		19.2
7	West.	130				130.0	00.1			130.0
			218.0	218.4	257.2	257.0	218.2	218.2	257.1	257.1
				218.0	257.0					
			Diff.	= .4	.2 =	Diff.				

The latitudes and departures being thus balanced, proceed to insert the meridian distances by the above method, where we still make use of the same field-notes, only changing chains and links into perches and tenths of a perch. Then, by looking along the column of departure, it is easy to observe, that in the columns of eastings opposite station 9, all the eastings may be added, and the westings subtracted, without altering the denomination of either. Therefore, by placing 46.0, the east departure belonging to this station, in the column of meridian distances, and proceeding to add the eastings and subtract the westings, according to the rule already mentioned, we shall find that at station 8, these distances will end in 0, 0, or a cipher, if the additions and subtractions be rightly made. Then multiplying the upper meridian distance of each station by its respective northing or southing, the product will give the north or south area, as in the examples already insisted on, and which is fully exemplified in the annexed specimen. When these products are all made out and placed in their respective columns, their difference will give double the area of the plot, or twice the number of acres contained in the survey. Divide this remainder by 2, and the quotient thence arising by 160, (the number of perches in an acre), then will this last quotient exhibit the number of acres and perches contained in the whole survey; which in this example may be called 110 acres, 103 perches, or 110 acres, 2 quarters, 23 perches.

Field-Notes of the two foregoing methods, cast up by perches and tenths of a perch.

N.	Courses.	Dist.	N.	S.	E.	W.	M. D.	N. Area.	S. Area.
1	N 75°.00′ E	54.8	14.2		52.9		235.3 288.2	3341.26	
2	N 20.30 E	41.2	38.6		14.4		302.6 317.0	11680.36	
3	East.	64.8			64.8		381.8 446.6		
4	S 33.30 W	141.2		117.7		77.9	368.7 290.8		43395.99
5	S 76.00 W	64.0		15.5		62.1	228.7 166.6		3544.85
6	North.	36.0	36.0				166.6 166.6	5977.60	
7	S 84.00 W	46.4		4.9		46.1	120.5 74.4		590.45
8	N 53.15 W	46.4	27 8			37.2	37.2 00.0	1034.16	
9	N 36.45 E	76.8	61.5		46.0		46.0 92.0	2829.00	
10	N 22.30 E	56.0	51.7		21.4		113.4 134.8	5862.78	
11	S 76.45 E	48.0		11.0	46.7		181.5 228 2		1996.50
12	S 15.00 W	43.4		41.9		11 2	217.0 205.8		9092.30
13	S 16 45 W	40.5		38.8		11.7	194.1 182.4		7531.08
			229.8	229.8	246.2	246.2		30745.16	66151.17 30745.16
								1 2	35406.01
							Area in perches		177030.05

Note.—In the foregoing methods, the first meridian passes through the map; but as it is more convenient to have it pass through the extreme east or west point of the same, I have given the following example to illustrate this method.

Of computing the area of a survey by having the bearings and distances given.

Let BCDEFGHA represent the boundary of a survey, of which the following field-notes are given; it is required to find the area.

EXAMPLE.

Sides of the Land.	Bearings.	Length in Chains.
BC	East.	4 00
CD	N 9° E	4.00
DE	S 69 E	5.56
EF	S 36 E	7.00
FG	S 42 W	4.00
GH	S 75 W	10.00
HA	N 39 W	7.50
AB	N 42 E	5.00

RULE I.

Find the difference of latitude and departure answering to each course and distance, by the traverse-table or right-angled plane trigonometry, according to the directions already given, and place them under the succeeding columns N. or South, E. or West; according as they are, North or South, East or West then if the survey does not close, correct the errors, by saying, as the sum of all the distances is to each particular distance, so is the whole error in departure to the correction of the corresponding departure; each correction being so applied as to diminish the whole error in departure: proceed the same way for the corrections in latitudes. These corrections being applied to their corresponding differences of latitude and departure, that is, add when of the same name, and subtract when of different names, then the corrected difference of latitude and departure will be obtained, and the table will stand thus:

TABLE I.

Sids.	Courses.	D. Ch.	N.	S.	E.	W.	C. S.	C. W.	N.	S.	E.	W.
BC	East.	4.00			4.00		.02	.02		.02	3.98	
CD	N 9° E	4.00	3.95		0.63		.02	.02	3.93		0.61	
DE	S 69 E	5.56		1.99	5.19		.03	.02		2.02	5.17	
EF	S 36 E	7.00		5.66	4.11		.05	.03		5.71	4.08	
FG	S 42 W	4.00		2.97		2.68	.02	.02		2.99		2.70
GH	S 75 W	10.00		2.59		9.66	.06	.05		2.65		9.71
HA	N 39 W	7.50	5.82			4.72	.05	.04	5.77			4.76
AB	N 42 E	5.00	3.72		3.35		.03	.02	3.69		3.33	
			13.49	13.21	17.28	17.06	.28	.22	13.39	13.39	17.17	17.17
			13.21		17.06							
			.28	Er. S.	.22	Er. W.						

The errors being corrected thus:

As 47 : 4 :: .28 : .02
As 47 : 4 :; .28 : .02
&c. &c.
} The corrections of difference of lat. as in Table 1.

As 47 : 4 :: .22 : .02
As 47 : 4 :: .22 : .02
&c. &c.
} The corrections of departure as in Table 1.

The latitudes and departures being thus balanced, it is necessary to calculate the several meridian distances, in order to compute the area of the survey.

As beginning at the most easterly, or most westerly point of the survey, admits of a continual addition of the one and subtraction of the other:

The most easterly or most westerly point can be easily discovered from the foregoing table, thus:

The first departure corrected is 3.98, which is the meridian distance of the second point of the survey from the first, to which add 0.61, the next dep. corrected, and their sum is 4.59, the meridian distance of the third point of the survey from the first, and in like manner $4.59 + 5.17 = 9.76 =$ the meridian distance of the fourth point from the first, and $9.76 + 4.08 = 13.84 =$ the meridian distance east of the fifth point from the first; after the same manner, continue to add the dep. when east; but subtract when west; the next dep. is west, therefore $13.84 - 2.70 = 11.14 =$ the meridian distance of the sixth point from the first, and $11.14 - 9.71 = 1.43 =$ the next. Now the next departure is 4.76, which is west, and 1.43 is the meridian distance of the seventh point from the first, which is east; therefore $4.76 - 1.43 = 3.33 =$ the meridian distance of the eighth point from the first; as, 3.33 is the greatest meridian distance west of the eighth point of the survey from the first, because the next departure is east 3.33; then, $3.33 - 3.33 = 0$, which closes the survey: consequently the eighth point of the survey is the most westerly point, and for the same reason, as 13.84 is the greatest meridian distance east, which is the meridian distance of the fifth point of the survey. In like manner, the most easterly or most westerly point of the survey can be found by beginning at any other point.

After the most easterly or most westerly point of the survey is discovered, call that point the first station, and proceed to find the meridian distances for the several lines in the order in which they were surveyed; that is, the first dep. will be the first meridian distance, which place in the column of meridian distances opposite the said departure; to the same meridian distance add the said departure, to which sum add the next departure, if it be of the same name with the foregoing departure; but subtract if it be of a different name, which sum or difference call the next meridian distance, and set it in the column of meridian distances opposite the departure last used; and in like manner, continue to add the departure twice when of the same name; but if of a different name, subtract twice, and the last meridian distance will be zero, if the additions and subtractions are rightly performed; because the sum of the northings is equal to the sum of the southings, after the survey is corrected, which is evident from Theo. 1, and the foregoing table. Then, multiplying the upper meridian distance of each station by the corresponding northing or southing, and placing the product in the north or south area, according as the latitude is north or south, the difference of the sum of these products will give twice the area, half of which gives the area of the survey.

The most westerly point of the survey being made the first station, and the several meridian distances being calculated, &c. the foregoing table will stand thus:

St.	Courss.	D.C.	N.	S.	E.	W.	C. S.	C. W.	N.	S.	E.	W.	M. D.	N. Area.	S. Area.
1	N 42°E	5	3.72		3.35		.03	.02	3.69		3.33		3.33 6.66	12.2875	
2	East.	4			4.00		.02	.02		.02	3.98		10.64 14.62		.2128
3	N 9 E	4	3.95		0.63		.02	.02	3.93		0.61		15.23 15.84	59.8539	
4	S 69 E	5.56		1.99	5.19		.03	.02		2.02	5.17		21.01 26.18		42.4402
5	S 36 E	7		5.66	4.11		.05	.03		5.71	4.08		30.26 34.34		172.7846
6	S 42 W	4		2.97		2.68	.02	.02		2.99		2.70	31.64 28.94		94.6036
7	S 75 W	10		2.59		9 66	.06	.05		2.65		9.71	19.23 9.52		50.9595
8	N 39W	7.50	5.82			4.72	.05	.04	5.77			4.76	4.76 0.00	27.4652	
														99.606 8	361.0007

```
              99.606  361.0007
                       99.6068
                      --------
                     2)261.3939
                      --------
Area 13 A. 0R. 11P.  10)130.69695  S. C.
                      --------
                      13.069695
                              4
                      --------
                       0.278780
                             40
                      --------
                      11.151200
```

RULE II.

The difference of latitude and departure being found and corrected, as in the preceding rule.

As beginning at the most northerly or most southerly point of the survey, admits of a continual addition of the one and subtraction of the other; make choice of either of these points, in order to calculate the area of the survey.

1. It is necessary to calculate the several latitudes in order to find the most northerly or most southerly point of the survey, which may be done from table 1, thus:

The first lat. is .02 south, which is the difference of latitude between the second point of the survey and the first, when the survey is corrected from the next departure 3.93 which is N. subtract .02, and their difference 3.91 is equal to the difference of latitude between the third point and the first, which is N. and $3.91 - 2.02 = 1.89 =$ the difference of latitude between the fourth point and the first; which is also N. But as the next difference of lat. is south, therefore $5.71 - 1.89 = 3.82 =$ the difference of lat. S. between the fifth point and the first; and $3.82 + 2.99 = 6.81 =$ the difference of lat. S. between the sixth point and the first; and $6.81 + 2.65 = 9.46 =$ the difference of lat S. between the seventh point and the first; and $9.46 - 5.77 = 3.69 =$ the difference of lat. S. between the eighth point and the first; and $3.69 - 3.69 = 0$; hence it is evident that 9.46 is the greatest lat. S. = the difference of lat. between the seventh point and the first; therefore, the seventh point of the survey is the most southerly point; and, in like manner, 3.91 = the difference of lat. between the third point and the first, is the greatest lat. north; hence, the third point is the most northerly point of the survey.

Now, by calling the most southerly point of the survey the first station, and proceeding to find the latitudes for the several lines in the order in which they were surveyed; that is, the first difference of lat. will be the first lat., which place in the column of latitudes, opposite the said difference of latitude; to the same lat. add the said difference of lat., to which sum add the next difference of lat., if it be of the same name, but subtract if of a different name, and place it on the column of latitudes; in like manner continue to add or subtract the difference of lat. twice, and the last lat. comes out nothing, if the additions and subtractions are rightly performed. Multiply each of the upper numbers in the column of latitudes by the corresponding dep., and place the products in the column of east or west area, according as the dep. is E. or W. The difference of these columns will be equal to twice the area, half of which will give the area of the survey, as in the following table.

TABLE III.

S.	Courss.	D.C.	N.	S.	E.	W.	C. S.	C. W.	N.	S.	E.	W.	Lat.	E. A.	W. A.
1	N39°W	7.50	5.82			4.72	.05	.04	5.77			4.76	5.77 11.54		27.4662
2	N 42 E	5	3.72		3.35		.03	.02	3.69		3.33		15.23 18.92	50.7159	
3	East.	4			4.00		.02	.02		.02	3.98		18.90 18.88	75.2220	
4	N 9 E	4	3.95		0.63		.02	.02	3.93		0.61		22.81 26.74	13.9141	
5	S 69 E	5.56		1.99	5.19		.03	.02		2.02	5.17		24.72 22.70	127.8024	
6	S 36 E	7		5.66	4.11		.05	.03		5.71	4.08		16 99 11.28	69.3192	
7	S 42 W	4		2.97		2.68	.02	.02		2.99		2.70	8.29 5.30		22.3530
8	N 75 W	10		2.59		9.66	.06	.05		2.65		9.71	2.65 0.00		25.7315
														336.9736	75.5797

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336.9736 75.5797
 75.5797
 ─────────
2)261.3939
 ─────────
10)130.69695
 ─────────
 13.069695 As.
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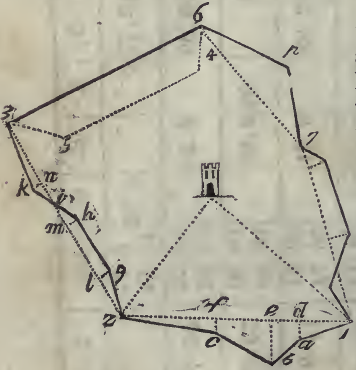
A. R. P.
Ans. 13 0 11

OF OFF-SETS.

In taking surveys it is unnecessary and unusual to make a station at every angular point, because the field-work can be taken with much greater expedition, by using off-sets and intersections, and with equal certainty; especially where creeks, &c. bound the survey.

Off-sets are perpendicular lines drawn or measured from the angular points of the land, that lie on the right or left hand to the stationary distance, thus:



Let the black lines represent the boundaries of a farm or township: and let 1 be the first station; then if you have a good view to 2, omit the angular points between 1 and 2, and take the bearing and length of the stationary line 1, 2, and insert them in your field-book: but in chaining from 1 to 2, stop at *d*, opposite the angular point *a*, and in your field-book insert the distance from 1 to *d*, which admit to be 4C. 25L. as well as the measure of the off-set *ad*, which admit to be 1C. 12L., thus: by the side of your field-book, in a line with the first station, say at 4C. 25L. L. 1C. 12L., that is, at 4C. 25L., there is an off-set to the left hand of 1C. 12L.

This done, proceed on your distance-line to *e*, opposite o the angle *b*, and measure *eb*, supposing then 1*e* to be

7C. 40L., and *eb* 3C. 40L., say (still in a line with the first station in your field-book) "at 7C. 40L. L. 3C. 40L." That is, at 7C. 40L. there is an off-set to the left of 3C. 40L.; proceed then with your distance-line to *f*, opposite to the angle *c*, and measure *fc*; suppose then 1 *f* to be 13C. and *fc* 1C. 25L., say, in the same line as before, at 13C. L. 1C. 25L. Then proceed from *f* to 2, and you will have the measure of the entire stationary line 1, 2, which insert in its proper column by the bearing.

In taking off-sets, it is necessary to have a perch-chain, or a staff of half a perch, divided into links, for measuring them; for by these means the chain in the stationary line is undisturbed, and the number of chains and links in that line from whence, or to which, the off-sets are taken, may be readily known.

Having arrived at the second station, if you find your view will carry you to 3, take the bearing from 2 to 3, and in measuring the distance-line, stop at *l*, opposite *g*; admit 2*l* to be 4C. 10L., and the off-set *lg* 1C. 20L.; then, in a line with the second station in your field-book, say at 4C. 10L. R. 1C. 20L., that is, the off-set is a right-hand one of 1C. 20L. Again at *m*, which suppose to be 10C. 25L. from 2: take the off-set *mh* of 1C. 15L., and in a line with the second station, say at 10C. 25L. R. 1C. 15L. In the same line, when you come to the boundary at *i*, insert the distance 2*i*, 13C. 10L., thus, at 13C. 10L. 0; that is, at 13C. 10L. there is no off-set. At *n*, which is 15C. from 2, take the off-set *nk* 45L; and still opposite to the second station say at 15C. L. 45L.

Let the line 3, 6, represent the boundary, which by means of water, briers, or any other impediment, cannot be measured. In this case make one or more stations within or without the land, where the distances may be measured, and draw a line from the beginning of the first to the end of the last distance, thus: make stations at 3, 4, and 5; take the bearings, and measuring the distances as usual, which insert in your field-book, and draw a mark like one side of a parenthesis, from the third to the fifth station, to show that a line drawn from the third station to the farthest end of the fifth stationary line will express the boundary. Thus,

No. Sta.	Deg.	Ch. L.
⎧ 3	$172\frac{1}{2}$	5.45
⎨ 4	200	13.25
⎩ 5	250	3.36

Suppose the point *p* of the boundary to be inaccessible by means of the lines 6*p* or *p*7 being overflowed, or that a quarry, swamp, &c. might prevent your taking their lengths: in this case take the bearing of the line 6, 7, which insert opposite to the sixth station in your field-book with the other bearing then direct the index to the point *p*, and insert its bearings on the left side of the field-book, opposite to the sixth station, annexing thereto the words *Int. for boundary*; and having measured and inserted the distance 6, 7, set the index in the direction of the line 7*p*, and insert its bearing on the left of the seventh station of the field-book, annexing thereto the words *Int. for boundary*: the crossing or intersection of these two bearings will determine the point *p*, and of course the boundary 6*p*7 is also determined.

If your view will then reach to the first station, take its bearing, stationary line, and off-sets as before, and you have the field-book completed. Thus,

The Field-Book.

Remarks and intersection.	N. St.	Deg.	C. L.	OFF-SETS.
318 Int. to a tower	1	358	22.12	At 4c. 25L. L. 1c. 12L. at 7c. 40L. L. 3c. 40L. at 13c. L. 1c. 25L.
231½ Int to ditto.	2	297¾	22.12	At 4c. 10L. R. 1c. 20L. at 10c. 25L. R. 1c. 51L. at 13c. 10L. 0. at 15c. L. 45L.
	3	172¼	5.45	
	4	200	13.25	
	5	250	3.36	
155½ Int. for bound.	6	125	15.15	At 1c. 20L. L. 2c. 20L.
274 Int. for ditto	7	105¼	15.10	at 7c. 45L. L. 2c. 32L. at 11c. 25L. 0. at 12c. 25L. R. 36L.

Close at the first station.

If you would lay down a tower, house, or any other remarkable object in its proper place; from any two stations, take bearings to the object, and their intersection will determine the place where you are to insert it, in the manner that the tower is set out in the figure, from the intersection taken at the first and second stations of the above field-book.

A protraction of this will render all plain, on which lay off all your off-sets and intersections, and proceed to find the content by any of the methods in the preceding section.

The foregoing field-book may be otherwise kept, thus:

Remarks and intersection.	No. St.	Deg.	L. han. Off-set. Ch. L.	Dist. Ch. L.	R. han. Off-set. Ch. L.
318 Int. to a tower	1	358	1.12	4.25	
			3.40	7.40	
			1.25	13.00	
				22.12	
232½ Int. for ditto	2	297¾		4.10	1.20
				10.25	1.15
				13.10	
			0.45	15.00	
				21.21	
	3	172½		5.45	
	4	200		13.25	
	5	250		3.36	
155½ Int. for bound.	6	125		15.15	
274 Int. for bound'y	7	105	2.20	1.20	
			2.32	7.45	
				11.25	
				12.25	
				15.10	0.36

How to cast up off-sets by the pen.

$$1, 2 - 1f = 2f,\ 1f - 1e = fe,\ 1e - 1d = ed.$$

Then $1d \times \frac{1}{2}\, da = 1da$, by prob. 2, page 53, and $ed \times \frac{1}{2}\,(da + eb) = beda$, $\frac{1}{2}\,(eb + fc) \times fe = befc$, and $2f \times \frac{1}{2}\, fc = cf2$; the sum of all which will be $1abc21$; the area contained between the stationary line 1, 2, and the boundary, 1 *abc* 2.

In the same manner you may find the area of *2ihg2*, of *ik3i*, as well as what is without and withinside of the stationary line 7, 1.

If, therefore, the left-hand off-sets exceed the right-hand ones, it is plain, the excess must be added to the area within the stationary lines; but if the right-hand off-sets exceed the left-hand ones, the difference must be deducted from the said area; if the ground be kept on the right hand, as we have all along supposed; or in words, thus:

To find the contents of Off-sets.

1. From the distance-line, take the distance to the preceding off-set, and from that the distance of the one preceding it, &c. in four-pole chains; so will you have the respective distances from off-set to off-set, but in a retrograde order.

2. Multiply the last of these remainders by ½ the first off-set, the next by ½ the sum of the first and second, the next by ½ the sum of the second and third, the next by ½ the sum of the third and fourth, &c. The sum of these will be the area produced by the off-sets.

Thus, in the foregoing field-book, the first stationary line is 22C. 12L. or 11C. 12L of four-pole chains. See the fig.

	Ch. L.	Ch. L.	Ch. L.
From	$11.12 = 1, 2$	$6.50 = 1f$	$3.90 = 1e$
Take	$6.50 = 1f$	$3.90 = 1e$	$2.25 = 1d$
	$4.62 = 2f$	$2.60 = ef$	$1.65 = ed$

Ch. L.	
$1d = 2.25 \times 32L.$ ½ the first off-set =	.7200
$ed = 1.65 \times 1C.\ 26L.$ ½ the sum of the 1st and 2d	2.0790
$ef = 2.60 \times 1C.\ 32L.$ ½ the sum of 2d and 3d =	3.4320
$2f = 4.62 \times 37L.$ ½ the last off-set =	1.7094
Content of left off-sets on the first distance in square four-pole chains	7.9404

In like manner the rest are performed.

The sum of the left hand off-sets will be	14.0856
And the sum of the right hand ones	3.6825
Excess of left hand off-sets in sq. four-pole chains	10.4031
Acres	1.04031
	.16124
	4
Perches	6.4496

Excess of left hand off-sets above the right hand ones, 1A. 0R. 6P. to be added to the area within the stationary lines.

EXAMPLES.

1. Two sides of a triangle are respectively 20 and 40 perches; required the third, so that the content may be just one acre. *Ans.* Either 23.099 or 58.876 perches.

2. What difference is there between a lot 28 perches long by 20 broad, and two others, each of half the dimensions? *Ans.* 1 acre 3 roods.

3. In a pentangular field, beginning with the south side, and measuring round towards the east, the first or south side is 2735 links, the second 3115, the third 2370, the fourth 2925, and the fifth 2220; also the diagonal from the first angle to the third is 3800 links, and that from the third to the fifth 4010; required the area of the field.
Ans. 117 acres 2 roods 28 perches.

4. Required the content of a field in the form of a trapezium, one of the diagonals being 64.4 perches, and the perpendiculars upon this diagonal from the opposite angles, 13.6 and 27.2 respectively. *Ans.* 8 acres 33¾ perches.

5. Required the area of a triangular field, two sides of which are 49.2 and 40.8 perches, and their contained angle 144½ degrees. *Ans.* 3 acres 2 roods 22 perches.

6. Required the area of a farm in Oysterbay, belonging to Mr. Daniel Cock, which was surveyed by *Gerardus B. Docharty*, Esq.: the following are field-notes.

Beginning at the house by the road: 1st S. 8°W. 3.5.2 ch. S. 5½ E. 0.43 ch. S. 23°E. 5.50 ch. S. 2°¾ W. 6.91 ch. S. 4°½ E. .8 ch. S. 5°E. 2.85 ch. S. 8°W. 4.64 ch. S. 12° W. 2.41 ch. S. 11°W. 5.64 ch. N. 85°E. 10.01 ch. N. 13° E. 15.51 ch. N. 88°E. 15.79 ch. N. 13°½ E. 13.09 ch. N. 83°W. 3 ch. N. 15°W. 19.87 ch. N. 9°E. 12 ch. N. 65°½ W. 1.2.8 ch. S. 89°W. 4. 23 ch. S. 68°W. 0.93 ch. S. 47° W. 5.42 ch. S. 53°½ W. 3.89 ch. S. 68°½ W. 6.05 ch. S. 65° W. 3.52 ch. S. 51°½ W. 1.98 ch. S. 33°½ W. 1.97 ch. S. 15°W. 1.52 ch. S. 6°¼ W. 7.60 ch.

Answer, 122 acres, 1 rood, 21.7 perches.

7. Required the area of a farm lying in Oysterbay, and belonging to Mr. Benjamin Townsend, which was surveyed by *Gerardus B. Docharty*, Esq. The following are his field-notes:

1st S. 50°½ W. 1.95 ch. S. 50°¼ W. 1.74 ch. S. 54°¾ W. 0.57 ch. S. 70°¼ W. 3.43 ch. S. 42°½ W. 2.47 ch. S. 50°W. 3.11 ch. S. 60°½ W. 4.30 ch. S. 79°¼ W. 1.67 ch. N. 82°¾ W. 1.71 ch. N. 72°½ W. 2.28 ch. N. 67°¾ W. 2.37 ch. N. 64°W. 1.95 ch. N. 74°½ W. 4.64 ch. N. 64°¾ W. 4.54 ch. N. 78°¾ W. 5.50 ch. N. 61°W. 0.60 ch. N. 24°¾ W. 0.30 ch. N. 4°½ E. 5.93 ch. N. 9°½ E. 4.37 ch. N. 81°½ E. 4.25 ch. N. 83°E. 7 ch. N. 53°E. 3.90 ch. N. 39°¾ E. 7.31 ch. N. 30′ E. 10.40 ch. N. 14°½ E. 1.16 ch. N. 47°½ E. 4.62 ch. N. 59°E. 13.83 ch. N. 64°½ E. 1.50 ch. S. 71°E. 1.57 ch. South 11 ch. S. 1°¼ E. 2.76 ch. S. 3°¼ E. 8.85 ch. S. 9°¼ E. 3.37 ch. S. 2°½ W. 12.19 ch. S. 53°W. 1.10 chains.

Ans. 109 acres, 0 roods, and 26 perches, nearly.

CASK-GAUGING.

The meaning of the word *gauging* is restricted to the measuring of casks, and other things falling under the cognizance of the excise; and it has received its name from a gauge or rod used by the practitioners of the art.

The business being performed, or the calculations made, commonly by means of the instrument called the gauging or diagonal rod, and the sliding rule or gauging rule, it will be necessary to treat of these instruments, which we shall do as below.

The description and use of the Sliding Rule.

This is a square rule, having consequently four sides or faces, three of which are furnished with sliding pieces running in grooves. The lines on them are mostly logarithmic ones, or distances which are proportional to the logarithms of the numbers placed at the ends of them; which kind of lines was placed on rules by Mr. *Edmund Gunter*, for expeditiously performing arithmetical operations; in which business he used a pair of compasses for taking the several logarithmic distances: but instead of the compasses, sliding pieces were added, by Mr. *Thomas Everard*, as being more convenient and certain in practice.

On the first face are three lines, namely, two marked A, B, for multiplying and dividing; and the third M D, for malt depth, because it serves to gauge malt. The middle one, B, is upon the slider, and it is a kind of double line, being marked at both the edges of the slider, for applying it to both the lines A and M D. These three lines are all of the same radius, or distance, from 1 to 10, each containing twice the length of the radius, A and B are placed and numbered exactly alike, each beginning at 1, which may be either 1, or 10, or 100, &c. or ·1, or ·01, or ·001, &c.; but whatever it is, the middle division, 10, will be 10 times as much, and the last division 100 times as much. But one on the line MD is opposite 215, or more exactly 2150·4 on the other lines, which number 2150·4 denotes the cubic inches in a malt bushel, and its divisions numbered retrograde to those of A and B. On these two lines are also several other marks and letters: thus, on the line A, are M B, for malt bushel, at the number 2150·4; and A for ale, at 282, the cubic inches in an ale gallon; and on the line B, is W, for wine, at 231, the cubic inches in a wine gallon; also *si*, for square inscribed, at ·707, the side of a square inscribed in a circle whose diameter is 1: *se*, for square equal, at ·886, the side of a square which is equal to the same circle: and C for circumference, at 3·1416, the circumference of the same circle.

On the second face, or that opposite the first, are a slider and four lines, marked D, C, D, E, at one end, and root, square, root, cube, at the other; the lines C and E containing respectively the squares and cubes of the opposite numbers on the lines D, D; the radius of D being double to that of A, B, C, and triple to that of E: so that whatever the first 1 on D denotes, the first on C is the square of it, and the first on E the cube of it; so if D begin with 1, C and E will begin with 1; but if D begin with 10, C will begin with 100, and E with 1000; and so on. On the line C, are marked *oc* at ·0796, for the area of a circle whose circumference is 1; and *od* ·7854, for the area of the circle whose diameter is 1. Also, on the line D, are WG, for wine gauge, at 17·15: and AG, for ale gauge, at 18·95; and MR, for malt round, at 52·32; these three being the gauge points for round or circular measure, and are found by dividing the square roots of 231, 282, and 2150·4 by the square root of ·7854: also MS, for malt-square, are marked at 46·37, the malt gauge-point for square measure, being the square root of 2150·4.

On the third face are three lines, one on a slider marked N; and two on the stock, marked SS, and SL, for segment standing and segment lying, which serve for ullaging standing and lying casks.

On the fourth, or opposite face, are a scale of inches and three other scales, marked spheriod or 1st variety, 2d variety, 3d variety; the scale for the 4th, or conic variety, being on the inside of the slider in the third face. The use of these lines is to find the mean diameters of casks.

Besides all those lines, there are two others on the insides of the first two sliders, being continued from the one slider to the other. The one of these is a scale of inches, from $12\frac{1}{2}$ to 36; and the other is a scale of ale-gallons, between the corresponding numbers ·435 and 3·61; which form a table to show, in ale-gallons, the contents of all cylinders whose diameters are from $12\frac{1}{2}$ to 36 inches, their common altitude being 1 inch.

As the sliding rule is for performing, very expeditiously, any operations of multiplication, division, and extraction of roots, which may be required by any precept proposed in words, &c.; so the manner of making these operations will appear in the following problems.

PROBLEM I.

To find the product of two given numbers by the Sliding Rule.

To either of the given numbers on A set 1 on B, then against the other number on B is the product on A.

EXAMPLES.

1. Required the product of 12 and 25.

By placing 1 on B under 12 on A, above 25 on B stands 300 on A; which is the product required.

Note.—When the 1 on B has been set to the one factor on A, if it happen that the other factor on B falls beyond the division, on either A or B, divide it by 10, or 100, &c. till the quotient found on B fall under some division on the line A, and multiply this said division by the same 10, 100, &c. for the product required.

2. So when 250 is to be multiplied by 56: having set 1 on B to 250 on A, although 56 be found on B, it is beyond the end of A; therefore dividing it by 10, I find that opposite to the quotient 5·6 on B, is the division 1400 on A; which being multiplied by 10, we obtain 14000 for the product required.

3. But if 250 were to be multiplied by 1120: having set 1 to 250 as before, 1120 is beyond the end of B, but being divided by 100, opposite to the quotient 11·2 on B I find 2800 on A, which being multiplied by 100, we have 280000 for the product required.

PROBLEM II.

To find the Quotient of two numbers.

Set 1 on B to the divisor on A, then against the dividend on A, is the quotient on B.

EXAMPLES.

1. To divide 300 by 25: having set 1 on B to 25 on A, opposite 300 on A I find 12 on B, the quotient required.

Note.—When the dividend falls beyond the end of the line A, let it be divided by 10, 100, or some other power of 10, till it fall within the line, and use the quotient instead of it, multiplying the result by the same power of 10 as before.

2. So, if 14000 must be divided by 56: having set 1 to 56, the dividend cannot be found on A till it is divided by 100, the quotient being 140, opposite to which I find 2·5 on B, which being multiplied by 100, we obtain 250 for the quotient required.

PROBLEM III.

To work the Rule-of-Three on the Sliding Rule: or having three numbers given, to find a fourth, which shall be to the third as the second is to the first.

Set the first term on B, to either the second or third on A; then against the remaining term on B, stands the fourth term required on A.

EXAMPLE.

If 8 yards of cloth cost 24 shillings, what will 96 yards cost at the same rate?

Having set 8 on B to 24 on A, opposite 96 on B, I find, on A, 288 shillings, or 14*l.* 8*s.*, which is the answer.

PROBLEM IV.

To extract the Square Root by the Sliding Rule.

The first one on C standing against the first 1 on D, on the stock, opposite the given number on C is its root on D.

EXAMPLE.

To find the side of a square, which shall be equal to a triangle, or circle, &c., whose area is 225; or, to extract the root of 225.

Here, opposite 225 on C stands 15 on D, which is the answer required.

PROBLEM V.

To extract the Cube Root by the Sliding Rule.

The line D on the slide being set straight with E: find the given number on E, and opposite to it will be its cube root on D.

EXAMPLE.

To find the side of a cube equal to any other solid whose content is 3375; or to find the cube root of 3375.

Here, opposite 3375 on E, stands 15 on D, which is the answer required.

Note.—It is evident that the same lines as are used in these two last problems, will serve to find the square or the cube of any given number, by taking the given number on the contrary lines.

PROBLEM VI.

To find a Mean Proportional between two given Numbers.

Set one of the given numbers on C, to the like or same number on D; then against the other given number on C, is the number required on D.

EXAMPLE.

To find the side of a square whose area shall be equal to that of a parallelogram whose length is 9, and its breadth 4 feet; or, to find a mean proportional between 4 and 9.

Having set 4 on C to 4 on D, against 9 on C stands 6 on D, which is the number sought.

PROBLEM VII.

To find a number which shall be to a given number, in a given duplicate proportion; or having given three numbers, to find a fourth, which shall be to the third, as the square of the second is to the square of the first.

Set the third number on C, to the first on D; then against the second on D, will be found, on C, the fourth required.

EXAMPLE.

If the area of a parallelogram, or any other figure, be 120; it is required to find the area of a similar figure, their like dimensions or sides being as 2 to 3.

Similar figures being as the squares of their like dimensions, by setting 120 on C, to 2 on D; against 3 on D, stands 270 on C, for the number sought.

PROBLEM VIII.

To find a number which shall be to a given number, in a given subduplicate proportion; or having given three numbers, to find a fourth, which shall be to the third, as the root of the second is to the root of the first.

Set the first number on C, to the third on D; then against the second on C, will be found the fourth on D.

EXAMPLE.

The side of a regular figure is 2, and its area 120; it is required to find the side of a similar figure whose area is 270.

The roots of the areas of similar figures being as their sides, we must find a number which shall be to 2, as the root of 270 is to the root of 120. Therefore, having set 120 on C to 2 on D, against 270 on C, will be found 3 on D, which is the number sought.

PROBLEM IX.

To find a number in a given triplicate proportion to a number given; or, having three numbers given, to find a fourth, which shall be to the third, as the cube of the second is to the cube of the first.

Set the first number on the slide D, to the third number on E; then against the second on D, is the fourth required on E.

EXAMPLE.

If a cask, whose length is 40 inches, contain 100 gallons, what will be the content of a similar cask, whose length is 36 inches?

Similar solids being as the cubes of their like sides, the content required must be to 100 gallons, as 36^3 is to 40^3. Therefore setting 40 on D, to 100 on E; against 36 on D, will be found 72.9 gallons on E, which is the content required.

PROBLEM X.

To find a number in a given subtriplicate proportion to a given number; or, having three numbers given, to find a fourth, which shall be to the third, as the cube root of the second is to the cube root of the first.

Set the third number on D, to the first on E; then against the second on E, will stand the fourth on D.

EXAMPLE.

What is the length of a cask whose content is 72.9 gallons, supposing the length of a similar cask to be 40 inches, and its content 100 gallons?

Since the dimensions of similar solids are as the cube roots of their contents, we must find a number which shall be to 40 as the cube root of 72.9 is to the cube root of 100. Therefore having set 40 on D, to 100 on E; against 72.9 on E, will be found 36 on D, which is the length required.

PROBLEM XI.

The length and breadth of a Parallelogram being given, to find its area in malt bushels by the line MD.

Set either of the given dimensions on B, to the other on MD; then against 1 on A, is the required area on B.

EXAMPLE.

How many malt bushels can be contained on every inch of the depth of a cistern whose length is 180, and breadth 72 inches?

By setting 72 on B, to 180 on MD; against 1 on A, will appear nearly 6 bushels on B, which is the quantity sought.

PROBLEM XII.

To find, by the line MD, *the malt bushels which may be contained in a couch, floor, or cistern, whose length, breadth, and depth are given.*

Set one of the dimensions on B, to another on MD; then against the third on A, will appear the content on B.

EXAMPLE.

Required the number of bushels in the cistern whose length is 230, breadth 58.2, and depth 5.4 inches.

Having set 230 on B, to 5.4 on MD; against 58.2 on A, is found 33.6 bushels on B, which is the content, nearly.

The use of the other parts or marks on the rule will appear in the examples further on.

Of the Gauging or Diagonal Rod.

The diagonal rod is a square rule having four sides or faces, being generally four feet long, and folding together by means of joints.

It takes its name from its use in measuring the diagonals of casks, and computing the contents from the said diagonal only: where it may be noted, that by the diagonal of a cask is meant the line from the bung to the intersection of the head with the stave opposite to it, and is commonly the longest line that can be drawn from the middle of the bung to any part within the cask.

And, accordingly, on one face of the rule is a scale of inches, for taking the measure of the diagonal; to which are adapted the area, in ale gallons, of circles to the corresponding diameters, like the lines on the under sides of the three slides in the sliding rule.

On the opposite face are two scales, of ale and wine gallons, expressing the contents of casks having the corresponding diagonals; and these are the lines which chiefly constitute the difference between this instrument and the sliding rule; for all other lines on it are the same with those on that instrument; and are to be used in the same manner.

EXAMPLE.

Let it be required to find the content of a cask, whose diagonal measures 34.4 inches, which agrees with the cask in the following chapter, whose head and bung diameters are 32 and 24, and length 40 inches; for if to the square of 20, half the length, be added the square of 28, half the sum of the diameters, the square root of the sum will be 34.4, nearly.

Now, to this diagonal 34.4, corresponds, on the rule, the content $90\frac{3}{4}$ ale, or 111 wine gallons; which differs from all the contents, in the next chapter, obtained by considering the cask as belonging to each of the four proposed varieties; being, indeed, a kind of medium among them all, and falling in between the second and third variety; and so answering to the most common form of casks.

Of Casks considered as divided into several varieties.

According to the custom of most writers on this subject, casks are distinguished into four forms, or varieties, viz.

1. The middle frustum of a spheroid.
2. The middle frustum of a parabolic spindle.
3. The two equal frustums of a paraboloid.
4. The two equal frustums of a cone.

The middle frustum of circular, elliptic, and hyperbolic spindles, are here omitted, on account of the difficulty of their rules, which renders them unfit for the purpose of practical gauging. And, indeed, some of the above four forms are of very little real use; for very few, if any casks are to be met with which will hold so much as the first form, or so little as the third or fourth; so that the second is the most generally, if not the only, useful one of the four varieties.

Note. 282 cubic inches make one ale gallon.
231 - - - - wine gallon.
2150.42 - - - a malt bushel.

It is also to be noted that the dimensions are supposed to be inches, in the following rules.

PROBLEM I.

To find the content of a Cask of the first or Spheroidal variety.

To the square of the head diameter add double the square of the bung diameter, and multiply the sum by the length of the cask. Then let the product be multiplied by $.0009\frac{1}{4}$, or divided by 1077, for ale gallons; or multiplied by $.0011\frac{1}{3}$, or divided by 882, for wine gallons.

BY THE SLIDING RULE.

Set the length on C, to 32.82 for ale, or to 29.7 for wine, on D; and on D find the bung and head diameters, noting the numbers opposite to them on C; then if the latter of these two numbers be added to the double of the former, the sum will be the measure in gallons.

EXAMPLE.

Required the content of a spheroidal cask, whose bung and head diameters are 32 and 24, and length 40 inches.

By the Pen. Here $(2 \times 32^2 + 24^2) \times 40 \times \left\{ \begin{matrix} \cdot0009\frac{1}{4} \\ \cdot0011\frac{1}{3} \end{matrix} \right\} = \left\{ \begin{matrix} 97\cdot44 \text{ ale} \\ 118\cdot95 \text{ wine} \end{matrix} \right\}$ gallons, the content required.

By the Sliding Rule. Having set 40 on C, to 32·82 on D; against 32 and 24 on D, stand 38 and 21·3, as near as can be judged, on C; then $2 \times 38 + 21\cdot3 = 76 + 21\cdot3 = 97\cdot3$ ale gallons.

And having set 40 on C, to 29·7 on D; against 32 and 24 on D, stand 46·5 and 26·1 on C; then $2 \times 46\cdot5 + 26\cdot1 = 93 + 26\cdot1 = 119\cdot1$ wine gallons.

PROBLEM II.

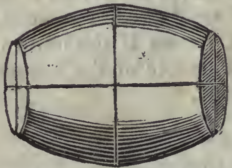
To find the content of a Cask of the second or Parabolic Spindle form.

To the square of the head diameter add double the square of the bung diameter, and from the sum take $\frac{2}{5}$ or $\frac{4}{10}$ of the square of the difference of the said diameters; then multiply the remainder by the length, and the product, multiplied or divided by the same numbers as in the rule to the last problem, will give the content.

BY THE SLIDING RULE.

As in the last problem, se the length on C, to 33.82 or 2.97 on D; and on D find both the bung and head diameters, and also their difference, taking out the three numbers opposite to them on C; then if to twice the first be added the second, and $\frac{4}{10}$ of the third be taken from the sum, the remainder will be the content.

EXAMPLE.



Required the content of a cask of the second variety, whose bung and head diameters are 32 and 24 and length 40 inches.

By the Pen. Here $(2 \times 32^2 + 24^2 - \frac{2}{5} \times 8^2) \times 40 \times \left\{ \begin{matrix} \cdot0009\frac{1}{4} \\ \cdot0011\frac{1}{3} \end{matrix} \right\} = \left\{ \begin{matrix} 96\cdot49 \text{ ale} \\ 117\cdot79 \text{ wine} \end{matrix} \right\}$ gallons the content required.

By the Sliding Rule. Having set 40 on C, to 32·82 on D; against 32, 24, and 8, on D, stand 38, 21·3, and 2·4; then $2 \times 38 + 21\cdot3 - \frac{2}{5} \times 2\cdot4 = 76 + 21\cdot3 - 0\cdot9 =$ 96·4 ale gallons.

And having set 40 on C, to 29·7 on D, against 32, 24, and 8, on D, stand 46·5, 26·1, and 2·9; then $2 \times 46\cdot5 + 26.1 - \frac{2}{5} \times 2\cdot9 = 93 + 26\cdot1 - 1\cdot2 = 117\cdot9$ wine gallons.

PROBLEM III.

To find the content of a Cask of the third or Paraboloidal variety.

To the square of the bung diameter, add the square of the head diameter, and multiply the sum by the length; then if the product be

$$\text{mult by} \begin{Bmatrix} \cdot 0014 \\ \cdot 0017 \end{Bmatrix} \text{ or div. by } \begin{Bmatrix} 718 \\ 588 \end{Bmatrix} \text{ for } \begin{Bmatrix} \text{ale} \\ \text{wine} \end{Bmatrix},$$

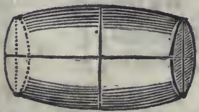
the product or quotient will be the content.

BY THE SLIDING RULE.

Set the length on C, to $\begin{Bmatrix} 26\cdot 8 \\ 24\cdot 25 \end{Bmatrix}$ on D, $\begin{Bmatrix} \text{for ale} \\ \text{for wine} \end{Bmatrix}$; then find the bung and head diameters on D, noting the two opposite numbers on C, whose sum will be the content required.

EXAMPLE.

Required the content of a cask of the third variety, whose bung and head diameters are 32 and 24 and length 40 inches.



By the Pen. Here $(32^2 + 24^2) \times 40 \times$

$$\begin{Bmatrix} \cdot 0014 \\ \cdot 0017 \end{Bmatrix} = \begin{Bmatrix} 89\cdot 1 \text{ ale} \\ 108\cdot 8 \text{ wine} \end{Bmatrix} \text{gallons, the content.}$$

By the Sliding Rule. Having set 40 on C, to 26·8 on D; against 32 and 24 on D, stand 57·3 and 32; whose sum is 89·3 ale gallons.

And having set 40 on C, to 24·25 on D; against 32 and 24 on D stand 69·8 and 39·1, whose sum is 108·9 wine gallons.

PROBLEM IV.

To find the content of a Cask of the fourth or Conical variety.

To three times the square of the sum of the diameters, add the square of the difference of the diameters; then if the sum be multiplied by the length, and the product be

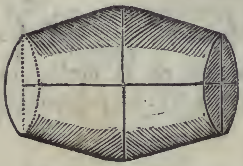
$$\text{mult. by} \left\{ \begin{array}{l} \cdot00023\frac{1}{5} \\ \cdot00028\frac{1}{3} \end{array} \right\} \text{ or div. by} \left\{ \begin{array}{l} 4308 \\ 3529 \end{array} \right\} \text{ for } \left\{ \begin{array}{l} \text{ale,} \\ \text{wine,} \end{array} \right.$$

the product or quotient will be the content required.

BY THE SLIDING RULE.

Set the length on C, to $\left\{ \begin{array}{l} 65\cdot64 \\ 59\cdot41 \end{array} \right\}$ for $\left\{ \begin{array}{l} \text{ale} \\ \text{wine} \end{array} \right\}$ on D; and on D find the sum and the difference of the diameters, noting their opposite numbers on C; then if the second be added to three times the first, the sum will be the content.

EXAMPLES.



1. Required the content of a cask of the fourth variety, whose bung and head diameters are 32 and 24, and length 40 inches.

By the Pen. Here $(3 \times 56^2 + 8^2) \times 40 \times$

$$\left\{ \begin{array}{l} \cdot00023\frac{1}{5} \\ \cdot00028\frac{1}{3} \end{array} \right\} = \left\{ \begin{array}{ll} 87\cdot9342 & \text{ale} \\ 107\cdot348 & \text{wine} \end{array} \right\} \text{gallons, the content.}$$

By the Sliding Rule. Having set 40 on C, to 65·64 on D; against 56 and 8 on D, stand 29·1 and 0·6; then $3 \times 29\cdot1 + 0\cdot6 = 87\cdot3 + 0\,6 = 87.9$ ale gallons.

And, having set 40 on C, to 59·41 on D; against 56 and 8 on D, stand 35·6 and 0·7; then $3 \times 35.6 + 0\cdot7 = 107\cdot5$ wine gallons.

2. Suppose the bung diameter of a cask to be 26·5 inches, head diameter 23 inches, and length 28·3 inches; the content in ale gallons, for each variety, is required.

Ans. $\left\{ \begin{array}{ll} 50\cdot8 & \text{for the first variety.} \\ 50 & \text{for the second.} \\ 48\cdot8 & \text{for the third.} \\ 48\cdot28 & \text{for the fourth.} \end{array} \right.$

Of Gauging Casks by their Mean Diameters.

PROBLEM I.

To find the Mean Diameter of a Cask of any of the four varieties, having given the bung and head diameters.

Divide the head diameter by the bung diameter, and find the quotient in the first column of the following table, marked Qu. Then if the bung diameter be multiplied by the number on the same line with it, and in the column answering to the proper variety, the product will be the true mean diameter, or the diameter of a cylinder of the same content with the cask proposed, cutting off four figures for decimals.

Qu	1 Var.	2 Var.	3 Var.	4 Var.	Qu	1 Var.	2 Var.	3 Var.	4 Var.
50	8660	8465	7905	7637	76	9270	9227	8881	8827
51	8680	8493	7937	7681	77	9296	9258	8944	8874
52	8700	8520	7970	7725	78	9324	9290	8967	8922
53	8720	8548	8002	7769	79	9352	9320	9011	8970
54	8740	8576	8036	7813	80	9380	9352	9055	9018
55	8760	8605	8070	7858	81	9409	9383	9100	9066
56	8781	8633	8104	7902	82	9438	9415	9144	9114
57	8802	8662	8140	7947	83	9467	9446	9189	9163
58	8824	8690	8174	7992	84	9496	9478	9234	9211
59	8846	8720	8210	8037	85	9526	9510	9280	9260
60	8869	8748	8246	8082	86	9556	9542	9326	9308
61	8892	8777	8282	8128	87	9586	9574	9372	9357
62	8915	8806	8320	8173	88	9616	9606	9419	9406
63	8938	8835	8357	8220	89	9647	9638	9466	9455
64	8962	8865	8395	8265	90	9678	9671	9513	9504
65	8986	8894	8433	8311	91	9710	9703	9560	9553
66	9010	8924	8472	8357	92	9740	9736	9608	9602
67	9034	8954	8511	8404	93	9772	9768	9656	9652
68	9060	6983	8551	8450	94	9804	9801	9704	9701
69	9084	9013	8590	8497	95	9836	9834	9753	9751
70	9110	9044	8631	8544	96	9868	9867	9802	9800
71	9136	9074	8672	8590	97	9901	9900	9851	9850
72	9162	9104	8713	8637	98	9933	9933	9900	9900
73	9188	9135	8754	8685	99	9966	9966	9950	9950
74	9215	9166	8796	8732	100	10000	10000	10000	10000
75	9242	9196	8838	8780					

EXAMPLE.

Supposing the diameters to be 32 and 24, it is required to find the mean diameter for each variety.

Dividing 24 by 32, we obtain ·75; which being found in the column of quotients, opposite thereto stand the numbers

·9242	which being each	29·5744	for the corres-
·9196	multiplied by 32,	29·4272	ponding mean
·8838	produce respec-	28·2816	diameters re-
·8780	tively	28·0960	quired.

BY THE SLIDING RULE.

Find the difference between the bung and head diameters on the fourth face of the rule, or inside of the third slider; and opposite thereto is, for each variety, a number to be added to the head diameter, for the mean diameter required.

So, in the above example, against 8, the difference of the diameters, are found the numbers

5·60	which being	29·60	for the respective mean di-
5·10	added to24,	29·10	ameters; all of which are too
4·56	there result	28·56	great, except the second,
4·12		28·12	which is too little.

So that this method does not give the true mean diameter.

PROBLEM II.

To find the content of a Cask by the mean diameter on the Sliding Rule.

Set the length on C, to the gauge point, 18·95 for ale, or 17.15 for wine, on D; then against the mean diameter on D, is the content on C.

EXAMPLE.

If the bung diameter be 32, the head 24; and the length 40 inches,

Having found the mean diameters, as in the last problem, and set 40 on C, to 18·95 or 17·15 on D.

against	29·57	on D, is	97·4	or	119·5	on C, as near as can
	29·43		96·5		118·0	be judged; which
	28·28		89·1		108·8	agree nearly with the
	28·10		88·0		107·3	contents computed in

the preceding chapter.

Of the Gauging of all Casks in general by means of four dimensions, viz. the length, the bung and head diameters, and the diameter taken in the middle, between the bung and head.

GENERAL PROBLEM.

To find the content of any Cask, in ale or wine gallons, by four dimensions.

Add into one sum the square of the bung diameter, the square of the head diameter, and the square of double the middle diameter, and multiply that sum by the length of the cask; then the product

mult. by $\left\{ \begin{array}{l} \cdot 0004\frac{7}{11} \\ \cdot 0005\frac{2}{3} \end{array} \right\} \left\{ \begin{array}{l} \text{for ale} \\ \text{for wine} \end{array} \right\}$, will give the content.

BY THE SLIDING RULE.

Set the length on C, to $\left\{ \begin{array}{l} 46\cdot 4 \text{ for ale} \\ 42\cdot 0 \text{ for wine} \end{array} \right\}$ on D; and find the bung, head, and middle diameters, on D, noting the three numbers against them on C; then the sum of the first and second, with four times the third, will be the content required.

EXAMPLE.

Let the length of a cask be 40 inches, the bung diameter 32, the head 24, and middle diameter 30·2 inches nearly $= \sqrt{912}$, which is taken on the supposition that the cask is spheroidal.

Then $(32^2 + 24^2 + 4 \times 912) \times 40 \times \cdot 0004\frac{7}{11} =$ 97·44 ale gallons; or multiplied by $\cdot 0005\frac{2}{3}$, gives 118·95 wine gallons, for the content.

By the Sliding Rule. Having set 40 on C, to 46·4 on D; against 32, 24, and 30·2, on D, stand 19, 10·5, and 17 on C; then $19 + 10\cdot 5 + 4 \times 17 = 97\cdot 50$ ale gallons.

And, having set 40 on C, to 42 on D; against 32, 24, and 30·2 on D, stand 23·2, 13, and 20.7 on C; then $23.2 + 13 + 4 \times 20.7 = 119$ wine gallons for the content, as before, nearly.

A new and very exact method of computing the content of a Cask of any form, from three dimensions only.

Add into one sum 39 times the square of the bung diameter, 25 times the square of the head diameter, and 26 times the product of the diameters; multiply the sum by the length, and the product by ·00034; then the last product divided by 9 will give the wine gallons, and divided by 11 will give the ale gallons.

EXAMPLES.

1. Required the content of a cask whose length is 40, bung diameter 32, and head diameter 24 inches.

Here $(39 \times 32^2 + 66 \times 32 \times 24 + 25 \times 24^2) \times 40 \times \cdot 00034 = 1010\cdot 5$; which being divided by 9 and by 11, we obtain 112·3 wine gallons, or 91·9 ale gallons, for the content required. Agreeing nearly with the content found by the Diagonal Rod, in page 288.

2. The head diameter of a cask is 34·8 inches, the bung diameter 44.8 inches, and the length of the cask 54 inches: required the content in ale and wine gallons.

Ans. { 248·8374 ale gallons.
304·1346 wine gallons.

Of the Ullage of Casks.

The ullage of a cask is now generally understood to be either the empty part or the part filled, of a cask which is not quite full.

In this business, casks are considered as in two positions, viz. with their axes either parallel or perpendicular to the horizon, that is, either lying along, or standing upright on one end. The ullage of a cask, either standing or lying, may be determined for any particular variety, by the preceding parts of this work; but the rules thence obtained are too complex for ordinary practice, especially for a lying cask; I shall not, therefore, introduce those rules into this place, but shall content myself with setting down such approximations as have been found to agree best with expedition and accuracy; together with the universal method by three diameters, which is not only sufficiently easy, but also the most accurate for this purpose.

PROBLEM I.

To find the Ullage of a Standing and Lying Cask, by the lines SS *and* SL *on the Sliding Rule.*

By some of the preceding chapters, find the whole content of the cask.

Then set the length on N, to 100 on SS, for a seg. standing; or set the bung diameter on N, to 100 on SL, for the seg. lying; then against the wet or dry inches on N, is a number to be reserved. Next, set 100 on B, to the reserved number on A; and against the whole content on B, will be found the ullage on A. And this ullage will be either the empty part, or the part filled, according as the dry or wet inches were used, in finding the reserved number.

EXAMPLE I.

Required the ullage of a standing cask, whose length is 40, bung diameter 32, head diameter 24, wet inches 10, and consequently the dry inches 30.

By the last problem the content was nearly 92 ale gallons: hence, having set 40 on N, to 100 on SS; and against 10 on N, found 23 on SS; then set 100 on B, to 23 on A; and against 92 on B, is found 21·2 ale gallons on A, for the quantity remaining in the cask.

If the dry inches 30 be used, the reserved segment will be found to be 76·7; and then the corresponding ullage is 70·2, for the gallons drawn off.

And the sum of these two parts is 91·4 gallons, which is about half a gallon less than the whole content; which error would be inconsiderable if it were to be divided between the two parts; but instead of that, commonly the one part is too little, and the other too great, by which it happens that the error in each part mostly exceeds that of their sum.

EXAMPLE II.

Let the dimensions and content be the same in the case of the lying cask, also the wet inches 8, and consequently the dry inches 24.

Having set 32 on N, to 100 on SL; against 8 and 24 on N, are the reserved segments 17·8 and 92.5 on SL. Then, having set 100 on B, to 92 on A; against 17·8 and 82.5 on

B, are 16.4 and 76 on A ; which are the parts filled and empty respectively, and whose sum is 92·4, near half a gallon too much.

PROBLEM II.

To find the Ullage of a Standing Cask by the Pen.

RULE I.

Add all together, the square of the diameter at the surface of the liquor, the square of the diameter of the nearest end, and the square of double the diameter taken in the middle between the other two ; then multiply the sum by the length between the surface and nearest end, and the product again by $\cdot 0004\frac{2}{3}$ for ale gallons, or by $\cdot 0005\frac{2}{3}$ for wine gallons, in the less part of the cask, whether empty or filled.

EXAMPLE.

Taking the above diameters 24, 27, and 29 inches, and the wet inches 10 ; we shall have $(24^2 + 54^2 + 29^2) \times 10 \times \cdot 0004\frac{2}{3} = 4333 \times \cdot 004\frac{2}{3} = 20\cdot 2$ gallons for the ullage required.

RULE II.

As the square of the length of the cask is to the square of the difference between the said length and wet or dry inches, viz. the less of them ; so is the difference between the bung and head diameters to a number, which, being taken from the bung diameter, will leave the diameter of a cylinder of the same length with, and nearly equal to, the part filled or empty, viz. the less of them.

EXAMPLE.

Taking the same example as in problem 1, we have, as $40^2 : (40-10)^2 :: 32 - 24 : 4\frac{1}{2}$; and $32 - 4\frac{1}{2} = 27\frac{1}{2}$ the mean diameter.

Then $27\cdot 5^2 \times 10 \times \cdot 002\frac{7}{9} = 21$ ale gallons is the ullage required.

Note. The number ·0028751 or .002$\frac{7}{9}$ nearly, is the constant multiplier $\dfrac{\cdot 785398, \&c.}{282}$

PROBLEM III.

To find the Ullage of a Lying Cask by the Pen.

Divide the wet inches by the bung diameter; find the quotient in the first column of the table of circular segments, at the end of the book, and take out the segment opposite to it; then multiply this segment by the whole content of the cask, and the product again by $1\frac{1}{4}$, for the ullage, nearly.

EXAMPLE.

Taking the same cask as before, whose length is 40, bung diameter 32, head diameter 24; and supposing the wet inches to be 8.

The whole content is nearly 92 ale gallons. Then $\frac{8}{32} = \frac{1}{4} = \cdot 25$; opposite to which, in the table of areas, is the segment ·15354621; hence $92 \times \cdot 15354621 \times 1\frac{1}{4} = 18$ ale gallons, the ullage required.

SCHOLIUM.

Having delivered the necessary rules for measuring casks, &c., I do not suppose that any thing more of the subject of gauging is wanted to be given in this book. For, as to cisterns, couches, &c. tuns, coolers, &c. coppers, stills, &c. which are first supposed to be in the form of some of the solids in the former parts of this work, and then measured accordingly, no person can be at a loss concerning them, who knows any thing of such solids in general; and to treat of them here, would induce me to a long and tedious repetition, only for the sake of pointing out the proper multipliers or divisors; which is, I think, a reason very inadequate to so cumbersome an increase of the book.

I shall only just observe, that when tuns, &c. of oval bases are to be gauged; as those bases really measure more than true ellipses of the same length and breadth, they ought to be measured by the equi-distant ordinate method.

And that when casks are met with which have different head diameters, they may be deemed incomplete casks, and their contents considered and measured as the ullage of a cask.

TO FIND THE TONNAGE OF A SHIP.

The length is taken in a straight line along the rabbet of the keel, from the back of the main stern-post to a perpendicular from the fore part of the main stem, under the bowsprit, from which subtract $\frac{3}{5}$ of the breadth, the remainder is the length. The breadth is taken at the broadest part of the ship, from the outside to the outside.

RULE.—Multiply the square of the breadth by the length, and divide the product by 188, the quotient will be the tonnage.

Ex. 1.—Required the tonnage of a ship, of which the length is 75 feet, and the breadth 26 feet.

Ans. $26 \times 26 \times 75 \div 188 = 270$ tons, nearly.

Ex. 2.—Length 96, and breadth 33 feet? *Ans.* 556 tons.

Note. This rule is very erroneous, and no other general rule can be given which is perfectly accurate; the best way is to find the quantity of water displaced by the ship when she is loaded; but as this must be done by means of ordinates, the operation is laborious. It is easier to load her with ballast, weighing the load as it is put on board.

The following rule is a near approximation.

1st. *For men of war.*—Take the length of the gun-deck, from the rabbet of the stem to that of the stern-post, subtract $\frac{1}{24}$ of it, the remainder is the length. Take the extreme breadth from outside to outside of the plank, and add it to the length, $\frac{1}{24}$ of the sum is the depth. Set up this height from the limber strake, and at that height take a breadth from outside to outside, where the extreme breadth was taken, and take another breadth in the middle, between this and the limber strake, add the extreme and these two breadths, and take $\frac{1}{3}$ of the sum for the breadth. Then multiply the length, breadth, and thickness, and divide the product by 49.

2d. *For ships of burden.*—Take the length of the lower deck, from the rabbet of the stem to that of the stern-post, and from it subtract $\frac{1}{32}$ of it, for the length. Take the extreme breadth from outside to outside, and add it to the length of the lower deck, $\frac{3}{55}$ of the sum is the depth. Set up this depth from the limber strake, where the extreme breadth was taken, and at this height take a breadth from outside to outside, take another breadth at $\frac{2}{3}$ of this height, and a third at $\frac{1}{3}$ of the height, add these three to the extreme breadth, and $\frac{1}{4}$ of the sum is the mean breadth. Multiply the length, breadth, and depth, and divide three times the product by 110 for the tonnage.

SPECIFIC GRAVITY.

The specific gravity of a body is the proportional weight between that body and another of a known density; and water is admirably adapted to be the standard, as a solid foot of it weighs 1000 ounces Avoirdupois.

To find the specific gravity of a body.

PROBLEM I.

When the body is heavier than water.

RULE. Weigh it both in and out of water, and take the difference, which will be the weight lost in water; then say,

As the weight lost in water,
Is to the whole or absolute weight;
So is the specific gravity of water,
To the specific gravity of the body.

EXAMPLE.

What is the specific gravity of a stone which weighs 10 libs. but in the water only $6\frac{3}{4}$ libs. water being 1000?

$10 - 6\frac{3}{4} = 3\frac{1}{4}$ weight lost in water.
$3\frac{1}{4} : 10 :: 1000 : 3077$ specific gravity.

PROBLEM II.

When the body is lighter than water.

RULE. Annex to it a piece of another body heavier than water, so that the mass compounded of the two may sink together. Weigh the denser body and the compound mass separately, both in and out of water, then find how much each loses in water, by subtracting its weight in water from its weight in air, and subtract the less of these remainders from the greater; then say,

As the last remainder,
Is to the weight of the light body in air;

So is the specific gravity of water,
To the specific gravity of the body.

EXAMPLE.

What is the specific gravity of a piece of elm which weighs in air 15 libs; attached to it is a piece of copper, weighing 18 libs. in air, and 16 libs. in water, and this compounded weighs in water 6 libs.?

$$
\begin{array}{l}
33 = 18 + 15 \\
\;\;6 \quad\; 16 \\
\overline{\;\;\;}\quad \overline{\;\;\;} \\
27 - \;\; 2 = 25 \textit{ last remainder.}
\end{array}
$$

As 25 : 15 :: 1000 : 600, specific gravity of the elm.

PROBLEM III.

For a fluid of any sort.

RULE. Take a piece of a body of known specific gravity, weigh it both in and out of the fluid, finding the loss of weight by taking the difference of the two; then say.

As the whole or absolute weight,
Is to the loss of weight;
So is the specific gravity of the solid,
To the specific gravity of the fluid.

EXAMPLE.

What is the specific gravity of a fluid in which a piece of cast iron weighs 34.51 oz. and 40 oz. out of it?
40 : 5.39 :: 7425 : 1000, specific gravity of the fluid.

Another method.

RULE. Weigh any convenient body, (a bubble of glass is best for the purpose) in air, in water, and in the fluid whose specific gravity is required; then say,

As the loss of the weight in water,
Is to the loss of weight in the other fluid;
So is the specific gravity of water,
To the specific gravity of the fluid.

EXAMPLE.

A body weighs 1000 grs. in air, 750 grs. in water, and 770 in a liquid whose specific gravity is required.

250 : 230 :: 1000 : 920, specific gravity of the liquid.

PROBLEM IV.

To find the quantities of two ingredients in a given compound.

Rule. Take the three differences of every pair of the three specific gravities, namely, the specific gravities of the compound and each ingredient; and multiply each specific gravity by the difference of the other two; then say,

As the greatest product,
Is to the whole weight of the compound;
So is each of the other two products,
To the weights of the two ingredients.

EXAMPLE.

A composition of 112 libs. being made of tin and copper, whose specific gravity is found to be 8784; required the quantity of each ingredient, the specific gravity of tin being 7320, and copper 9000.

8784, . . .	Composition,
9000, . . .	Copper,
7320, . . .	Tin,

$$9000 - 7320 = 1680 \times 8784 = 14757120$$
$$8784 - 7320 = 1464 \times 9000 = 13176000$$
$$9000 - 8784 = 214 \times 7320 = 1566480$$

As 14757120 : 112 :: 13176000 : 100 = Copper } Weight of
112 − 100 = 12 = Tin } ingred'ts.

A TABLE OF SPECIFIC GRAVITIES OF BODIES.

Platina (pure)	23000	Tin	7320
Fine Gold	19400	Clear Crystal Glass	3150
Standard Gold	17724	Granite	3000
Quicksilver (pure)	14000	Marble and Hard Stone	2700
Quicksilver (common)	13600	Common Green Glass	2600
Lead	11325	Flint	2570
Fine Silver	11091	Common Stone	2520
Standard Silver	10535	Clay	2160
Copper	9000	Brick	2000
Copper Halfpence	8915	Common Earth	1984
Gun Metal	8784	Nitre	1900
Cast Brass	8000	Ivory	1825
Steel	7850	Brimstone	1810
Iron	7645	Solid Gunpowder	1745
Cast Iron	7425	Sand	1520
Coal	1250	Ash	800
Boxwood	1030	Maple	755
Sea-water	1030	Elm	600
Common water	1000	Fir	550
Oak	925	Charcoal	400
Gunpowder close shaken	922	Cork	240
Do. in a loose heap	836	Air at a mean state	$1\frac{2}{9}$

Note.—The several sorts of wood are supposed to be dry. Also, as a cubic foot of water weighs just 1000 ounces avoirdupois, the numbers in this table express not only the specific gravities of the several bodies, but also the weight of a cubic foot of each in avoirdupois ounces; and therefore, by proportion, the weight of any other quantity, or the quantity of any other weight, may be known. Also, 100 cubic inches of common air weigh nearly $31\frac{1}{2}$ grains troy, or $1\frac{1}{7}$ drams avoirdupois.

It is more usual to take the specific gravity of water as unity.—The preceding table will give the specific gravity of the substances mentioned in it, by placing the decimal point before the last three figures of each of the numbers.

PROBLEM V.

To find the magnitude of a body from its weight being given.

Rule.—As the tabular specific gravity of the body is to its weight in avoirdupois ounces,

So is one cubic foot, or 1728 cubic inches, to its content in feet, or inches, respectively.

EXAMPLES.

1. Required the content of an irregular block of common stone which weighs 1 cwt. or 112 lbs.

Here 113 × 16 = 1792
Whence 2520 : 1792 :: 1728

```
                 1728
                 ----
                14336
                3586
               12544
               1792
               -------
         2520)3096576(1228⅘ cubic in. the ans.
               252
               ---
                576
                504
                ---
                 725
                 504
                 ---
                 2217
                 2016
                 ----
                 5016
```

2. How many cubic inches of gunpowder are there in one pound weight? *Ans.* 30, nearly.

3. How many cubic feet are there in a ton weight of dry oak? *Ans.* $38\frac{138}{185}$.

PROBLEM VI.

To find the weight of a body from its magnitude being given.

Rule. As one cubic foot, or 1728 cubic inches, is to the content of the body,

So is its tabular specific gravity, to the weight of the body.

EXAMPLES.

1. Required the weight of a block of marble, whose length is 63 feet, and its breadth and thickness each 12 feet; these being the dimensions of one of the stones in the walls of Balbec.

Here 63
12

756
12

Whence 1 : 9072 :: 2700
2600

6350400
18144

16 { 4	24494400 } *oz.*	
4	6123600	
112	1530900*lbs.*	
20	13678*cwts.*	
	$683\frac{9}{10}$ *tons.*	

Ans. $683\frac{9}{10}$ tons.

2. What is the weight of a pint of gunpowder, ale-measure? *Ans.* 19 oz. nearly.

3. What is the weight of a block of dry oak, which measures 10 feet in length, 3 feet in breadth, and $2\frac{1}{2}$ feet deep? *Ans.* $4335\frac{15}{16}$ lbs.

As water varies in bulk at different temperatures, it becomes necessary in some calculations to take into view this circumstance. For this purpose, the approximate specific gravity obtained by the foregoing rules must be multiplied by the specific gravity of water of the temperature at which the experiment is performed, the specific gravity of water at the standard temperature being taken as the unit. The best temperature for a standard is one pointed out by a physical fact in the constitution of water independent of thermometrical indications. Water contracts only until it is cooled to the temperature of 40°, it then remains of the same bulk until cooled to 38°, and afterwards begins to expand, which it continues to do until it freezes. To facilitate reductions to this temperature a table is subjoined.

TABLE of the Specific Gravity of Water, at various temperatures, by Fahrenheit's Thermometer.

Temperature.	*Spec. Gravity.*	*Exp. for 1°.*
30 dg.	999.82	0.04
32	999.89	0.03
34	999.95	0.02
38° to 40	1000.00	0.00
45	999.91	0.02
50	999.77	0.04
60	999.08	0.08
70	998.02	0.12
80	996.66	0.16
90	994.91	0.20
100	992.89	0.24
120	988.04	0.29
140	982.44	0.31
160	976.29	0.34
180	969.72	0.36
200	962.62	0.37
212	958.60	0.38

FALLING BODIES.

The motion described by Bodies freely descending by their own gravity is, viz.—The velocities are as the times, and the spaces as the squares of the times. Therefore, if

the times be as the numbers	1	2	3	4	&c.
The velocities will be also as	1	2	3	4	&c.
The spaces as their squares	1	4	9	16	&c.
and the spaces for each time, as . . .	1	3	5	7	&c.

namely, as the series of the odd numbers, which are the differences of the squares, denoting the whole spaces; so that if the first series of numbers be seconds of time:

i. e.	1″	2″	3″	&c.
Velocities in feet will be	$32\frac{1}{6}$	$64\frac{1}{3}$	$96\frac{1}{2}$	&c.
Spaces in the whole time will be . .	$16\frac{1}{12}$	$64\frac{1}{3}$	$144\frac{3}{4}$	&c.
Spaces for each second will be . .	$16\frac{1}{12}$	$48\frac{1}{4}$	$80\frac{5}{12}$	&c.

The following TABLE *shows the Spaces fallen through, and the Velocities acquired at the end of each* 20 *seconds.*

Time in Seconds.	SPACE.				VELOCITY.		
	Each Time.	As the Squares of the Time.	Fallen through in Feet and Inches.		As the Times.	Acquired in Feet and Inches.	
1	1	1	16	1	1	32	2
2	3	4	64	4	2	64	4
3	5	9	144	9	3	96	6
4	7	16	257	4	4	128	8
5	9	25	402	1	5	160	10
6	11	36	579	0	6	193	0
7	13	49	788	1	7	225	2
8	15	64	1029	4	8	257	4
9	17	81	1302	9	9	289	6
10	19	100	1608	4	10	321	8
11	21	121	1946	1	11	353	10
12	23	144	2316	0	12	386	0
13	25	169	2718	1	13	418	2
14	27	196	3152	4	14	450	4
15	29	225	3618	9	15	482	6
16	31	256	4117	4	16	514	8
17	33	289	4648	1	17	546	10
18	35	324	5211	0	18	579	0
19	37	361	5806	1	19	611	2
20	39	400	6433	4	20	643	4

EXAMPLE I.

To find the space descended by a body in 7″ and the velocity acquired.

$$16\ 1 \times 49 = 788\ 1 \text{ of space.}$$
$$32\ 2 \times 7'' = 225\ 2 \text{ of velocity.}$$

Look into the Table at 7″ and you have the answers.

EXAMPLE. II.

To find the time of generating a velocity of 100 feet per second, and the whole space descended.

$$\frac{100 \quad \times 12}{32\ 2 \times 12} = 3''\tfrac{21}{193} \text{ Time.}$$

$$\frac{3''\tfrac{21}{193} \times 100}{2} = 155\tfrac{85}{193} \text{ Space descended}$$

EXAMPLE IV.

To find the time of descending 400 feet, and the velocity at the end of that time.

$$\sqrt{\frac{400 \times 12}{16.1 \times 12}} = 4'' 987 \text{ Time.}$$

$$\frac{400 \times 2}{4'' 987} = 169.662 \text{ Velocity.}$$

Or these answers can be found from the Table by Proportion.

PENDULUM.

The vibrations of Pendulums are as the square roots of their lengths; and as it has been found by many accurate experiments, that the pendulum vibrating seconds in the latitude of London, is $39\frac{1}{8}$ inches long nearly, the length of any other pendulum may be found by the following rule, viz.—As the number of vibrations given is to 60, so is the square root of the length of the pendulum that vibrates seconds, to the square root of the length of the pendulum that will oscillate the given number of vibrations; or, as the square root of the length of the pendulum given, is to the square root of the length of the pendulum that vibrates seconds, so is 60 to the number of vibrations of the given pendulum.

Since the pendulum that vibrates seconds, or 60, is $39\frac{1}{8}$ inches long, the calculation is rendered simple; for $\sqrt{39\frac{1}{8}} \times 60 = 375$, a constant number, therefore 375, divided by the square root of the pendulum's length, gives the vibrations per minute, and divided by the vibrations per minute, gives the square root of the length of the pendulum.

EXAMPLE I.

How many vibrations will a pendulum of 49 inches long make in a minute?

$$\frac{375}{\sqrt{49}} = 53\tfrac{4}{7} \text{ vibrations in a minute.}$$

EXAMPLE II.

What length of a pendulum will it require, to make 90 vibrations in a minute?

$$\frac{375}{90} = 4.16, \text{ and } \overline{4.16}^2 = 17.3056 \text{ inches long.}$$

EXAMPLE III.

What is the length of a pendulum, whose vibrations will be the same number as the inches in its length?

$$\sqrt[3]{(375)^2} = 52 \text{ inches long, and 52 vibrations.}$$

It is proposed to determine the length of a pendulum vibrating seconds, in the latitude of London, where a heavy body falls through $16\frac{1}{12}$ feet in the first second of time?

3.1416 circumference, the diameter being 1.
$16\frac{1}{12}$ feet = 193 inches fall in the 1″ of time.

$$\begin{array}{l} 193 \times 2 = 386.00000000 \\ 3.1416^2 = \quad 9.86965056 \end{array} = 39.109 \text{ inches,}$$

or 39.11 inches.
By experiment this length is found to be $39\frac{1}{8}$ inches.

What is the length of a pendulum vibrating in 2 seconds, and another in half a second?

$$\sqrt{39\tfrac{1}{8}} = 6.25 \times 60 = 375.$$

$\frac{375}{30} = 12.5$ squared $= 156.25$ inches the length of a 2 [seconds' pendulum.

$\frac{375}{120} = 3.125$ squared $= 9.765625$ inches the length of a $\frac{1}{2}$ second's pendulum.

MECHANICAL POWERS, &c.

The Science of Mechanics is simply the application of Weight and Power, or Force and Resistance. The weight is the resistance to be overcome ; the power is the force requisite to overcome that resistance. When the force is equal to the resistance, they are in a state of equilibrium, and no motion can take place ; but when the force becomes greater than the resistance, they are not in a state of equilibrium, and motion takes place ; consequently, the greater the force is to the resistance, the greater is the motion or velocity.

The Science of Equilibrium is called *Statics ;* the Science of Motion is called *Dynamics.*

Mechanical Powers are the most simple of mechanical applications to increase force and overcome resistance. They are usually accounted six in number, viz. *The Lever,—The Wheel* and *Axle,—The Pulley,—The Inclined Plane,—The Wedge,*—and the *Screw.*

LEVER.

To make the principle easily understood, we must suppose the lever an inflexible rod without weight ; when this is done, the rule to find the equilibrium between the power and the weight, is,—Multiply the weight by its distance from the fulcrum, prop, or centre of motion, and the power by its distance from the same point : if the products are equal, the weight and power are in equilibrio, if not, they are to each other as their products.

EXAMPLE I.

A weight of 100 libs on one end of a lever, is 6 inches from the prop, and the weight of 20 libs at the other end, is 25 inches from the prop—What additional weight must be added to the 20 libs, to make it balance the 100 libs ?

$\frac{100 \times 6}{25} = 24 - 20 = 4$ libs. weight to be added.

EXAMPLE II.

A block of 960 libs. is to be lifted by a lever 30 feet long, and the power to be applied is 60 libs.—on what part of the lever must the fulcrum be placed?

$\frac{960}{60} = 16$, that is, the weight is to the power as 16 is to 1; therefore the whole length $\frac{30}{16+1} = 1\frac{13}{17}$, the distance from the block, and $30 - 1\frac{13}{17} = 28\frac{4}{17}$, the distance from the power.

EXAMPLE III.

A beam 32 feet long, and supported at both ends, bears a weight of 6 tons, 12 feet from one end,—What proportion of weight does each of the supports bear?

$\frac{12 \times 6}{32} = 2\frac{1}{4}$ tons, support at end farthest from the weight.

$\frac{20 \times 6}{32} = 3\frac{3}{4}$ tons, support at end nearest the weight.

EXAMPLE IV.

A beam supported at both ends, and 16 feet long, carries a weight of 6 tons, 3 feet from one end, and another weight of 4 tons, 2 feet from the other end. What proportion of weight does each of the supports bear?

$\frac{3 \times 6}{16} + \frac{14 \times 4}{16} = \frac{74}{16} = 4\frac{10}{16}$ tons, end at the 4 tons.

$\frac{2 \times 4}{16} + \frac{13 \times 6}{16} = \frac{86}{16} = 5\frac{6}{16}$ tons, end at the 6 tons.

WHEEL AND AXLE.

The nature of this machine is suggested by its name. To it may be referred all turning or wheel machines of different radii: as well-rollers and handles, Cranes, Capstans, Windlasses, &c.

The mechanical property is the same as in the lever: that is, the product of the weight into the distance at which it acts is equal to the product of the power into the distance at which it acts, *the distances being estimated in directions perpendicular to those,* in which the weight and power act respectively, because the wheel and axle is only a kind of perpetual lever.

And hence also this property: *The product of the power applied, multiplied by its velocity, is equal to that of the weight to be raised into its velocity.*

When a series of wheels and axles act upon each other, so as to transmit and accumulate a mechanical advantage, whether the communication be by means of cords and belts, or of teeth and pinions, *the weight will be to the power, as the continual product of the radii of the wheels to the continual product of the radii of the axles.*

Thus, if the radii of the axles a, b, c, d, e, be each 3 inches, while the radii of the wheels A, B, C, D, E, be 9, 6, 9, 10 and 12 inches, respectively: then $W : P :: 9 \times 6 \times 9 \times 10 \times 12 : 3 \times 3 \times 3 \times 3 \times 3$, or as 240 : 1. A computation, however, in which the effect of friction is disregarded.

A train of wheels and pinions may also serve for the augmentation of velocities. Thus, in the preceding example, whatever motion be given to the circumference of the axle e, the rim of the wheel A will move 240 times as fast.

And if a series of 6 wheels and axles, each having their diameters in the ratio of 10 to 1 were employed to accumulate velocity, the *produced* would be to the *producing* velocity as 10^6 to 1; that is, as 1,000,000 to 1.

Note. A man's power producing the greatest effect, is 31 libs at a velocity of 2 feet per second, or 120 feet per minute.

The Rule to find the power of Cranes is, viz.

Divide the product of the driven by the product of the drivers, and the quotient is the relative velocity, as $1 : v$, which multiplied by the length of winch, and by the power applied (in libs.) and divided by the radius of the barrel, the quotient will be the weight raised.

EXAMPLE I.

A weight of 94 tons is to be raised 360 feet in 15 minutes, by a power, the velocity of which is 220 feet per minute : —What is the power required ?

$\frac{360}{15} =$ 24 feet per minute, velocity of weight

$24 \times 94 = \frac{2256}{220} =$ 10.2545 tons power required.

EXAMPLE II.

A Stone weighing 986 libs, is required to be lifted : What power must be applied, when the power is to the weight as 9 is to 2 ?

$\frac{986 \times 2}{9} = \frac{1972}{9} = 219\frac{5}{9}$ tons power.

EXAMPLE III.

A Power of 18 libs is applied to the winch of a crane, the length of which is 8 inches ; the pinion makes 12 revolutions for 1 of the wheel, and the barrel is 6 inches diameter.

$\frac{8 \times 2 \times 22}{7} =$ 50.28 circumference of the winch's circle.

$50.28 \times 12 = 603.36$ inches velocity of power on winch to 1 revolution of the barrel.

$603.36 \times 18 = \frac{10860.48}{19} =$ 571.604 libs weight,

$\frac{6 \times 22}{7} = 18.857$ . . 19.

that can be raised by a power of 18 libs on this crane.

PULLEY.

There are two kinds of Pulleys, the *fixed* and the *moveable*. From the fixed Pulley no power is derived ; it is as a common beam used in weighing goods, having the two ends of equal weight, and at the same distance from the centre of motion ; the only advantage gained by the fixed pulley, is in changing the direction of the power.

From the movable pulley power is gained ; it operates as a lever of the second order; for if one end of a string be fixed to an immovable stud, and the other end to a movable power, the string doubled and the ends parallel, the pulley that hangs between is a lever; the fixed end of the string being the fulcrum, and the other the movable end of the lever: hence the power is double the distance from the fulcrum, than is the weight hung at the pulley; and therefore the power is to the weight as 2 is to 1. This is all the advantage gained by one movable pulley, for two, twice the advantage; for three, thrice the advantage; and so on for every additional movable pulley.

From this the following rule is derived:—Divide the weight to be raised by twice the number of movable pulleys, and the quotient is the power required to raise the weight.

EXAMPLE I.

What power is requisite to lift 100 libs, when two blocks of three pulleys, or sheives each, are applied, the one block movable and the other fixed?

$\frac{100}{6} = 16\frac{4}{6}$ libs, the power required, 3 sheives $\times$ 2 = 6.

EXAMPLE II.

What weight will a power of 80 libs lift, when applied to a 4 and 5 sheived block and tackle, the 4 sheived block being movable?

$80 \times 8 = 640$ libs weight raised.

INCLINED PLANE.

When a body is drawn up a vertical plane, the whole weight of the body is sustained by the power that draws or lifts it up: hence the power is equal to the weight.

When a body is drawn along an horizontal (truly level) plane, it takes no power to draw it (save the friction occasioned by the rubbing along the plane.)

From these two hypotheses, if a body is drawn up an inclined plane, the power required to raise it is as the inclination of the plane; and hence when the power acts parallel to the plane, the length of the plane is to the weight, as the height of the plane is to the power; for the greater the angle, the greater the height.

EXAMPLE I.

What power is requisite to move a weight of 100 libs up an inclined plane, 6 feet long and 4 feet high?

If 6 : 4 :: 100 : $66\frac{2}{3}$ libs power.

EXAMPLE II.

A power of 68 libs, at the rate of 200 feet per minute, is applied to pull a weight up an inclined plane, at the rate of 50 feet per minute—When the plane is 37 feet long and 12 feet high, how much will be the weight drawn?

As 12 : 37 :: 68 × 200 : 50 × $838_{\overline{6}}$

$$\frac{68 \times 200 \times 37}{12 \times 50} = \frac{503200}{600} = 838\tfrac{4}{6} \text{ libs weight.}$$

WEDGE.

The Wedge is a double inclined plane, and therefore subject to the same Rules; or the following Rule, which is particularly for the Wedge; but drawn from its near connexion to the inclined plane, is,—If the power acts perpendicularly upon the head of the wedge, the power is to the pressure which it exerts perpendicularly on each side of the wedge, as the head of the wedge is to its side: hence, it is evident, that the sharper or thinner the wedge is, the greater will be the power.

But the power of the Wedge being not directly according to its length and thickness, but to the length and width of the split, or rift, in the wood to be cleft, the rule therefore is of little use in practice; besides, the wedge is very seldom used as a power; for these reasons, the nature of its properties and effects need not be here discussed.

SCREW.

The screw is a cord wound in a spiral direction round the periphery of a cylinder, and is therefore an inclined plane, the length being the circumference of the cylinder, and the height, the distance between two consecutive cords, or threads of the Screw, hence, the Rule is derived;

—As the circumference of the Screw is to the Pitch, or distance between the threads; so is the Weight to the Power.

When the Screw turns, the cord or thread runs in a continued ascending line round the centre of the cylinder, and the greater the radius of the cylinder, the greater will be the length of the plane to its height, consequently, the greater the power.—A lever fixed to the end of the screw will act as one of the second order, and the power gained will be as its length, to the radius of the cylinder; or the circumference of the circle described by it, to the circumference of the cylinder; hence, an addition to the rule is produced, which is,—If a lever is used, the circumference of the lever is taken for, or instead of, the circumference of the screw.

EXAMPLE I.

What is the power requisite to raise a weight of 8000 libs by a screw of 12 inches circumference and 1 inch pitch?

As 12 : 1 :: 8000 : $666\frac{8}{12}$ libs = power at the circumference of the screw.

EXAMPLE II.

How much would be the power if a lever of 30 inches was applied to the screw?

Circumference of 30 inches = $188\frac{4}{7}$

As $188\frac{4}{7}$: 1 :: 8000 : $42\frac{560}{1320}$ libs = power with a lever of 30 inches long.

VELOCITY OF WHEELS.

WHEELS are for conveying motion to the different parts of a machine, at the same, or at a greater or less velocity, as may be required.—When two wheels are in motion their teeth act on one another alternately, and consequently, if one of these wheels has 40 teeth, and the other 20 teeth, the one with 20 will turn twice upon its axis for one revolution of the wheel with 40 teeth.—From this the Rule is taken, which is,—As the velocity required is to the number of teeth in the driver, so is the velocity of the driver to the number of teeth in the driven.

Note. To find the proportion that the velocities of the wheels in a train should bear to one another, subtract the less velocity from the greater, and divide the remainder by the number of one less than the wheels in the train; the quotient will be the number rising in arithmetical progression, from the least to the greatest velocity of the train of wheels.

EXAMPLE I.

What is the number of teeth in each of three wheels to produce 17 revolutions per minute, the driver having 107 teeth; and making three revolutions per minute?

$$\frac{17-3=14}{3-1=2}=7,$$ therefore 3 10 17 are the velocities of the three wheels.

By the Rule

$$10 : 107 :: 3 : 32 = \frac{107 \times 3}{10} = 32 \text{ teeth.}$$

$$17 : 32 :: 10 : 19 = \frac{32 \times 10}{17} = 19 \text{ teeth.}$$

EXAMPLE II.

What is the number of teeth in each of 7 wheels, to produce 1 revolution per minute, the driver having 25 teeth, and making 56 revolutions per minute?

$$\frac{56-1=55}{7-1=6}=9,$$ therefore 56 46 37 28 19 10 1, are the progressional velocities.

46	:	25	::	56	:	30	Teeth.
37	:	30	::	46	:	37	——
28	:	37	::	37	:	49	——
19	:	49	::	28	:	72	——
10	:	72	::	19	:	137	——
1	:	137	::	10	:	1370	——

It will be observed that the last wheel, in the foregoing Example, is of a size too great for application; to obviate this difficulty, which frequently arises in this kind of training, wheels and pinions are used, which give a great command of velocity.—Suppose the velocities of last Example, and the train only of 2 wheels and 2 pinions.

$\frac{56-1=55}{4-1=3}=18$, therefore 56 19 1, are the progressional velocities.

19 : 25 : : 56 : 74 = teeth in the wheel driven by the first driver, and 1 : 10 : : 19 : 190 = teeth, in the second driven wheel, 10 teeth being in the driving pinion.

25 drivers	74 driven.
10 ———	190 ———

STEAM ENGINE.

Boilers—are of various forms, but the most general is proportioned as follows, viz. width 1, depth 1.1, length 2.5; their capacity being, for the most part, two horse more than the power of the engine for which they are intended.

Boulton and Watt allow 25 cubic feet of space for each horse power, some of the other engineers allow 5 feet of surface of water.

Steam—arising from water at the boiling point, is equal to the pressure of the atmosphere, which is in round numbers, 15 libs on the square inch; but to allow for a constant and uniform supply of steam to the engine, the safety valve of the boiler is loaded with three libs on each square inch.

The following Table exhibits the expansive force of steam, expressing the degrees of heat at each lib of pressure on the safety valve.

Degrees of Heat.	Libs of Pressure.	Degrees of Heat.	Libs of Pressure.	Degrees of Heat.	Libs of Pressure.
212°	0	268°	24	298°	48
216	1	270	25	299	49
219	2	271	26	300	50
222	3	273	27	301	51
225	4	274	28	302	52
229	5	275	29	303	53
232	6	277	30	304	54
234	7	278	31	305	55
236	8	279	32	306	56
239	9	281	33	307	57
241	10	282	34	308	58
244	11	283	35	309	59
246	12	285	36	310	60
248	13	286	37	311	61
250	14	287	38	312	62
252	15	288	39	313	63
254	16	289	40	313½	64
256	17	290	41	314	65
258	18	291	42	315	66
260	19	293	43	316	67
261	20	294	44	317	68
263	21	295	45	318	69
265	22	296	46	319	70
267	23	297	47	320	71

By the following Rule the quantity of steam required to raise a given quantity of water to any given temperature is found.

Rule. Multiply the water to be warmed by the difference of temperature between the cold water and that to which it is to be raised, for a dividend, then to the temperature of the steam add 900 degrees, and from that sum take the required temperature of the water: this last remainder being made a divisor to the above dividend, the quotient will be the quantity of steam in the same terms as the water.

EXAMPLE.

What quantity of steam at 212° will raise 100 gallons of water at 60° up to 212°?

$$\frac{212^\circ - 60^\circ \times 100}{212^\circ + 900^\circ - 212^\circ} = 17$$ gallons of water formed into steam.

Now, steam at the temperature of 212° is 1800 times its bulk in water; or 1 cubic foot of steam, when its elasticity is equal to 30 inches of mercury, contains 1 cubic inch of water. Therefore 17 gallons of water converted into steam, occupies a space of 4090¾ cubic feet, having a pressure of 15 libs on the square inch.

In boiling by steam, using a jacket instead of blowing the steam into the water, about 10.5 square feet of surface are allowed for each horse capacity of boiler; that is, a 14 horse boiler will boil water in a pan set in a jacket, exposing a surface of $10.5 \times 14 = 147$ square feet.

Horse Power.—Boulton and Watt suppose a horse able to raise 32,000 libs avoirdupois 1 foot high in a minute.

Desaguliers makes it 27,500 libs.
Smeaton do. 22,916 do.

It is common in calculating the power of Engines, to suppose a horse to draw 200 libs at the rate of 2½ miles in an hour, or 220 feet per minute, with a continuance, drawing the weight over a pulley—now, $200 \times 220 = 44000$, *i. e.* 44000 libs at 1 foot per minute, or 1 lib at 44000 feet per minute. In the following table 32,000 is used.

One horse power is equal to raise —— gallons or —— libs —— feet high per minute.

Feet High Per Minute	Ale Gallons.	Libs Avoirdupois.	Feet High Per Minute.	Ale Gallons.	Libs Avoirdupois.
1	3123	32000	20	156	1600
2	1561½	16000	25	125	1280
3	1041	10666	30	104	1066
4	780	8000	35	89	914
5	624	6400	40	78	800
6	520	5333	45	69	711
7	446	4571	50	62	640
8	390	4000	55	56	582
9	347	3555	60	52	533
10	312	3200	65	48	492
11	284	2909	70	44	457
12	260	2666	75	41	426
13	240	2461	80	39	400
14	223	2286	85	37	376
15	208	2133	90	34	355
16	195	2000	95	32	337
17	183	1882	100	31	320
18	173	1777	110	28	291
19	164	1684	120	26	267

LENGTH OF STROKE.—The stroke of an engine is equal to one revolution of the crank shaft, therefore double the length of the cylinder. When stating the length of stroke, the length of cylinder is only given, that is, an engine with a 3 feet stroke, has its cylinder 3 feet long, besides an allowance for the piston.

The following Table shows the length of stroke, (or length of cylinder,) and the number of feet the piston travels in a minute, according to the number of strokes the Engine makes when working at a maximum.

When calculating the power of Engines, the feet per minute are generally taken at 220.

Length of Stroke.	Number of Strokes.	Feet per Minute.
Feet 2	43	172
. . 3	32	192
. . 4	25	200
. . 5	21	210
. . 6	19	228
. . 7	17	238
. . 8	15	240
. . 9	14	250

CYLINDER.—When an engine in good order is performing its regular work, the effective pressure may be taken at 8 libs on each square inch of the surface of the piston.

To calculate the power of an Engine.

RULE 1. Multiply the area of cylinder by the effective pressure = say 8 lbs, the product is the weight the engine can raise.—Multiply this weight by the number of feet the piston travels in one minute, which will give the momentum, or weight, the engine can lift 1 foot high per minute; divide this momentum by a horse power, as previously stated, and the quotient will be the number of horse power the engine is equal to do.

RULE 2. 25 inches of the area of cylinder is equal to one horse power, the velocity of the engine being constantly 220 feet per minute.

EXAMPLE I.

What is the power of an engine, the cylinder being 42 inches diameter, and stroke 5 feet?

$$\frac{42^2 \times .7854 \times 10 \times 210}{44000}=66.12 \text{ horse power.}$$

EXAMPLE II.

What size of cylinder will a 60 horse power engine require, when the stroke is 6 feet?

$$\frac{44000 \times 60}{228 \times 10}=1158 \text{ inch. area of cylinder.}$$

Note. To find the power to lift a weight at any velocity, multiply the weight in libs by the velocity in feet, and divide by the horse power; the quotient will be the number of horse power required.

TABLE.

When the effective pressure on each inch of piston is	The area equal to one horse power will be
53 lbs.	3.7 inches.
48 —	4.17 —
43 —	4.65 —
38 —	5.26 —
33 —	6.06 —
28 —	7.14 —
23 —	8.7 —
18 —	11.11 —
13 —	15.46 —
8 —	25. —

Nozles.—The diameter of the valves of nozles ought to be fully one-fifth of the diameter of cylinder.

Air-Pump.—The solid contents of the air-pump is equal to the fourth of the solid contents of cylinder, or when the air-pump is half the length of the stroke of the engine, its area is equal to half the area of the cylinder.

Condenser—is generally equal in capacity to the air-pump; but when convenient, it ought to be more; for when large, there is a greater space of vacuum, and the steam is sooner condensed, Cold Water Pump.—The capacity of the Cold Water Pump depends on the temperature of the water. Many Engines return their water, which cannot be so cold as water newly drawn from a river, well, &c.; but when water is at the common temperature, each horse power requires nearly $7\frac{1}{2}$ gallons per minute. Taking this quantity as a standard, the size of the pump is easily found by the following Rule, viz.—Multiply the number of horse power by $7\frac{1}{2}$ gallons, and divide by the number of strokes per minute; this will give the quantity of water to be raised each stroke of pump. Multiply this quantity by 231, (the number of cubic inches in a gallon,) and divide by the length of effective stroke of pump, the quotient will be the area.

EXAMPLE.

What diameter of pump is requisite for a 20 horse power Steam Engine having a three feet stroke, the effective stroke of pump to be fifteen inches?

$$20 \times 7\tfrac{1}{2} = \frac{150}{32} = 4.6875$$ gallons the pump lifts each troke.

$$\frac{4.6875 \times 231}{15} = 72.1875$$ inches area of pump.

Hot Water Pump.—The quantity of water raised at each stroke ought to be equal in bulk to the 900th part of the capacity of the cylinder.

Proportions.—The length of stroke being 1, the length of beam to centre will be 2, the length of crank .5 and the length of connecting rod 3.

The following table shows the force which the connecting rod has to turn round the crank at different parts of the motion.

Col. A. Decimal proportions of descent of the Piston, the whole descent being 1.

Col. B. Angle between the connecting Rod and Crank.

Col. C. Effective length of the Lever upon which the connecting Rod acts, the whole Crank being 1.

Col. D. Decimal proportions of half a revolution of the Fly-Wheel.

Col. C. Also shows the force which is communicated to the Fly-Wheel, expressed in decimals, the force of the Piston being 1.

A	B	C	D
.0	180°	.0	.0
.05	151½	.46	.128
.10	141	.62	.158
.15	131½	.74	.228
.2	123½	.830	.271
.25	117¼	.892	.308
.3	110¾	.94	.342
.35	104	.976	.377
.4	97½	.986	.41
.45	91¾	1.	.441
.5	85½	1.	.473
.55	80	.986	.507
.6	75	.956	.538
.65	69	.92	.572
.7	62½	.83	.607
.75	57½	.824	.642
.8	49	.746	.68
.85	42	.66	.723
.9	34	.546	.776
.95	23½	.390	.84
1.0	0	.000	1.0

Fly Wheel—Is used to regulate the motion of the Engine, and to bring the crank past its centres. The rule for finding its weight, is,—Multiply the number of horses' power of the Engine by 2000, and divide by the square of the velocity of the circumference of the wheel per second, the quotient will be the weight in cwts.

EXAMPLE.

Required the weight of a fly-wheel proper for an Engine of 20 horse-power, 18 feet diameter, and making 22 revolutions per minute?

18 feet diameter = 56 feet circumference, × 22 revolutions per minute = 1232 feet, motion per minute ÷ 60 = $20\frac{1}{8}$ feet motion per second; then $20\frac{1}{2}^2 = 420\frac{1}{4}$ the divisor.

20 horse power × 2000 = 40000 dividend.

$$\frac{40000}{420\frac{1}{4}} = 90.4 \text{ cwt. weight of wheel.}$$

Parallel Motion.—The radius and parallel bars are of the same dimensions; their length being generally 1-4 of the length of the beam between the two glands, or one half of the distance between the fulcrum and gland. Both pairs of straps are the same length between the centres, and which is generally three inches less than the half of the length of stroke.

GOVERNOR OR DOUBLE PENDULUM.—If the revolutions be the same, whatever be the length of the arms, the balls will revolve in the same plane, and the distance of that plane from the point of suspension, is equal to the length of a pendulum, the vibrations of which will be double the revolutions of the balls. For example; suppose the distance between the point of suspension and plane of revolution be 36 inches, the vibrations that a pendulum of 36 inches will make per minute is, $=\frac{375}{\sqrt{36}}=$ 62 vibrations, and $\frac{62}{2}=31$ revolutions per minute the balls ought to make.

WATER WHEEL.

WATER. (*Hydrostatics*)

Hydrostatics is the science which treats of the pressure, or weight, and equilibrium of water, and other fluids, especially those that are non-elastic.

Note 1. The pressure of water at any depth, is as its depth: for the pressure is as the weight, and the weight is as the height.

Note 2. The pressure of water on a surface any how immersed in it, either perpendicular, horizontal or oblique, is equal to the weight of a column of water, the base being equal to the surface pressed, and the altitude equal to the depth of the centre of gravity, of the surface pressed, below the top or surface of the fluid.

PROBLEM I.

In a vessel filled with water, the sides of which are upright and parallel to each other, having the top of the same dimensions as the bottom, the pressure exerted against the bottom will be equal to the area of the bottom multiplied by the depth of water.

EXAMPLE.

A vessel 3 feet square and 7 feet deep is filled with water; what pressure does the bottom support?

$$\frac{3^2 + 7 + 1000}{16}=3937\tfrac{1}{2} \text{ libs. Avoirdupois.}$$

PROBLEM II.

A side of any vessel sustains a pressure equal to the area of the side multiplied by half the depth, therefore the sides and bottom of a cubical vessel sustain a pressure equal to three times the weight of water in a vessel.

EXAMPLE I.

The gate of a sluice is 12 feet deep and 20 feet broad; what is the pressure of water against it?

$$\frac{20\times12\times6\times1000}{16}=90000=40\tfrac{1}{5}\text{ tons nearly.}$$

From Note 2d. The pressure exerted upon the side of a vessel, of whatever shape it may be, is as the area of the side and centre of gravity below the surface of water.

EXAMPLE. II.

What pressure will a board sustain, placed diagonally through a vessel, the side of which is 9 feet deep, and bottom 12 feet by 9 feet?

$\sqrt{12^2+9^2}=15$ feet, the length of diagonal board.

$$\frac{15\times9\times4\tfrac{1}{2}\times1000}{16}=37969\text{ libs nearly.}$$

Though the diagonal board bisects the vessel, yet it sustains more than half of the pressure in the bottom, for the area of bottom is 12 × 9, and the half of the pressure is $\frac{1}{2}$ of 60750=30375.

The bottom of a conical or pyramidical vessel sustains a pressure equal to the area of the bottom and depth of water, consequently, the excess of pressure is three times the weight of water in the vessel.

WATER (*Hydraulics.*)

Hydraulics is that science which treats of fluids considered as in motion; it therefore embraces the phenomena exhibited by water issuing from orifices in reservoirs, projected obliquely, or perpendicularly, in *jet-d'eaux*, moving in pipes, canals, and rivers, oscillating in waves, or opposing a resistance to the progress of solid bodies.

It would be needless here to go into the minutiæ of hydraulics, particularly when the theory and practice do not agree. It is only the general laws, deduced from experiment, that can be safely employed in the various operations of hydraulic architecture.

Mr. Banks, in his treatise on Mills, after enumerating a number of experiments on the velocity of flowing water, by several philosophers, as well as his own, takes from thence the following simple rule, which is as near the truth as any that have been stated by other experimentalists.

RULE. Measure the depth (of the vessel, &c.) in feet, extract the square root of that depth, and multiply it by 5.4, which gives the velocity in feet per second; this multiplied by the area of the orifice in feet, gives the number of cubic feet which flows out in one second.

EXAMPLE.

Let a sluice be 10 feet below the surface of the water, its length 4 feet, and open 7 inches; required the quantity of water expended in one second?

$\sqrt{10}=3.162 \times 5.4=17.0748$ feet velocity.

$\frac{4\times 7}{12}=2\frac{1}{3}$ feet $\times$ 17.0748=39.84 cubic feet of water per second.

If the area of the orifice is great compared with the head, take the medium depth, and two thirds of the velocity from that depth, for the velocity.

EXAMPLE.

Given the perpendicular depth of the orifice 2 feet, its horizontal length 4 feet, and its top 1 foot below the surface of water. To find the quantity discharged in one second:

The medium depth is $=1.5 \times 5.4=8.10$ $\frac{2}{3}$ of $8.10 = 5.40$, and $5.40 \times 8 = 43.20$ cubic feet.*

The quantity of water discharged through slits, or notches, cut in the side of a vessel or dam, and open at the top, will be found by multiplying the velocity at the bottom by the depth, and taking $\frac{2}{3}$ of the product for the area; which again multiplied by the breadth of the slit or notch, gives the quantity of cubic feet discharged in a given time.

EXAMPLE.

Let the depth be 5 inches, and the breadth 6 inches; required the quantity run out in 46 seconds?

---

*The square root of the depth is not taken in this example, but when the depth is considerable, it ought to be taken.

The depth is .4166 of a foot.
The breadth is .5 of a foot.

$\sqrt{.4166} = .6445 \times 5.4 \times \frac{2}{3} = 2.3238 \times .4166 = .96825 \times .5 = .48412$ feet per second.

Then $.48412 \times 46 = 222.69$ cubic feet in 46 seconds.

There are two kinds of water wheels, Undershot and Overshot. Undershot, when the water strikes the wheel at, or below the centre. Overshot when the water falls upon the wheel above the centre.

The effect produced by an *undershot* wheel, is from the impetus of the water. The effect produced by an *overshot* wheel, is from the gravity or weight of the water.

Of an undershot wheel, the power is to the effect as 3 : 1. —Of an overshot wheel, the power is to the effect as 3 : 2 —which is double the effect of an undershot wheel.

The velocity at a maximum is = 3 feet in one second.

Since the effect of the overshot is double that of the undershot, it follows that the higher the wheel is in proportion to the whole descent, the greater will be the effect.

The maximum load for an overshot wheel is that which reduces the circumference of the wheel to its proper velocity, = 3 feet in one second; and this will be known, by dividing the effect it ought to produce in a given time, by the space intended to be described by the circumference of the wheel in the same time; the quotient will be the resistance overcome at the circumference of the wheel, and is equal to the load required, the friction and resistance of the machinery included.

The following is an extract from Banks on Mills.

The effect produced by a given stream in falling through a given space, if compared with a weight, will be directly as that space; but if we measure it by the velocity communicated to the wheel, it will be as the square root of the space descended through, agreeably to the laws of falling bodies.

Experiment 1. A given stream is applied to a wheel at the centre; the revolutions per minute are 38.5.

Ex. 2. The same stream applied at the top, turns the same wheel 57 times in a minute.

If in the first experiment the fall is called 1, in the second it will be 2: then the $\sqrt{1} : \sqrt{2} :: 38.5 : 54.4$, which are in the same ratio as the square roots of the spaces fallen through, and near the observed velocity.

In the following experiments a fly is connected with the water wheel.

Ex. 3. The water is applied at the centre, the wheel revolves 13.03 times in one minute.

Ex. 4. The water is applied at the vertex of the wheel, and it revolves 18.2 times per minute.

As 13.03 : 18.2 :: $\sqrt{1} : \sqrt{2}$ nearly.

From the above we infer, that the circumferences of wheels of different sizes may move with velocities which are as the square roots of their diameters without disadvantage, compared one with another, the water in all being applied at the top of the wheel, for the velocity of falling water at the bottom or end of the fall is as the time, or as the square root of the space fallen through; for example, let the fall be 4 feet, then, As $\sqrt{16} : 1'' :: \sqrt{4} : \frac{1}{2}''$, the time of falling through 4 feet:—Again; let the fall be 9 feet, then, $\sqrt{16} : 1'' :: \sqrt{9} : \frac{3}{4}''$, and so for any other space, as in the following Table, where it appears that water will fall through one foot in a quarter of a second, through 4 feet in half a second, through 9 feet in 3 quarters of a second, and through 16 feet in one second. And if a wheel 4 feet in diameter moved as fast as the water, it could not revolve in less than 1.5 second, neither could a wheel of 16 feet diameter revolve in less than three seconds; but though it is impossible for a wheel to move as fast as the stream which turns it; yet, if their velocities bear the same ratio to the time of the fall through their diameters, the wheel 16 feet in diameter may move twice as fast as the wheel 4 feet in diameter.

TABLE.

Height of the fall in feet.	Time of falling in Seconds.	Height of the fall in feet.	Time of falling in Seconds.
1	.25	14	.935
2	.352	16	1.
3	.432	20	1.117
4	.5	24	1.22
5	.557	25	1.25
6	.612	30	1.37
7	.666	36	1.5
8	.706	40	1.58
9	.75	45	1.67
10	.79	50	1.76
12	.864		

Power and effect.—The power water has to produce mechanical effect, is as the quantity and fall of perpendicular height.—The mechanical effect of a wheel is as the quantity of water in the buckets and the velocity.

The power is to the effect as 3 : 2, that is, suppose the power to be 9000, the effect will be

$$=\frac{9000\times 2}{3}=\frac{18000}{3}=6000$$

Height of the wheel.—The higher the wheel is in proportion to the fall, the greater will be the effect, because it depends less upon the impulse, and more upon the gravity of the water; however, the head should be such, that the water will have a greater velocity than the circumference of the wheel; and the velocity that the circumference of the wheel ought to have, being known, the head required to give the water its proper velocity, can easily be known from the rules of Hydrostatics.

Velocity of the Wheel.—Banks, in the foregoing quotation, says, That the circumferences of overshot wheels of different sizes may move with velocities as the square roots of their diameters, without disadvantage. Smeaton says, Experience confirms that the velocity of 3 feet per second is applicable to the highest overshot wheels, as well as the lowest; though high wheels may deviate further from this rule, before they will lose their power, by a given aliquot part of the whole, than low ones can be admitted to do; for a 24 feet wheel may move at the rate of 6 feet per second, without losing any considerable part of its power.

It is evident that the velocities of wheels, will be in proportion to the quantity of water and the resistance to be overcome:—if the water flows slowly upon the wheel, more time is required to fill the buckets than if the water flowed rapidly; and whether Smeaton or Banks is taken as a data, the mill-wright can easily calculate the size of his wheel, when the velocity and quantity of water in a given time is known.

EXAMPLE I.

What power is a stream of water equal to, of the following dimensions, viz. 12 inches deep, 22 inches broad; velocity, 70 feet in $11\frac{3}{4}$ seconds, and fall, 60 feet?—Also, what size of a wheel could be applied to this fall?

$\frac{12\times 22}{144}=1.83$ square feet:—area of stream.

$11\frac{3}{4}''$: 70 : : 60″ : 357.5 lineal feet per min.—velocity.
357.5 × 1.83 = 654.225 cubic feet per minute.
654.225 × 62.5 = 40889.0625 avoir. libs per minute.
40889.0625 × 60 = 2453343.7500 momentum at a fall of 60 feet.

$\frac{2453343.7500}{44000} = 55.7$ horse power.

3 : 2 : : 55.7 : 37.13 effective power.

The diameter of a wheel applicable to this fall, will be 58 feet, allowing one foot below for the water to escape, and one foot above for its free admission.
58 × 3.1416 = 182.2128 circumference of wheel.
60 × 6 = 360 feet per minute, = velocity of wheel.

$\frac{654.225}{360} = 1.8$ sectional area of buckets.

The bucket must only be half full, therefore 1.8 × 2 = 3.6 will be the area.

To give sufficient room for the water to fill the buckets, the wheel requires to be 4 feet broad.

now, $\frac{3.6}{4} = .9$, say 1 foot depth of shrouding.

$\frac{360}{182.2128} = 1.9$ revolutions per minute the wheel will make.

Power of water . .		= 55.7 H. P.	Ans.
Effective power of do.		= 37.13 H. P.	
Dimensions of Wheel.	Diameter . .	= 58 feet.	
	Breadth . .	= 4 feet.	
	Depth of shrouding	= 1 foot.	

EXAMPLE II.

What is the power of a water wheel, 16 feet diameter, 12 feet wide, and shrouding 15 inches deep.
16 × 3.1416 = 50.2656 circumference of wheel.
$12 \times 1\frac{1}{4} = 15$ square feet, sectional area of buckets.
60 × 4 = 240 lineal feet per minute, = velocity.
240 × 15 = 3600 cubic feet water, when buckets are full; when half full, 1800 cubic feet.
1800 × 62.5 = 112500 avoir. libs of water per minute.
112500 × 16 = 1800000 momentum, falling 16 feet.

3: 2 : : 1800000 : $\frac{1200000}{44000} = 27$ horse power.

Buckets.—The number of buckets to a wheel should be as few as possible, to retain the greatest quantity of water; and their mouths only such a width as to admit the requisite quantity of water, and at the same time sufficient room to allow the air to escape.

The Communication of Power.—There are no prime movers of machinery, from which power is taken in a greater variety of forms than the water wheel; and among such a number there cannot fail to be many bad applications.

Suffice it here to mention one of the worst, and most generally adopted. For driving a cotton mill in this neighborhood, there is a water wheel about 12 feet broad, and 20 feet diameter; there is a division in the middle of the buckets, upon which the segments are bolted round the wheel, and the power is taken from the vertex; from this erroneous application, a great part of the power is lost; for the weight of water upon the wheel presses against the axle in proportion to the resistance it has to overcome, and if the axle was not a very large mass of wood, with very strong iron journals, it could not stand the great strain which is upon it.

The most advantageous part of the wheel, from which the power can be taken, is that point in the circle of gyration horizontal to the centre of the axle; because, taking the power from this part, the whole weight of water in the buckets acts upon the teeth of the wheels; and the axle of the water wheel suffers no strain.

The proper connexion of machinery to water wheels is of the first importance, and mismanagement in this particular point is often the cause of the journals and axles giving way, besides a considerable loss of power.

EXAMPLE.

Required the radius of the circle of gyration in a water wheel, 30 feet diameter; the weight of the arms being 12 tons, shrouding 20 tons, and water 15 tons.

30 feet diameter, radius = 15 feet.

S $20\times15^2 = 4500\times2=9000$
A $\frac{12\times15^2}{3} = 900\times2=1800$
} The opposite side of the water wheel must be taken.

W $15\times15^2=3375 \quad =3375$

$2\times(20+12)=64 \qquad 14175$

W $\frac{15}{79}$ $\qquad \frac{14175}{79}=179$, the square root of which is $13\frac{4}{10}$ feet, the radius of the circle of gyration.

PUMPS.

There are two kinds of Pumps, Lifting and Forcing. The Lifting, or Common Pumps, are applied to wells, &c. where the depth does not exceed 32 feet; for beyond this depth they cannot act, because the height that water is forced up into a vacuum, by the pressure of the atmosphere, is about 34 feet.

The Force Pumps are those that are used on all other occasions, and can raise water to any required height.—Bramah's celebrated Pump is one of this description, and shows the amazing power that can be produced by such application, and which arises from the fluid and non-compressible qualities of water.

The power required to raise water any height is equal to the quantity of water discharged in a given time, and the perpendicular height.

EXAMPLE.

Required the power necessary to discharge 175 ale gallons of water per minute, from a pipe 252 feet high?

One ale gallon of water weighs $10\frac{1}{4}$ libs avoir. nearly.

$$175\times10\frac{1}{4}=1799\times252=\frac{453348}{44000}=10.3 \text{ horse power.}$$

The following is a very simple Rule, and easily kept in remembrance.

Square the diameter of the pipe in inches, and the product will be the number of libs of water avoirdupois contained in every yard length of the pipe. If the last figure of the product be cut off, or considered a decimal, the remaining figures will give the number of ale gallons in each yard of pipe; and if the product contains only one figure, it will be tenths of an ale gallon. The number of ale gallons multiplied by 282, gives the cubic inches in each yard of pipe; and the contents of a pipe may be found by Proportion.

EXAMPLE.

What quantity of water will be discharged from a pipe 5 inches diameter, 252 feet perpendicular height, the water flowing at the rate of 210 feet per minute?

A TABLE OF THE

AREAS OF CIRCULAR SEGMENTS.

Height	AREA.	Height	AREA	Height	AREA	Height	AREA
.001	.000042	.038	.009763	.075	.026761	.112	.048262
.002	.000119	.039	.010148	.076	.027289	.113	.048894
.003	.000219	.040	·010537	.077	027821	.114	.049528
.004	.000337	.041	.010931	.078	.028356	.115	.050165
.005	.000470	.042	.011330	.079	.028894	116	.050804
.006	.000618	.043	.011734	.080	.029435	.117	.051446
.007	.000779	.044	.012142	.081	.029979	.118	.052090
.008	.000951	.045	.012554	.082	.030526	.119	.052736
.009	.001135	.046	.012971	.083	.031076	.120	.053385
.010	.001329	.047	.013392	.084	.031629	.121	.054036
.011	.001533	.048	.013818	.085	.032186	.122	.054689
.012	.001746	.049	.014247	.086	.032745	.123	.055345
.013	.001969	.050	.014681	.087	.033307	·124	.056003
.014	.002199	.051	.015119	.088	.033872	.125	.056663
.015	.002438	.052	.015561	.089	.034441	.126	.057326
.016	.002685	.053	.016007	.090	.035011	.127	.057991
.017	.002940	.054	.016457	.091	.035585	.128	.058658
.018	.003202	.055	.016911	.092	.036162	.129	.059327
.019	.003471	.056	.017369	.093	.036741	.130	.059999
.020	.003748	057	.017831	.094	.037323	.131	.060672
.021	.004031	.058	.018296	.095	.037909	.132	.061348
.022	.004322	.059	.018766	.096	.038497	.133	.062026
.023	.004618	.060	.019239	.097	.039087	.134	.062707
.024	.004921	.061	.019716	.098	.039680	.135	.063389
.025	.005230	.062	.020196	.099	.040276	.136	.064074
.026	.005546	.063	.020691	.100	.040875	.137	.064760
.027	.005867	.064	.021168	.101	.041476	.138	.065449
.028	.006194	.065	.021659	.102	.042080	.139	.066140
.029	.006527	.066	.022154	.103	.042687	.140	.066833
.030	.006865	.067	.022652	.104	.043296	.141	.067528
.031	.007209	.068	.023154	.105	.043908	.142	.068225
.032	.007558	.069	.023659	.106	.044522	.143	.068924
.033	.007913	.070	.024168	.107	.045139	.144	.069625
.034	.008273	.071	.024680	108	.045759	.145	.070328
.035	.008638	.072	.025195	.109	.046381	.146	.071033
.036	.009008	.073	.025714	.110	.047005	.147	.071741
.037	.009383	.074	.026236	.111	.047632	.148	.072450

Areas of Circular Segments.

Height	AREA.	Height	AREA.	Height	AREA.	Height	AREA.
.149	.073161	.197	.109430	.245	.149230	.293	.191775
.150	.073874	.198	.110226	.246	.150091	.294	.192684
.151	.074589	.199	.111025	.247	.150953	.295	.193596
.152	.075306	.200	.111823	.248	.151816	.296	.194509
.153	.076026	.201	.112624	.249	.152680	.297	.195422
.154	.076747	.202	.113426	.250	.153546	.298	.196337
.155	.077469	.203	.114230	.251	.154412	.299	.197252
.156	.078194	.204	.115035	.252	.155280	.300	.198168
.157	.078921	.205	.115842	.253	.156149	.301	.199085
.158	.079649	.206	.116650	.254	.157019	.302	.200003
.159	.080380	.207	.117460	.255	.157890	.303	.200922
.160	.081112	.208	.118271	.256	.158762	.304	.201841
.161	.081846	.209	.119084	.257	.159636	.305	.202761
.162	.082582	.210	.119897	.258	.160510	.306	.203683
.163	.083320	.211	.120712	.259	.161386	.307	.204605
.164	.084059	.212	.121529	.260	.162263	.308	.205527
.165	.084801	.213	.122347	.261	.163140	.309	.206451
.166	.085544	.214	.123167	.262	.164019	.310	.207376
.167	.086289	.215	.123988	.263	.164899	.311	.208301
.168	.087036	.216	.124810	.264	.165780	.312	.209227
.169	.087785	.217	.125634	.265	.166663	.313	.210154
.170	.088535	.218	.126459	.266	.167546	.314	.211083
.171	.089287	.219	.127285	.267	.168430	.315	.212011
.172	.090041	.220	.128113	.268	.169316	.316	.212940
.173	.090797	.221	.128942	.269	.170202	.317	.213871
.174	.091554	.222	.129773	.270	.171089	.318	.214802
.175	.092313	.223	.130605	.271	.171978	.319	.215733
.176	.093074	.224	.131438	.272	.172867	.320	.216666
.177	.093836	.225	.132272	.273	.173758	.321	.217599
.178	.094601	.226	.133108	.274	.174649	.322	.218533
.179	.095366	.227	.133945	.275	.175542	.323	.219468
.180	.096134	.328	.134784	.276	.176435	.324	.220404
.181	.096903	.229	.135624	.277	.177330	.325	.221341
.182	.097674	.230	.136465	.278	.178225	.326	.222277
.183	.008447	.231	.137307	.279	.179122	.327	.223215
.184	.099221	.232	.138150	.280	.180019	.328	.224154
.185	.099997	.233	.138995	.281	.180918	.329	.225093
.186	.100774	.234	.139841	.282	.181818	.330	.226033
.187	.101553	.235	.140688	.283	.182718	.331	.226974
.188	.102334	.236	.141537	.284	.183619	.332	.227915
.189	.103116	.237	.142387	.285	.184521	.333	.228858
.190	.103900	.228	.143238	.286	.185425	.334	.229801
.191	.104685	.239	.144091	.287	.186329	.335	.230745
.192	.105472	.240	.144944	.288	.187234	.336	.231689
.193	.106261	.241	.145799	.289	.188140	.337	.232634
.194	.107051	.242	.146655	.290	.189047	.338	.233580
.195	.107842	.243	.147512	.291	.189955	.339	.234526
.196	.108636	.244	.148371	.292	.190864	.340	.235473

THE END.

W. J. Johnston
Berlin C.W.
1858
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