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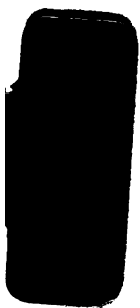
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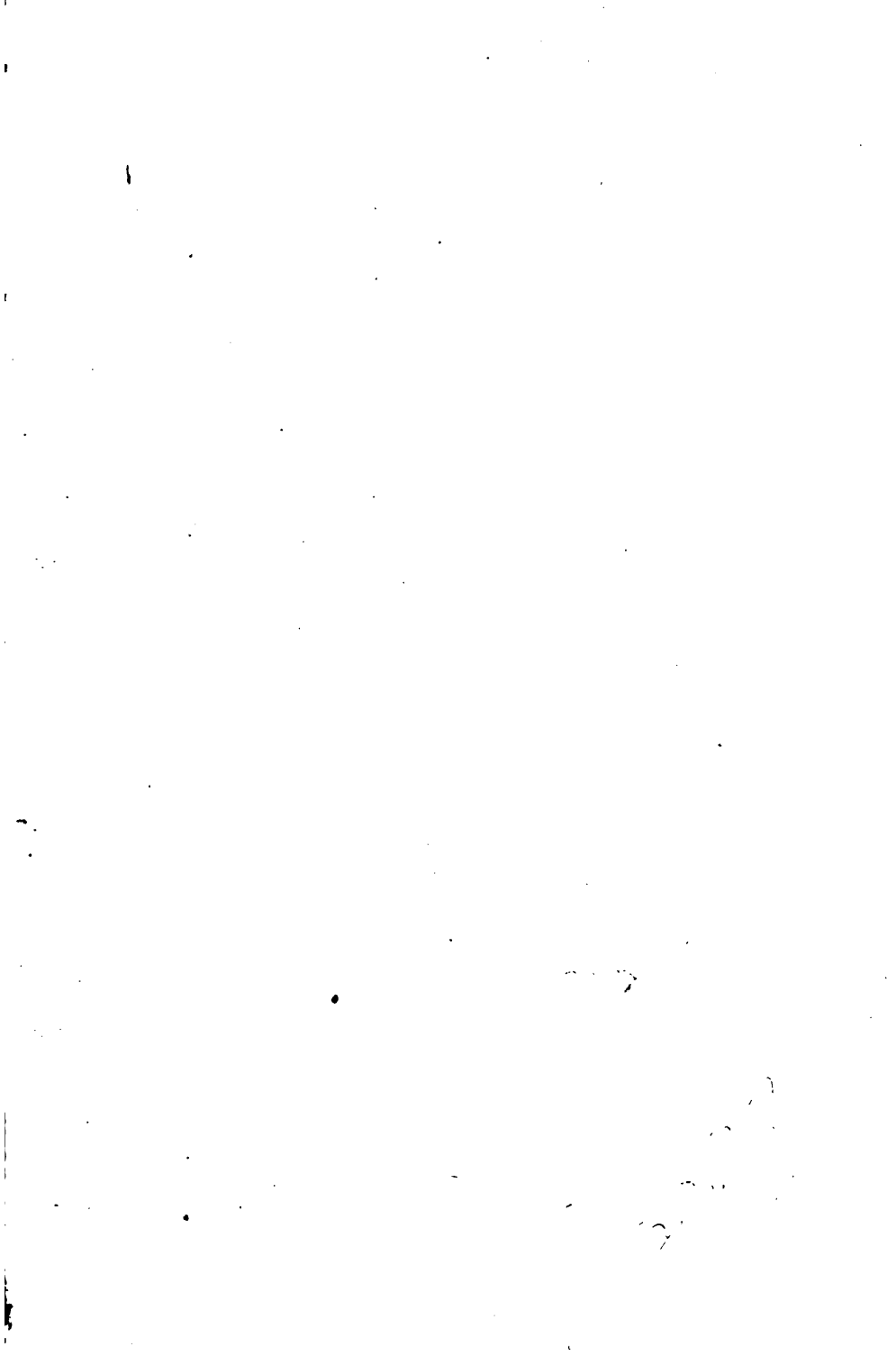
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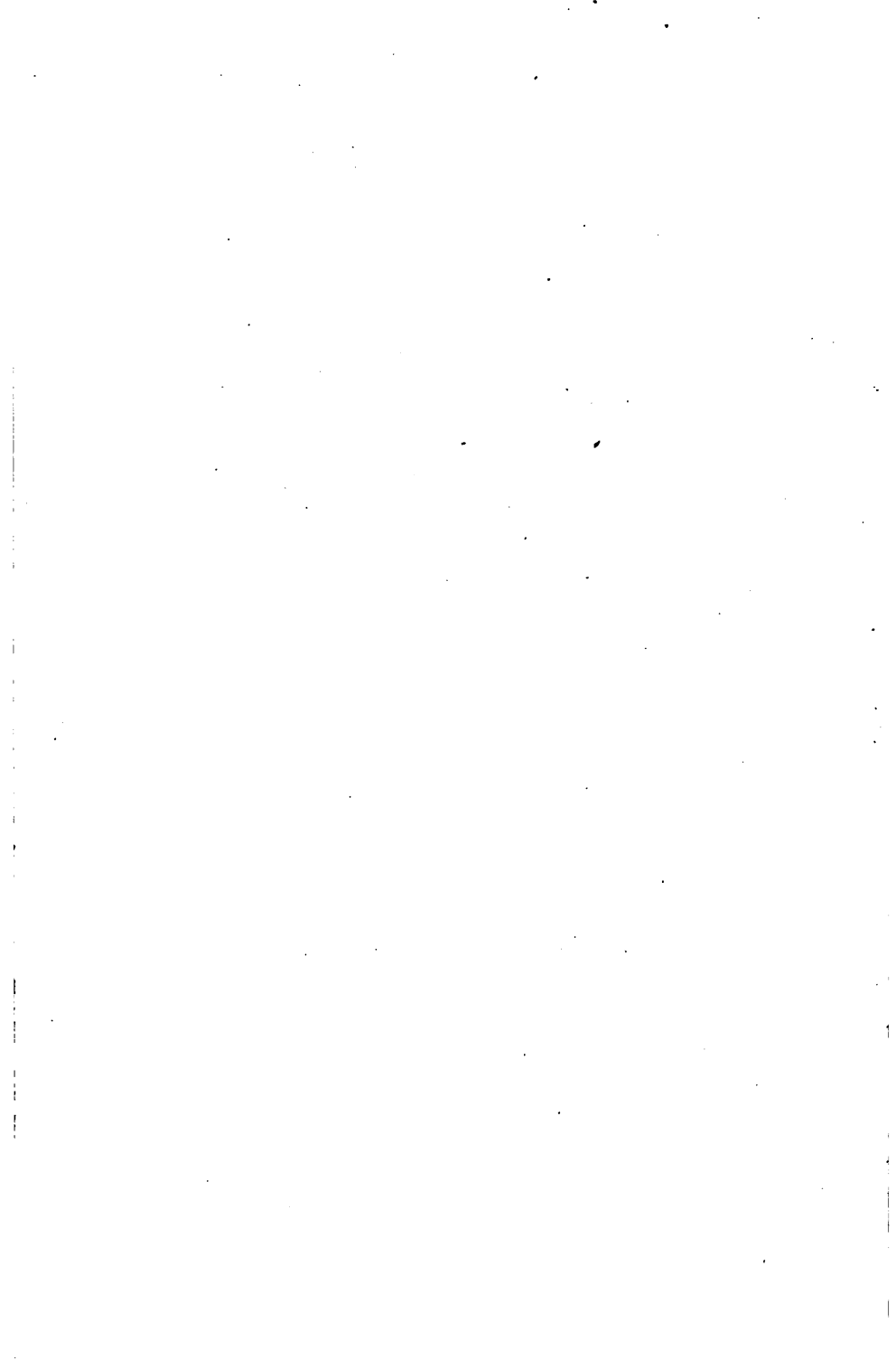
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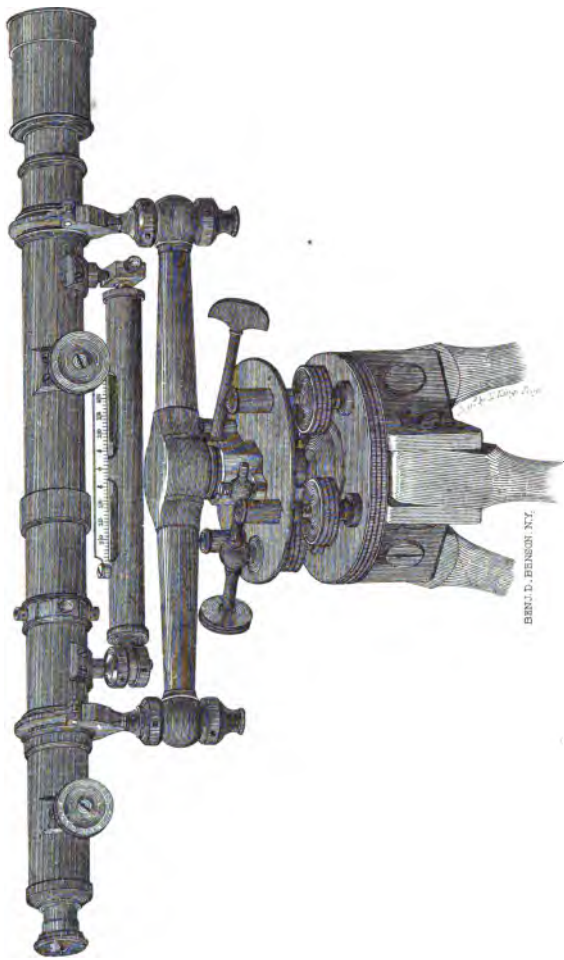
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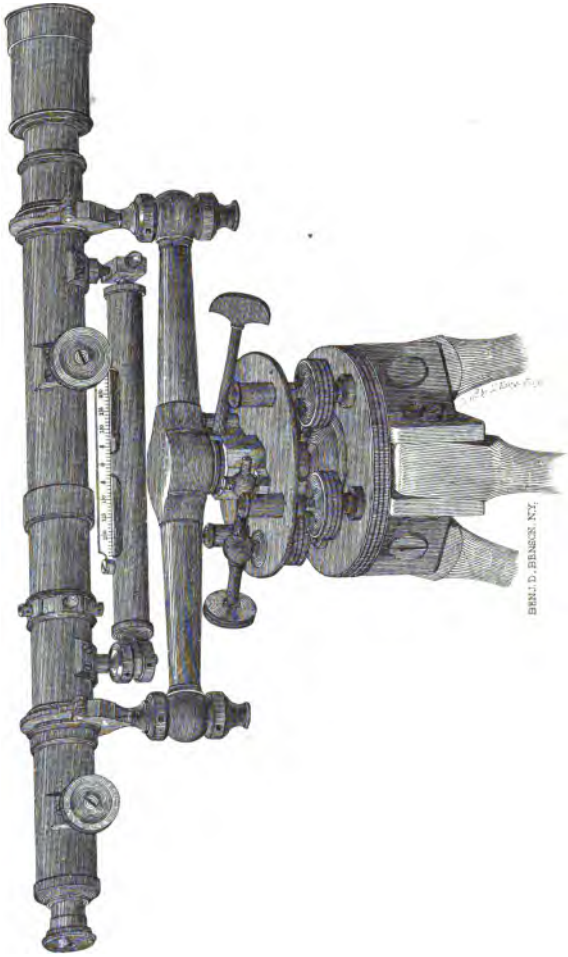
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LAND SURVEY

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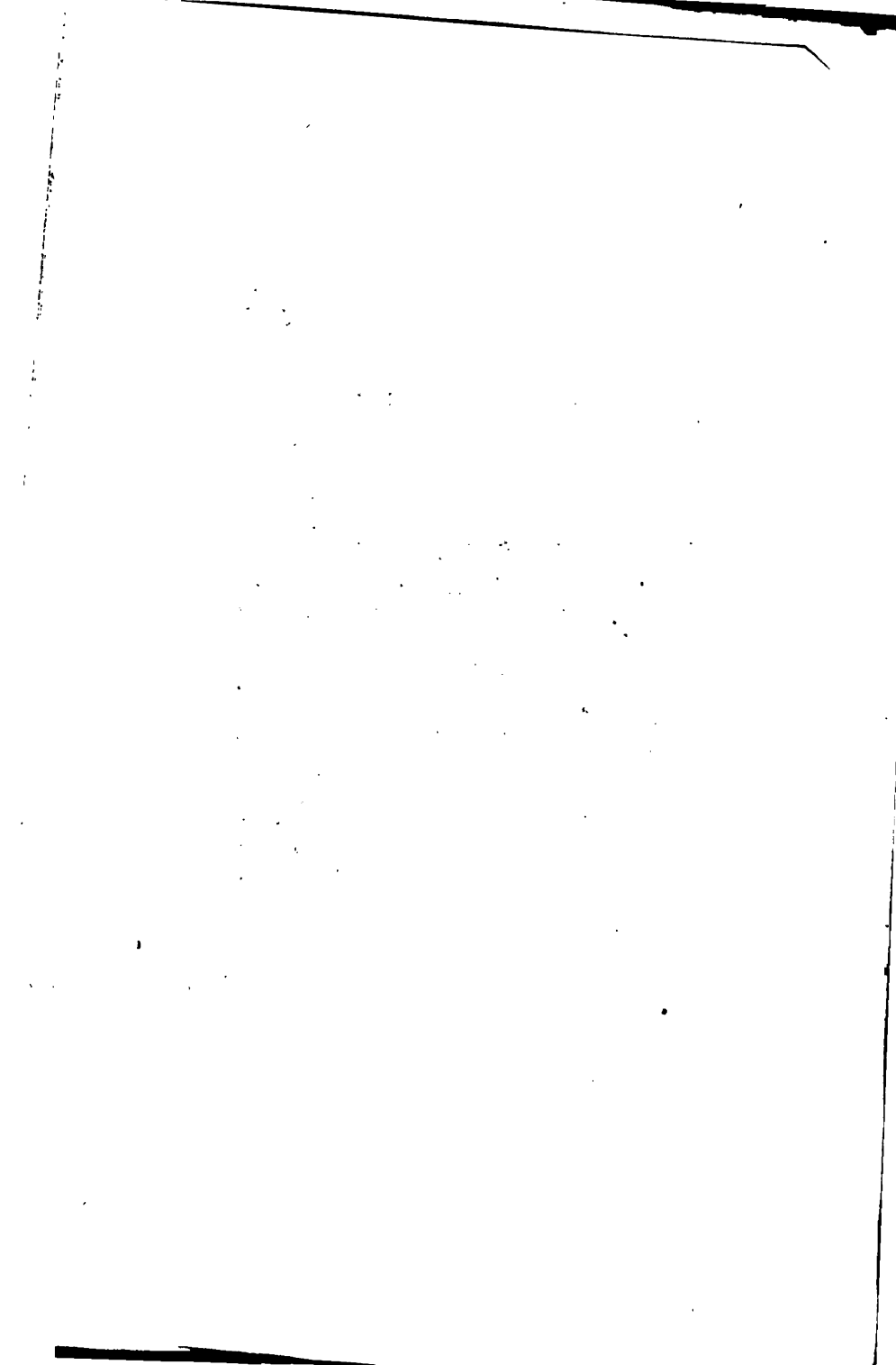
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MANUAL
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WITH
TABLES.

BY
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NEW-YORK:
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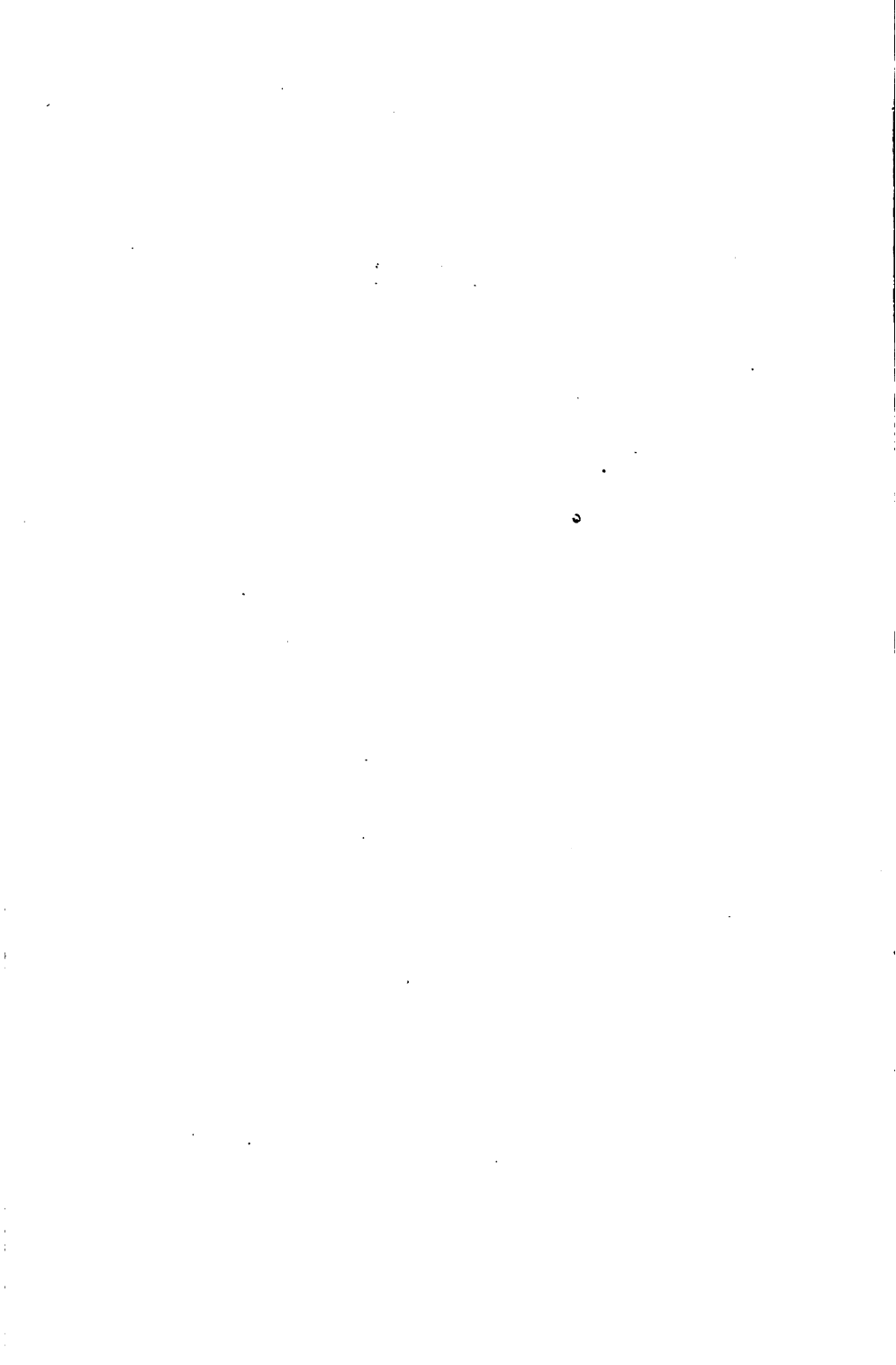
PREFACE.

THE following little work has been prepared with the double object of furnishing a text-book in this branch of practical mathematics, and also a manual for the use of the surveyor. No attempt has been made to extend the work beyond the bounds indicated in the title. It will be found to be simply a treatise on Land Surveying accompanied with such practical directions and tables as the experience of American surveyors has shown to be most useful. The methods and instruments described are mainly those which prevail in American practice, and which have been found best adapted to the peculiar wants of this country. At the same time, in order to adapt the practice to the demand for more rigorous methods, and to give to the measurement of land the precision which its increase in value requires, greater prominence has been given to the use of the Transit in surveying, and improved modes of computing the areas introduced.

The admirable plates of field instruments with which the work is embellished, and which contribute so materially to the value of the book, have been kindly placed at the disposal of the author by the eminent manufacturers of Engineers' and Surveyors' Instruments, Messrs. W. & L. E. Gurley, of Troy, N. Y.

The author acknowledges his obligations to his associates, Professor George H. Cook and Professor E. A. Bowser for valuable suggestions and assistance in the preparation of the work, and also to the officers of the United States Coast Survey for many courtesies.

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UNIVERSITY
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MANUAL OF LAND SURVEYING.

INTRODUCTION.

1. LAND SURVEYING has for its object to determine the length of lines, the area and figure of parcels of land, and to represent the results in plots and maps.

2. The surface of the earth, considered in proportion to the extent of the measurements in surveying, being exceedingly irregular, we conceive a perfectly smooth and regular spherical surface to be formed by extending the surface of the quiet ocean beneath the uneven land. On this regular surface we conceive all points, lines, and surfaces to be projected by perpendiculars let fall. The points, lines, and areas considered in Land Surveying are not the real points, lines, and areas on the earth's surface, but their projections on this imaginary regular surface.

When a farm is surveyed and its contents computed, the boundaries which are measured are not the irregular and undulating lines on the surface of the ground, but the horizontal lines included between the same limits; and the area obtained is not the actual ground-surface, but its horizontal equivalent. To obtain the real lines and real areas would, in most cases, be impossible, and the representation of them upon flat maps equally so. Thus, suppose two adjoining plots of ground, ABCD and CBEF (Fig. 1)

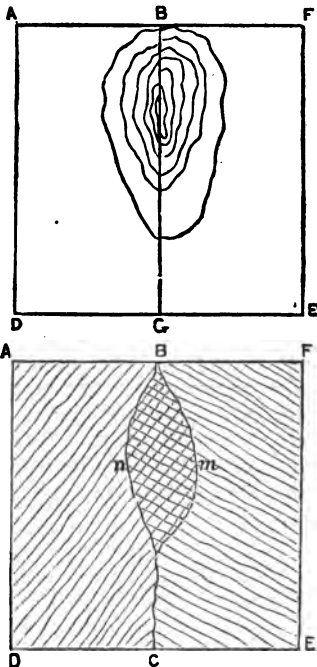


FIG. 1.

having the lines AB, AF, EF, and BE all horizontal, but the line CB,

which divides the plots, to run over an elevation. If the surveyor, having obtained the true area, should attempt to map these two plots in their true position, side by side, in order to give the first its full area, he must curve its boundary-line to the right, giving the figure ABmCD, and in the second he must curve its boundary line to the left, giving the figure BnCEF. The two would overlap each other by the space CmBn.

Fortunately, in the transfer and partition of land, it has always been considered equitable to regard the loss of area in uneven land as compensated by the greater value of level land. And in practical agriculture it is plain that no more stems of grain can grow upon a slope than could grow on the corresponding horizontal surface.

3. The surface on which surveys are, therefore, actually to be made, is a spherical surface. But in **Plane Surveying** the extent of surface is so small that the lines may be regarded as straight lines, and the areas as planes. In **Geodetical Surveying** the operations are so extensive and so precise as to require the curvature of the earth to be taken into account.

In a triangle having sides each one mile in length, the difference in area, whether it be regarded as a plane triangle, or a triangle having sides curved with the earth's surface, is only $2\frac{1}{2}$ square feet. Each of the angles of such a spherical triangle would only exceed that of the plane triangle by 0''.0019. And the angles of a spherical triangle, whose area is 75.5 square miles, would only exceed those of the plane triangle, having sides of the same length, by 1''.

Hence it is safe to assume that the divisions of land included in the operations of an ordinary land-survey may be treated without appreciable error as plane figures.

4. There are two kinds of quantities to be measured in Land Surveying, **lines** and **angles**. From the direct measurement of certain lines and angles, in a figure, we are able to obtain, by computation, other lines and angles, and their included areas. It will be the object of this manual to explain how to measure these lines and angles, and how to compute the required areas. The instruments which are required in the operations will be described, and their uses explained.

INSTRUMENTS.

5. Besides the field instruments employed in Land Surveying, a description of which is given in their appropriate chapters, there are a number of smaller instruments, which are used in making diagrams and plots of the executed work. A brief enumeration of them is here given.

Drawing-Board.—This is a plane, smooth board of dry pine, made perfectly square at the corners. The ends should have strips tongued and

grooved into them, with the grain running in the opposite direction, to prevent warping. The board should be about 24 by 30 inches.

T-Square.—This consists of two pieces of wood fastened firmly together, at right angles to each other.

The long arm is not less than 24 inches long, and the short one about 10 inches. The short arm is thicker than the other, giving a shoulder on one or both sides. This instrument, laid on the drawing-board, with the shoulder against the edge, furnishes an easy method of drawing lines parallel and perpendicular to each other.

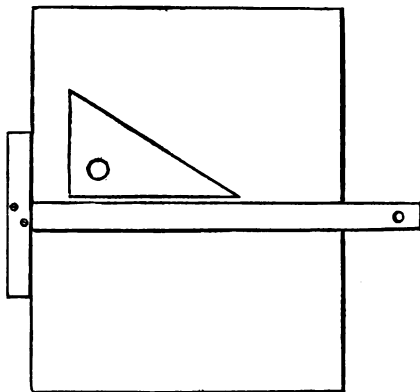


FIG. 2.

Triangle.—A thin triangle of wood, made right-angled at one corner and the other angles 30° and 60° , is useful in drawing parallel lines.

Scales.—The distances measured upon the ground are to be laid down

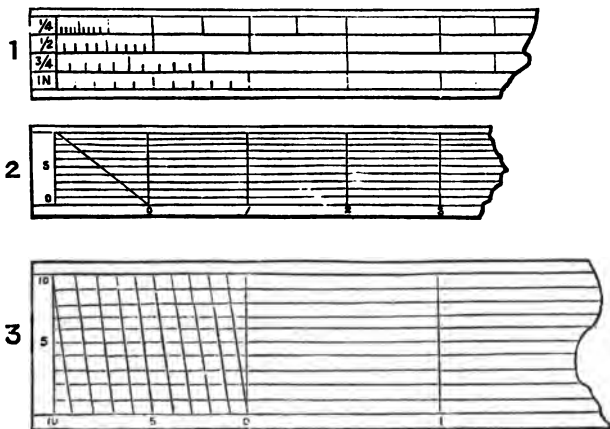


FIG. 3.

on paper on a reduced scale. Thus, we represent the chain by an inch, or

by half-an-inch. The instrument with which these reduced measurements are laid off is called a **scale**. Fig. 3 represents three different varieties of scales. The simplest one consists of a small unit, as one inch, laid off on a wood or ivory bar about 6 inches long. The unit at the left end is divided into smaller equal parts, usually 10 or 12. Such a scale measures inches and tenths, or inches and lines. If an inch is taken to represent one chain, then each tenth would represent ten links.

The second is called a **Diagonal Scale**. The unit at the left end has a line diagonally across the ten equal spaces into which the scale is divided, by parallel lines. Since the diagonal advances one unit in crossing the entire ten spaces, it must advance one-tenth in crossing one space, two-tenths in crossing two spaces, etc. Hence, if we want a distance of seven spaces, for example, we must measure the distance the diagonal has advanced when it has reached the line marked 7. Thus, a distance of 1.7 would extend from the point where the diagonal cuts the seventh parallel to the point where the cross-line marked 1 cuts the same parallel.

When we want a scale which will measure to hundredths, it can best be arranged as shown in the third part of the figure. The scale is divided longitudinally into ten spaces. The upper and lower lines of the left unit are divided each into ten equal parts. Diagonals are then drawn from 0 in the lower line, to 1 in the upper; from 1 in the lower, to 2 in the upper, etc. It is evident that one of these diagonals in crossing one space advances one-tenth of one-tenth of the unit; and hence the scale can be used for measuring to hundredths.

6. Dividers.—This instrument is used for laying off on paper the distances measured. Two forms are shown in the figure. The one has shifting legs, to which may be fitted, when wanted, a pencil-point, a pen-point, or a needle-point. When very large circles are to be drawn, a beam compass is used. It consists of a beam of wood or metal to which are attached two points; one of which, carrying a pencil or pen, may be clamped at any required distance.

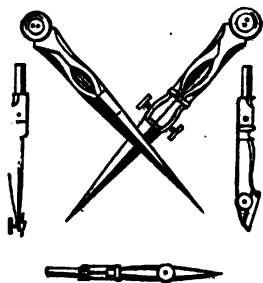


FIG. 4.

7. Drawing-Pen.—A drawing-pen consists of two blades of steel commanded by a small screw, by which their distance apart can be regulated to produce any thickness of line. It may be used without blotting to draw lines along a ruler. In such pens **India-ink** should be used instead of common ink. It does not corrode the pen, and gives a peculiarly smooth and brilliant line.

8. Protractor.—It consists generally of a semicircle divided into degrees, and is used for laying off a given angle from any point of a given line. Lay the protractor so that the straight edge coincides with the given line, and the centre point with the given point. The angle may then be found on the divided circle, and a straight line drawn through the centre, and this point will make the angle required.

A protractor may be made by marking off the degrees on three sides of the rectangle composing any scale.

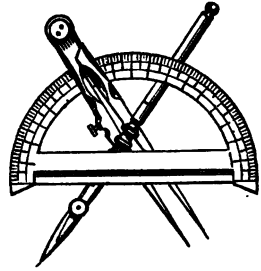
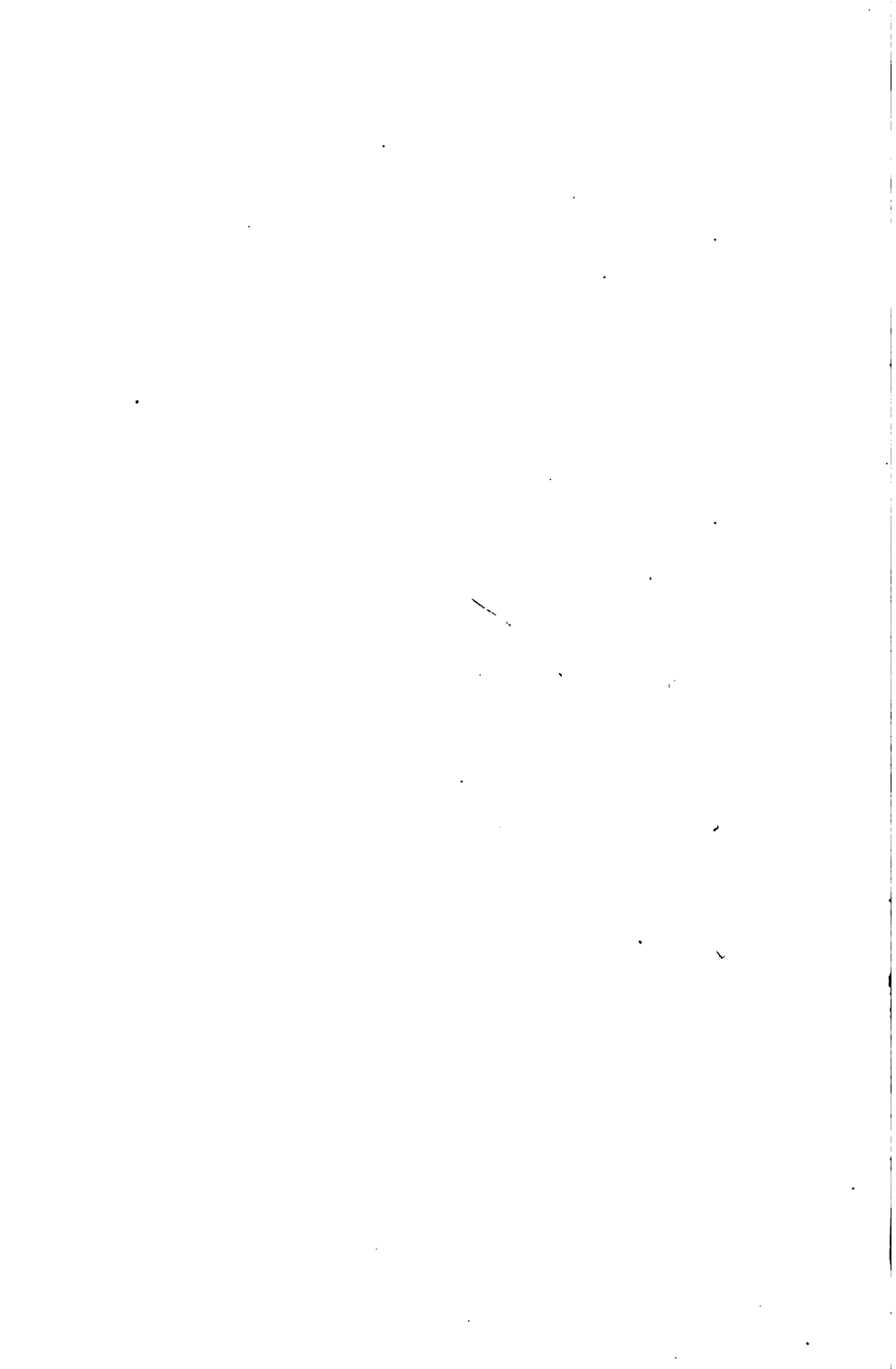


FIG. 5.





CHAPTER I.

PLANE TRIGONOMETRY.

9. A **TRIANGLE** has three sides and three angles, which are called its **parts**. These parts are so related to each other that, if three of them are given—one being a side—the remainder may be found. Trigonometry is the science of triangles; and plane trigonometry treats of triangles in a plane. Only so much of plane trigonometry is here given as is essential to the subject of land surveying.

10. The angle between two lines is measured by the arc of a circle included between its sides. Thus BC , in Fig. 6, is the measure of the angle A .

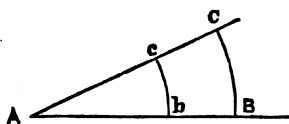


FIG. 6.

For the purpose of expressing this measurement, we conceive the circumference of a circle to be divided into 360 equal parts, called degrees, and each degree into 60 equal parts, called minutes, and each minute into 60 equal parts, called seconds. They are expressed by the characters $^{\circ} ' ''$. The magnitude of the angle A is thus expressed by the number of degrees, minutes, and seconds contained in the arc BC .

It is plain that the arc bc contains the same number of degrees, minutes, and seconds as the arc BC ; hence, either arc may be taken as the

measure of the angle A , and the measurement of angles is independent of the magnitude of the radius of the measuring arc.

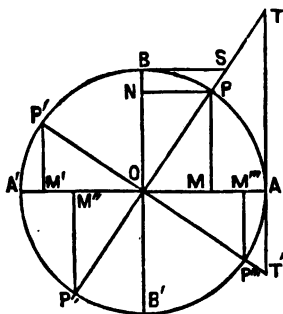


FIG. 7.

11. A **quadrant** is the fourth part of the circumference of a circle. The **complement** of an arc is what remains after subtracting the arc from a quadrant. The **supplement** of an arc is what remains after subtracting it from half the circumference. Since a quadrant is 90° , the complement of an arc, or the angle which it measures, will be obtained by subtracting the number of degrees from 90° . And the supplement will be obtained by subtracting from 180° . Thus, let an arc contain

$70^\circ 30'$, its complement will be $19^\circ 30'$, and its supplement $109^\circ 30'$. In Fig. 7 AB is a quadrant, BP is the complement of the arc AP, and PA' is its supplement. AP' being an arc, minus BP' is its complement, and P'A' its supplement.

12. The **sine** of an arc is a perpendicular let fall from one end of an arc upon a diameter drawn through the other end. Thus, PM is the sine of the arc AP, BO the sine of AB, P'M' the sine of ABP', etc.

The **tangent** of an arc is a line drawn touching one extremity of an arc, and terminated by a line drawn through the centre and the other extremity. Thus, AT is the tangent of the arc AP, AT' the tangent of ABP', AT the tangent of ABA'P'', etc.

The **secant** of an arc is a line drawn from the centre through one extremity of the arc, and terminated by a tangent drawn through the other extremity. Thus, OT is the secant of the arc AP, OT' the secant of ABP', etc.

The sine of the complement of an arc is called its **cosine**. In Fig. 7, PN, which is the sine of BP, is called the cosine of AP. It is evident that in all cases the cosine is equal to the distance from the centre to the foot of the sine; thus NP, the cosine of AP, is equal to OM.

The **cotangent** and **cosecant**, in like manner, are the tangent and secant of the complement of an arc. BS and OS, Fig. 7, are the tangent and secant of BP, and consequently the cotangent and cosecant of AP.

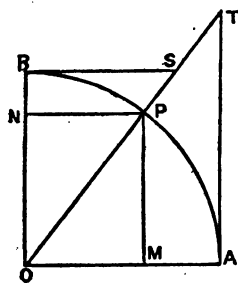


FIG. 8.

13. The mutual relations of the above lines, which are called the functions of an arc, and of the radius of the circle, are expressed in the following proportions:

In Fig. 8 let the arc AP be represented by a , and the radius OA by R, then from the similar triangles OPM, OTA, and ORS, we have

$$OM : PM :: OA : AT$$

$$\cos a : \sin a :: R : \tan a = \frac{R \sin a}{\cos a}. \quad (1.)$$

$$OM : OP :: OA : OT$$

$$\cos a : R :: R : \sec a = \frac{R^2}{\cos a}. \quad (2.)$$

$$PM : OM :: OR : RS$$

$$\sin a : \cos a :: R : \cot a = \frac{R \cos a}{\sin a}. \quad (3.)$$

$$PM : OP :: OR : OS$$

$$\sin a : R :: R : \operatorname{cosec} a = \frac{R^2}{\sin a}. \quad (4.)$$

$$AT : OA :: OR : RS$$

$$\tan a : R :: R : \cot a = \frac{R^2}{\tan a}. \quad (5.)$$

Also, from the right-angled triangle OPM we have

$$PM^2 + OM^2 = OP^2$$

$$\sin^2 a + \cos^2 a = R^2 \quad (6.)$$

$$\sin a = \sqrt{R^2 - \cos^2 a}$$

$$\cos a = \sqrt{R^2 - \sin^2 a}$$

14. The algebraic signs of the functions of an arc are determined thus :

All lines measured upward from the horizontal diameter are positive, all measured downward are negative.

All lines measured from the vertical diameter to the right are positive, and all measured to the left are negative.

In Fig. 7, PM, the sine of AP, is positive; P'M', the sine of AP', is positive. PN, the cosine of AP, is positive, and OM', the cosine of AP', is negative. From an inspection of this figure it will be apparent that the sines are positive for arcs in the first and second quadrant, and negative for the third and fourth. Also that the cosines are positive in the first and fourth quadrants, and negative in the second and third.

The signs of the tangents, secants, cotangents and cosecants may be

determined from the equations in article 13. Since $\tan a = \frac{R \sin a}{\cos a}$ and

$\cot a = \frac{R \cos a}{\sin a}$, it is evident that both tangent and cotangent will be

positive when the sine and cosine have the same signs, and negative when their signs are unlike. From equations (2) and (4) it is also evident that the secant will have the same sign as the cosine, and the cosecant as the sine. These results may be summed up in the following table :

	FIRST QUAD.	SECOND QUAD.	THIRD QUAD.	FOURTH QUAD.
Sine.....	+	+	-	-
Cosine.....	+	-	-	+
Tangent.....	+	-	+	-
Cotangent.....	+	-	+	-
Secant.....	+	-	-	+
Cosecant.....	+	+	-	-

15. **Natural sines, tangents, etc.**, are so named when the radius of the circle is equal to one. In Fig. 7, if the radius of the circle were one foot or one yard, the sine PM would then be the natural sine, AT would be the natural tangent, etc.

A table of natural sines, tangents, etc., gives the values of these functions for all arcs from 0° to 90° , usually at intervals of one minute. By interpolation the value for intermediate seconds may be obtained. In such a table the sines would begin with a value of zero for an arc of 0° , and increase to a value equal to the radius or a unit for 90° . The cosines would begin with a value of a unit for 0° , and decrease to a value of zero for 90° .

Between the same limits the tangents would vary between zero and infinity; the secants between radius and infinity; the cotangents between infinity and zero, and the cosecants between infinity and radius.

16. In Fig. 7 we see that PM is the sine of the arc AP and also of the arc A'P, and P'M' is the sine of both the arcs AP' and A'P'. And, in general, the sine of an arc is always equal to the sine of its supplement. The sine of 100° is the same as the sine of 80° .

Similarly we see that the cosine of an arc is equal to the cosine of its supplement, but is measured in an opposite direction, and therefore has an opposite sign. The tangent of an arc is equal to minus the tangent of its supplement, and the cotangent equal to minus the cotangent of its supplement.

A table of sines, tangents, etc., is only required, therefore, to extend to 90° . For arcs greater than 90° the sines, tangents, etc., may be obtained by employing their supplements.

$$\begin{aligned} \text{Thus,} \quad \sin 144^\circ 36' &= \sin 35^\circ 24' = 0.57928 \\ \cos 144^\circ 36' &= -\cos 35^\circ 24' = -0.81513 \end{aligned}$$

17. **A table of logarithmic sines, tangents, etc.**, gives the logarithms of the sines, tangents, etc., of all arcs from 0° to 90° . By the use of these

the computations may be much abbreviated, using additions instead of multiplications, and subtractions instead of divisions.

The natural sines and cosines being always, and the other functions sometimes, less than one, their logarithms would in these cases have negative characteristics. To avoid this inconvenience in computation, all the functions are multiplied by 10 billions, or, in other words, their logarithms are increased by 10. This is the same as taking the functions in a circle whose radius is 10 billions.

The method of finding the sine, tangent, etc., for a given arc, and also the reverse method of finding the arc for a given sine, tangent, etc., both from the table of natural sines and from the table of logarithmic sines, will be found in the explanations prefixed to the tables. To these the student is referred.

18. The principles involved in the solution of plane triangles may be stated in six theorems. Throughout we shall use the large letters A, B, C, to represent the angles, and the small letters a, b, c , to represent their corresponding opposite sides.

Theorem I. In any right-angled triangle, radius is to the hypotenuse as the sine of either acute angle is to the side opposite, or as the cosine of the angle is to the side adjacent.

Let ABC be a triangle right-angled at A. With a radius equal to R describe the arc DE around the angle C, and draw the sine DF. CF will be the cosine.

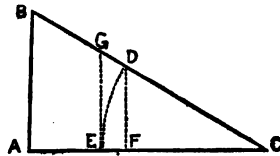


FIG. 9.

By similar triangles we have

that is,	$CD : CB :: DF : AB$
	$R : a :: \sin C : c$
also,	$CD : CB :: CF : CA$
	$R : a :: \cos C : b$

These proportions may be expressed in two equations:

$$R \times c = a \times \sin C \quad (7.)$$

$$R \times b = a \times \cos C \quad (8.)$$

19. **Theorem II.** In any right-angled plane triangle, radius is to either side as the tangent of the adjacent acute angle is to the side opposite, or as the secant of the same angle is to the hypotenuse.

In Fig. 9, draw the line EG a tangent to the arc DE, then will CG be the secant.

By similar triangles we have

$$\begin{aligned} \text{that is,} \quad & \text{CE : CA :: EG : AB} \\ \text{also,} \quad & \text{R : } b \text{ :: tan C : } c \\ & \text{CE : CA :: CG : CB} \\ & \text{R : } b \text{ :: sec C : } a \end{aligned}$$

By changing these proportions into equations, we have

$$\begin{aligned} \text{R} \times c &= b \times \tan C \quad (9.) \\ \text{R} \times a &= b \times \sec C \quad (10.) \end{aligned}$$

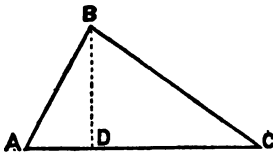


FIG. 10.

20. Theorem III. In any plane triangle, the sides are proportional to the sines of the opposite angles.

Let ABC be any triangle. Draw BD perpendicular to one side; it divides the triangle into two right-angled triangles.

In the triangles ABD and BDC we have from equation (7)

$$\begin{aligned} \text{R} \times \text{BD} &= \text{AB} \times \sin A = c \times \sin A \\ \text{R} \times \text{BD} &= \text{BC} \times \sin C = a \times \sin C \\ a \times \sin C &= c \times \sin A \\ a : c &:: \sin A : \sin C \end{aligned}$$

or,

$$\frac{a}{c} = \frac{\sin A}{\sin C} \quad (11.)$$

21. Theorem IV. In any plane triangle the sum of two sides is to their difference as the tangent of half the sum of the angles opposite is to the tangent of half their difference.

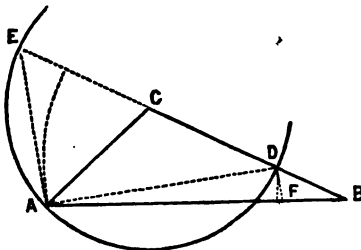


FIG. 11.

Let ABC be a triangle. With C as a centre and CA, the shorter of two sides, as a radius, describe a circle, cutting CB in D, and BC extended in E. Join AE, and draw DF parallel to AE.

$$\text{BE} = \text{BC} + \text{CA} = a + b$$

$$\text{BD} = \text{BC} - \text{CA} = a - b$$

also,

$$\text{ACE} = \text{CAB} + \text{CBA} = A + B$$

But ADE being an angle at the circumference, while ACE is at the centre, both intercepting the same arc,

$$ADE = \frac{1}{2}ACE = \frac{1}{2}(A + B)$$

By subtracting half the sum of two quantities from the greater, we obtain half the difference. Hence,

$$DAF = CAB - CAD = A - \frac{1}{2}(A + B) = \frac{1}{2}(A - B)$$

The angle EAD, being inscribed in a semicircle, is a right angle, and AE and DF are perpendicular to AD. If, with AD as radius, arcs be described, measuring the angles ADE and DAF, AE will be the tangent of the first and DF of the last.

Then, by similar triangles,

$$BE : BD :: AE : DF$$

that is, $a + b : a - b :: \tan \frac{1}{2}(A + B) : \tan \frac{1}{2}(A - B)$

or,
$$\frac{a + b}{a - b} = \frac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)} \quad (12.)$$

22. Theorem V. In any triangle, if a perpendicular be let fall from either angle upon the opposite side, the sum of the segments is to the sum of the sides as the difference of the sides is to the difference of the segments.

Let ACB, Fig. 12, be a triangle, and CD a perpendicular let fall upon AB. With C as a centre, and a radius AC, describe a circle cutting BC in G and BC extended in E.

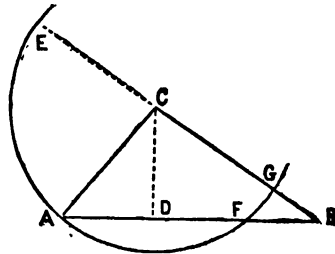


FIG. 12

then,

$$\begin{aligned} BE &= BC + CA = a + b \\ BG &= BC - CA = a - b \\ BA &= BD + AD = m + n \\ BF &= BD - AD = m - n \end{aligned}$$

From the relation of secants to their external segments we have

$$BA : BE :: BG : BF$$

$$m + n : a + b :: a - b : m - n$$

or,
$$(m + n)(m - n) = (a + b)(a - b) \quad (13.)$$

23. Theorem VI. To determine the angles of a triangle when the

three sides are given, we have the following formulæ, for proof of which we refer to works on analytical trigonometry :

$$\left. \begin{aligned} \sin \frac{1}{2}A &= R \sqrt{\frac{(s-b)(s-c)}{bc}} \\ \sin \frac{1}{2}B &= R \sqrt{\frac{(s-a)(s-c)}{ac}} \\ \sin \frac{1}{2}C &= R \sqrt{\frac{(s-a)(s-b)}{bc}} \end{aligned} \right\} (14.)$$

in which $s = \frac{1}{2}(a + b + c)$.

From the above theorems, with the aid of ordinary algebra and geometry, all problems arising under triangles may be solved.

24. RIGHT-ANGLED TRIANGLES.

I. Given the hypotenuse a , and one of the acute angles, to find the two sides b and c , and the remaining angle.

1. The angle may be found from the equation

$$B + C = 90^\circ.$$

2. To find b and c , we have equations (7) and (8)

$$R \times b = a \times \sin B; \quad R \times c = a \times \cos B.$$

II. Given one of the acute angles, as B , and one of the legs, as c , to find the other angle and leg, and the hypotenuse.

1. Find the remaining angle from the equation

$$B + C = 90^\circ.$$

2. Find the other leg from equation (9)

$$R \times b = c \times \tan B.$$

3. Find the hypotenuse from equation (10)

$$R \times a = c \times \sec B.$$

III. Given the hypotenuse and one leg, as b , to find the angles and other leg.

1. Find B from equation (7)

$$R \times b = a \times \sin B.$$

2. Find C from the equation

$$B + C = 90^\circ.$$

3. Find c from the equation

$$R \times c = a \times \cos B.$$

IV. Given the two legs, to find the angles B and C, and the hypotenuse.

1. Find B and C from equations

$$R \times b = c \times \tan B, \text{ and } R \times c = b \times \tan C$$

From which

$$\tan B = \frac{R \times b}{c}, \text{ and } \tan C = \frac{R \times c}{b}$$

2. Find a from the equation

$$R \times b = a \sin B, \therefore a = \frac{R \times b}{\sin B}$$

Or from equation

$$a^2 = b^2 + c^2$$

EXAMPLES.

1. In a right-angled triangle, given the angle B $30^\circ 45'$, and the hypotenuse 100 yards. Required the remaining parts.

$$C = 90^\circ - B = 59^\circ 15'$$

$R \times b = a \times \sin B$ $\text{Log } a = 2.000000$ $+ \text{Log } \sin B = 9.708670$ $- \text{Log } R = 10.000000$ <hr style="width: 100%;"/> $\text{Log } b \ 51.13 = 1.708670$	$R \times c = a \times \cos B$ $\text{Log } a = 2.000000$ $+ \text{Log } \cos B = 9.934199$ $- \text{Log } R = 10.000000$ <hr style="width: 100%;"/> $\text{Log } c \ 85.04 \ 1.934199$
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Or, by natural sines and cosines

$$b = 100 \times .51129 = 51.129 \text{ Ans.}$$

$$c = 100 \times .85941 = 85.941 \text{ Ans.}$$

2. In a right-angled triangle, given the two legs 130 and 150 yards, to find the remaining parts.

Ans. Hypotenuse 198.49, angles $49^\circ 5' 8''$ and $40^\circ 54' 52''$.

3. In a right-angled triangle, given one acute angle $48^\circ 12'$, and the side opposite to it 166 feet, required the remaining parts.

Ans. Hypotenuse 222.67, side 148.42, angle $41^\circ 48'$.

4. In a right-angled triangle, given the hypotenuse 200 yards, and one leg 100 yards, to find the remaining parts.

Ans. Side 173.20, angles 60° and 30° .



25. OBLIQUE-ANGLED TRIANGLES.

V. Given two angles and one side, to find the remaining parts.

Let A, B, and a be given.

1. Find the third angle from the equation

$$A + B + C = 180^\circ$$

2. Find the remaining sides from equation (11)

$$\frac{a}{c} = \frac{\sin A}{\sin C}$$

$$c = \frac{a \sin C}{\sin A} \quad \text{and} \quad b = \frac{a \sin B}{\sin A}$$

EXAMPLE.—Let two angles of a triangle be A $41^\circ 38'$, and B $68^\circ 12'$, respectively, and the side included between them be c 150 yards.

$$C = 180^\circ - (A + B) = 70^\circ 10'$$

$$a = \frac{c \sin A}{\sin C} \quad \text{and} \quad b = \frac{c \sin B}{\sin C}$$

$$\begin{aligned} \text{Log } c &= 2.176091 \\ + \text{Log } \sin A &= 9.822404 \\ - \text{Log } \sin C &= 9.973444 \end{aligned}$$

$$\begin{aligned} \text{Log } a &= 2.025051 \\ a &= 105.93 + \text{Ans.} \end{aligned}$$

$$\begin{aligned} \text{Log } c &= 2.176091 \\ + \text{Log } \sin B &= 9.967775 \\ - \text{Log } \sin C &= 9.973444 \end{aligned}$$

$$\begin{aligned} \text{Log } b &= 2.170422 \\ b &= 148.05 + \text{Ans.} \end{aligned}$$

VI. Given two sides and the angle opposite to one of them, to find the remaining parts.

Let a , b , and A be given.

1. Find the angle B by equation (11)

$$\frac{a}{b} = \frac{\sin A}{\sin B} \quad \therefore \sin B = \frac{b \sin A}{a}$$

2. Find the angle c from the equation

$$A + B + C = 180^\circ$$

3. Find c from equation (11)

$$c = \frac{a \sin C}{\sin A}$$

Since the sine of an angle and the sine of its supplement are the same, in finding the angle B, from the table, either the acute angle or its supplement will be a proper result. There must, therefore, in this case, be

two triangles, both answering to the given conditions. Thus ABC , Fig. 13, is a triangle whose angle is A , and whose sides are a and b . The angle B is an acute angle.

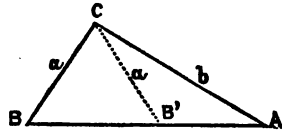


FIG. 13.

If, with the point C as a centre and the side a as a radius, we describe an arc, cutting the side AB in B' , it is plain the triangle $AB'C$ has the same angle A , and the same two given sides a and b .

Either triangle is, therefore, a proper solution of the problem. It is further evident that the angle $CB'A = 180^\circ - B$, and, therefore,

$$\sin CB'A = \sin B$$

When A is acute, and the side a is greater than b , or when A is obtuse, it is evident there can be but one triangle, and then but one solution.

EXAMPLE.—Given $A 44^\circ 27'$, and the side a 160 yards, and b 200 yards, required the remaining parts.

$$\sin B = \frac{b \sin A}{a}$$

$$\log b = 2.301030$$

$$+ \log \sin A = 9.845276$$

$$- \log a = \underline{2.204120}$$

$$\log \sin B = 9.942186$$

$$\therefore B = 61^\circ 5' 14'' \text{ or } B' 118^\circ 54' 46''$$

$$C = 180^\circ - (A + B) = 74^\circ 27' 46'' \text{ or } C' = 16^\circ 38' 14''$$

$$c = \frac{a \sin C}{\sin A}$$

$$\log a = 2.204120$$

$$+ \log \sin C = 9.983832$$

$$- \log \sin A = \underline{9.845276}$$

$$\log c = 2.342676$$

$$c = 220.128 \text{ Ans.}$$

or

$$\log a = 2.204120$$

$$+ \log \sin C' = 9.456839$$

$$- \log \sin A = \underline{9.845276}$$

$$\log c' = 1.815683$$

$$c' = 65.416 \text{ Ans.}$$

VII. Given two sides and the included angle, to find the remaining parts.

Let A , b , and c be the parts given.

1. Find the angles B and C by equation (12)

$$\frac{b + c}{b - c} = \frac{\tan \frac{1}{2}(B + C)}{\tan \frac{1}{2}(B - C)}$$

$$(B + C) = 180^\circ - A.$$

$$\tan \frac{1}{2}(B - C) = \frac{(b - c) \tan \frac{1}{2}(B + C)}{b + c}$$

$$B = \frac{1}{2}(B + C) + \frac{1}{2}(B - C)$$

$$C = \frac{1}{2}(B + C) - \frac{1}{2}(B - C)$$

2. Find a by equation (11)

$$\frac{a}{b} = \frac{\sin A}{\sin B} \therefore a = \frac{b \sin A}{\sin B}$$

EXAMPLE.—Given in a triangle $A 54^\circ 20'$, $b 180$ yards, and $c 150$ yards. Required B , C , and a .

$$B + C = 180^\circ - A = 125^\circ 40'$$

$$\frac{1}{2}(B + C) = 62^\circ 50'$$

$$b + c = 330 \text{ and } b - c = 30$$

$$\log(b - c) = 1.477121$$

$$+ \log \tan \frac{1}{2}(B + C) = 10.289718$$

$$- \log(b + c) = \underline{2.518514}$$

$$\log \tan \frac{1}{2}(B - C) = 9.248325$$

$$C \frac{1}{2}(B - C) = 10^\circ 2' 43''$$

$$B = 72^\circ 52' 43'' \quad C = 52^\circ 47' 17''.$$

$$\log b = 2.255273$$

$$+ \log \sin A = 9.909782$$

$$- \log \sin B = \underline{9.980314}$$

$$\log a = 2.184741$$

$$\therefore a = 153.017$$

VIII. Given the three sides, to find the angles.

First Method.

I. Find the segments m and n by equation (13)

$$(m + n)(m - n) = (a + b)(a - b)$$

$$m - n = \frac{(a + b)(a - b)}{m + n}$$

$$m = \frac{1}{2}(m + n) + \frac{1}{2}(m - n)$$

$$n = \frac{1}{2}(m + n) - \frac{1}{2}(m - n)$$

2. Find A and B by equation (8)

$$R \times n = b \cos A \therefore \cos A = \frac{R \times n}{b}$$

$$R \times m = a \cos B \therefore \cos B = \frac{R \times m}{a}$$

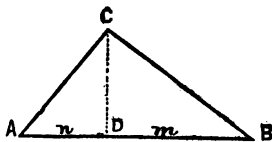


FIG. 14.

Second Method.

Find each of the angles by the equations (14)

$$\sin \frac{1}{2}A = R \sqrt{\frac{(s-b)(s-c)}{bc}}, \text{ etc.}$$

When all the angles are thus found, their sum should make 180° . When two have been thus found, the third may be obtained by subtracting their sum from 180° .

EXAMPLE.—Given the three sides of a triangle, a , b , c , 100, 150, and 200 yards, to find the angles.

$$2s = a + b + c = 450 \therefore s = 225$$

$$s - a = 125, s - b = 75, s - c = 25$$

To find A,

$$\begin{aligned} \log R^2 &= 20.000000 \\ + \log (s - b) &= 1.875061 \\ + \log (s - c) &= 1.397940 \\ - \log b &= 2.176091 \\ - \log c &= 2.301030 \\ & \underline{18.795880} \\ \sin \frac{1}{2}A &= 9.397940 \\ \frac{1}{2}A &= 14^\circ 28' 39'' \\ A &= 28^\circ 57' 18'' \end{aligned}$$

To find B,

$$\begin{aligned} \log R^2 &= 20.000000 \\ + \log (s - a) &= 2.096910 \\ + \log (s - c) &= 1.397940 \\ - \log a &= 2.000000 \\ - \log c &= 2.301030 \\ & \underline{19.193820} \\ \sin \frac{1}{2}B &= 9.596910 \\ \frac{1}{2}B &= 23^\circ 17' 1'' \\ B &= 46^\circ 34' 2'' \end{aligned}$$

To find C,

$$\begin{aligned} \log R^2 &= 20.000000 \\ + \log (s - a) &= 2.096910 \\ + \log (s - b) &= 1.875061 \\ - \log a &= 2.000000 \\ - \log b &= 2.176091 \\ & \underline{19.795880} \end{aligned}$$

$$\begin{aligned}\sin \frac{1}{2} C &= 9.897940 \\ \frac{1}{2} C &= 52^{\circ} 14' 20'' \\ C &= 104^{\circ} 28' 40''\end{aligned}$$

Sum of the angles,

$$\begin{aligned}A &= 28^{\circ} 57' 18'' \\ B &= 46^{\circ} 34' 2'' \\ C &= 104^{\circ} 28' 40'' \\ \text{Sum} &= 180^{\circ}\end{aligned}$$

MISCELLANEOUS EXAMPLES.

1. In the triangle ABC, given AB 153 yards, AC 137 yards, and the angle A $40^{\circ} 33' 12''$; required the remaining parts.

$$\text{Ans. } B 61^{\circ} 13' 47'', C 78^{\circ} 13' 1'', BC 101.62.$$

2. In the triangle ABC, given AB 70 miles, AC 60 miles, and BC 50 miles: required the angle A. Ans. $44^{\circ} 24' 56''$.

3. In the triangle ABC, given AB 500 yards, the angle A $105^{\circ} 30'$, and the angle B 47° ; required AC. Ans. 791.9.

4. In a right-angled triangle ABC, given B 90° , A 30° , and AB 200 feet; required BC. Ans. 115.4.

5. In the right-angled triangle ABC, given the hypotenuse AC 480, and the angle A $53^{\circ} 8'$; required the two legs.

$$\text{Ans. } BC 384.0, AB 287.9.$$

6. In the triangle ABC, given A $44^{\circ} 13' 24''$, B $55^{\circ} 59' 58''$, and AC 368; required the remaining parts.

$$\text{Ans. } C 79^{\circ} 46' 38'', AB 436.84, BC 309.60.$$

CHAPTER II.

MEASUREMENT OF LINES.

26. THE unit of length commonly employed in Land Surveying is the **chain**, called, after its inventor, **Gunter's chain**. It is 66 feet, or 4 rods, in length. It was so chosen because 10 square chains were exactly equal to one acre. The chain is divided into 100 parts, called links; hence, in recording distances measured with the chain, the number of links may be written as decimals of a chain, and so used in computations. The link is 7.92 inches, or nearly two-thirds of a foot.

Other units of length are often used. In measuring small plots of ground, as city-lots, or streets and roads, the dimensions are nearly always expressed in **feet** and inches. The **rod** and **yard** are also used. All of these units are easily converted into each other. In the United States Coast Survey it has been found convenient to employ the **metre**, the French unit of measure. It is equal to 3.280899 feet, or 39.370799 inches.

27. The Gunter's chain is made of iron or steel wire, joined together in links by small rings. At the ends handles of brass are attached, and swivels are inserted at the middle and quarter points to prevent kinking. Brass tags are inserted at intervals of ten links to facilitate counting; those at ten links from each end having one point, those at twenty, two, those at thirty, three, those at forty, four, and that at the middle being rounded.

Chains should also be made so that they may be taken apart at the middle and quarter points, and used in sections for chaining on steep ground.

The best chains are now made of steel wire, which, being stronger than iron, permits the chain to be made much lighter.

A chain measuring 100 feet is used in preference to the Gunter's chain for many kinds of work. It is divided into 100 links, each being, there-

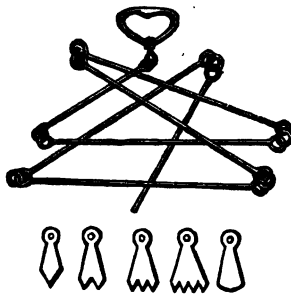


FIG. 15.

fore, one foot long. It is used in the engineering work of railroads, canals, sewers, etc. Being longer, work with it is more rapid and liable to fewer errors. The formulas for railroad curves, and for embankments and excavations, are all constructed on the basis of a hundred feet chain.

28. The chain is liable to changes of length, arising from stretching the rings, or bending the links. A change of temperature from 32° to 100° Far.—a change to which the chain is often exposed—makes a difference of about one-third of an inch in the length of a Gunter's chain. The surveyor requires, therefore, the means of **testing** the length of his chain. He should have a standard accurately laid off on some permanent, smooth surface, such as a flag-walk, a curb-stone, or the stone water-table of a building. On frequent occasions the chain should be compared with the standard, and, if found inaccurate, be corrected.

29. If a line has been measured by an inaccurate chain, the true distance may be determined by the following proportion :

66 feet : the length of the inaccurate chain in feet ::
the measured distance : the correct distance.

Or, 66 feet : the error of the chain in feet ::
the measured distance : the correction required.

Or, the length of the standard given by the incorrect chain, in links :
100 links :: the length of the measured line : the true length.

If the area of a plot of ground has been computed upon measurements with an incorrect chain, the true area may be obtained by proportions similar to those above ; thus,

66 feet^2 : $\frac{\text{length of the inaccurate chain in feet}^2}{10000}$::
the computed area : the true area.

30. Except in measuring perfectly level ground, the chain does not rest on the ground, but is suspended between the two ends. The sagging of the middle must, of course, have the effect of shortening the chain. It is customary in compass surveying, where the character of the operations does not admit of great accuracy, to disregard this error. If, how-

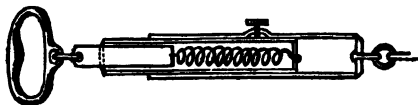


FIG. 16.

ever, precision is required, as in surveying city property, and in engineer-

ing work, means must be used to obviate it. The most effectual means is the insertion, at the handle, of a spring balance or dynamometer, Fig. 16, the index of which shows the amount of tension. The standard length of the chain is obtained by swinging the chain freely at the ends, and stretching it until the index of the dynamometer marks the standard point.

The spring of such a chain is liable to become weakened by use. The chain must, therefore, be frequently tested by means of a standard line, and the index adjusted.

31. Tape-lines are frequently used for the measurement of lines, instead of the chain. The best are made of a thin ribbon of steel, jointed at intervals, and wound up in a leather case. A cheaper measure is made of linen tape, with a fine brass wire interwoven through its length. The common linen tape must be used with caution, as its length is materially affected by being wet.

32. Two men are required for measuring a line with a chain, the one in advance being called the **leader**, and the other the **follower**. The leader is provided with ten iron pins, bent into rings at the top and pointed at the bottom. Bright tags are tied to the top rings, to render the pins easily seen. An improved pin is made with the pointed end so heavy that when dropped it will fall plumb below the point where it is held. A few thin rods or staves are also often useful for ranging out lines, and marking their terminations.

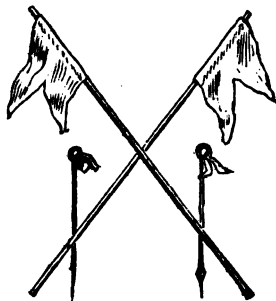


FIG. 17.

The leader, taking an end of the chain and the ten marking-pins, walks directly toward the point to be reached. The follower, holding his end of the chain at the starting-point, by word or gesture ranges the leader exactly in line. Then the leader, assuring himself that the chain is straight and taut, puts down a pin. They then walk on one chain's length, and repeat the same operation. The follower gathers up the pins put down by the leader. In thus measuring a line the number of pins taken up by the follower will be the number of full chains in the line. If all the pins are put down before the line is completed, they are passed back to the leader, and a tally kept for every ten chains.

Care and skill are required to make the measurement trustworthy. If the pins are not truly in line, or the chain is not drawn taut, the distance

will be exaggerated. With all the precautions possible, measurements made over the ordinary rough surface of the ground with the surveyor's chain can only be relied on within very considerable limits. It may be fairly claimed, however, that they are within the limits of accuracy attainable in the other parts of compass surveying. When greater accuracy is required, a chain with a dynamometer attachment should be used and the line should be measured in both directions. Methods for the very precise measurement of a line, which is required in surveys on a large scale, will be given subsequently.

33. Since the distances used in surveying are horizontal distances, it is necessary that the chain, in measuring over uneven ground, should be held horizontal. In going down-hill, as in Fig. 18, the leader must hold his end of



FIG. 18.

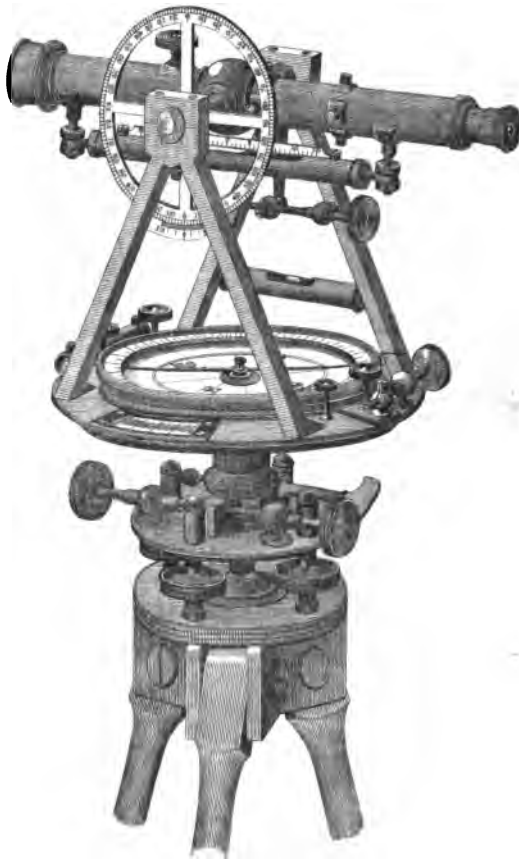
the chain elevated, and in going up-hill the follower. The chainmen depend upon the eye to determine when the chain is level. A small spirit-level is sometimes attached for this purpose. The chain is drawn taut, an allowance is made for sag, and by means of a staff, or by dropping a pin, the point immediately below the elevated end is marked. If the chain provided with a dynamometer is used, no allowance for sag is required.

In case the ground is too steep to admit of using the full chain, the half or quarter chain may be used. The best chains admit of being taken apart for this purpose.

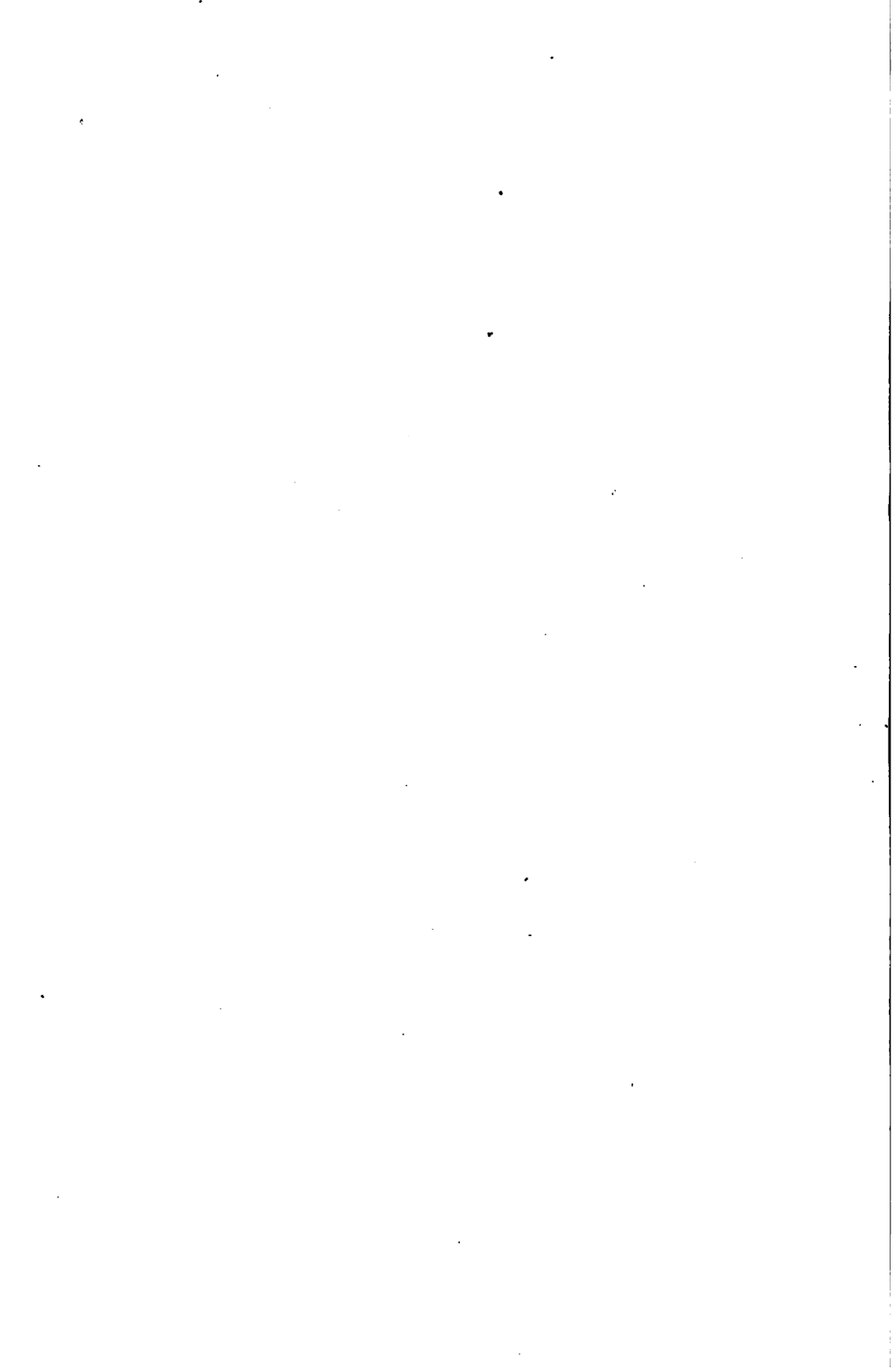
34. In chaining across a valley, as in Fig. 18, the follower sometimes is unable to see at the same time the staff at B, and the pin in the leader's hand at D. In this case a plumb-line may be held at A, and when the eye is so held that the line covers B it will also cover the point D. It will be found convenient in such cases, where the difficulty is liable to recur at each chain, to set up, by means of a plumb-line, a stake, at C, the lowest point, and then to range the intermediate points by this.

35. If one end of a line cannot be seen from the other because of an intervening elevation, it is necessary to fix intermediate points. This may be done as follows: Let it be required to find intermediate points between A and B, Fig. 19, separated by an intervening elevation. Plant

SURVEYOR'S TRANSIT.



BENJ. D. BRINSON, N.Y.



a staff at m'' , approximately in the desired line, as shown in the horizontal plan of the figure. Plant another at n'' , in line with A and m'' . Sight from m'' toward B; if n'' is not in line, have it moved to n' in line. Sight from n' toward A, and bring the staff at m'' into line at m' . Thus proceed, bringing the staves more and more nearly into line, until at last they are found exactly in range at m and n .

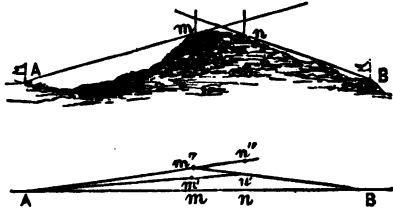


FIG. 19.

36. When an obstacle interferes with the ranging and measurement of a line, the following expedients may be employed to find the length indirectly.

By setting off perpendiculars. Let it be required to find the length of the line AX, Fig. 20, of which the part BC is covered by an obstacle. Measure the part AB. At B erect a perpendicular, BD, of such length as to be clear of the obstacle. At D erect a perpendicular, DE, and

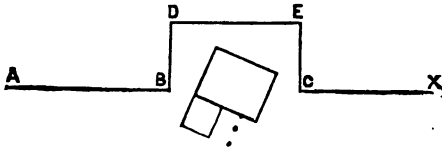


FIG. 20.

measure past the obstacle. At E erect a perpendicular, EC, of the same length as BD. Finally, at C erect a perpendicular, CX; it will be the continuation of the line AB, and DE will be the length of the wanting part.

The right angle in the above problem may be set off by the transit or compass. A small instrument called the surveyor's cross may also be conveniently used. It consists of a staff, on which is mounted a head, having two sights at right angles. The staff is set at the point where the right angle is to be constructed. One set of sights is pointed in the direction of the given line; the other will then point in the direction of a perpendicular. Two slits sawed in a board, at right angles to each other, will, for ordinary cases, take the place of the sights.

A perpendicular may also be set off on the ground by means of the chain only. Let it be required to erect a perpendicular at B, Fig. 21, to the line AB. Measure back from B, 40 links to A. Fix one end of the chain at A, by means of a pin, and also fix the 80th link at B.

Then, holding the middle point of the chain, stretch it taut; it will take the position D, which will be the third angle of a right-angled triangle, AD being the hypotenuse, and AB and BD being the two legs.

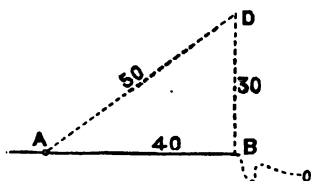


FIG. 21.

length. At two points, B and C, erect perpendiculars BD and CE, of equal length, and so long that a line running through DE will clear the

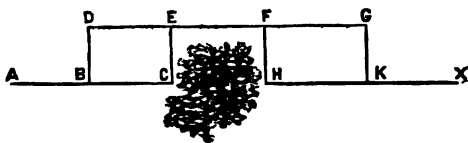


FIG. 22.

obstacle. At two points in this line, F and G, erect perpendiculars FH and GK, equal in length to those on the other side. The extremities of these perpendiculars, H and K, will be two points in the continuation of the line ABC, and EF, when measured, will give the length of the wanting part, CH.

38. By equilateral triangles. Let it be required to extend the line ABC, Fig. 23, beyond an obstacle, and also to obtain its length. Set off

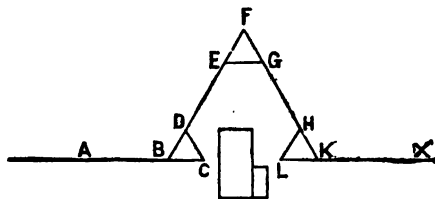


FIG. 23.

BC equal to 50 links. Fix one end of the chain at B, and the other at C; the middle of the chain, when drawn taut, will give D the third angle of an equilateral triangle. Extend BD sufficiently far to clear the obstacle, and construct in the same way the equilateral triangle EFG. Extend FG until FK is equal in length to BF. Construct the equilateral triangle

HLK ; LK will be the continuation of AC, and BK will be equal in length to BF or FK.

39. By symmetrical triangles. If the obstacle be such as not to prevent ranging across it, but only prevents measurement, the distance may be obtained as follows :

Let AX, Fig. 24, be a line of which the part BF cannot be directly

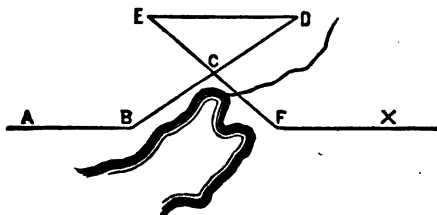


FIG. 24.

measured. Select some convenient point, C. Range and measure a line BCD, so that CD shall be equal to BC. Also range and measure the line FCE, making CE equal to FC. Then measure the distance ED, it will be equal to the wanting part BF.

Other methods for obtaining the length of inaccessible lines will be given under Chapter IV.

40. The surveyor will have occasion sometimes to determine the length of a line more accurately than is possible by any of the methods of measurement heretofore described. To accomplish this, some variety of **measuring-rod** must be used.

Rods of iron or brass are often used ; and, when a strict estimate is made of the changes of length resulting from temperature, they are capable of almost any degree of precision.

Iron expands, between 32° and 212° Far., .0012575 of its own length.

Brass " " " " .0019062 " "

The rods are usually made to measure a certain standard length at a given standard temperature. Thus, a rod may be made five and a half yards in length, at a temperature of 60° Far., and in measuring with such a rod, any variations above or below this must be noted and taken into account. Let the temperature, for example, be 80° Far., the correction for the length of an iron rod will be found by the following formula, *l* representing the standard length of the rod :

$$\text{Correction} = \frac{20^\circ \times .0012575 \times l}{180^\circ} \text{ to be added.}$$

A small thermometer may be attached to the rod, which can be read as occasion requires.

Wooden rods may be advantageously used instead of metal, in ordinary cases. They are very little affected by temperature, and the effect of moisture may be mostly prevented by thoroughly soaking the rods in linseed oil, and afterward covering them with shellac. The best material is yellow pine, selected of straight grain and even texture.

In the rigorous operations required in trigonometrical surveys on a large scale, such as the United States Coast Survey, the best results have been attained by using rods made of several metals combined, on the principle of the compensation pendulum. In the report of the United States Coast Survey for 1854 will be found a detailed description of the rods devised for measuring the base-lines, by Professor A. D. Bache, late superintendent.

41. To measure a **base-line** accurately with rods, the first step is to clear it of obstructions, so that it may be seen from end to end. Two rods are required for the measurement. Place one end of the first rod precisely at the beginning of the line, and by means of a transit align it in the required direction, and level it with a spirit-level. Bring the second rod into line by the transit, level it, and bring the extremities into contact. Care must be taken not to disturb the position of the first rod by the jar of the contact. The first rod may now be moved from its position and aligned in contact with the second. Light tripods are used for bringing the rods level, and a light framework to prevent the rods from sagging.

When it is not convenient to measure the whole line on a level, it may

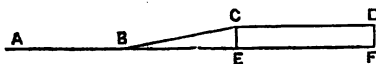


FIG. 25.

be measured on a slope, and a correction made to reduce the distance to a horizontal. Thus, if we desired to find the horizontal distance between A and D, Fig. 25, the part

BC having been measured on a slope, we would have

$$AF = AB + BE + CD$$

in which

$$BE = BC \times \cos B$$

Hence,

$$AF = AB + BC \times \cos B + CD$$

In this case it is necessary to measure the angle of inclination of the slope, either by the spirit-level or by the vertical circle of the transit or theodolite.

42. Methods of measuring lines with dispatch, although with inferior precision, are often of the greatest service to the surveyor. An apparatus called an **odometer** has been devised for this purpose. It consists of a

vehicle in which one wheel is connected with a train of wheelwork in such a way as to record the number of its revolutions by means of an index. Commonly the wheel is made 8 feet 3 inches in circumference, so that two revolutions cover one rod. On level ground such an instrument will give tolerably correct results; but where it is undulating the distance recorded will be the surface-distance, and not the required horizontal distance. It is also supplied with a magnetic compass, so that the surveyor may record the bearings of the line while the wheel records the distances.

This instrument has been much used in making rough surveys for county and township maps, and, in the hands of a skillful surveyor who knows how to make proper allowances, it has proved a very useful invention. It has been used in the United States Coast Survey for filling up details of roads, etc.

43. The **micrometer telescope** furnishes a beautiful method of obtaining distances with considerable precision. A pair of parallel threads, Fig. 26, is inserted in the focus of the telescope of an ordinary surveyor's transit. These threads are commanded by a pair of fine thread mill-head screws. They can be made to coincide at the centre of the field, or by the screws withdrawn from each other any distance. A rod divided into feet, etc., is placed at a convenient distance, say 100 feet. The telescope is then pointed at the rod and focused. The threads of the micrometer are now made to cover a given space in the rod, say one foot.

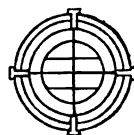


FIG. 26.

It is evident if the rod be removed to twice the distance, the threads will cover two feet, and so on in the same proportion.

Let it now be required to determine the distance across a lake, or some other distance not easily accessible. Adjust the threads, as stated, to the space of one foot on the rod; send the rod to the desired point, and set it

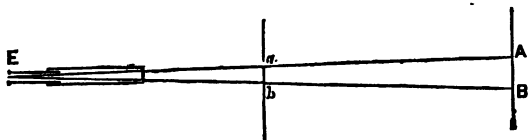


FIG. 27.

up perpendicular to the line of sight; bring the telescope so that one thread shall coincide with an even foot. Signal to the assistant until he slides a mark or target on the rod, so that it shall exactly coincide with the other thread. The space covered on the rod will enable us to determine the distance. Let E, Fig. 27, be the position of the eye, ab the

space covered on the rod by the threads when at the distance 100 feet, and $AB (= m)$ the space covered by the threads at the unknown distance EA . Then, by similar triangles, we have

$$ab : AB :: Ea : EA$$

$$1 : m :: 100 : EA$$

$$EA = m \times 100$$

Let the space covered on the rod be, for example, 5.45 feet :

$$EA = 5.45 \times 100 = 545$$

In the hands of a skillful observer this apparatus will give results nearly or quite equal to the best measurements which can be made with a chain. The manufacturers of transits for surveyors now insert in the telescopes the micrometer, at little or no additional cost.

CHAPTER III.

MEASUREMENT OF ANGLES.

44. ANGLES are the second class of magnitudes which, in surveying, admit of direct measurement, and which, together with the measured lines, enable us to determine the unknown lines and areas which we require.

An **angle** is the inclination of two lines. A **horizontal angle** is one in which the lines are horizontal, and which consequently lies in a horizontal plane. A **vertical angle** is one which lies in a vertical plane. If

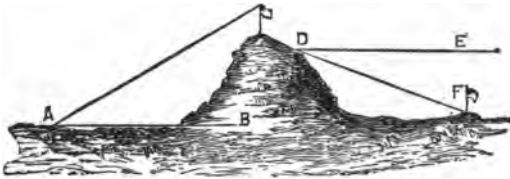


FIG. 28.

one side of a vertical angle is horizontal, and the other lies above it, it is called an **angle of elevation**. If one side is horizontal and the other lies below it, it is called an **angle of depression**. In Fig. 28, BAC is an angle of elevation, and EDF an angle of depression.

An **oblique angle** is one whose sides are neither both in a horizontal

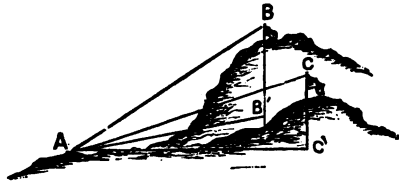


FIG. 29.

plane, nor both in a vertical plane. The angle formed by two lines meet-

ing at some point, as A, Fig. 29, and running to the tops of two mountains of unequal height, would be an oblique angle.

The two kinds of angles used in Land Surveying are horizontal and vertical, and the Chapter will be restricted to describing the measurement of these.

45. A circle divided into 360 equal parts is universally employed for the measurement of angles. One of these parts is called a degree, which

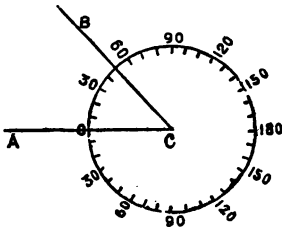


FIG. 30.

is subdivided into 60 equal parts called minutes, and these again into 60 equal parts called seconds. If two lines, as AC and BC, Fig. 30, intersect each other in the centre of such a circle, the number of these equal parts which they include between them in the circumference of the circle (in this case 50) will give the angle which they make with each other. As shown in Art. 10, Chap. I, such measurement of angles is independent

of the size of the circle on which it is made.

The chief instruments employed in surveying, according to American practice, for the measurement of angles, are **The Transit**, **The Surveyor's Compass**, and **The Theodolite**. The last of these, however, has nearly disappeared in American surveying, and its place is supplied by attaching a vertical circle to the Transit. A description of these instruments will now be given.

SURVEYOR'S COMPASS.

46. The essential parts of a Surveyor's Compass are the divided circle, the magnetic needle, and the sights. The form and arrangement of the complete instrument will be seen in the plate opposite.

The **magnetic needle** is a small bar of steel, the length of which nearly equals the diameter of the divided circle. At its middle is a small cap with a steel or jewel centre which serves to suspend it upon a steel pivot at the centre of the circle. It is made to balance after being magnetized,



FIG. 31.

by a small coil of fine wire around the south end, which may be moved until it be exactly poised. The delicacy of the needle may be tested by the number of vibrations which it will make after being disturbed, before

coming to rest. The greater the number of vibrations before coming to rest, the greater the magnetic force, and delicacy of suspension.

The needle, when suspended freely, assumes a direction nearly north and south, which is termed the **Magnetic Meridian**.

47. The circle of the ordinary Surveyor's Compass is divided into degrees and half-degrees, and is numbered 0 at the north and south points, right and left up to 90° at the east and west points. This circle is attached to a plate provided with two sights placed on opposite sides of the divided circle. A straight line running through the two sights will pass through the centre of the circle at which the needle is suspended, and through the north and south points. They are placed precisely perpendicular to the plate to which they are attached, and are capable of being removed when the instrument is to be packed. Each sight has one narrow slit and one wide slit in which a thread is inserted. The narrow slit in one is opposite to the wide slit in the other. When pointed at an object the eye is placed at the narrow slit, and the instrument turned until the thread in the wide slit crosses the target.

48. Two small **spirit-levels** are attached to the plate of the compass for the purpose of determining when the instrument is level. They are placed at right angles to each other, and their axes are made parallel to the plate. When the two spirit-levels are horizontal the plate is horizontal in every position.

The spirit-level consists of a glass tube, slightly but uniformly curved, and closed at both ends. The tube is nearly filled with spirits, leaving a small bubble of air. When placed with the curvature upward, and the two ends exactly in the same level, the air-bubble will run to the middle. If one end is raised, the air-

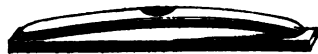


FIG. 32.

bubble indicates the fact by running toward the elevated end. To make the plate of the compass, therefore, horizontal, we have only to move it until the bubbles are brought to the middle point of the two tubes.



FIG. 33.

49. The Compass is supported on a **ball-and-socket joint** or on a **tripod-head**.

The construction of the ball-and-socket joint will be apparent from Fig. 33. The ball is shown by the dotted line enclosed in the socket. This joint permits motion in any direction, and is made to move with sufficient friction to maintain the compass in its place. For the purpose of levelling the compass, the plate is grasped in the hands and moved on this joint until the bubbles occupy the middle of the tubes.

In sighting the instrument the level of the plate is liable to be disturbed, by moving it by hand into the required line. To obviate this difficulty, as well as to render the pointing of the instrument more accurate, a clamp and tangent screw are attached to the ball of the joint. It is shown in the plate of the compass, just below the brass plate bearing the sights.

A staff with an iron-shod point, commonly called a **Jacob's staff**, is used to support the ball-and-socket joint and the compass. The compass, joint, and staff may all be detached from each other when desired. Instead of the Jacob's staff, a **tripod** may be used to support the instrument. It gives greater steadiness to it while it is levelled and used.

50. To facilitate the exact levelling of the instrument, a contrivance called a **tripod-head**, Fig. 34, is used instead of the ball-and-socket joint above described. It consists of two brass plates connected together by a ball-and-socket joint. The lower plate is connected by means of a large screw to a brass plate, which forms the top of the tripod. Between the two plates of the tripod-head are four thumb-screws placed at opposite points, resting on the lower plate, and passing up through the upper plate. By means of these we can change at pleasure the position of the upper plate and the instrument supported on it.

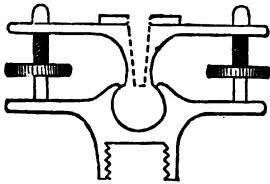


FIG. 34.

The spindle or axis of the compass fits into a conical opening which runs down into the neck of the ball-and-socket joint.

51. When supplied with a tripod-head, the compass may be **levelled** as follows:

1. Bring the plate as nearly level as possible by moving the legs of the tripod and watching the effect upon the levels.
2. Turn the compass so that one of the spirit-levels shall be parallel to one pair of levelling screws, and the other to the other pair.
3. If the bubble stands in the middle of either level, the plate is level in that direction. If the bubble is not at the middle of a tube, bring it there by means of the pair of screws parallel to it. Grasp both screws with the fingers and thumbs, and turn both at the same time an equal amount, one to the right and the other to the left. This motion raises the plate on one side and depresses it on the opposite. By watching the bubble the motion can be arrested at the right moment. When the one tube has been brought level, the same process may be used with the other. If, in levelling the second, the first has been disturbed, it must again be brought level; and this must be continued till

both tubes are brought level. Since two lines determine the position of a plane, when the plate is level in these two directions it is horizontal.

52. To measure a horizontal angle by means of the compass, proceed as follows :

1. Place the compass precisely over the angular point A, Fig. 35. The tripod is provided with a plumb-bob for this purpose.

2. Level the instrument in the manner described.

3. Turn the sights of the instrument along one of the lines forming the angle, as AB, pointing it carefully upon some well-defined signal. It is customary with most surveyors, in using the compass, to make the north end of the plate always point towards the signal. In that case the angle should always be read from the north end of the needle. The E and W letters on the face of the compass are usually put on in the reverse from their natural position, in order that the direction of the line of sight may always be correctly read.

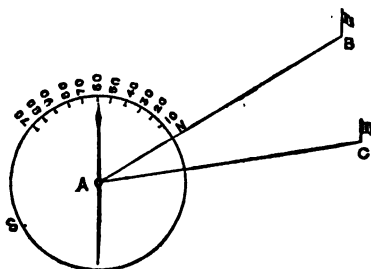


FIG. 35.

4. Note the reading indicated on the circle by the north end of the needle. If it lies between the N and E points, the line is northeast, etc. The north end of the needle is always distinguished by some peculiarity of shape or color. The circle is never divided nearer than to half degrees, but, by eye, the surveyor can easily estimate to the nearest quarter. This being the degree of precision usually employed in the traverse tables, no nearer reading is generally attempted with the compass.

5. Turn the line of sights toward the second signal, AC, and having accurately pointed it, read as before.

These two readings, when **properly combined**, will give the angle included between the lines AB and AC.

53. The following obvious rules for combining the readings to obtain the included angle will be found convenient. The propriety of the directions will be apparent from considering the annexed figure.

1. When the readings are both north or south, and both east or west, the included angle is equal to the difference between the readings. Thus,

$$AOB = NOB - NOA$$

2. When the readings are both north or south, and one east and the

other west, the included angle is equal to the sum of the two readings.

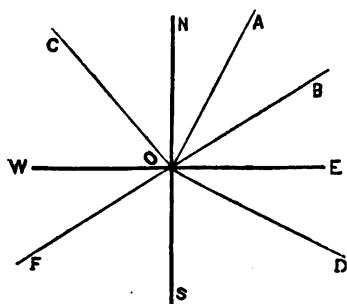


FIG. 36.

tracting the difference of the two readings from 180° . Thus,

$$AOF = 180^\circ - (SOF - NOA)$$

Thus,

$$AOC = NOA + NOC$$

3. When the readings are both east or west, and one north and the other south, the included angle is obtained by subtracting the sum of the two readings from 180° . Thus

$$AOD = 180^\circ - (NOA + SOD)$$

4. When one reading is north and the other south, and the one east and the other west, the included angle is obtained by subtracting the difference of the two readings from 180° . Thus,

EXAMPLES.

1. The line AC reads N $54^\circ 30'$ E,
the line AB reads N $25^\circ 45'$ E: what is the included angle?
2. The line AB reads S $48^\circ 15'$ E,
the line AC reads S $19^\circ 30'$ W: what is the included angle?
3. The line AB reads N $60^\circ 30'$ E,
the line AC reads S $16^\circ 0'$ E: what is the included angle?
4. The line AB reads S $73^\circ 45'$ W,
the line AC reads N $43^\circ 30'$ E: what is the included angle?

54. The angle which a line makes with a true North and South line is called the **bearing** of that line. Thus we say that a line from the City Hall, New York, to Sandy Hook Light-house, bears South $0^\circ 47'$ East. The angle $0^\circ 47'$ is the bearing of the line.

When we reckon the bearing from the magnetic needle, instead of the true meridian, it is called the magnetic bearing. The angles read from the surveyor's compass are the magnetic bearings. If the magnetic needle pointed due north and south, the magnetic bearings and true bearings would coincide. Or, if the magnetic needle always varied from the true meridian by a constant amount, the angle between two lines measured by referring them to the magnetic needle, would be the same as by referring them to the true meridian.

In Chapter VII will be found an account of the variation of the needle,

and the methods by which it may be ascertained, and the errors arising from it in practical surveying may be obviated.

VERNIER.

55. The object of a **vernier** is to enable us to read smaller divisions on any scale than would be possible with a simple index. It may be applied to the divisions on a straight scale as a levelling-rod, or of a curved scale as a graduated circle of a compass or transit.

Let Fig. 37 represent part of the circumference of a circle in an angular instrument divided into degrees and half-degrees. This is com-

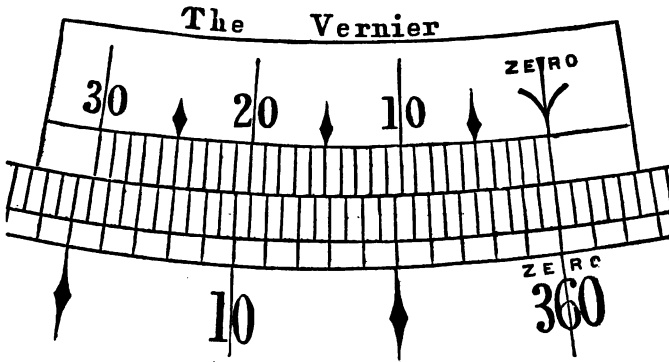


FIG. 37.

monly called the **limb**. And let the line marked 0, or zero, on the inner concentric circle, called the **vernier**, represent a movable index, which may point successively at all points of the outer divided circle. With this arrangement only, it is plain that the smallest division of the circle which could be indicated with accuracy would be half-degrees.

Suppose a space corresponding to 29 divisions on the limb were marked off on the vernier circle, to the right and left of the index-mark 0. Let this space now be divided into 30 equal divisions.

Since 29 divisions on the limb = 30 divisions on the vernier,
30 divisions on the limb exceed 30 divisions on the vernier by one division of the limb.

Hence, 1 division on the limb exceeds 1 division on the vernier by $\frac{1}{30}$ of one division of the limb.

If the divisions of the limb are half-degrees,

1 division on the limb exceeds 1 division on the vernier by $\frac{1}{10}$ of half a degree, that is, by 1 minute.

If now the 0 point of the vernier be made to correspond with 0 degrees on the limb, the first division of the vernier will fall short of reaching the first division on the limb by 1 minute; the second division, 2 minutes, etc. And if the vernier circle be moved until the first divisions correspond, it must have been moved 1 minute; if the second divisions correspond, 2 minutes, etc.

Let us suppose, now, that in pointing the instrument the index of the vernier falls somewhere between two divisions of the limb, as shown in

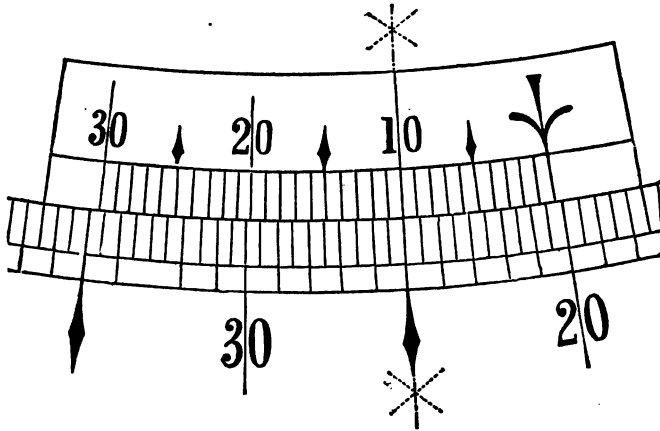


FIG. 38.

Fig. 38. First, we note at how many degrees and half-degrees from the 0 point of the limb the index falls; in this case 20°. Then we trace along the vernier until we find one of its division lines, which matches exactly a line of the limb; in this case we find that the (10th) line of the vernier matches a line of the limb. This indicates that the index of the vernier stands 10 minutes beyond the last half-degree; and hence the reading of the instrument will be 20° 10'.

In circular instruments the verniers are made to read in both directions, that is, to the right and left from the index-point. In all cases, in reading verniers constructed as above described, we use the forward end, that is, the end toward which the numbering on the limb proceeds.

Verniers may be constructed in which 31 divisions of the limb are equivalent to 30 divisions on the vernier. In these it is plain that one division on the limb will be less than one division on the vernier by $\frac{1}{30}$ of one division on the limb; that is, if the divisions are half-degrees, by one minute. In reading such a vernier, we read from the rear end instead of the forward end.

56. On the same principle verniers may be made to read to any desired degree of precision. Thus, a circle may be divided into thirds of degrees, and a vernier having 60 divisions used. One division of the limb would differ from one division on the vernier by $\frac{1}{60}$ of a third of a degree, that is, by $\frac{1}{2}$ of a minute, or 20 seconds. Such an arrangement therefore would enable us to read angles to 20 seconds.

Upon levelling-rods, scales, and other straight instruments, the verniers are made to indicate divisions of a foot, or an inch.

VERNIER COMPASS.

57. David Rittenhouse, of Philadelphia, introduced an improvement in the common surveyor's compass, by attaching a vernier to the graduated circle. The object of the improvement was to facilitate the running of lines when it was necessary to take into account the variation of the magnetic needle. In the plate of the compass opposite Art. 46, the vernier will be seen at the left end of the figure. An arc of a circle reading half-degrees is attached to the plate which carries the sights, and is concentric with the graduated circle and the needle. A vernier is arranged to move along this arc by a slow-motion screw, shown in the figure. This vernier is connected with the graduated circle, so that when the vernier is moved by the screw the whole graduated circle is carried along with it. The arc through which it is moved can be read in the manner described.

58. This vernier attachment of the compass subserves two objects:

First: In determining the bearing of any line by means of the compass, if the needle does not point exactly at any division line of the circle, the additional arc may be determined by turning the vernier-screw until the last previous division coincides exactly with the end of the needle; the arc through which the index-point has been moved, read off from the vernier, will give the additional arc required.

Since the precision with which the common compass-sights can be pointed on any object, is ordinarily not greater than can be estimated directly from the pointing of the needle on the graduated circle, this use of the vernier attachment is but little resorted to.

Second: The more important use of the vernier in the compass, is to

enable us to set off by means of it the variation of the needle, and thus to measure the bearings of lines from the true meridian instead of the magnetic meridian.

Suppose the variation of the needle at any place to be $5^{\circ} 13'$ west. Turn the vernier-screw until the vernier shall indicate $5^{\circ} 13'$ on the side toward which the needle varies. It is plain that when the line of sights is turned so that the needle shall read zero on the circle, it must point in a direction $5^{\circ} 13'$ east from the needle, and, therefore, must be due north and south. The compass, when thus set, will give the true bearings of all lines. If, however, the compass is removed to another locality, where the variation of the needle is different, it will require to be set anew to correspond to the changed variation.

The vernier compass will also facilitate the re-running of the lines of an old survey. Place the instrument on some well-defined line of the old survey, and turn the vernier-screw until the needle of the compass indicates the same bearing as that given in the old field notes. The reading of the vernier, when thus set, would give the variation of the needle at the time of the old survey. All the other lines of the survey may now be run without further alteration, and the new bearings ought to agree with the old.

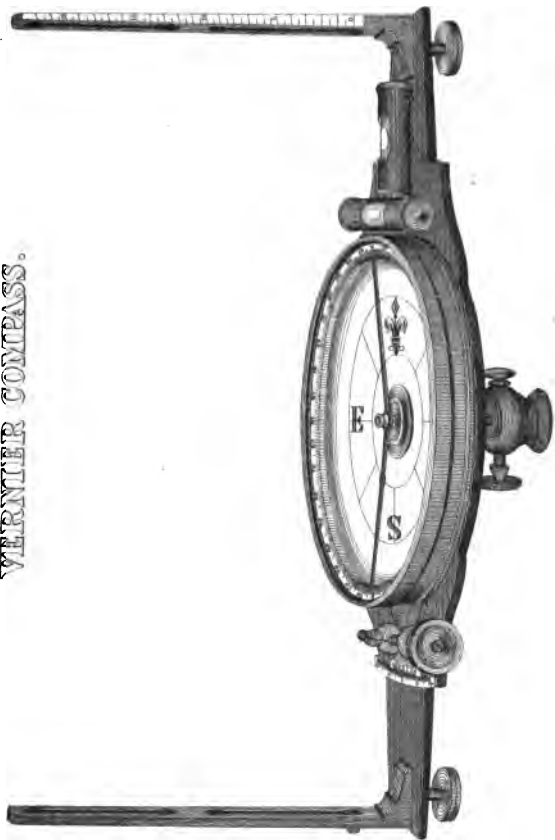
59. To increase the precision with which the compass may be pointed, a **telescope** is now often made to take the place of the sights. The construction of the telescope and the manner in which it is supported will be shown in full in the description of the transit. The advantage of the telescope in the compass consists in the fact that with it the target can be more distinctly seen, and seen at a much greater distance, and, by the apparatus for pointing the line of sight, can be more exactly directed toward it than is possible with the ordinary sights.

THE TRANSIT.

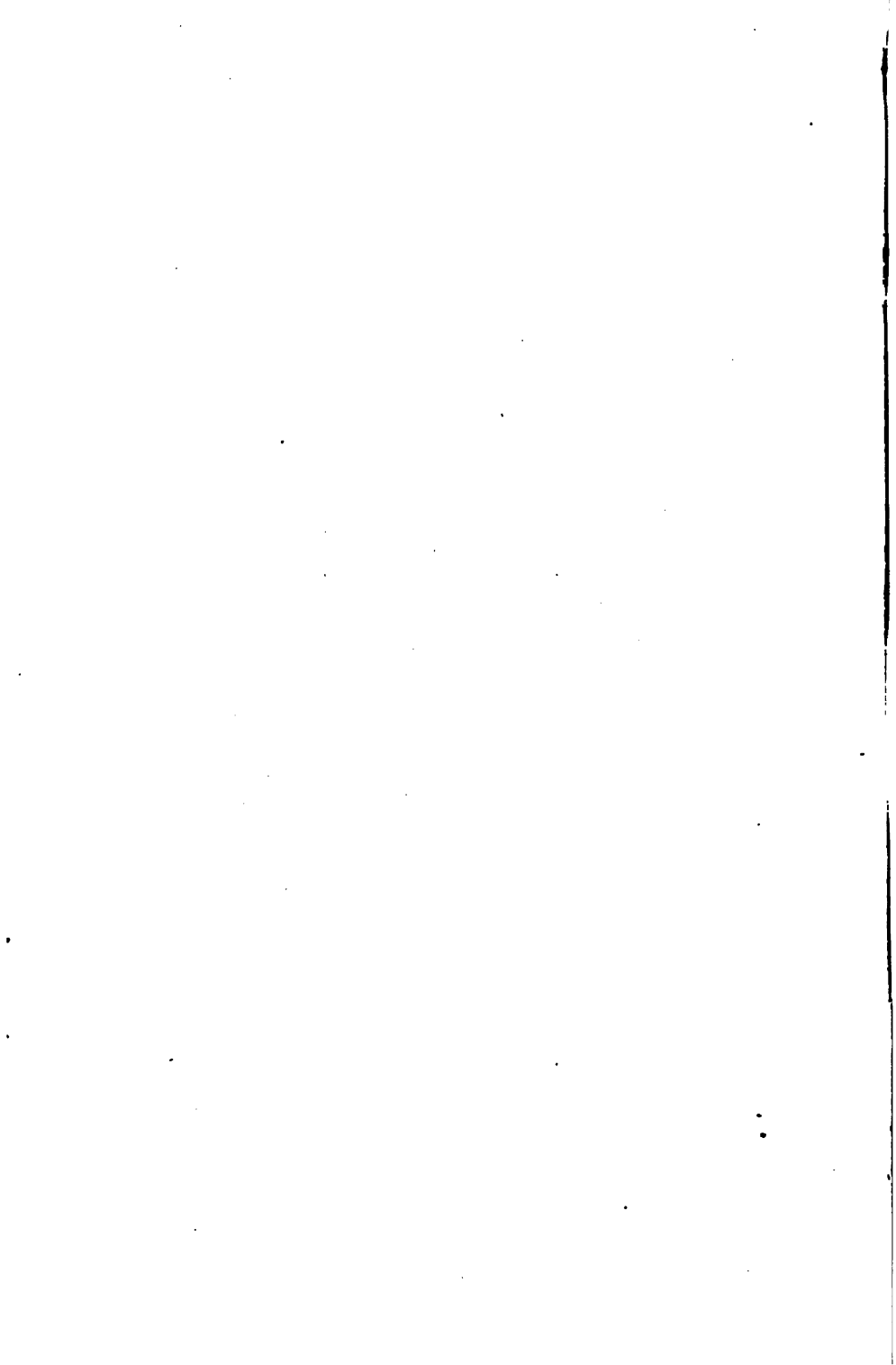
60. The plate opposite represents a transit of such form as is commonly employed by American surveyors. It is used to measure angles with greater accuracy than can be done by the surveyor's compass above described. To this end it is supplied with a telescope, and the angles are read, not from the end of the needle, but from the indications of a vernier upon a divided circle. It is supported on a tripod, and the arrangement for levelling it by means of a tripod-head is the same as has been described under the compass.

61. The part of this instrument which serves to measure horizontal angles consists of two concentric horizontal plates. The lower plate, called the **limb**, turns on a spindle, or axis, which fits into a socket in the

VERNIER COMPASS.



BENJ. D. LEBRON, N.Y.



tripod-head. By means of a clamp and tangent-screw, shown in the figure, it may be clamped fast in any position, and made to move slowly through a small arc. The circumference of this plate is graduated in divisions, usually of halves or thirds of degrees. In the transits used by surveyors these divisions are numbered from some one point on the limb in both directions around to the opposite point, which will be 180° . The graduation is generally concealed beneath the plate above it, except at those parts where the verniers are for the time turned.

62. The upper plate, called the **vernier-plate**, turns on a spindle fitted into a socket in the lower plate. It is also provided with a clamp, by means of which it can be clamped fast at any point, and with a tangent-screw, by which it can be turned through a small arc. These are shown in the figure at the outer rim of the vernier-plate.

This plate has two verniers at opposite sides, each of which enables us to read smaller subdivisions of the divisions on the limb. The principle of the vernier in the transit is the same as that described under the head of the compass. Thus, let the limb of the transit be divided into half-degrees, the vernier, by having 30 divisions, equivalent to 29 or 31 divisions of the limb, will indicate a subdivision of the limb equivalent to one-thirtieth of half a degree, or 1 minute.

The advantage attained by having two verniers located at opposite points of the circle lies in the fact that by this means is obviated any error which may arise from a want in the exact coincidence of the centres of the vernier-plate and limb. Thus, let C , in Fig. 39, be the centre of the limb, and C' the centre of the vernier, and let VV' be a line running through the 0-points of the two verniers. It is plain that the distance of V from 0° is just as much too small as the distance of V' from 180° is too large. Hence, if we take the reading of the vernier V and the supplement of the reading of V' , and add them together, and divide the sum by two, the result would be the true reading of the vernier V , freed from the error caused by the displacement of the centre C' .

63. A needle and a divided circle, as shown in the plate, are a common but not an essential accompaniment of the transit. It may serve as a rough check on the readings of the angles by the verniers; it will give us at any step the magnetic bearing of a line; and, finally, it will enable us to use the transit as a common compass for the surveying of land and the re-establishment of lost lines.

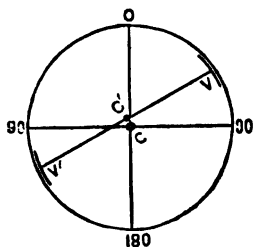


FIG. 39.

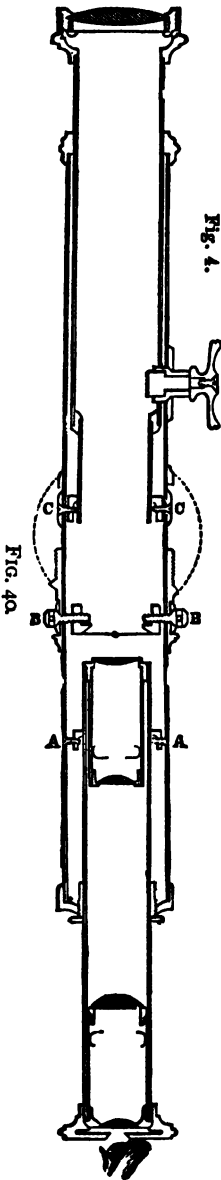


FIG. 4.

64. The telescope is supported on two firm columns, which are attached to the vernier-plate. The columns are sufficiently high to permit the telescope to be reversed by revolving on its axis. The axis of the telescope is made parallel to the vernier-plate, and at right angles to the centre line or axis of the telescope.

Most transits used in surveying have a **vertical circle** attached to one end of the axis of the telescope. It is divided into degrees and parts of degrees, and, by means of a vernier, may be read in the same manner as the horizontal circle. This circle is set so that when the telescope is horizontal the vernier shall read 0, and enables the surveyor to measure vertical angles.

Two **spirit-levels** are attached to the vernier-plate for the purpose of levelling the instrument. They are placed at right angles to each other, and are used as explained for levelling the compass.

65. The **interior mechanism** of the telescope used in the transit can be seen from the adjoining figure. The lens placed in the end of the telescope, which is turned toward the object, is called the **object-glass**. The four lenses placed at the opposite end in a smaller tube compose what is called the **eye-piece**. By means of the object-glass, rays of light coming from some distant object upon which the instrument is turned, are converged, and form a minute but distinct image of the object at the focus B. The eye-piece shows this image to the eye enlarged. In the telescope used for astronomical purposes, the eye-piece consists of two lenses, which shows the image inverted; but, in telescopes used for surveying, an erecting eye-piece consisting of four lenses is used.

An imaginary line running through the optical centre of the object-glass and of the eye-piece is called the **line of collimation**, or the **axis** of the telescope.

66. At the focus of the telescope, marked BB in the figure, is placed a ring carrying two **cross-threads**, one placed horizontal and the other vertical. The ring is so placed in the tube that the intersection of the two threads is exactly in the line of collimation, and forms a point by

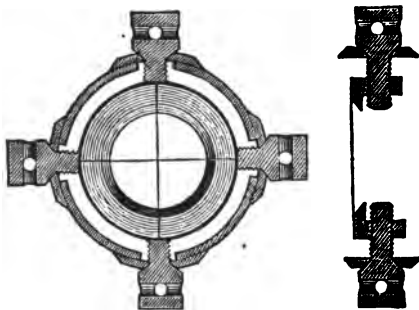


FIG. 41.

means of which the telescope may be directed upon any object. The ring is held in its place by four screws, as shown in Fig. 41, in which is given a cross-section of the telescope tube at the cross-threads. The material which experience has shown to be most suitable for these cross-threads is the natural thread spun by the spider. It is exceedingly fine, strong, and elastic, and when once drawn tense remains unchanged. The threads are fastened to the brass ring by beeswax or varnish.

67. Both the object-glass and the eye-piece are placed in separate tubes, which may be moved in or out as required, by a rack-and-pinion movement. Before making an observation with the telescope, the eye-piece should be moved in or out until the cross-threads are distinctly seen, and until, when the head is slightly moved from side to side, no motion appears to take place in the threads. Then the object-glass must be moved in or out until the object is distinctly visible. Each change of distance will require a change in the focusing of the object-glass; and, if different observers use the same instrument, the focusing of both the eye-piece and object-glass may require slight alteration.

68. A **spirit-level** is attached to the under side of the telescope, and is made parallel to the centre line of the telescope. It serves to indicate when the telescope is horizontal.

ADJUSTMENTS OF THE TRANSIT.

69. From the preceding description of a transit, it must be apparent that the various parts of the instrument require to be brought into certain

exact and definite relations with each other, in order that its functions may be satisfactorily performed. The detection of any deviations from this just relation of the parts, and their rectification, compose what are termed the **adjustments** of the transit. Aside from those which are made by the manufacturer, and which are not liable to be disturbed in the ordinary use of the instrument, there are five principal adjustments.

1. The plane of the levels on the horizontal limb must be perpendicular to the axis on which it revolves.

2. The line of collimation of the telescope must be at right angles to the axis of the telescope; so that when the telescope rotates on its axis the line of collimation will generate a plane surface.

3. The axis of the telescope must be parallel to the plane of the levels on the horizontal limb; so that when the horizontal limb is levelled the axis of the telescope shall also be level.

4. The spirit-level attached to the telescope must be parallel to the line of collimation; so that when the level tube is made horizontal, the line of collimation shall also be horizontal.

5. The vertical circle must be so placed that the vernier shall read zero when the telescope is horizontal.

70. To adjust the levels of the horizontal limb. Turn the horizontal limb so that one of the spirit-levels shall stand parallel to a pair of levelling screws. By means of the screws bring the bubble to the middle of the tube. Turn the instrument half-way around; if the bubble still remains in the middle of the tube, the axis of the tube must be at right angles to the axis of the limb. If the bubble runs to one end, that end of the tube must be too high. By means of the screws for the purpose, lower this end or raise the opposite, until the bubble recedes half-way back towards the middle. By the levelling screws bring the bubble once more to the middle. Repeat the operation until reversing the position of the limb does not disturb the position of the bubble.

Each of the level tubes must be adjusted separately.

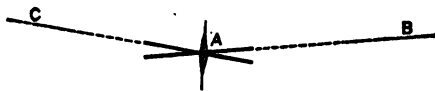


FIG. 42.

71. To adjust the line of collimation. If the line of collimation is not perpendicular to the axis, the direction in which the telescope points, when looking forward, and the direction looking backward, will not be in the same straight line. Thus, in Fig. 42, when the telescope is directed to the right it may point in the line AB; but when reversed, by revolving it on its axis and directed to the left, it may point in the line AC. If

the line of collimation were perpendicular to the axis, these lines would coincide; if not, they will, as in the figure, diverge. To make the adjustment, proceed as follows:

Set the instrument on some convenient ground and level it carefully by the levelling screws. Adjust the focus of the eye-piece to the cross-threads, and point the telescope on some well-defined object, making the intersection of the threads exactly coincide. Clamp the limb and vernier-plate of the instrument in this position. Revolve the telescope on its axis, and place some well-defined mark at about the same distance as the first, and in the direction indicated by the reversed telescope. Unclamp the vernier-plate and turn the instrument half-way round, pointing the cross-hairs carefully upon the first mark. Again clamp the vernier-plate in this position, and revolve the telescope on its axis toward the second mark. If the line of collimation is perpendicular to the axis, when reversed this second time, the intersection of the threads will point exactly upon the second mark. If, however, when reversed, the intersection of the threads falls to one side of the mark, a point half-way between this new direction and the mark must be

the direction in which it ought to point. Thus, in Fig. 43, let A represent the position of the instrument. Point first at a mark, B; then revolve



FIG. 43.

and set a mark, at the same distance, at C, in the direction of the threads. Unclamp and turn back toward B. Clamp and revolve the telescope on its axis. Let D represent the direction in which it now points. It is plain that neither AC nor AD is the continuation of AB, but a line, AE, lying midway between them, is its continuation.

The cross-threads must, therefore, be moved so that the intersection shall point toward E. This is done by the screws shown at BB, in Fig. 40. Loosen one of these and tighten the other until the intersection is made to point toward E; remembering that on account of the inversion of the position of the wires by the eye-piece, we must move the wires in a direction opposite to that apparently required. The operation must be repeated until no deviation can be observed.

72. To make the axis of the telescope parallel to the plane of the spirit-levels. If the standards which support the axis are perfectly equal the axis will be parallel. If the axis is parallel to the spirit-levels, whenever the plane of the levels is made horizontal, the axis will be horizontal, and the line of collimation of the telescope, when revolved on this axis, will describe a vertical plane. This may be tested by observing whether, when the telescope is revolved up and down, the intersection of the threads will follow a vertical line.

Level the instrument in some position where the top and base of some high object, as a steeple, can be seen. Direct the telescope at some definite point at the top of the steeple, as B, Fig. 44. Clamp the horizontal limb and vernier-plate. Revolve the telescope down and mark the point C, indicated by the intersection of the threads near the bottom of the steeple; the line BC ought to be vertical. Turn the vernier-plate of the instrument half-way round and direct the telescope again to B. Clamp the vernier-plate and revolve the telescope down. If the axis of the telescope were not horizontal, when revolved down, its point of direction, D, would now err as

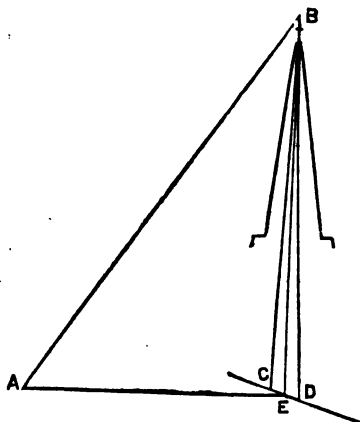


FIG. 44.

far to one side of a true vertical, BE, as the point C had done to the other side. The end of the axis on that side toward which the telescope deviates from a vertical is too high.

Instruments are usually made with one of the standards adjustable, so that the end of the axis may be slightly raised or lowered. By trial we ascertain when this has been sufficiently done to make the adjustment perfect.

73. To adjust the level attached to the telescope. This level tube requires to be made parallel to the line of collimation, so that when the level tube is horizontal, the line of collimation of the telescope will also be horizontal.

To effect this, select a space of ground nearly horizontal. Plant the instrument near the middle, and level the horizontal limb. Bring the telescope into such a position that the bubble of the attached level shall be in the middle, and clamp it fast. Set a stake in the ground at a distance of one or two hundred feet, and on this stake set a staff. Note the height on this staff indicated by the cross-threads of the telescope. Set another stake in the ground in an exactly opposite direction, and at the same distance. Set the staff on this stake, and note the height indicated on it by the cross-threads of the telescope when turned upon it. The difference between the two heights will be the difference of level between the two stakes.

Move the instrument and plant it by means of a plumb-line directly

over the lower of the two stakes. Level the horizontal limb, and also the spirit-level attached to the telescope. Measure carefully the height of the axis of the telescope above the stake over which it stands. From this subtract the difference of level between the two stakes, and note the remainder upon the staff. Set the staff on the higher stake and point the telescope at the mark above obtained. The telescope must then be level.

By means of the screws provided for this purpose, move the spirit-level, until, in this position of the telescope, the bubble stands in the middle. The level tube must now be horizontal, and therefore parallel to the line of collimation of the telescope.

74. To adjust the 0-point of the vertical circle. When the telescope stands horizontal, the index of the vertical circle ought to read exactly zero.

Having adjusted the spirit-level attached to the telescope, bring the horizontal circle to a level by the levelling screws, and also make the telescope and its attached level horizontal. The vernier of the vertical circle ought now to read zero. If not, its real reading is called the **index error**. Loosen the screws which hold the vertical circle, and carefully turn it until the reading is made precisely zero.

It is not always easy to make this adjustment so that no index error will remain. The index error should in that case be carefully noted, and whenever a vertical angle is measured this correction must be applied to it.

MEASUREMENT OF ANGLES.

75. To measure a horizontal angle with the transit. Let it be required to measure the horizontal angle BAC. Place the transit directly over the angular point by means of a plumb-line. Level the horizontal limb, and clamp one of the verniers at the 0-point of the circle. Turn the telescope upon the target at B, and clamp the limb. Unclamp the vernier-plate and turn the telescope upon the other target at C. Read the vernier which had been set at zero, and the reading will be the horizontal angle through which the telescope turned from B to C.

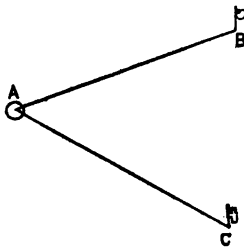


FIG. 45.

It is not necessary to set the vernier at zero before pointing at the first target. The result will be the same if the vernier is read when pointed at the first target, and then read when pointed at the second target. The difference between the readings will be the angle required. Care must be taken, however, in

this method, that when, in turning from the first to the second target, the vernier passes the 0-point or the 180°-point, we must add the readings in the first case, and add the supplements of the readings in the second case.

When precision is required in the measurement of these angles, both verniers should be read for each object. A mean taken between the results got from the two verniers, will, as was shown in explaining the vernier, be free from any error caused by inaccurate centering of the vernier circle and the limb.

76. To eliminate as far as possible errors of observation in measuring angles, the operation must be repeated several times, and the mean of the measurements taken. On the principle that the measurements have precisely the same probability of being too large as too small, the mean of many observations will be more accurate than any one.

This repetition is best performed with the transit in the following manner: Point the telescope at the first target B, and read.

Clamp the limb, unclamp the vernier, and turn the telescope on the second target C, and read (1).

Clamp the vernier, unclamp the limb, and turn back to the first target B; clamp the limb, unclamp the vernier, and turn to C, and read (2).

Proceed thus to turn back to B, retaining each time the reading at C.

It is plain that the difference between the successive readings at B and C must give successive determinations of the required angle; and that, omitting the intermediate readings, the difference between the first reading on B, and the last on C, divided by the number of repetitions, must give the mean of all the readings. This method has the advantage of bringing into use different portions of the graduated circle, so that errors from any imperfections in the graduation will be to a certain extent eliminated.

77. It will add to the accuracy of the method above described to continue it in a reverse order. That is, after noting the last reading at C, unclamp the vernier and turn back to B. Then clamp the vernier and unclamp the limb and turn to C. So proceed until the process has been exactly retraced. Lastly, read the vernier when pointed at B. The difference between this reading and the last in the direct process will be the sum of all the repetitions, which, being divided by the number of repetitions, will give another determination of the angle.

The advantage gained by this reversion is, that thereby the error arising from a slight dragging of the limb by the vernier-plate at each turn is eliminated. In the direct series of measurements this dragging would tend to decrease the measured angle; but in the reverse process the dragging of the limb would tend to increase the measured angle. Hence, the mean of the direct and reverse results must be free from this error.

78. It will be observed that the angles measured by the method above described are **horizontal angles**, because they are read from a horizontal circle. Since the telescope, in revolving on its axis, moves in a vertical plane, the angle indicated on the horizontal circle will be the same when the telescope is pointed at B and C, Fig. 46, as when pointed at B' and C', which lie vertically beneath them.

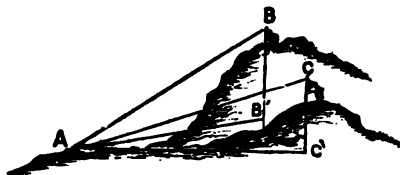


FIG. 46.

The angles, therefore, which are measured between lines on the earth's surface by the transit or compass are not the real angles, except when the lines are horizontal, but are the projections of the real angles on a horizontal plane. As, in surveying, we have to deal with horizontal distances between points and horizontal angles between lines, it is an advantage that the angles measured are horizontal angles.

In case the sextant or reflecting circle is used for measuring the angles, the measurements require a correction to reduce them to their corresponding horizontal angles.

79. To measure a vertical angle with the transit. Set the instru-

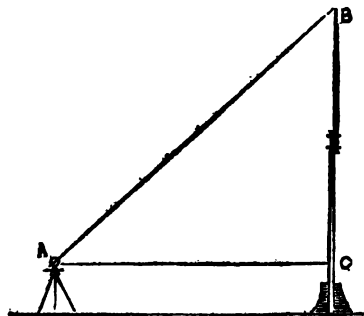


FIG. 47.

ment, by means of a plumb-line, directly over the station, Fig. 47, and level the horizontal limb. Examine the vertical circle to see whether it is in adjustment. If not, note its index error. Point the telescope at B, bringing the intersection of the cross-threads upon the point. The angle indicated on the vertical circle by the vernier, corrected, if necessary, for the index error, will be the angle BAC, which is called an angle

of elevation. If the object, B, is lower than the instrument, the line AB will lie beneath the horizontal line AC, and the angle would be an angle of depression.

THE THEODOLITE.

80. The principles involved in the construction and use of the theodolite do not differ essentially from those of the transit, above described. In American practice it is rarely used, having been displaced by the lighter, more convenient, and equally accurate, transit. It still holds its place, however, in English practice, and we give a figure of it. As seen from Fig. 48, it is supported on a tripod and tripod-head. For measuring horizontal angles it is provided with a horizontal circle, graduated to degrees and fractions of a degree, and read by two verniers. The vernier-plate, verniers, clamp and tangent-screws, spirit-levels, and standards, do not differ from those already described as belonging to the transit.

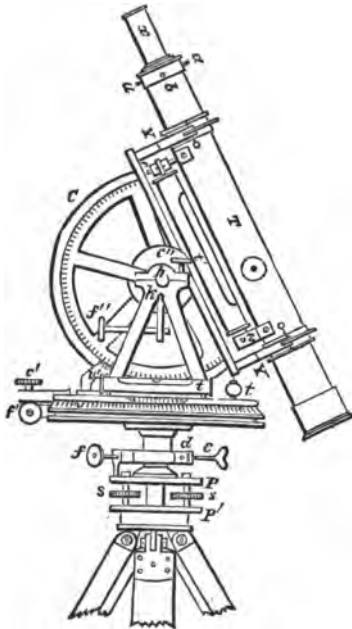


FIG. 48.

The vertical limb consists of a half-circle, C, graduated like the horizontal circle, and read by the vernier, *v*.

The telescope, T, is supported by two supports, YY, and from their shape called wyes. Collars and pins serve to hold the telescope in its supports. A spirit-level, *v*, is attached to the telescope to indicate when it is horizontal.

The principal difference between the theodolite and the transit is, that in the former the supports of the telescope are shorter and do not permit the telescope to be revolved entirely over on its axis. Instead of this, however, by unclamping the col-

lars which hold the telescope in the wyes, we may lift the telescope out of its supports and replace it in a reversed position.

The measurements of both horizontal and vertical angles are executed in precisely the same manner in both, and the directions do not need to be here repeated.

ADJUSTMENTS OF THE THEODOLITE.

81. The adjustment of the levels on the horizontal limb, and of the vertical limb, is executed in the same way as explained for the transit. But since the telescope cannot be reversed by revolving on its axis as in the transit, the adjustment of the line of collimation must be made in a different way.

82. To adjust the line of collimation. Level the horizontal limb, and point the telescope so that the intersection of the threads shall fall

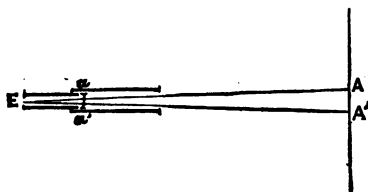


FIG. 49.

upon some distinct distant object, A, in the figure. If now the intersection of the threads is precisely in the axis of the telescope-tube, when the tube is revolved over in the wyes the intersection of the threads will still remain pointed upon the same object, A. If, however, the intersection of the threads be out of this line, as at a , when the telescope is revolved over in the wyes, the point a will fall as far on the one side of the true axis, at a' , as it was before on the opposite, at a . Hence, if the line of intersection before pointed to A, it will now point to a different one, A' . It is plain that the intersection of the threads ought to be so changed that it will point to a spot which should be exactly half-way between the two points A and A' . The same test will also determine whether the intersection of the threads is too high or too low in the tube.

The adjustment is made by means of small screws shown in the figure of the theodolite at $a\ b\ b$. By loosening one of these and tightening the opposite, the ring which carries the threads may be moved in any direction, until the test shows it to be correct.

83. To adjust the standards, so that the axis of the telescope shall be parallel to the plane of the levels.

Set up the instrument and level it. Hang up near it a long plumb-line, so protected that it shall not be disturbed by the wind. Point the intersection of the threads upon the plumb-line near the top. Revolve the tel-

escape down along the plumb-line ; if it follows it precisely the standards must be in adjustment ; if it diverges off on one side, the standard on that side is too high.

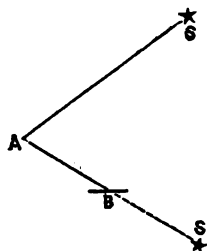


FIG. 50.

A more exact test will be afforded by pointing the telescope at some distant elevated object, as a star, and then at the reflection of the same object in the surface of a fluid at rest. A plate of mercury at B, Fig. 50, is the best for this purpose. From the known principles of optics the point A, the star S, and its reflection S', are all in the same vertical plane. Hence, when revolved down, the intersection of the threads of the telescope must point at the image S' in the surface of the fluid, and any deviation must be corrected by adjusting the standards.

CHAPTER IV.

INACCESSIBLE DISTANCES.

84. ONE of the principal objects in surveying is to obtain, by indirect measurement, the length of lines which cannot be measured directly. Thus, by the measurement of certain lines and angles, it is possible to obtain the distance across an impassable river, the altitude of a mountain above a horizontal plane, or the distance apart of two inaccessible mountain-peaks. Since the lines whose length is thus sought are commonly distances on a horizontal plane, or the altitude of points above a horizontal plane, the questions arising under this head may be termed **problems of heights and distances.**

85. The practical solution of all such problems is effected by two distinct processes :

1. The measurement of the necessary lines and angles.
2. The computation, from these measured lines and angles, of the unknown lines and angles which are required.

Trigonometry teaches what parts of a triangle are necessary to be known to enable us to find the remaining parts, and also explains the methods by which these computations may be made. From these principles it will be seen that in order to find the unknown parts of a triangle it will be necessary to measure

1. One side and two angles.
2. Two sides and the included angle.
3. Two sides and any other angle ; or,
4. The three sides.

By means of the methods of trigonometry, therefore, we may solve the following problems, which are those most frequently met with by the surveyor in his practice.

86. Problem I. To determine the distance of an inaccessible point.

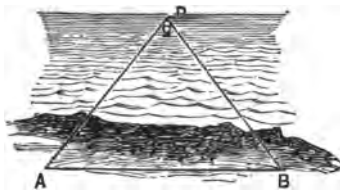


FIG. 51.

Let P be a point whose distance from A is desired. Measure a base-line, AB , in some convenient direction, and measure with the transit or com-

pass the angles PAB and PBA. These are sufficient to solve the triangle and obtain the distance AP. As a test of the accuracy of the measurement of the angles, it is desirable, when practicable, to measure also the angle P. In a plane triangle the sum of the three angles must equal 180° .

Hence $P = 180^\circ - (A + B)$

and by Theorem III., Plane Triangles, we have

$$\sin P : \sin B :: AB : AP = \frac{AB \times \sin B}{\sin P}$$

EXAMPLE.—Desiring to know the distance to an enemy's battery, an engineer measured a base-line, AB, 250 yards, and at its extremities the angle A, $68^\circ 44'$, and the angle B, $84^\circ 17'$. Required the distance AP.

By the formula

$$AP = \frac{AB \times \sin B}{\sin P}$$

$$P = 180^\circ - (A + B) = 180^\circ - 153^\circ 1' = 26^\circ 59'$$

$$\text{Log } AB = 2.397940$$

$$+ \log \sin B = 9.997835$$

$$- \log \sin P = 9.656799$$

$$\log AP = 2.738976$$

$$AP = 548.25$$

EXAMPLE.—A surveyor, desiring to ascertain the distance across a river, measured a base-line, AB, 178 yards, and at A measured an angle between the base and a point, P, opposite, 90° ; at B he measured an angle $58^\circ 30'$. Required the distance AP. *Ans.*

87. Problem II. To determine the distance apart of two objects separated by an impassable barrier.

Let A and B be two points whose distance from each other cannot be directly measured. Select some point, P, from which both points may be seen. Measure the angle APB, and the distances BP (= a) and AP (= b).

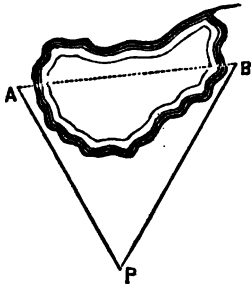


FIG. 52.

From Theorem IV., Plane Triangles, we have

$$a + b : a - b :: \tan \frac{1}{2}(A + B) : \tan \frac{1}{2}(A - B),$$

in which $A + B = 180^\circ - P$

This proportion, when solved, will give $\frac{1}{2}(A - B)$. Add together half the sum of

A and B, and half their difference, the result will be the greater of the two angles; subtract the same quantities, the result will be the less. Then, by Theorem III., we have

$$\sin A : \sin P :: a : AB = \frac{a \times \sin P}{\sin A}$$

EXAMPLE.—In order to obtain the distance, AB, across a swamp, a surveyor measured at a point, P, an angle, APB, $64^{\circ} 10'$, and the distances PA, 1048.3 yards, and PB, 848.7 yards.

$$\begin{aligned} B + A &= 180^{\circ} - P = 115^{\circ} 41' & \frac{1}{2}(B + A) &= 57^{\circ} 50' 30'' \\ b + a &= 1897.0 & b - a &= 199.6 \end{aligned}$$

$$\begin{aligned} \log(b + a) &\dots\dots\dots 3.278067 \\ : \log(b - a) &\dots\dots\dots 2.300161 \\ :: \log \tan \frac{1}{2}(B + A) &\dots\dots\dots 10.201544 \\ &\hline : \log \tan \frac{1}{2}(B - A) &\dots\dots\dots 9.223638 \\ &\frac{1}{2}(B - A) &= 9^{\circ} 30' 2'' \end{aligned}$$

$$A = 48^{\circ} 20' 28''$$

$$\begin{aligned} \log \sin A &\dots\dots\dots 9.873387 \\ : \log \sin P &\dots\dots\dots 9.954823 \\ :: \log a &\dots\dots\dots 2.928754 \\ &\hline : \log AB &\dots\dots\dots 3.010190 \end{aligned}$$

$$AB = 1023.61. \text{ Ans.}$$

EXAMPLE.—For the purpose of determining the distance, AB, through a group of buildings, a surveyor measured an angle, P, 60° , and the distances PA 4.48 chains, and PB 5 chains. *Ans.* AB = 5.314.

88. Problem III. To determine the distance apart of two inaccessible objects.

Let P and Q be two objects which are inaccessible from each other and from the station of the observer. On some convenient ground measure a base-line AB, and at its extremities measure the angles PAQ, QAB, and ABP, PBQ. To compute PQ proceed as follows :

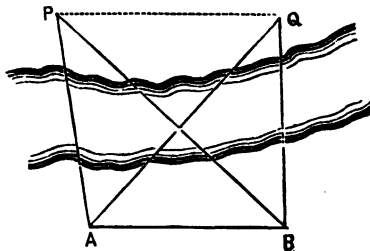


FIG. 53.

1. In the triangle ABP compute AP (Theorem III.).

2. In the triangle ABQ compute AQ (Theorem III.).

3. In the triangle APQ compute PQ (Theorems III. and IV.).

EXAMPLE.—A traveller wishes to determine the distance from himself and from each other of two headlands, P and Q. He measures a base-line, AB, 1000 yards, and the angles PAQ $21^{\circ} 36'$, PAB $78^{\circ} 11'$, ABP $71^{\circ} 30'$, and ABQ $95^{\circ} 41'$.

1. In the triangle ABP, AB = 1000 yards, PAB = $78^{\circ} 11'$, ABP = $71^{\circ} 30'$, APB = $30^{\circ} 19'$.

Therefore, $\log \sin P \dots \dots \dots 9.7031011$
 $\quad \quad \quad : \log \sin B \dots \dots \dots 9.9769566$
 $\quad \quad \quad :: \log AB \dots \dots \dots \underline{3.0000000}$
 $\quad \quad \quad : \log AP \dots \dots \dots 3.2738555$

$$AP = 1878.69$$

2. In the triangle ABQ, AB = 1000 yards, QAB = $56^{\circ} 35'$, ABQ = $95^{\circ} 41'$, AQB = $26^{\circ} 44'$.

Therefore, $\log \sin Q \dots \dots \dots 9.6215871$
 $\quad \quad \quad : \log \sin B \dots \dots \dots 9.9978599$
 $\quad \quad \quad :: \log AB \dots \dots \dots \underline{3.0000000}$
 $\quad \quad \quad : \log AQ \dots \dots \dots 3.3762728$

$$AQ = 2378.33$$

3. In the triangle APQ, AP = 1878.69, AQ = 2378.33, PAQ = $21^{\circ} 36'$.

Hence AQ + AP = 4257.02, AQ - AP = 499.64, $\frac{1}{2}(P + Q) = 79^{\circ} 12'$.

$\log (AQ + AP) \dots \dots 3.6291057$ $\quad \quad \quad \log \sin P \dots \dots 9.9707234$
 $: \log (AQ - AP) \dots \dots 2.6986572$ $\quad \quad \quad : \log \sin A \dots \dots 9.5659948$
 $:: \log \tan \frac{1}{2}(P + Q) \dots \dots \underline{10.7195122}$ $\quad \quad \quad :: \log AQ \dots \dots \underline{3.3762728}$
 $: \log \tan \frac{1}{2}(P - Q) \dots \dots \underline{9.7890637}$ $\quad \quad \quad : \log PQ \dots \dots \underline{2.9715442}$

$$\frac{1}{2}(P - Q) = 31^{\circ} 36' 9''$$

$$PQ = 936.57$$

hence
and

$$P = 110^{\circ} 48' 9''$$

$$Q = 47^{\circ} 35' 51''$$

EXAMPLE.—A surveyor wishes to ascertain the distance apart of two islands in a lake. For this purpose he measures a base-line, AB, 500 yards, and the angles PAQ $58^{\circ} 15'$, PAB $93^{\circ} 48'$, ABP $46^{\circ} 30'$, ABQ 120° . Required the distance.

Ans. 1113.72.

89. Problem IV. To determine the distance apart of two inaccessible objects, when no point can be occupied from which both can be seen.

Let P and Q be two points whose distance apart is required. Measure the distance between two points, A and B, from one of which P may be seen, and from the other, Q. Measure also a line AD to a point from which P and A can be seen; and a line BC to a point from which Q and B can be seen. Measure the following angles, PDA, DAP, PAB, and ABQ, QBC, BCQ. The computation may then be made as follows:

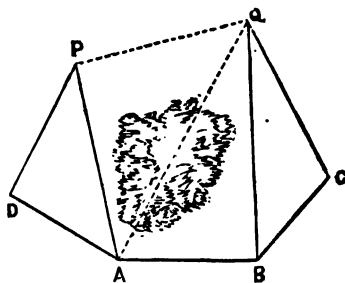


FIG. 54.

1. In the triangle ADP compute AP (Theorem III.).
2. In the triangle BCQ compute BQ (Theorem III.).
3. In the triangle ABQ compute AQ (Theorems IV. and III.).
4. In the triangle APQ compute PQ (Theorems IV. and III.).

EXAMPLE.—To determine the distance apart of two spires, P and Q, a surveyor measured a base-line, AB, 600 yards, and two auxiliary lines, AD and BC, each 500 yards. At the extremities of these lines he measured the following angles: PDA 109° , DAP $44^\circ 30'$, PAB $99^\circ 30'$, ABQ $103^\circ 20'$, QBC $46^\circ 45'$, BCQ $101^\circ 50'$. Required PQ. *Ans.*

90. Problem V. To determine the distance of a point from three objects whose distances from each other are known.

This problem is one of frequent occurrence in maritime surveying, for the location on charts and maps of rocks, reefs, buoys, etc. A number of points are first surveyed on the shore and mapped down in their proper location on the chart. The surveyor then occupies any point whose position is desired, and with a transit (or sextant, when the observations must be made from a boat) measures the angles formed by lines to any three known points. These two angles, together with the known distances of the points from each other, are sufficient to determine the position of the point.

MATHEMATICAL SOLUTION.

Let P be a point whose distances from three points, A, B, and C, are required, the mutual distances of A, B, and C being known. At the point

P measure the angles APB and BPC. Suppose the circumference of a circle constructed which shall pass through P, A, and C. It will intersect the line PB in some point D. Join AD and DC. The angles APB and ACD, being both angles inscribed in the same segment, are equal; in like manner the angles BPC and DAC are equal. The computation may then be conducted as follows:

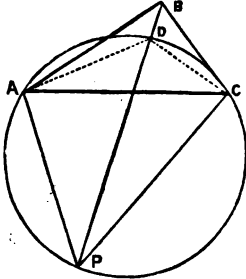


FIG. 55.

1. In the triangle DAC compute DC (Theorem III.).

2. In the triangle ABC compute the angle ACB (Theorem V.).

Then,

$$BCD = ACB - ACD.$$

3. In the triangle BCD compute the angle CBD (Theorem IV.).

4. In the triangle PBC compute PB and PC (Theorem III.).

5. In the triangle APB compute AP (Theorem III.).

It may happen in this problem that the point P is located on the same side of AC with the point B, instead of the opposite side as in the figure; or the point P may be located within the triangle formed by the lines AB, AC, and BC; or the circumference of the circle may cut the line PB in a point beyond B. In each of these cases the same solution will apply, provided that the proper changes are made in the signs of the quantities employed.

GRAPHICAL SOLUTION.

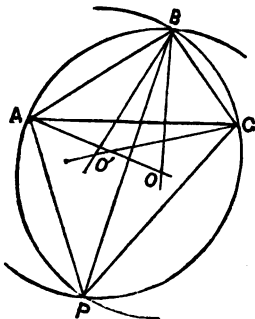


FIG. 56.

91. When it is required to locate the point P on a map or chart on which the points A, B, and C are already located, a graphical solution only is required, and may conveniently be executed as follows:

Let A, B, C, be the three fixed points, already plotted in their true position on the chart. Let the angles measured at the point P be APB 20° and BPC 30° . Since an angle at the centre of a circle is twice as great as an angle at the circumference, including the same arc, the angle at O, in Fig. 56, must be twice the angle APB, that is, 40° ; and since the tri-

angle AOB is isosceles, the angles at its base must each be $\frac{1}{2}(180^\circ - O)$. Hence, draw at A and B two lines, making with AB angles each equal to 70° ; they will intersect each other at some point O. With O as a centre, and a radius AO or BO, describe a circumference; it must pass through the point P. In the same way construct angles at B and C, each equal to 60° , giving the point O'. With this centre and O'B as radius, describe a circumference. It must also pass through the point P. Hence the intersection of the two circumferences must give on the chart the location of the point P.

EXAMPLE.—Desiring to know the position of a rock in a harbor, the positions of three points on the shore were ascertained, and their distances found to be, AB = 623.5 yards, AC = 1216 yards, and BC = 836.4 yards. The angles from the point P were also measured, viz., APB = $26^\circ 18'$, and BPC = $32^\circ 52'$. The point, P, lay on the opposite side of the line AC from B. Required the distances PA, PB, and PC.

Ans.

92. Problem VI. To determine the altitude of a vertical line above a horizontal plane.

Let PQ be a vertical line whose altitude above the horizontal plane AQ is desired. Measure from the base of the object any convenient distance QA, and at A measure the angle of elevation PAQ. Then, in the right-angled triangle PAQ we can find the side PQ by Theorem II.

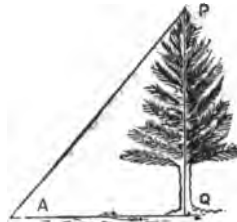


FIG. 57.

In this and the succeeding problems the angles of elevation are measured by an instrument situated above the horizontal surface. The computed result will be the altitude above the level of the instrument, to which must be added in each case the height of the instrument.

EXAMPLES.

1. Required the height of a flag-staff standing on a horizontal plane, the distance QA being 100 feet, and the angle PAQ $68^\circ 45'$.

$$\begin{array}{r}
 R \dots\dots\dots 10.000000 \\
 : \tan A \dots\dots\dots 10.4101868 \\
 \therefore AQ \dots\dots\dots 2.000000 \\
 \hline
 : PQ 257.15 \dots\dots\dots 2.4101868
 \end{array}$$

2. An explorer, wishing to determine the height of one of the mammoth trees of California, measured from its base 550 feet, and at that



point measured the angle of elevation to its extreme top, $37^{\circ} 30'$. Required its height. *Ans.*

93. Problem VII. To determine the altitude of an inaccessible object above a horizontal plane.

Let P be a point whose altitude, PQ, is required. Measure a horizontal line, AB, running directly toward the object, and at the points A and B

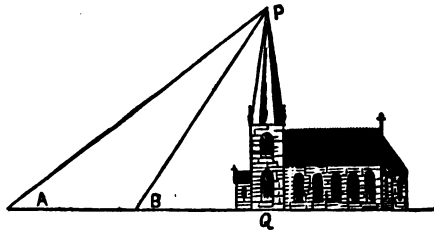


FIG. 58.

measure the angles of elevation PAQ and PBQ. The altitude may then be computed as follows :

$$APB = PBQ - PAQ$$

1. In the triangle ABP compute BP (Theorem III.).
2. In the triangle PBQ compute PQ (Theorem I.).

EXAMPLE.—The altitude of a church spire is required. A line, AB, is measured 100 feet, and the angles PBQ $62^{\circ} 45'$, and PAQ 40° .

Ans.

Second Method.

94. When a horizontal line in the direction of the object cannot be obtained, we may proceed thus :

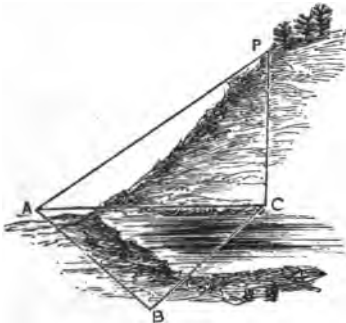


FIG. 59.

Measure a base-line, AB, in any direction. If the ground is not horizontal on which it is measured, the horizontal distance between A and B must be measured. At A and B measure the horizontal angles CAB and CBA, and also at one of the points the vertical angle PAC. Then,

1. In the triangle CAB compute AC (Theorem III.).
2. In the triangle PAC compute PC (Theorem II.).

EXAMPLE.—To find the height of a precipice, a surveyor measured a base-line, AB, 250 feet, and the angles CBA $73^{\circ} 15'$, CAB $50^{\circ} 48'$, and the angle PAC $48^{\circ} 56'$.
Ans.

95. Problem VIII. To determine the altitude of an object situated on an inaccessible elevated point.

Let PQ be an object whose height above the elevation, Q, is required. Measure a base-line, AB, directly toward the object. Measure the angles of elevation QAR, PAR, and PBR. The height of PQ can then be computed thus :

1. In the triangle ABP compute AP (Theorem III.).

2. In the triangle PQA compute PQ (Theorem III.) in which

$$PAQ = PAR - QAR$$

and $AQP = 90^{\circ} + QAR$

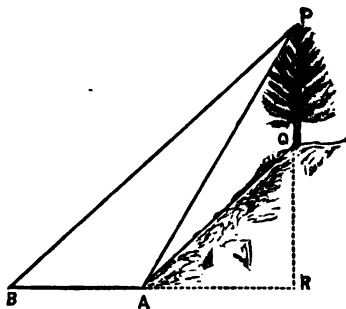


FIG. 60.

EXAMPLE.—A surveyor, wishing to ascertain the height of a tall tree standing upon the top of an elevation, measured a base-line, AB, 600 feet, and the angles QAR $34^{\circ} 10'$, PAR $45^{\circ} 45'$, and PBR 31° . Required the height PQ.
Ans.

MISCELLANEOUS EXAMPLES.

1. Having measured a distance of 200 feet in a direct horizontal line from the bottom of a steeple, the angle of elevation of its top, taken at that point, was found to be $47^{\circ} 30'$. Required the height. *Ans.* 218.26 feet.

2. Wanting to know the distance between two trees situated on a plain, in a straight line from the bottom of a tower 120 feet high, I measured the angles of depression to each from the top of the tower, to the nearest $64^{\circ} 30'$, and to the most distant 33° . Required the distance between the trees. *Ans.* 173.66 feet.

3. To get the distance across a river, a surveyor measured 200 yards along the shore, and the horizontal angles $68^{\circ} 2'$ and $73^{\circ} 15'$, at the extremities of the line, to a house on the opposite shore. Required the distances to the house. *Ans.* 296.54 yards and 306.19 yards.

4. To determine the distance of a fort from two ships of war, the vessels sail directly from each other 440 yards. The horizontal angle at each, between the other and the fort, is then measured, viz., $83^{\circ} 45'$ and $85^{\circ} 15'$. Required the distances from the fort to each vessel. *Ans.* 2292.26 yards and 2298.05 yards.

5. Desiring to know the distance between two headlands, I measured from each, to a certain point inland, 735 and 840 yards; also, the horizontal angle included between these lines $55^{\circ} 40'$. What was the distance required?

Ans. 741.2 yards.

6. There are on shore three points, whose distances I know to be AC 848 yards, AB 426 yards, and BC 524 yards. From a boat off shore I measure the angles, from my position, P, APB $13^{\circ} 30'$, CPB $29^{\circ} 50'$. The point B was observed to lie on the same side of AC as the boat. Required the distances from my position to the three points.

Ans. AP 1211.42, BP 859.36, CP 1048.47.

CHAPTER V.

AREAS.

96. **THE area** of a surface is the number of times it will contain some known unit of area. Thus, a square measuring 10 feet on each side, is said to contain 100 square feet. That is, if a square, measuring one foot on each side, were applied 100 times to the given surface, it would exactly cover it. If the figure ABDC be a rectangle whose length, AB, is 8 feet, and height, AC, 6 feet, it is plain, if we divide the base and height into feet, and draw parallel lines through the points of division, the rectangle will be divided into as many squares of one foot each as the product of the base and the altitude. To find the area of any figure, therefore, as a triangle, parallelogram, or circle, is to determine to what number of units of area its surface is equivalent. From the principles of geometry and trigonometry, rules are deduced for determining the areas of regular figures from their linear dimensions.

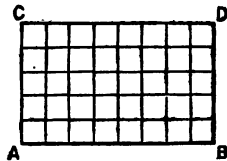


FIG. 61.

97. The **units of area** used in surveying are the square foot, the square yard, the square rod, the square chain, the rood, and the acre. All these, except the last two, are squares having sides of a length equal to the linear unit of the same name. These units all bear to each other such simple ratios that they may easily be translated into each other.

The following table shows these ratios :

EQUIVALENT AREAS.

ACRE.	ROOD.	SQUARE CHAIN.	SQUARE ROD.	SQUARE YARD.	SQUARE FOOT.
		1	16	1	9
		1	16	30 $\frac{1}{4}$	272 $\frac{1}{4}$
	1	2 $\frac{1}{2}$	40	484	4356
	1	10	160	1210	10890
1	4	10	160	4840	43560

98. In surveying farm-lands, the area is usually expressed in acres, roods, and square rods. Thus, a farm in the form of a rectangle, one

side being 34.26 chains, and the other 21.87 chains, will contain 749 acres, 1 rood, 3 square rods.

When small plats of ground are measured, the area is expressed in square feet, or square yards, or square rods.

Ten square chains are exactly equivalent to one acre. Hence, when land has been measured with Gunter's chain, and the lines expressed in chains and links, the area obtained will be expressed in square chains and decimals. From this, the area in acres and decimals may be obtained by dividing by 10. The roods and square rods equivalent to the decimals are obtained by multiplying successively by 4 and by 40.

In Table I. will be found a formula for computing the areas of most regular geometrical figures. From these may be derived rules for solving all problems involving areas in surveying. The solution requires two distinct processes: *first*, the measurement in the field of the necessary lines and angles; and, *second*, the computation from these of the areas required. ●

99. Problem I. To determine the area of a rectangular field.
Measure two adjacent sides, and the area is equal to their product.

EXAMPLES.

1. A rectangular field has one side 10.14 chains, and the other 24.46 chains. Required the area in acres, roods, and perches.

Ans.

2. A square field measures 11.38 chains on each side. Required the area.

Ans.

100. Problem II. To determine the area of a triangular field.

First Method.

Measure one side, and a perpendicular let fall on that side from the opposite angle. Half the product of the two dimensions is the area required.

EXAMPLE.—What is the area of a triangular field whose base is 15.34 chains, and its altitude 6.73 chains? *Ans.*

Second Method.

Measure two sides of the triangle, and the angle included between them. The area is equal to half the product of the two sides into the sine of the included angle.

EXAMPLES.

1. Two sides of a triangle are 34.18 chains and 22.16 chains, and the included angle is $53^{\circ} 30'$. Required the area.

$$\begin{aligned} \text{Area} &= \frac{1}{2}(34.18 \times 22.16 \times \sin 53^{\circ} 30') \\ \log 17.09 &\dots\dots\dots 1.232742 \\ \log 22.16 &\dots\dots\dots 1.345570 \\ \log \sin 53^{\circ} 30' &\dots\dots\dots 9.905179 \\ \log \text{area} &\dots\dots\dots 2.483491 \end{aligned}$$

Ans. 304.43.

2. Two sides of a triangular lot were 200 feet and 325 feet, and the included angle was $79^{\circ} 48'$. Required the area. Ans.

Third Method.

Measure the three sides of the triangle. The area may then be computed as follows:

1. Take half the sum of the three sides.
2. Subtract from the half sum each side separately.
3. Multiply together the half sum and the three remainders.
4. Extract the square root of the product.

This rule may be expressed in the formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

in which a , b , and c are the three sides, and

$$s = \frac{1}{2}(a + b + c)$$

EXAMPLES.

1. Given the three sides of a triangle, 200, 150, and 100 yards. Required the area.

$$s = \frac{1}{2}(a + b + c) = 225; s - a = 25; s - b = 75; s - c = 125.$$

$$\begin{aligned} \log s &\dots\dots\dots 2.352183 \\ \log (s - a) &\dots\dots\dots 1.397940 \\ \log (s - b) &\dots\dots\dots 1.875061 \\ \log (s - c) &\dots\dots\dots 2.096910 \end{aligned}$$

$$2) 7.722094$$

$$\hline 3.861047$$

Ans. 7261.85.

2. What is the area of a triangular field whose sides are 25.69, 49, and 50.25 chains?
Ans. 61 acres, 1 rood, 39 perches.

101. Problem III. To determine the area of a field in the form of a parallelogram.

First Method.

Measure one of the parallel sides and a perpendicular let fall on that side from the opposite. The area is equal to the product of the two measurements.

EXAMPLES.

1. The base and altitude of a parallelogram were measured, 40 rods and 4 rods. Required its area in acres. *Ans.*

2. How many square feet of plank will cover the roadway of a skew-bridge whose length is 218 feet 6 inches, and its perpendicular width 22 feet 6 inches? *Ans.*

Second Method.

Measure two adjacent sides of the parallelogram and the angle between them. The area is then equal to the product of the two sides into the sine of the included angle.

EXAMPLE.—A plat of ground in the form of a parallelogram is 228 feet and 140 feet on its two sides, and their included angle is $75^{\circ} 15'$. Required the area. *Ans.* 30868.3 square feet.

102. Problem IV. To determine the area of a trapezoid.

Measure the length of the two parallel sides and a perpendicular between them. The area is equal to the product of half the sum of the parallel sides by the perpendicular. Or,

$$\text{Area} = p \times \frac{a + b}{2}$$

in which a and b are the two sides, and p the perpendicular.

EXAMPLE.—The two parallel sides of a trapezoid are 5.42 chains and 7.89 chains, and the perpendicular 4.34 chains. Required the area.

Ans.

103. Problem V. To determine the area of any quadrilateral figure.

Measure the two diagonals and the angle which they make with each other. Then

$$\text{Area} = \frac{1}{2} ab \sin A$$

in which a and b are the diagonals, and A the angle made by them. Since the angle A' is the supplement of the angle A , and the sines of both are equal, we may measure whichever angle is most convenient.

EXAMPLE.—In a quadrilateral field there were measured the diagonals 9.48 chains and 6.78 chains, and the included angle $98^\circ 56'$. Required the area. *Ans.*

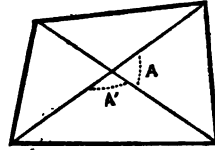


FIG. 62.

104. Problem VI. To determine the area of any figure bounded by straight lines.

Let $ABCDE$, etc., be any figure bounded by straight lines. Divide the field into triangles by diagonal lines in any convenient manner. Measure the lines and angles necessary to determine the areas of these triangles by either of the methods explained. The sum of the triangles must be equal to the area of the figure required. Thus, let the diagonals $d, d', d'',$ etc., and the perpendiculars $p, p', p'',$ etc., as shown in the figure, be measured. Then

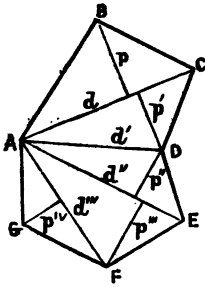


FIG. 63.

$$\text{Area} = \frac{1}{2}(pd + p'd + p''d'' + p'''d''' + p''d''')$$

EXAMPLE.—Wishing to ascertain the area of a field bounded by five sides, a surveyor measured the sides AB 4.24 chains, AE 5.10 chains, and the diagonals AC 11.36 chains and AD 11.09 chains; also,

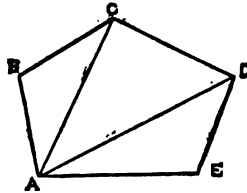


FIG. 64.

the angles BAC $46^\circ 48'$, CAD $30^\circ 50'$, and DAE $38^\circ 15'$. Required the area. *Ans.*

105. Problem VII. To determine the area of a piece of ground bounded on one or more of its sides by an irregular line.

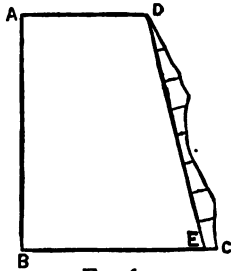


FIG. 65.

Let ABCD be a figure in which DC, one of its boundary-lines, is irregular. Run a straight line, DE, agreeing as nearly as convenient, with the irregular line. At convenient intervals along this line, measure, perpendicular to it, offsets to the irregular line.

Compute the area of the regular figure, ABED, by the usual rules. Compute the area of the small figures between the offsets on the hypothesis that they are triangles and trapezoids. If these areas are without the

regular figure, add them; if within, subtract them.

If the offsets are measured at equal intervals, the area of the whole irregular part may be obtained by the following rule:

RULE.—Add together the intermediate offsets and half the extreme offsets, and multiply the sum by the interval between the offsets.

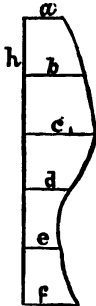


FIG. 66.

This rule will be evident from the following demonstration:

$$\text{First trapezoid} = h \left(\frac{a}{2} + \frac{b}{2} \right)$$

$$\text{Second trapezoid} = h \left(\frac{b}{2} + \frac{c}{2} \right)$$

$$\text{Third trapezoid} = h \left(\frac{c}{2} + \frac{d}{2} \right)$$

$$\text{Fourth trapezoid} = h \left(\frac{d}{2} + \frac{e}{2} \right)$$

$$\text{Fifth trapezoid} = h \left(\frac{e}{2} + \frac{f}{2} \right)$$

Hence, $\text{whole area} = h \left(\frac{a}{2} + b + c + d + e + \frac{f}{2} \right)$

CHAPTER VI.

SURVEYING WITH THE COMPASS.

106. THE **surveyor's compass**, described in Chapter III., furnishes one of the most valuable methods of surveying land. Although deficient in accuracy, the simplicity and rapidity of the field-operations with the compass give it great advantages when great precision is not required. Nearly all the early surveys in the United States have been made with the compass, and the necessity which constantly arises for the surveyor to trace out old boundary-lines, and to settle the location and contents of lands described in old title-deeds, renders a thorough knowledge of the practice of compass-surveying essential to the land-surveyor.

107. In compass-surveying the directions of the boundary-lines are all referred to the **magnetic meridian**, as indicated by the direction of the magnetic needle. The magnetic needle makes an angle with the true north and south line, which is called the declination or variation of the needle. The laws of this variation will be discussed in the succeeding chapter. The relative direction of the boundary-lines, or the area of plots of ground, will not differ, whether referred to the true meridian or to the magnetic meridian.

108. The **bearing** of a line or course is the angle which that line makes with the meridian. MAB , in Fig. 67, is the bearing of the course AB . The **reverse bearing** is the angle which the course, BA , makes with the meridian at B , that is, ABS' . Since the meridians passing through points not far distant are parallel, the bearing and reverse bearing of a course must be equal.

The bearing of a course is denominated **north-east** when it runs to the east of the meridian and north of an east and west line; **northwest** when it runs west of the meridian and north of the east and west line, etc.

109. The **difference of latitude** of a course is the distance by which its second extremity is farther north or south than the first. For brevity

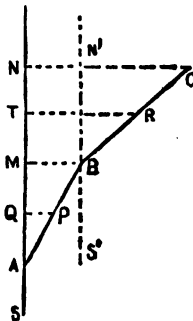


FIG. 67.

this is usually called the **latitude** of the course. It is called a northing if the course runs northward, and a southing if southward. AM is the latitude of the course AB, and is a northing; BN' is the latitude of the course BC.

The **departure** of a course is the distance by which its second extremity is farther from a given meridian than the first. It is called an easting if the course runs eastward, and a westing if westward. MB is the departure of the course AB, and N'C of the course BC, both being eastings.

The **meridian distance** of a point is the distance of that point from a meridian. MB and N'C are the meridian distances of the points B and C respectively. The meridian distance of a line is the distance of its middle point from a meridian. PQ and RT are respectively the meridian distances of the lines AB and BC from the meridian NS.

110. From the right-angled triangle ABM, we have

$$\begin{aligned}AM &= AB \times \cos MAB \\BM &= AB \times \sin MAB\end{aligned}$$

That is, for any course

$$\begin{aligned}\text{The latitude} &= \text{distance} \times \cos \text{bearing} \\ \text{The departure} &= \text{distance} \times \sin \text{bearing}\end{aligned}$$

From these formulæ the latitude and departure for any course may be computed.

Look out in a table of natural sines and cosines, the cosine and sine corresponding to the given bearing.

Multiply these numbers respectively by the length of the course. The first will give the latitude, the second the departure.

Thus, required the latitude and departure of a course whose distance is 11.25 chains, and bearing $38^{\circ} 45'$:

$$\begin{aligned}\text{The latitude} &= 11.25 \times \cos 38^{\circ} 45' = 8.7737 \\ \text{The departure} &= 11.25 \times \sin 38^{\circ} 45' = 7.0416\end{aligned}$$

111. A **traverse table** is a table of latitudes and departures for different distances, and bearings. It is computed from the formulæ above. The traverse table in this work is computed for every quarter of a degree of bearings, and for distances from 1 to 10. Bearings are usually not measured with the compass nearer than quarter degrees. The latitudes and departures for other distances than those given in the table can be obtained by addition and multiplication. Thus, to obtain the latitude and departure for a course whose bearing is N. $44^{\circ} 30'$ W, distance 39.44 chains: under the proper angle we find in the table,

For distance 3, latitude = $\underline{2.1398}$, departure = $\underline{\quad 27}$
 \therefore For distance 30, latitude = $\underline{21.398}$, departure = $\underline{21.027}$
 For distance 9, latitude = $\underline{6.419}$, departure = $\underline{6.308}$
 For distance .4, latitude = $\underline{0.285}$, departure = $\underline{0.280}$
 For distance .04, latitude = $\underline{0.029}$, departure = $\underline{0.028}$
 \therefore For distance 39.44, latitude = $\underline{28.131}$, departure = $\underline{27.643}$

When greater accuracy is required than a table calculated to quarter degrees will furnish, the table of natural sines and cosines may be used. This table may indeed be regarded as a traverse table in which the cosines give the latitudes and the sines the departures for a distance 1. For any other distance we multiply the latitudes and departures by the given distance.

112. Let ABCDE, Fig. 68, represent a piece of ground bounded by straight lines whose distances have been measured, and whose bearings have each of them been determined by the compass. Let perpendiculars be let fall from the angles upon a meridian, NS, running through the most western angle. A series of triangles and trapezoids will be formed, by combining which the area of the field may be obtained.

$$\text{Area} = C'CDD' + D'DEE' - (ABB' + B'BCC' + AEE')$$

By inspecting this formula it will be found that the area of the field is the difference between the sum of the triangles and trapezoids constructed on those sides running north, and those constructed on the sides running south.

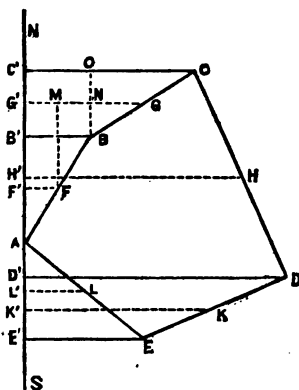


FIG. 68.

113. To determine the areas of the several triangles and trapezoids which enter this formula, we proceed thus:

Let perpendiculars be drawn from the middle points of all the courses upon the meridian NS. These will constitute the meridian distances (M. D.) of the sides. Twice these lines will be the double meridian distances (D. M. D.) of the sides.

$$\begin{aligned} 2 \times \text{area of } ABB' &= AB' \times 2FF' = \text{latitude of } AB \times (\text{D. M. D.}) \\ 2 \times \text{area of } B'BCC' &= B'C' \times 2GG' = \text{latitude of } BC \times (\text{D. M. D.}) \\ 2 \times \text{area of } C'CDD' &= C'D' \times 2HH' = \text{latitude of } CD \times (\text{D. M. D.}) \\ \text{etc.} & \qquad \qquad \qquad \text{etc.} & \qquad \qquad \qquad \text{etc.} \end{aligned}$$

The triangle or trapezoid, therefore, constructed on any side, has for its double area the product of the latitude of that side multiplied by its double meridian distance.

The double meridian distance is used in this computation instead of the meridian distance, because the method of deriving the former can be expressed in a simpler rule than the latter.

114. From the figure it is apparent that $2FF'$ is equivalent to BB' . That is, the double meridian distance of the first course is equal to its departure. Again,

$$GG' = G'M + MN + NG.$$

$$GG' = (\text{M. D.}) \text{ of } AB + \frac{1}{2}\text{dep. of } AB + \frac{1}{2}\text{dep. } BC.$$

$$2GG' = (\text{D. M. D.}) \text{ of } AB + \text{dep. of } AB + \text{dep. } BC.$$

That is, the double meridian distance of the second course is equal to the double meridian distance of the first course + the departure of the first course + the departure of the second course.

By similar reasoning we could show that—

The double meridian distance of any course is equal to the double meridian distance of the preceding course + the departure of the preceding course + the departure of the course itself.

Having now found the double meridian distances of the several courses, the areas of the several triangles and trapezoids can be computed by article 113; and by combining these in the formula of article 112, the area of the whole figure may be found.

115. The most convenient mode of combining all these operations may be summed up in the following rule, recording the successive results in a table :

STA.	DIS.	BEARING.	LATITUDES.		DEPARTURES.		D. M. D.	N. AREAS.	S. AREAS.
			N.	S.	E.	W.			

1. Measure the sides of the field and their bearings, and record them in the columns headed distance and bearing.

2. Seek out in the traverse table the latitudes and departures of each course, and record them in the appropriate columns, according as they are north or south, east or west.

3. Compute the double meridian distance of each course, and record it in the column headed D. M. D., remembering that—

The double meridian distance of the first course is equal to its departure :

The double meridian distance of the second course is equal to the double meridian distance of the first course + the departure of the first course + the departure of the second course ;

The double meridian distance of any course is equal to the double meridian distance of the preceding course + the departure of the preceding course + the departure of the course itself.

In adding for double meridian distances, the departures to the east are to be treated as positive, and those to the west as negative. If worked correctly, the double meridian distance of the last course will be equal to its departure.

4. Multiply the quantities in column D. M. D. by those in columns N. and S. of latitude, and place the products in the corresponding columns of areas.

5. Sum up the areas in the columns N. and S., and half the difference between them will be the area required.

FIELD-WORK OF COMPASS-SURVEYING.

116. From the preceding discussion of the theory of a compass-survey it will be plain that the field-work must consist in measuring the lengths and bearings of the boundary-lines, and of such other lines whose locations and dimensions we desire to ascertain. It will best be understood from an example.

Begin at any prominent corner of the farm, as A, Fig. 69, and measure the distance and bearing of each side successively. It is immaterial in which direction the surveyor proceeds, but the explanation which follows is formed on the supposition that he goes round the farm keeping it on his right hand.

117. The **distances** are measured with the chain, as explained in Chapter II. A signal is established at stations B, C, etc., toward which the chainmen direct their measurement. Sometimes the real line to be measured is occupied by a wall or hedge, and cannot be followed with the chain. In this case, set off on one side, by signals placed at equal distances from the two ends, a line which is parallel and equal to the real line, and free from obstruction.

In measuring the sides, the surveyor notes, not only the lengths of the sides, but also the distances to any noteworthy objects, as the crossing of the small stream from the spring, by the course CD, etc. Prominent objects near the line are sometimes determined by measuring from some point in the line, as the distance of the bridge from the corner B.

118. The bearing of the line AB is determined by placing the compass directly over the point A and sighting toward a signal at B. In this case he finds the line bears by the needle $S. 73^{\circ} 15' E.$ A signal is then left at A and the compass moved to B. It is sighted back toward A and the reverse bearing read. The **direct** and **reverse** bearing ought to agree. If the disagreement is small, it may be considered as resulting from the imperfection of the instrument, and the mean between the two taken as the true reading. If the disagreement is considerable, it indicates either

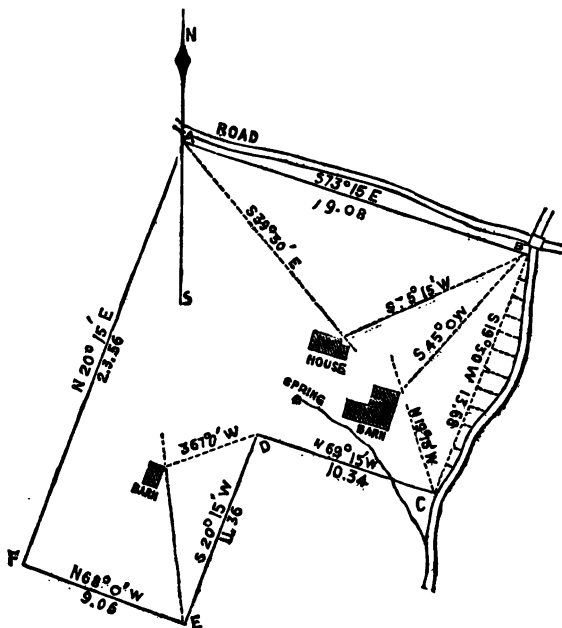


FIG. 69.

an error in the measurement, or some disturbance of the needle caused by the attraction of iron. The error in measurement can be remedied by a second trial. The fact of the local attraction will be shown by an irreconcilable difference between the direct and reverse bearing. The station at which the local attraction causes the error will be that one at

which the reverse bearing on the preceding station and the direct bearing on the succeeding station are both irreconcilable. In this case, the bearings must be assumed to be those which are found at the unaffected ends of the lines.

When the line is occupied by a fence or other obstructions, the compass may be set at any convenient distance from the true line, and sighted at a signal set at the same distance from the other end of the line. The bearing of this parallel must be the same as that of the true line. The compass may be placed also at any point on the line, as well as at its extremity.

119. If there are prominent objects on either side of the line whose position it is desired to fix, the bearings of lines running to these from any two stations must also be measured. Thus, the bearings of the lines from the two stations, A and B, to the house, will determine its position. It would be sufficient to measure the bearing of one such line, provided its distance also is measured.

120. The surveyor must keep a record of the bearings and distances which are measured, and also of any other points noted in the course of the survey. The following table will show the mode of keeping such a record. An additional record of the offsets and of the bearings upon prominent objects must also be kept.

STA.	DIS.	BEARING.	REMARKS.
A	19.08	S. 73° 15' E.	Along road.
B	13.68	S. 19° 30' W.	Offsets to stream.
C	10.34	N. 69° 15' W.	I.II crosses small rivulet.
D	11.36	S. 20° 15' W.	
E	9.06	N. 68° 0' W.	
F	23.56	N. 20° 15' E.	

121. Since in many surveys much detail must be recorded concerning offsets, and intersection of streams, fences, etc., a mode of keeping field-notes, represented in Fig. 70, is preferable.

Draw in the middle of a page of the note-book two parallel lines. These may be considered as representing the continuous boundary-line of the farm, split into two, so that the distances may be recorded between them.

Begin at the bottom of the page, in order that the line in the note-book, when held in front of the face, may correspond in position to the

line in the field. Insert between the lines the letter A, representing the first station. To the right set the bearing of the first course.

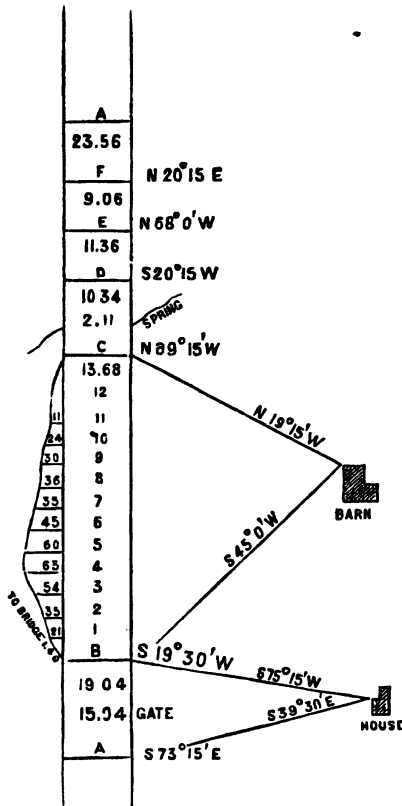


FIG. 70.

direction of the offsets being determined with sufficient accuracy by eye.

From such a record the surveyor could put down on paper an exact representation of the farm surveyed, and could make the necessary computations for determining the area.

122. In making a survey of land, it is important to fix carefully the position of the corners, so that they may be readily found subse-

If the bearing of any other object from A is taken, as the line to the house, draw the line to the right or left as the object stands, and write on it the observed bearing.

Above A insert the measured distance along the boundary-line to any objects noted, and to the next stations. Thus, 15.94 shows the distance from A to the gate, and 19.04 the distance to B.

To the right at B write the bearing of the next course BC, and also note the bearing of any other line from B; in this case a line from B to the house, and a line from B to the barn.

When offsets are measured from a course, as along BC, mark between the lines the distance from the previous station, and on the proper side record the offsets. In this example the offsets are measured at intervals of one chain, and recorded in links. They are measured by a rod or tape at right angles to the course, the di-

quently. A stone, or a stone monument, or a stake with stones heaped around it, should be placed at the corners, and the location with reference to surrounding objects described in the field-notes of the survey.

If a line runs through timber-land, it is well to mark it by "blazing" the trees lying in or near its direction. These marks on trees remain for many years, and may aid in identifying the line. The names of the surveyor and his assistants should be inserted in the notes of the survey, and in the maps of the land, so that witnesses in regard to the location of the lines may be available at any subsequent date.

COMPUTATION OF AREA.

123. Having finished the field-work of the survey, the next step is to compute the area. For this purpose construct a table having twelve vertical columns, and insert in them the stations, distances, bearings, taken from the notes of the survey.

STA.	BEARING.	DIS.	Latitudes		Dep'tures		Corr'd Lat's.	Corr'd Dep'te.	D.M.D.	N. AREAS.	S. AREAS.
			N.	S.	E.	W.					
A	S. 73° 15' E.	19.06		5.49	18.26		- 5.53	+ 18.31	34.67		191.3794
B	S. 19° 30' W.	13.68		13.80		4.57	- 12.91	- 4.55	48.43		635.2113
C	N. 69° 15' W.	10.34	3.66			9.67	+ 3.65	- 9.65	34.23	194.9395	
D	S. 20° 15' W.	11.26		10.66		3.93	- 10.68	- 3.91	20.67		230.7556
E	N. 68° 0' W.	9.06	3.39			8.40	+ 3.38	- 8.38	8.88	28.3344	
F	N. 20° 15' E.	23.56	22.11		8.15		+ 22.06	+ 8.18	8.18	180.6144	
		87.08	29.16	29.04	26.43	26.57				333.8783	1087.3453
											333.8783
											2)708.467
											351.733

Error in latitude = .19 ; error in departure = .14.

Seek out in the traverse table the latitudes and departures of the several courses, and insert them in their appropriate columns. If the survey were absolutely correct, the sum of the northings must be equal to the sum of the southings, and the sum of the eastings to the sum of the westings. With such measurements as can be made with the chain and compass such accuracy cannot be attained. Generally, there is a difference between the northings and southings, called the **error in latitude**; and a difference between the eastings and westings, called the **error in departure**. On the theory that these errors are in proportion to the distances run, the required corrections may be made by the following proportions:

The sum of all the distances : any one course ::
the error in latitude : the correction.

The sum of all the distances : any one course ::
the error in departure : the correction.

These corrections must be added to the latitudes and departures which are too small, and subtracted from those which are too large.

In the above example these proportions will be

$$87.08 : 19.08 :: .12 : \text{correction for latitude, 1st course,} = .026$$

$$87.08 : 13.68 :: .12 : \text{correction for latitude, 2d course,} = .019$$

etc.,

$$87.08 : 19.08 :: .14 : \text{correction for departure, 1st course,} = .030$$

$$87.08 : 13.68 :: .14 : \text{correction for departure, 2d course,} = .022$$

etc.

In most cases it is not necessary to compute the corrections by these proportions. They may be obtained sufficiently nearly by observing how much the error is for each chain, and then distributing the corrections in proportion to the number of chains.

When the latitudes and departures have been thus corrected, record them in columns of "corrected" latitudes and departures. These ought to balance exactly. In the table of the example given, we have placed the northings and southings in the same column, marking the former + and the latter - ; so also with the eastings and westings.

124. Beginning at the most westerly station, in this example at station F, compute the double meridian distances of each course according to the rule given in article 114, and record them in the column D. M. D. It is not absolutely essential to begin at the most westerly station ; any other point of beginning might be used. But our explanation has been based on the supposition that the meridian to which the departures and meridian distances are referred passed through the most western station. This station may be determined by inspecting the notes of the survey, or by constructing a hand-sketch of the field surveyed. Of course, the most western station is not always the station from which the survey began, and which stands first in the notes.

In this example, beginning at F,

$$D. M. D. \text{ of } FA = 8.18$$

To get D. M. D. of AB, we have, by the rule,

$$D. M. D. \text{ of } FA = 8.18$$

$$\text{Departure of } FA = 8.18$$

$$\text{Departure of } AB = \underline{18.31}$$

$$D. M. D. \text{ of } AB = 34.67$$

To get D. M. D. of BC, we have, by the rule,

$$\begin{aligned} \text{D. M. D. of AB} &= 34.67 \\ \text{Departure of AB} &= 18.31 \\ \text{Departure of BC} &= -4.55 \\ \text{D. M. D. of BC} &= \underline{48.43} \\ &\text{etc.} \end{aligned}$$

125. The columns of areas are computed by multiplying the number found in the D. M. D. column by the number in the latitude column. If the latitude is north or +, set the product in the column of north or + areas; if south or -, set it in the south or - column.

Add up these two columns and take their difference. Divide this difference by two, and the quotient must be the area in square chains and decimals. Divide by ten, and the area will be in acres and decimals. For our example,

$$\text{Area} = 35.1733.$$

126. The area of the part outside of the course BC may be found by the method explained in Problem VII., Chapter V.

$$\text{Area} = A \left(\frac{a}{2} + b + c + d \dots \frac{x}{2} \right)$$

$$\text{Area} = 100 \left(\frac{0}{2} + 21 + 35 + 54 + 63 + 60 + 45 + 35 + 36 + 30 + 24 + 11 + \frac{0}{2} \right)$$

$$\text{Area} = 41400 \text{ square links} = 4.1400 \text{ square chains} = .414 \text{ acres.}$$

$$\text{Whole area} = 35.1733 + .414 = 35.5873.$$

$$\text{Whole area} = 35 \text{ acres, 2 roods, 14 perches.}$$

DRAWING A PLOT.

127. The completion of a survey generally includes the construction of a **plot** or map of the land surveyed. This is of value, not only as a representation of the ground, showing its form and natural features, but also as a check upon the accuracy of work. We give here merely the method of making a diagram of the ground.

A plot of a field is a miniature representation of its boundary, and other lines, given in their true proportion and making their proper angle with each other.

128. The first step is to determine upon what **scale** the plot shall be drawn; that is, what ratio the lines of the real field shall bear to their

representatives upon the paper. We may, for example, make an inch upon the paper represent a chain upon the ground; that would be called a scale of 1 chain to an inch. Or we may make an inch represent 2 chains, or 5 chains, or 10 chains. To facilitate the construction of plots, the scales employed for making the drawings have usually a number of small spaces marked off into tenths, such as an inch, a half-inch, a quarter-inch, etc. If one of these is taken to represent a chain, each

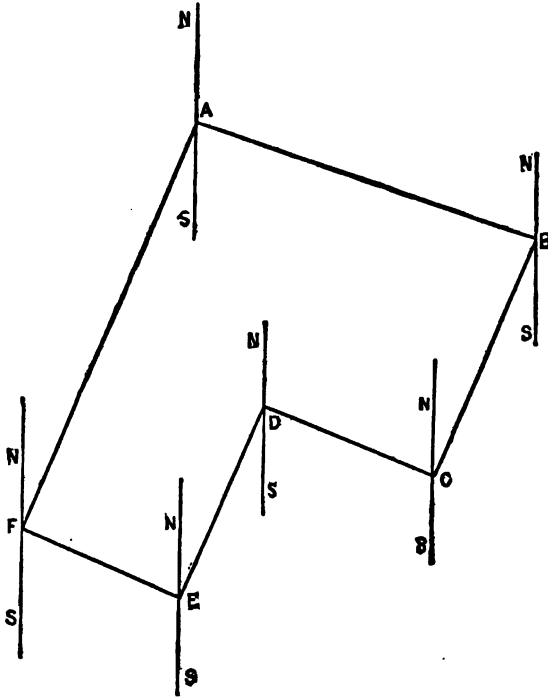


FIG. 71.

tenth would represent ten links. In any particular case the scale to be used must be determined by the dimensions of the ground and the size of the plot which it is proposed to make.

129. Having determined upon the scale, select upon the paper some point, A, Fig. 71, as a starting-point, to correspond with the first station of the survey. Through this point draw a straight line, NS, to represent the

direction of the meridian, and with a protractor lay off the direction of the first course; in this case $S. 73^{\circ} 15' E.$ Draw an indefinite line in this direction, and on it lay off with the proper scale the length of the first course, 19.08 chains. Mark this point B, the second station in the field. At B draw a line parallel to NS, which will be the meridian at B. From this lay off the bearing of the second course, and draw a line to the same scale, 13.68 chains.

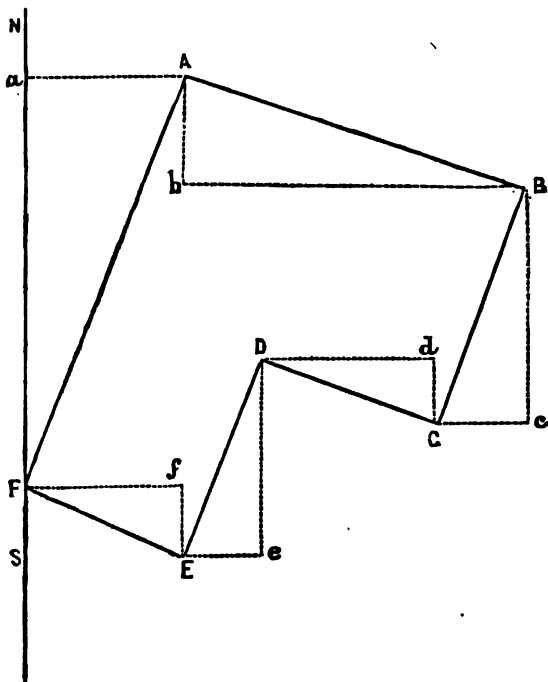


FIG. 72.

At each station draw a meridian, and from it lay off the next course in its given length and direction. The last course should end at the starting-point. Generally, even with the most careful drawing, there will be found a discrepancy. Any very marked discrepancy will show defects in field operations or in making the drawing.

130. The above method of plotting employs the bearings and distances as measured in the field, and whatever errors they contained are

followed. By using the latitudes and departures after they have been corrected, we can construct a more accurate plot. Draw at the left side of the paper, Fig. 72, a line, to represent the meridian, through the most western station. Select a point, F, on this line, to represent the most westerly station of the field. Lay off on this line Fa 22.08 chains, the latitude (north) of the course FA, and perpendicular to this aA 8.18 chains, its departure (east). Join F and A; the result will be the true plot of the course FA. At the point A draw Ab parallel to NS 5.52 chains, the latitude (south) of the course AB; and perpendicular to this bB 18.31 chains, its departure (east); and join A and B.

Proceed thus to lay off the latitude and departure of each course; when finally, the latitude and departure of the course FE being measured off, the point of beginning, F, will be reached.

Instead of beginning at F, the most western station, it would be equally easy to begin at A, the first station of the survey.

This method of plotting employs chiefly lines running either north and south, or east and west. By using a drawing-board and a T-square (Art. 5), these lines can be drawn with great facility and correctness.

131. After having plotted the boundary-lines of the field, the prominent objects which were observed in the survey may be located by drawing the lines in their proper bearing by which their position in the field was determined.

Thus, to locate the house, draw from A a line bearing S. $39^{\circ} 30'$ E. from the meridian at A, and from B a line bearing S. $75^{\circ} 15'$ W. from the meridian at B; they will intersect at the location of the house on the plot.

The offsets must also be set off along the side BC to the same scale, and an irregular line drawn through their extremities will be the adjacent bank of the stream.

CHAPTER VII.

VARIATION OF THE NEEDLE.

132. WHEN a small steel bar, which has been carefully balanced so as to hang horizontally, is magnetized, it assumes a position which makes an angle both with the horizon and with the meridian. The angle which it makes with a horizontal line is called the **dip** or **inclination** of the needle. In the United States the north end of the needle dips downward. In the surveyor's compass, to overcome this **dip**, a small counterpoise is placed on the south end.

The angle which the magnetic needle makes with the true meridian, at any place, is called its **declination**, or more commonly its **variation**. When the north end of the needle lies on the east side of the meridian, it is called **east variation**; when upon the west side, **west variation**.

133. In the United States the needle varies both east and west; in the eastern portion, varying to the west; and in the western portion, varying to the east. A line drawn through those places where the needle varies neither east nor west, is called a **line of no variation**. Such a line runs through the United States in a northwest and southeast direction, passing through Michigan, Lake Erie, near Cleveland, through the eastern part of Ohio, and through the Virginias and North Carolina. In all places east of this line the variation is west, and in all places west of it the variation is east; the amount of variation increasing as we recede from this line in either direction. In the extreme eastern part of Maine the variation is 18° west, and in the State of Oregon it is more than 20° east.

134. The accompanying **chart**, Fig. 73, is copied by permission from the Report of the United States Coast Survey for 1865, and represents the lines of equal variation for a part of the United States. It was compiled by Charles A. Schott, from numerous data, and reduced to the epoch of 1870, according to the known rates of change which the variation undergoes. The surveyor can, by inspecting this chart, readily determine within small limits the variation of the needle at any place for 1870. Then, by applying a correction for the annual change, he can determine the variation for any preceding or subsequent year.

135. Observations continued through a series of years, at any place, show a continual change to be going on in the variation of the needle.

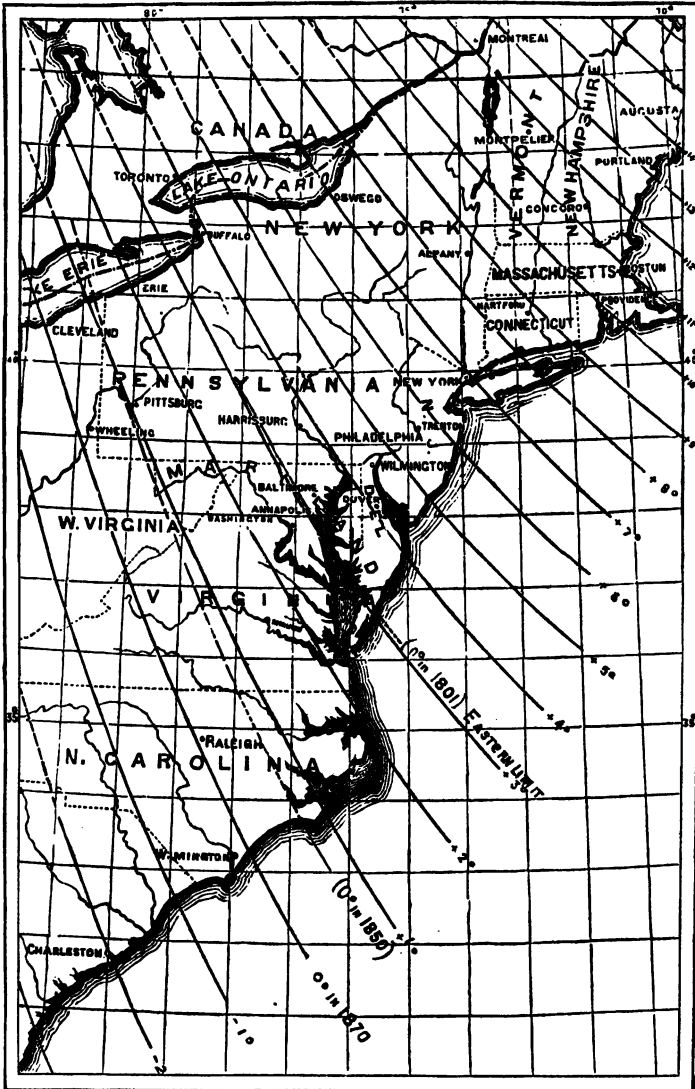


FIG. 73.—LINES OF EQUAL VARIATION OF THE NEEDLE.



VARIATION OF THE NEEDLE OF CALIFORNIA

At the present time this change consists in a steady **annual movement of the needle** to the westward ; so that on the east side of the line of no variation, the variation has been increasing, and on the west side decreasing. An examination of the oldest records in regard to changes in the variation of the needle, indicate that from about 1670 to 1795, the needle changed eastward annually ; but from about 1795, the annual change has been westward. This would indicate an oscillatory motion of the needle, requiring about 125 years to complete one oscillation. If this be true, the present annual movement westward will continue till about A.D. 1920, when it will be reversed, and continue eastward.

136. The table given below, shows the annual changes of variation for a number of places in the eastern portion of the United States. It is taken from the United States Coast Survey for 1865, from a chart prepared by Charles A. Schott, and gives the annual changes between 1860 and 1870. It will be approximately true for several years to come.

WESTWARD ANNUAL CHANGE FROM 1860 TO 1870.

PLACE.	ANNUAL CHANGE.	PLACE.	ANNUAL CHANGE.
Albany, N. Y.	3'.9	Key West, Fla.	3'.0
Baltimore, Md.	3.4	Mobile, Ala.	1.1
Bath, N. Y.	3.6	New Haven, Ct.	4.7
Boston, Mass.	3.7	New Orleans, La.	0.2
Buffalo, N. Y.	3.6	New York, N. Y.	3.8
Burlington, Vt.	4.5	Oxford, N. Y.	4.9
Cairo, Ill.	2.0	Portland, Me.	3.2
Charleston, S. C.	2.0	Providence, R. I.	3.0
Eastport, Me.	1.6	Quebec, Can.	5.5
Eric, Pa.	3.0	Rutland, Vt.	5.7
Harrisburg, Pa.	1.5	Savannah, Ga.	1.8
Hartford, Ct.	4.6	Washington, D. C.	3.1
Havana, Cuba.	1.3	Williamsburgh, Va.	3.7

137. If curves were drawn on a chart through those points in the eastern part of the United States included in the above table, which have the same annual change, we should find that :

The curve of 2' annual change would pass near Eastport, Me., through the Atlantic near Charleston and Savannah, through Florida into the Gulf of Mexico.

The curve of 3' would pass near Portland, Me., through Martha's Vine-

yard, through the Atlantic south of Long Island, through southern New Jersey and Maryland into Virginia; thence bending northwest, through Baltimore, through Pennsylvania, and entering Lake Erie between Buffalo and Erie.

The curve of 4' would pass through New Hampshire, Massachusetts, Connecticut into Long Island Sound, nearly through New York City and Philadelphia; thence bending northward through central Pennsylvania and New York.

The curve of 5' would pass between Burlington and Rutland, Vt., into New York west of Albany, and in Otsego County, bending northward, and running through northern New York.

138. Besides the annual westward change, the needle is subject, also, to **daily fluctuations**, depending apparently upon the movements of the sun. In the northern hemisphere the needle has a westward movement from about 7 or 8 A.M., reaching a maximum of about 10' to 15' at 2 P.M., from which time it recedes to its former position. In the summer this diurnal change of variation is greater than in winter.

Slight **disturbances** are also not unfrequently observed in the position of the needle, especially during thunder-storms, auroras, and other electrical phenomena. These, however, do not interfere with the use of the needle in surveying.

139. It is evident, from the above facts, that the magnetic needle, although of the utmost value in surveying as well as navigation, must be employed with great caution, in order that the various irregularities to which it is liable may not invalidate the operations depending upon it. In re-running the lines of old surveys, made when the variation of the needle was different from its present variation, we must not expect to find the magnetic bearings correspond until we have made an allowance for the change during the interval. In surveying with the compass in places distant from each other, it is necessary to take into account the difference in the amount of variation prevailing at these places. In making a survey with the compass, it is important that the surveyor should note upon his field-book and upon the map the date of the survey and the variation of the needle prevailing at the time, and see that it is inserted in any deed or conveyance drawn up by him.

To enable the surveyor to determine the variation of the needle, he must be provided with the means of establishing the direction of a true meridian. This is done by observations on the heavenly bodies.

140. If the position of the **north pole** of the heavens were exactly marked by a star, by pointing the telescope of the transit or the sights of the compass upon it, we would have the direction of a true north and south line. No star occupies this position, but the north star (Polaris) revolves

around it in a small circle at a distance of about one and a half degrees. It makes the circuit in 23h. 56m. Twice during this interval it crosses the meridian, once above the true pole, at A, Fig. 74, and once below, at B. These are called respectively the upper and lower culminations of the star. An observation on the north star at either of these points would give the direction of a north and south line.

Almanacs prepared for the use of astronomers and navigators, such as the British and American Nautical Almanacs, give the time of the upper culminations for each day in the year. The time of the lower culmination will be 11h. 58m. later. Several smaller almanacs give the means of knowing these times with sufficient exactness for the purposes of the surveyor.

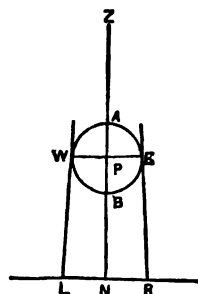


FIG. 74.

141. The following table gives the times of the upper culmination of the north star for each fifth day. For intermediate days the times may be obtained by interpolation. When the upper culmination occurs in daylight, it is necessary to use the lower culmination instead. The times printed in black-faced type in the table indicate those occurring between 6 P.M. and 6 A.M.

UPPER CULMINATION OF POLARIS, 1872.

	1st Day.	6th Day.	11th Day.	16th Day.	21st Day.	26th Day.
	H. M. S.	H. M. S.	H. M. S.	H. M. S.	H. M. S.	H. M. S.
January....	6 29 3	6 9 19	5 49 35	5 29 51	5 10 6	4 50 22
February....	4 26 41	4 6 57	3 47 14	3 27 30	3 7 47	2 48 4
March.....	2 32 19	3 12 36	1 52 54	1 33 13	1 13 31	12 49 55
April.....	12 26 30	12 6 49	11 47 0	11 27 22	11 7 43	10 48 6
May.....	10 28 27	10 8 51	9 49 14	9 29 38	9 10 1	8 50 26
June.....	8 26 54	8 7 19	7 47 43	7 28 7	7 8 33	6 48 57
July.....	6 29 23	6 9 47	5 50 12	5 30 37	5 11 1	4 51 26
August....	4 27 56	4 8 20	3 48 44	3 29 9	3 9 32	2 49 57
September.	2 26 24	2 6 48	1 47 10	1 27 33	1 7 56	12 48 18
October....	12 28 38	12 9 0	11 49 21	11 29 41	11 10 2	10 50 21
November..	10 26 45	10 7 4	9 47 23	9 27 41	9 8 0	8 48 17
December..	8 28 35	8 8 52	7 49 8	7 29 25	7 9 41	6 49 58

142. Instead of observing the north star at its upper and lower culmination, we may observe it at the points E or W, Fig. 74. These are called its greatest eastern or western elongation, respectively. 5h. 59m. before its upper culmination it is at its greatest eastern elongation, and 5h. 59m. after, at its greatest western elongation.

Vertical lines through E and W cut the horizon at R and L to the right

and left of the north point N. When these points are observed, therefore, for determining the meridian, a correction of the amount of LN or RN must be made. This is the azimuth of the star, and may be obtained by the formula,

$$\sin NR = \frac{\sin PE}{\cos PN} = \frac{\sin \text{polar distance}}{\cos \text{latitude}}$$

The polar distance of the star is given in the nautical almanacs. For January 1, 1872, it is $1^{\circ} 22' 23''$, and changes very slowly. The following are the values of the azimuth for latitudes from 30° to 49° .

AZIMUTH OF POLARIS, 1872.

LAT.	AZIMUTH.	LAT.	AZIMUTH.	LAT.	AZIMUTH.	LAT.	AZIMUTH.
30°	1° 35' 8"	35°	1° 40' 34"	40°	1° 47' 33"	45°	1° 56' 31"
31	1 36 7	36	1 41 51	41	1 49 10	46	1 58 36
32	1 37 9	37	1 43 9	42	1 50 52	47	2 0 48
33	1 38 14	38	1 44 33	43	1 52 37	48	2 3 08
34	1 39 22	39	1 46 1	44	1 54 32	49	2 5 35

143. The observations for **establishing a meridian** are best made with a transit or theodolite. Select some point where a suitable permanent mark may be left, and directly over this mark place the instrument and level it. Set the vernier at zero and unclamp the lower limb. A few minutes before the expected culmination, or elongation, point the telescope on the star so that the vertical thread covers it. As the star moves away from the thread, follow it by the slow-motion screw attached to the limb. Continue the motion until the exact time arrives. If a culmination is observed, the instrument is exactly north and south, and a second mark may be set by it.

If an elongation is observed, the vertical thread must be made to follow the star, until the star, moving more and more slowly, begins to retrograde. The slow motion of the star at elongation, and the fact that a small error in the time does not affect it, render observations at elongation more easy and more reliable than at culmination.

The amount of the azimuth given in the table in the preceding article being now turned off on the horizontal limb, the telescope will point in the true meridian, and may be used to set a second mark.

In following a star with the telescope of a transit, some mode of making the threads visible must be employed. This is readily done by causing an assistant to hold a light near, but not quite in front of, the object-glass of the telescope. The light thrown into the tube, and reflected by its

sides, is generally sufficient to illuminate the threads. Manufacturers of instruments now provide a contrivance of a mirror and small lamp for this purpose.

144. The direction of the meridian may be obtained without a telescope, although less accurately.

To do this, suspend a long plumb-line from some projecting point in a building, and behind it, at a convenient distance, place on a steady support one of the compass sights, or any contrivance furnished with a fine hole, through which to look. Place the sight so that when the eye looks through it, the plumb-line covers the star. As the star moves away from the line in one direction, move the eye and the sight in the other, so as to keep the plumb-line covering the star. When the moment of culmination comes, as given by the table, stop the sight in the position when the star was covered. The sight and the plumb-line give two points in a true meridian, which may be permanently marked.

If the observation is made at an elongation, the star must be followed until it begins to retrograde. The position of the sight and of the plumb-line will be two points in a line varying from the true meridian by the azimuth given in the table. Place the compass on this line and sight along it. Observe the position of the needle, and then turn the compass in the proper direction the amount of the azimuth.

145. A close approximation to the true meridian may be obtained by observing when the north star is in the same vertical line with the star **Alioth** in the constellation of the Great Bear. If a plumb-line be suspended as in the preceding article, and it be made to follow the north star, when it has revolved into such a position that the plumb-line covers also the star **Alioth**, as shown in Fig. 75, it also coincides very nearly with the true pole.



FIG. 75.

At its lower culmination the figure would be inverted, but the same holds true. The above method may be made more accurate by remembering that the north star comes exactly upon the meridian 22 minutes after it has been in the same vertical line with **Alioth**; so that if, after we

have observed both stars covered by the plumb-line, we wait 22 minutes, and then cover the north star with the plumb-line, the direction will be exact.

146. Every surveyor should establish with as much accuracy as his instruments will permit, a **true meridian**, and mark it with permanent marks. With this line he can, whenever desirable, test the variation of the needle, and any changes which it undergoes would be detected.

It would greatly conduce to the reliability of surveys if **standard meridian lines** were established at various accessible points; and surveyors should be required to observe the variation of the needle in their instruments at designated intervals, and to insert in their surveys of land the variation which prevailed at the time. The State of New Jersey some years since authorized the officers of the various counties to secure the establishment at the county seat of each county of a standard meridian line. Many of the counties availed themselves of this authority, and erected stone monuments, giving the true meridian, and generally also the approximate latitude and longitude of the place. Such a system, if extended and improved, would do much to lessen land litigation, and to develop a knowledge of the laws of the variation of the needle.

CHAPTER VIII.

SURVEYING WITH THE TRANSIT OR THEODOLITE.

147. **The transit or theodolite** has two advantages over the compass as an instrument for surveying land. **First**, the angles, being read by a vernier, can be read more accurately than is possible with the needle. Instead of quarter degrees, it is easy to read to minutes or less. **Second**, the use of the telescope instead of the ordinary compass sights, enables the surveyor to see the target at longer distances, and to point upon it more precisely. The addition of the telescope to the magnetic compass, which is now made by many manufacturers, remedies the second of these objections to the compass. But it must be remembered that as we cannot read the angles on the compass to less than quarter degrees, there is little advantage in improving the method of pointing beyond the degree of precision which can be attained in reading the angles.

148. **To survey a line with the transit.** The line of a road or railroad, or the boundary-line between two estates or townships, has often to be surveyed. Let ABCD, etc., Fig. 76, be a broken line, whose position, directions, and distances are desired. Measure the distances AB, BC, CD, etc., the

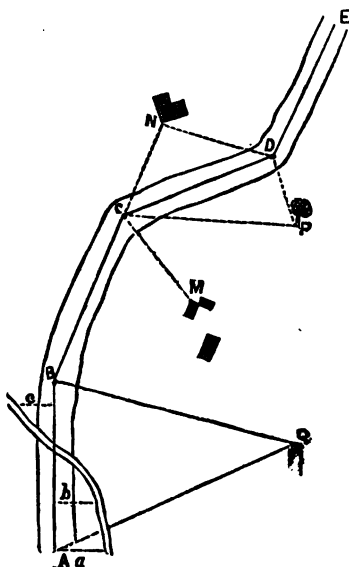


FIG. 76.

several parts of the broken line, and with the transit measure the angle ABC, BCD, etc., included between them. Whenever there are objects, as M, N, and P, whose positions are to be fixed, the angles to these objects

from the extremities of one of the lines, or one angle and the distance, must also be measured. Objects near to the line surveyed may be fixed by measuring offsets, as *a*, *b*, and *c*, from the line AB.

The notes for such a survey may best be kept in the method described for compass surveying, Art. 121. Begin at the bottom of the page of the

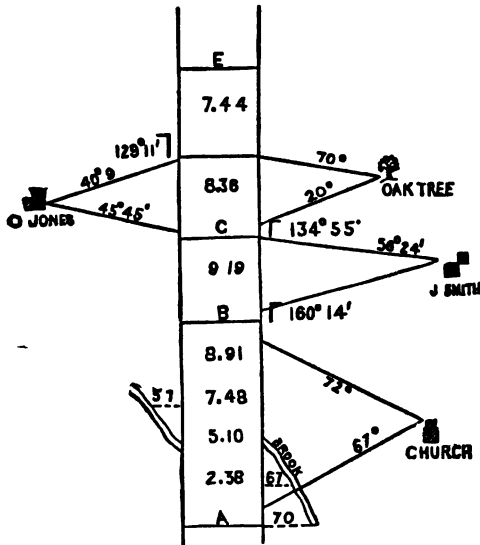


FIG. 77.

note-book, with station A, and record the distances, angles, offsets, etc., as shown in the annexed specimen. The character Γ or \lrcorner is used to indicate to which side the subsequent line turns.

149. For convenience in plotting the survey of such a line, we may, instead of measuring the angles included between the preceding and succeeding line, measure directly from the transit the bearings of all the lines from the first one. Thus, in Fig. 78, set the transit at B, and fix the vernier at zero. Turn the limb so that the telescope shall point to A. Revolve the telescope on its axis: it will now point in the direction BN, with the vernier at zero. Turn the vernier plate till the telescope points at C; the vernier will then give the angle NBC, which may be recorded as the bearing of the line BC with the line AB.

Clamp the vernier, and remove the instrument to C. Revolve the telescope on its axis, and then point it (the vernier still clamped) back at B. The horizontal limb now stands in the same position in the field as at B, viz., with its zero point in the direction CN'. Clamp the limb. Revolve the telescope back on its axis, and then, unclamping the vernier, point it toward the next station D. The angle indicated by the vernier will be the angle N'CD, which is the bearing of the line CD with CN', or its parallel AB.

Proceeding in this way, we obtain at each station the bearing of the succeeding course with the first course. These angles, when continued from 0° to 360°, measured from the forward end of the line, are called the **azimuth** of the line. In the above example these azimuths would be as follows:

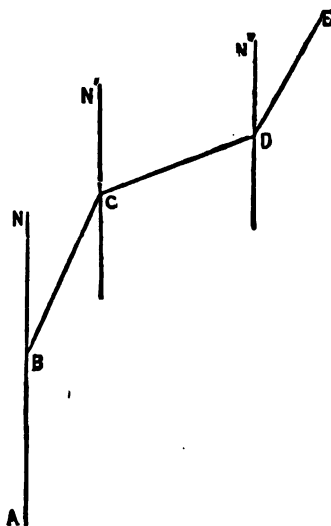


FIG. 78.

STATIONS.	AZIMUTH WITH AB.
A	
B	19° 46'
C	64 51
D	14 02

150. The survey of the streets of a city can best be made with the transit and chain, in the manner above explained. Run the centre line of the street, measuring the distances to each intersection, and at each bend of the street measure the change of direction. Also at the intersections measure the angles made by the cross streets with the one which is run, and measure offsets to the sidewalks, buildings, and other objects, whose position it is required to determine.

Since land under such circumstances is much more valuable than farm-

ing lands, greater care must be exercised, both in measurements of the angles and the distances. The chain should be well tested, and due allowance must be made for slopes. Important distances should be measured in both directions and made to agree. For city surveying, the foot is the most common unit of measure, and hence it is best to use a chain or tape divided into feet.

151. To survey a tract of land. For the methods used to determine the areas of tracts of land of regular geometrical form, we refer to the various problems of areas in Chapter V. The general method with the transit consists in dividing the ground into triangles, and measuring the lines and angles, which will suffice to determine the area of each.

When the tract is of such a form that all the angles can be seen from one of the corners, as A, Fig. 79, the simplest process is to place the transit at that point, and measure the angles BAC, CAD, and DAE. Then measure the sides AB and AE, and the diagonals AC and AD, which compose the lines running from the station occupied to the several corners of the field. The area can then be computed by Problem II., Chapter V.

152. The station occupied by the transit may be chosen within the field when more convenient. The angles at O, Fig. 80, being measured, and the distances OA, OB, etc., we can determine the areas of the triangles, and also the lengths of the sides AB, BC, etc. We have, in this method, a check upon the correctness of the measured angles around O, since their sum, if correctly measured, must be equal to 360° .

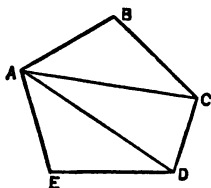


FIG. 79.

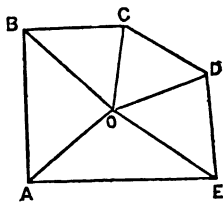


FIG. 80.

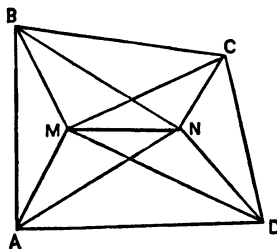


FIG. 81.

Since the measurement of angles can be more correctly made than the measurement of lines, it is obviously advantageous to reduce the measurement of lines as much as possible, and substitute for it the measurement

of angles. This can be done by measuring one line as a **base line**, MN, Fig. 81, and from its extremities measuring the angles to each corner. The base line must be so chosen that from its extremities all the stations can be seen, and its length must not be greatly disproportionate to the sides. It may be selected on the most favorable ground for measurement. One of the sides of the field may be used as a base line; or the base line may be chosen outside as well as inside the field.

The length of MN and two angles being known in each of the triangles, MNA, MNB, etc., the positions of A, B, C, etc., can either be plotted on paper or computed. The area of the entire tract may be computed by finding the area of all the triangles formed by lines radiating from M or N. Thus, MAB, MBC, MCD, and MDA are together equal to the whole figure.

153. It is sufficient for determining the figure and area of any tract of land, whatever its shape and dimensions, to **go entirely around it**, measuring the length of its sides, and the angles between the sides. In extended tracts of land, especially those partly covered with timber and otherwise encumbered, this method is almost the only one which can be employed.

Begin at any corner, as A, Fig. 82, and measure the sides, AB, BC, etc. With the transit measure the internal angles ABC, BCD, etc. When one of the angles is re-entrant, the internal angle will be greater than 180° . In traversing the boundary with the transit and the chain, take measurement and note of all objects whose position it is desired to preserve. The measurements will enable us to construct

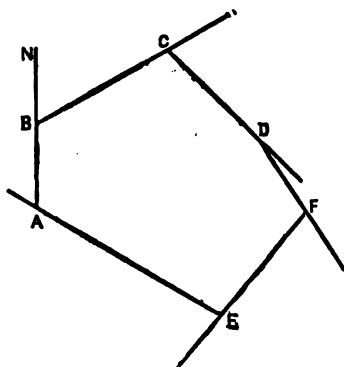


FIG. 82.

a plot of the tract, and also to compute the area. The field-notes may be kept in precisely the method explained in article 148.

154. The following example will illustrate the method of computing the area from the measurement of the sides and internal angles:

STA.	DISTANCE.	INTERNAL ANGLES.	EXTERNAL ANGLES.	AZIMUTH WITH AB.	BEARING WITH COURSE AB.
A	8.93	120° 30'	59° 30'	0	N.
B	15.64	118 30	61 30	61° 30'	N. 61° 30' E.
C	14.27	104 45	75 15	136 45	S. 43 15 E.
D	8.62	170 33	9 27	146 12	S. 33 48 E.
E	18.52	71 57	108 03	254 15	S. 74 15 W.
F	12.18	133 45	46 15	300 30	N. 59 30 W.
A				360	

Column I. gives the stations at which the angles were measured.

Column II. gives the distance from a station to the next.

Column III. gives the internal angles at each station.

Column IV. gives the external angle at each station; obtained by subtracting the internal angle from 180°. If the angle were re-entrant this remainder would be negative.

Column V. gives the azimuth of each course with the first course, measured from the forward end around toward the right. The azimuth of the first course must be 0. The azimuth of the second course must be equal to the external angle NBC. The azimuth of the third course must be equal to the azimuth of the second course + the external angle between this course and the second. The azimuth of any course will be equal to the azimuth of the preceding course + the external angle of the course itself.

When the angles are all correct, the azimuth of the last course added to the external angle of the first course must make 360°. The excess or deficiency may be distributed equally among the angles, giving the corrected azimuths.

Column VI. gives the bearings of the several courses with the first course. They are derived from the azimuths by considering that,

First. All azimuths between 0° and 90° must lie in the quadrant NOE, and may be marked, as in compass surveying, NE.

Second. All azimuths between 90° and 180° lie in the quadrant SOE, and the bearings may be got by subtracting the azimuths from 180°, and marking them SE.

Third. All azimuths between 180°

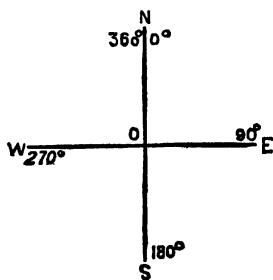


FIG. 83.

and 270° lie in the quadrant SOW, and the bearings may be got by subtracting 180° from the azimuths, and marking them SW.

Fourth. All azimuths between 270° and 360° lie in the quadrant NOW, and the bearings may be got by subtracting the azimuths from 360°, and marking them NW.

155. The second and sixth columns of the table may now be treated in the same manner as the distances and bearings in a compass survey.

Arranging the computation as there explained, the above example may be solved as follows :

STA.	DIS.	BEARING WITH AB.	LATITUDE.		DEPARTURE.		D. M. D.	N. AREAS.	S. AREAS.
			N.	S.	E.	W.			
A	8.98	N.	8.98				0		
B	15.64	N. 61° 30' E.	7.47		13.74		15.74	102.6378	
C	14.27	S. 43 15 E.		10.40	9.78		37.36		387.5040
D	8.63	S. 33 48 E.		7.15	4.80		51.84		370.6580
E	18.52	S. 74 15 W.		5.03		17.82	38.82		195.2648
F	12.18	N. 59 30 W.	6.18			10.50		64.8900	
								167.5378	953.4346
									167.5378
Area = 89 acres, 1 rood, 7.17 perches									
									785.8968

Since the traverse table does not give the bearings nearer than quarter degrees, it is necessary, generally, in these examples, to use the table of natural sines and cosines. Look out for each bearing the natural cosine, and the natural sine, and multiply them respectively by the distance ; the former will give the latitude, and the latter the departure.

The sum of the N. column should be equal to the sum of the S. column, and the E. column to the W. column. The errors in latitude and departure should always be small, and the correction can be made in the same way as in compass surveying. The corrected latitudes and departures may be put in the proper columns. The double meridian distances and areas are computed as in compass surveying. Half the difference between the N. areas and S. areas will give the area of the field in square chains and decimals.

156. When it is desired that the position of the tract of land with reference to the true or magnetic meridian should be indicated on the map, it is only necessary that the bearing of any one of the courses with the meridian should be observed, either with the magnetic needle or astronomically. With this the true north and south line can be drawn on the map, and the bearing of any other course with the meridian determined.

EXAMPLES.

1. Given a field, in which AB is 6.60 chains, BC 9.86 chains, CD 7.54 chains, and DA 9.93 chains. The internal angles measured were, A $89^{\circ} 15'$, B $96^{\circ} 0'$, C $84^{\circ} 30'$, D $90^{\circ} 15'$. Required the area.

2. In a six-sided field the following measurements were taken: A $92^{\circ} 30'$, B $94^{\circ} 30'$, C $155^{\circ} 15'$, D $179^{\circ} 30'$, E $94^{\circ} 15'$, and F 104° ; also the sides, AB 31.80 chains, BC 2.08, CD 2.21, DE 35.35, EF 20.90, and FA 31.30 chains. Required the area.

CHAPTER IX.

LAYING OUT AND DIVIDING LAND.

157. THE processes under this head are the reverse of those given in the preceding chapters. It is now required that we find methods for laying out upon the ground, lines and figures whose dimensions are known. In general, it must be left to the skill of the surveyor to devise methods for each particular case. Only a few cases of the most common occurrence are here given, which may serve as models of the methods to be pursued. It is most convenient to arrange them in a series of problems.

158. Problem I. To lay out a given quantity of land in the form of a square or rectangle.

In solving this problem it is presumed that the surveyor is limited to start from a given point, and run along some fixed line, as a road, stream, or fence. If the land is to be a square, the side is equal to the square root of the given area. If it is to be a rectangle, of which one side is given, the other side is equal to the area divided by the given side.

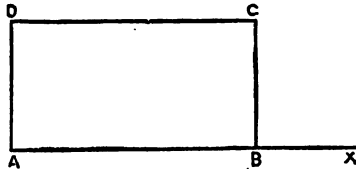


FIG. 84.

Measure off along the fixed line AX, Fig. 83, from the starting-point A, the side of the given square or rectangle. Then at A and B, with the transit, or otherwise, lay off right angles, and measure AD and BC equal to the other dimension.

EXAMPLE.—Lay out along a given line a square containing 5 acres.

Since

$$5 \text{ acres} = 50 \text{ square chains.}$$

$$\text{The side} = \sqrt{50} = 7.071 \text{ chains.}$$

159. Problem II. To lay out a piece of ground in the form of a parallelogram, the angle between the two sides and the length of one side being given.

Let it be required to lay off along the lines AX and AY, Fig. 85, a

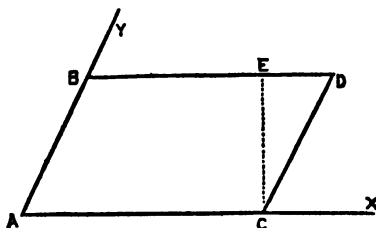


FIG. 85.

parallelogram whose area is m , and the side along AX to be equal to a .

The area of the parallelogram = $AC \times AB \times \sin A$.

Hence, $m = a \times \sin A \times AB$

$$AB = \frac{m}{a \times \sin A}$$

Hence, measure along AX a line equal to a , and along the side AY a line equal to $\frac{m}{a \cdot \sin A}$ and at the points B and C run lines parallel to AX and AY.

Otherwise,

The area of the parallelogram = $AC \times EC = a \times CE$

Hence, $m = a \times CE$

$$CE = \frac{m}{a}$$

At the point C erect a perpendicular to AC equal to $\frac{m}{a}$, and at E run a line at right-angles to CE meeting AY in B, and make BD equal to AC.

EXAMPLE.—Lay out a parallelogram having an area of 2 acres, along two roads, meeting at an angle of $78^\circ 30'$; one side of the parallelogram being 500 feet.

$$AB = \frac{m}{a \cdot \sin A} = \frac{2 \text{ acres}}{500 \cdot \sin 78^\circ 30'} = \frac{87120}{500 \cdot \sin 78^\circ 30'}$$

Log 87120.....	3.940118
- Log 500.....	1.699057
- Sin $78^\circ 30'$	9.991193
Log AB.....	2.249868

$$AB = 177.77 \text{ feet.}$$

Problem III. To lay out a given quantity of land when one or more of the boundary lines is an irregular line.

Required to lay off along a straight line AX, Fig. 85, and an irregular line AY, by a line perpendicular to AX, a quantity of land equal to m .

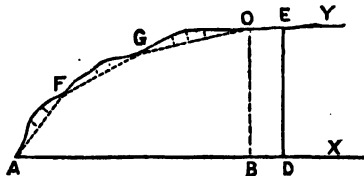


FIG. 86.

By estimate, assume that a perpendicular from B will cut off the required amount. Make a survey of the field AFGOB, in the usual way, including the offsets. Compute its area, and let it be represented by n .

$$m - n = \text{BOED} = \text{area to be added to the trial field.}$$

If the boundary OE is nearly parallel to BD, we may find the distance to be added in order to make the required area by the equation

$$BD = \frac{m - n}{BO}$$

If BOED is a trapezoid, we must divide, not by BO, but by a mean between BO and DE. This may generally be estimated with sufficient exactness. Hence,

$$BD = \frac{m - n}{\frac{1}{2}(BO + DE)}$$

Problem IV. To lay off a given quantity of land along an irregular line, and along a straight line, when a starting point for the dividing line is assigned.

Let it be required to lay off a given quantity of land m along the irregular line ABC, Fig. 86, and the straight line AX, by a line CF running from a given point C to some point in AX.

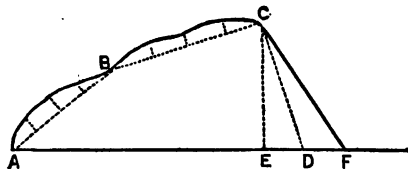


FIG. 87.

Estimate the position of the line cutting off the required amount to be

CD. Survey the field ABCD, including the offsets along AB and BC, and determine its area, and let it be represented by n .

$$m - n = CDF = \text{area to be added to the trial field.}$$

Measure a perpendicular CE drawn from C upon AX.

$$m - n = CDF = \frac{1}{2}CE \times DF$$

$$DF = \frac{m - n}{\frac{1}{2}CE}$$

The perpendicular CE can be got by computation, when in surveying the field the angle CDE was measured.

$$CE = CD \times \sin CDE,$$

$$DF = \frac{m - n}{\frac{1}{2}CD \sin CDE}$$

Problem V. To divide a rectangular tract of ground into parts bearing any proportion whatever to each other, by lines perpendicular to one side.

Let it be required to divide the rectangle ABCD into parts which shall

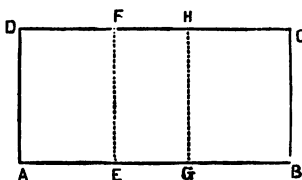


FIG. 88.

be to each other as m , n , and p , by lines perpendicular to AB. The areas of the several rectangles AEF, EGH, and GHB, into which the ground is to be divided, are to each other as the bases AE, EG, and GB. Hence, it is only necessary to divide the line AB into parts proportional to m , n ,

and p . That is,

$$AE = \frac{m}{m + n + p} \times AB$$

$$EG = \frac{n}{m + n + p} \times AB$$

$$GB = \frac{p}{m + n + p} \times AB$$

EXAMPLE. A rectangular plot of ground was bequeathed to three children. It was 10.24 chains on the side AB, and 4.93 chains on the side AD. It was valued at \$1,000, of which the first child was to receive \$400, the second \$350, and the third \$250. Required, the frontage of each along the side AB.

Ans. The first received 4.10 chains.
The second received 3.58 chains.
The third received 2.56 chains.

Problem VI. To divide a triangle by a line running from one angle to the opposite side.

Let it be required to divide the triangle ABC, Fig. 88, into parts which shall be to each other as m and n , by a line running from B to the side AC. Since the triangles ABD and BDC, into which the given triangle must be divided, have the same altitude, they must be to each other as their bases. Hence, it is only necessary to divide the base AC into parts which are to each other as m to n .

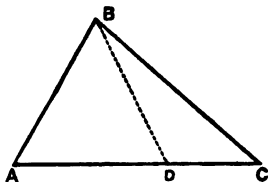


FIG. 88.

$$AD = \frac{m}{m+n} \times AC$$

$$DC = \frac{m+n}{n} \times AC \quad \leftarrow \text{WRONG}$$

EXAMPLE.—A triangular tract of land containing four acres is to be divided into two parts, which shall be to each other as 5 to 3, by a line running from one of the angles to a side 12.25 chains in length. Required the frontage of each part.

$$\text{Ans. } AD = \frac{5}{5+3} \times 12.25 = 7.66 \text{ chains.}$$

$$DC = \frac{3}{5+3} \times 12.25 = 4.59 \text{ chains.}$$

$$\text{Area ABD} = \frac{5}{8} \times 4 = 2.5 \text{ acres.}$$

$$\text{Area BDC} = \frac{3}{8} \times 4 = 1.5 \text{ acres.}$$

Problem VII. To divide off from a given tract of land in the form of a triangle, a given part by a line running parallel to one of the sides.

Let it be required to cut off from the triangular field ABC, Fig. 89, whose area is m , a part BDE, whose area is n , by a line DE running parallel to AC.

From similar triangles,

$$ABC : BDE :: AB^2 : BD^2$$

$$m : n :: AB^2 : BD^2$$

$$BD = AB \times \sqrt{\frac{n}{m}}$$

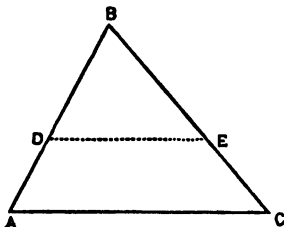


Fig. 89.



In the same way,

$$BE = CB \times \sqrt{\frac{n}{m}}$$

EXAMPLE.—Given a triangular field ABC, containing 5 acres, 2 roods, 20 perches; the side AB 8.64 chains, and the side BC 11.44 chains. It is required to cut off a triangle BDE, containing 2 acres 2 roods, by a line parallel to AC.

$$m = 5 \text{ acres } 2 \text{ roods } 20 \text{ perches} = 5.625 \text{ acres.}$$

$$n = 2 \text{ acres } 2 \text{ roods} = 2.5 \text{ acres.}$$

$$BD = 8.64 \sqrt{\frac{2.5}{5.625}} = 5.76 \text{ chains.}$$

$$BE = 11.44 \sqrt{\frac{2.5}{5.625}} = 7.63 \text{ chains.}$$

Problem VIII. To divide a quadrilateral field into 2 parts having a given proportion to each other by a line starting from a certain point on one side.

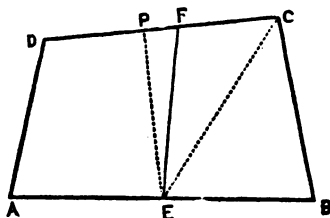


FIG. 90.

Let it be required to divide the quadrilateral ABCD, Fig. 90, into two parts which shall be to each other as m to n , by a line starting from E. Let A = area of the quadrilateral.

$$\text{Then, Area ADFE} = \frac{m}{m+n} \times A$$

$$\text{and, Area FCBE} = \frac{n}{m+n} \times A$$

Run a line from the point E to such a corner of the field that the line will divide the land most nearly into the parts required. Let EC be that line. Survey the part ADCE, and compute its area Q. The difference between this area and the area of ADFE will be the amount to be added to or subtracted from ADCE to make the right division.

If, as in the figure, the area ADCE is too great, the portion to be cut off must be a triangle, EFC having an altitude equal to the perpendicular EP, and an area equal to the difference stated. Hence, to find the base CF, we have

$$CF = \frac{ADCE - ADFE}{\frac{1}{2} EP} = \frac{Q - \frac{m}{m+n} A}{\frac{1}{2} EP}$$

In surveying the part ADCE, the angle ECP and the side EC would become known. From these we can compute the perpendicular EP.

$$EP = EC \sin ECP$$

$$CF = \frac{Q - \frac{m}{m+n} A}{\frac{1}{2} EC \sin ECP}$$

160. Problem IX. To divide any field into two parts having a given proportion to each other, by a line starting from a given point.

Let it be required to divide the field ABCDEFG, Fig. 91, into two equal parts by a line AM starting from A. Survey the entire field, and determine its area m . Suppose a line run from A to some corner D, to cut the field most nearly in the manner required. Compute the area of the part ABCD, and let it be represented by n . The difference between this area and half the entire field must be the portion to be subtracted from ABCD to give the required division. In the figure this is a triangle ADM, having a base DM and a perpendicular AP.

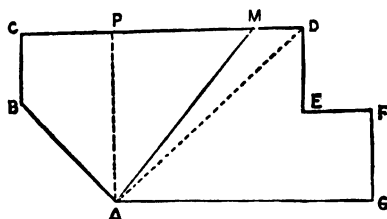


FIG. 91.

$$ADM = n - \frac{1}{2} m = \frac{1}{2} AP \times DM$$

$$DM = \frac{n - \frac{1}{2} m}{\frac{1}{2} AP}$$

To determine the perpendicular AP, when AD and the angle ADP are known, we have

$$AP = AD \cdot \sin ADP$$

$$DM = \frac{n - \frac{1}{2} m}{\frac{1}{2} AD \cdot \sin ADP}$$

CHAPTER X.

UNITED STATES PUBLIC LANDS.

161. THE vast territory which from time to time came into the possession of the United States, has all been, as the occasion required, surveyed, and laid out on a uniform system. This system was devised for the purpose of preparing for settlement the lands north of the Ohio River and east of the Mississippi, which had been ceded to the General Government by the States of New York, Virginia, Massachusetts, and Connecticut. It is first distinctly suggested in an ordinance reported by a committee of the old Congress, of which Mr. Jefferson was chairman, on the 7th May, 1784, but was not passed till May 20, 1785. Simeon DeWitt, when the report was presented, was geographer-in-chief of the Continental army, and it is believed that to him the committee owed the essential features of the method of dividing up the public lands, which has been developed into the most perfect system of land partition which has ever been devised. It is, in reality, the same method which was employed by the Roman land surveyors in dividing the conquered territory among the veterans of the army.

162. The system consists in dividing the land into equal squares by lines running north and south, east and west. One of these squares, the unit of the entire system, is called a **township**, and is six miles on each side. The township is divided, by lines one mile apart, into 36 **sections**, each of which is one mile square, and contains 640 acres. Each section is again subdivided into quarter sections, containing 160 acres. These townships, sections, and quarter sections are all run out upon the ground and marked by appropriate monuments, and correspondingly numbered upon the government maps.

163. This division of a territory into townships rests upon two principal lines, the one running east and west, called the **base line**; the other running north and south, termed the **principal meridian**. In making a survey, these two lines are first run, and the point of their intersection, or **initial point**, is always chosen at some prominent natural landmark, as the confluence of two rivers, or an isolated mountain.

164. The position of a township, with reference to these lines, is

designated by its number north or south of the base line, and its number east and west of the principal meridian. Thus, in Fig. 92, the township

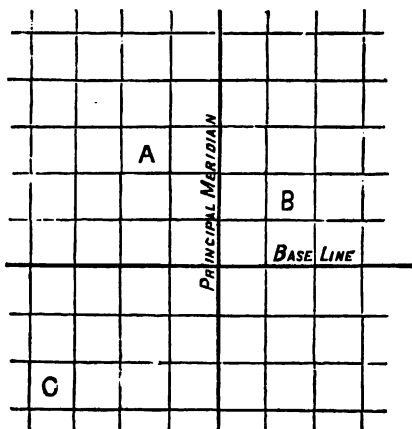


FIG. 92.

marked A would be designated as **township 3 N. range 2 W.**, that is, the third township north of the base line, and in the second range of townships, west of the principal meridian. The township marked B would be termed township 2 N., range 2 E., and that marked C would be township 3 S., range 4 W.

The sections into which a township is divided are designated by num-

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

FIG. 93.

bers beginning at the northeast corner, and running westward with the north tier of sections, eastward with the second tier, etc., as shown in Fig. 93.

165. Since the meridians in running north converge toward the earth's pole, it is plain they cannot be at the same distance apart at the north and south ends of a township. In latitude 45° , this convergence amounts to 109 feet in six miles. In a system of townships extending over a large territory, this convergence would materially modify the size of the townships and sections. To remedy this defect, it is provided in the government surveys that **standard parallels**, or correction lines, be run

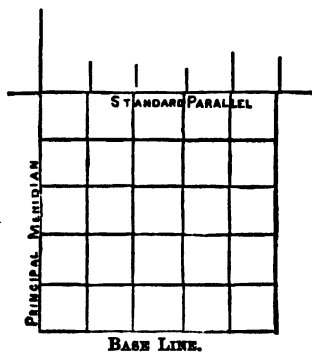


FIG. 94.

(since 1866) every thirty miles north and south of the base line. On these standard parallels the spaces of six miles are measured off anew. Fig. 94 shows how the meridians above and below the standard parallel do not coincide.

166. When a new territory is to be surveyed, it must be connected with some territory already surveyed, or a new base line and **principal meridian** established for it. Six **principal meridians** have been established, and are now connected with each other so as to form one continuous system. In addition to these, a number of independent meridians have been established in new territories, and have not yet been connected with the others by intermediate surveys. Thus, in Utah we have the Salt Lake meridian, in Oregon and Washington the Willamette meridian, etc. In the course of the progress of the surveys of the public lands, these independent meridians will all be connected with each other and with the more eastern meridians, thus making a complete network of rectangular townships from the Ohio River to the Pacific Ocean.

167. The first step in the survey of a new territory is the selection of an **initial point** from which the base line and principal meridian are to

start. This should be in the neighborhood of the lands whose survey is first desired. The principal meridian must then be run, the township corners and also the corners of the sections and quarter sections being marked upon it. It may extend both north and south of the initial point, and may be continued as the exigencies of the survey demand. The base line and the standard parallels are next run out, in both directions if required, and marked by monuments at the corners of the townships, sections, and quarter sections.

These fundamental lines of the survey must be run with the greatest precision possible. The instrument chiefly relied on is **Burt's solar compass**, a description of which is given in this chapter. The chain used is Gunter's 66 feet chain, and is to be frequently tested by comparison with a standard chain furnished by the Surveyor-General.

168. The township lines are always run in the order shown in the annexed diagram, commencing at the corner marked 1, and reaching in regular order the corners 2, 3, 4, etc. The object of this arrangement is

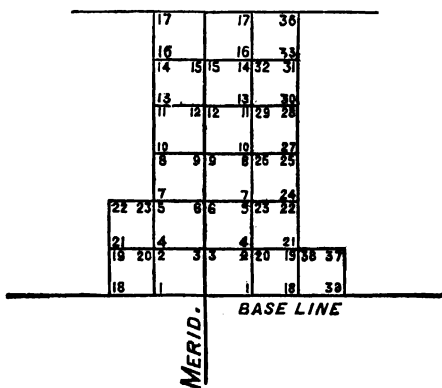


FIG. 95.

to throw all errors and corrections on the north and outside lines of the townships. In running the township lines, the section and quarter section corners are also determined and marked with monuments.

The lines for the sections are run in the same order as directed above for the townships; that is, the surveyor first runs No. 36, then No. 25, etc. Any errors are thus thrown on the upper and outside sections, and cannot extend beyond the next township boundary.

169. When a township or other line runs to some impassable barrier, as a river or lake, the surveyor is directed to establish at such point a permanent mark, called a **meander corner**, and then to run along such obstacle a line called a **meander line**. These meander lines determine the form and area of the space reserved from the lands surveyed. All navigable streams, all lakes of more than one mile in length are thus meandered, and reserved in the sale of the public lands.

170. The chief object of the Government survey is to establish the corners, and so to designate them that settlers can from them **locate** the lands which they may have purchased. The directions, therefore, issued to the surveyors, specify the mode of **marking the corners** so that they may be identified. The monuments are to be of stone, or of the most durable timbers in the vicinity. They are to be cut of specified figure and dimensions, and to be marked with appropriate devices, so that at any subsequent time it may be known to what township, section, or quarter section a given monument belongs. The meander corners are also to be appropriately marked.

On the standard parallels two sets of corners are required; one set facing the south, being the corners of the northern tier of townships, sections, etc., and one set facing the north, being the corners of the southern tier of townships, sections, etc. The former are called closing township corners, and the latter standard township corners.

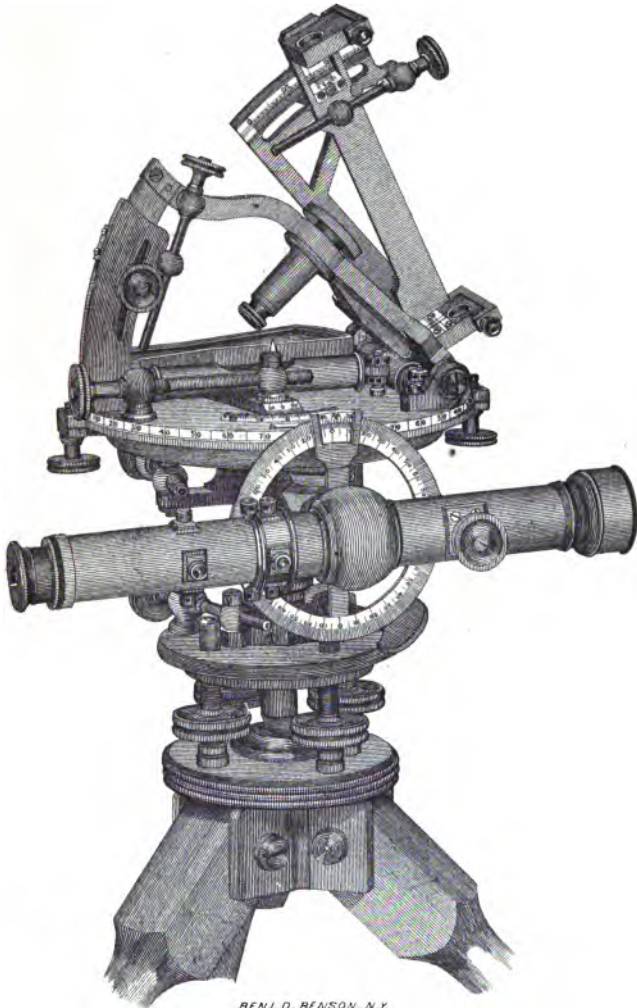
171. The surveyor is directed to make accurate note of the features of the land on which his lines are run. All timber lands, swamps, lakes, bayous, etc., are to be noted in his field-books. From these records of the surveys **maps** are constructed, on which the township lines are put down, and all the natural features, with the character of the soil, are indicated.

SOLAR COMPASS.

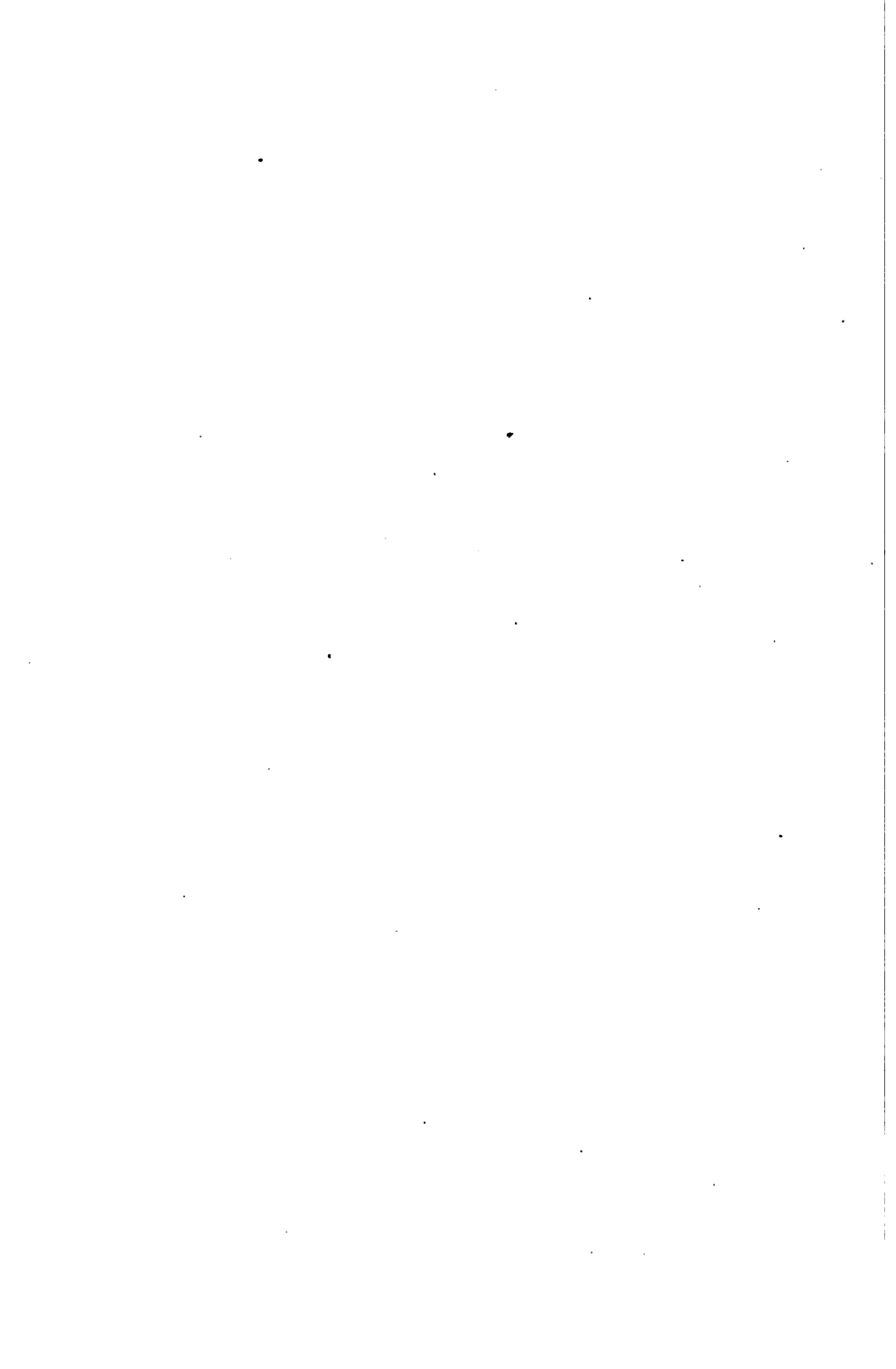
172. This instrument was invented by William U. Burt, of Michigan, and was patented by him in 1836. It was designed to furnish a ready and accurate method of determining a true meridian, and of running lines with reference to it. It has now come into general use in the surveys of the United States public lands, and in its present most approved form, as now manufactured by Messrs. W. & L. E. Gurley, of Troy, will be here described. The accompanying plate represents the instrument as now constructed by them.

173. Its essential parts are three arcs of circles, on which are set off the latitude of the place, the declination of the sun, and the hour of the day, and called respectively the **latitude circle**, the **declination circle**,

SOLAR TELESCOPE COMPASS,



BENJ. D. BENSON, N. Y.



and the **hour circle**. An examination of the plate will show how these parts are arranged.

The plane of the **latitude circle** is perpendicular to the horizontal limb of the instrument, and is shown in the plate at the left hand. It is connected with a curved bar, which may be called the latitude arm, and which has its centre of motion in two pivots, one of which is shown at the right hand of the figure. It is moved up and down by a tangent-screw, and is provided with a clamp-screw to secure it in any desired position. By means of a vernier it may be read to single minutes. To adapt it to the territory of the United States it has a range from 25° to 60° .

The **declination circle** is shown at the top of the plate, and is so divided as to read to single minutes, through a range of $23^{\circ} 28'$. Its vernier is attached to a straight movable arm, turning on a pivot at its lower end, and is moved by the tangent-screw, and may be clamped in position. The whole circle and its vernier arm are capable of being reversed by turning on an axis, so that the graduated arc may be at the lower end, and its pivot at the elevated end.

The **hour circle** is not shown in the plate, but lies at the right hand. It is used to set off the hour of the day. The middle of the arc is marked 12 o'clock, or 90 degrees. It is graduated on each side of this through an entire range of about 120 degrees, in divisions of half-degrees. The arc forms part of the movable arm of the latitude circle, and at its centre is the **polar axis**, on which the frame of the declination circle turns.

174. In addition to these distinguishing features, this instrument has sights like an ordinary compass, or, as in the improved form shown in the plate, has a telescope for more accurate vision; it has level tubes to set the plate horizontal, and it has a magnetic needle poised in a box, shown in the plate at the back of the figure. The needle has a range of about 36 degrees, graduated to half-degrees on an arc numbered from the middle point each way. The needle-box is moved by a projecting arm and a tangent-screw, and is used to set off the variation of the needle east or west.

The **horizontal limb** of the compass is divided so as to read by two opposite verniers to single minutes. The usual clamp and tangent screws serve to adjust this plate in position.

In addition to this, there is provided on the outer rim of the vernier plate a circle divided into parts of 5° or 10° , which may be used to make a rough estimate of any angle which does not require a more precise measurement.

175. At each end of the bar of the declination circle is a small convex lens set in a metal plate, having its focus adjusted so as to lie in the surface of the metal plate of the opposite end. Lines are drawn on this plate, two horizontal and two vertical, forming by their intersection a small square,

upon which the image of the sun, when adjusted to the meridian, is thrown. In the position of the instrument as shown in the plate, the image is formed by the lens at the upper end of the arm, upon the plate at its lower end. When the position is reversed, the image is formed by the other lens upon the plate opposite.

176. To explain the method of using this instrument, we will suppose, **first**, that the sun is on the equator, as it is on the 21st day of March and the 21st day of September, that is, the sun's declination is zero. **Secondly**, we will assume that it is exactly noon by the sun, that is, that the sun occupies a position precisely south. **Thirdly**, we will assume the latitude of the place occupied is 45° . Proceed as follows :

1. Set the latitude circle at 45° .
2. Set the declination circle at zero.
3. Set the hour circle at 12, the middle point.

It is evident now, that if the plane of the latitude circle of the instrument were set north and south, the plane of the hour circle would coincide with the plane of the equator, the polar axis would point directly to the north pole, and a line through the declination arm would point directly at the sun.

In that case, the sun's rays, passing through the lens at the upper end, would be brought to a focus in the little square on the opposite plate. If the image does not fall directly between the vertical lines which form this square, it would indicate that the instrument is not directed truly north and south, and must be moved right or left until the image is brought into position.

177. During the part of the year from March 21st to September 21st, the sun is north of the equator, that is, has north declination. From September 21st to March 21st, it is south of the equator, that is, has south declination. In the former case, the declination circle must be placed as shown in the plate, and then must be set to the proper north declination for the day. In the latter case, the declination circle must be reversed on its axis, and set to the proper south declination. The nautical almanac, which must be used in connection with this instrument, gives the proper declination for noon of each day at Greenwich, that is, for intervals of 24 hours. The change of declination from the noon of Greenwich to the hour of observation at any other place must be taken into account. Fifteen degrees of longitude correspond to one hour of time.

Thus, suppose the observation were made at 2 o'clock, in west longitude 90° . The difference in longitude is equivalent to a difference of 6 hours in time. Hence, the whole time subsequent to the Greenwich noon is 8

hours. The change per hour is given in the almanac, hence it is easy to find the change for 8 hours.

178. Assume now that the latitude circle has been set to the proper latitude of the place, and the declination circle set to the declination of the sun for that day, place, and hour.

When the instrument is north, and the polar axis is directed truly to the north pole, and the declination arm set to the sun's declination; by revolving the declination circle on its axis, we could follow the position of the sun in its path from the east to the west, keeping the declination arm constantly pointed toward it. At 6 hours before solar noon the sun would be due east, and when the arm is directed to the sun at that time, the hour angle reads 6; at 4 hours before noon it would read 4, etc. This enables us to use the solar compass at any hour of the day as well as at noon.

When the latitude circle is set, and the declination circle set, and the hour circle set to the proper hour before or after noon by the sun; then, in order that the solar image may be formed by the lens in the declination arm upon its opposite square, **the instrument must be north and south.** Hence, we must move it right or left until the solar image falls in the required place; then we know the instrument to be in the meridian.

179. The noon of a good watch or clock does not, in general, coincide with noon by the sun. They differ by an amount which is called the **equation of time**, and which is given for each day in the year in the nautical almanac.

Now, since the hour to be set off on the hour circle of the solar compass is the time before or after noon by the sun, the time given by the watch or clock will require to be corrected by the equation of time, before it can be used for this purpose.

Thus, let the observation about to be made take place at 3h. 18m. 30s. P.M., May 24, 1872. From the nautical almanac for May 24, we find

Clock time	=	3	18	30
Equation of time	=		3	24
Sun time	=	3	21	54

180. By the **refraction** of the atmosphere, the rays coming to the eye are slightly bent out of a straight line, so that all objects except those directly overhead appear higher than they really are. At the horizon this effect is greatest, and amounts to about 34', so that we see the whole discs of the sun and moon above the line of the horizon, when they are still

really below it. In using the solar compass, this displacement of the sun by refraction requires to be taken into account.

The correction requires in all cases to be added to the true altitude, when, as in using the solar compass, we wish to obtain the apparent altitude.

The amount of refraction varies with the altitude, being greatest at the horizon, and zero at the zenith. Tables of refraction are given in the nautical almanacs, and also in manuals of astronomy. For the precise operations of astronomy, the changes produced by variations in the thermometer and barometer require to be taken into account. This is not necessary in surveying. Below is given a table showing the refractions at a mean temperature of 60° Fahr.

TABLE OF REFRACTIONS.

APPARENT ALTITUDE.	MEAN REFRACTION.	APPARENT ALTITUDE.	MEAN REFRACTION.
0° 0'	34' 54"	11° 0'	4' 49"
0 20	30 52	12 0	4 25
0 40	27 23	13 0	4 5
1 0	24 25	14 0	3 47
1 30	20 51	15 0	3 32
2 0	18 9	20 0	2 37
2 30	16 1	25 0	2 3
3 0	14 15	30 0	1 40
3 30	12 48	35 0	1 22
4 0	11 39	40 0	1 9
5 0	9 47	45 0	0 58
6 0	8 23	50 0	0 48
7 0	7 20	60 0	0 33
8 0	6 30	70 0	0 21
9 0	5 49	80 0	0 10
10 0	5 16	90 0	0 0

181. The effect of refraction is to increase the north declination of the sun over that given in the nautical almanac, and to decrease the south declination. When the sun is in the meridian the effect of refraction is in the same line with the arcs of declination, and, therefore, the corrections for refraction may be taken directly from the tables of refraction. Take out the declination of the sun for the day, and correct it for the hour and the longitude. Then, if the declination is north, add the refraction; if south, subtract it.

Thus, let the observation about to be taken be in latitude $41^{\circ} 30' N.$, longitude $90^{\circ} W.$, at noon of May 24th, 1872.

Altitude of equator = $90^{\circ} - \text{lat.}$	= $48^{\circ} 30' 00''$
Declination of sun, Greenwich noon	= + $20 52 47$
Correction for 90° long.	= + $2 44$
	= $69 25 31$

Altitude of sun at observation	= $69 25 31$
--------------------------------	--------------

We then have

Correction for refraction	= 21
Declination at observation	= $20 55 31$
	= $20 55 52$
Corrected declination	= $20 55 52$

When the sun is either east or west of the meridian, the refraction, by changing its altitude, does not change its declination by its full amount. Thus, in figure 96, let s represent the true position of the sun, sD its declination, and ED its hour angle. Let ss' represent the effect of refraction upon the altitude; its effect upon the declination would be sm , and its effect on the hour angle DD' .

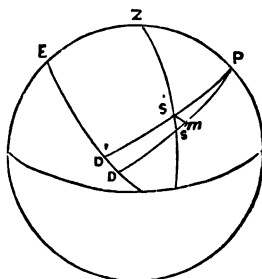


FIG. 96.

The values of sm and DD' for all hours from the meridian, and for different latitudes, have been computed, and are usually given in a table like the following. They are given as fractions of the value of ss' , which can be got from the table of refractions. Thus, in latitude 40° , for an observation three hours before or after the meridian, the effect of refraction on declination is $\frac{84}{100}$ of the refraction due to the altitude, and the effect on the hour circle is $\frac{54}{100}$. For example, if the altitude be 70° , we have

Refraction due to altitude	= $0' 21''$
Declination component	= $.84 = 0 17.6$
Hour component	= $.54 = 0 11.3$

In ordinary practice with the solar compass, the surveyor, by the aid of lines drawn on the plate, called refraction lines, estimates the effect of refraction, without having recourse to the tables.

PROPORTION OF REFRACTION TO BE ALLOWED IN HUNDREDTHS OF THE WHOLE.

LAT.	ON DECLINATION ARC.						ON THE HOUR ARC.					
	HOURS FROM THE MERIDIAN.						HOURS FROM THE MERIDIAN.					
	1.	2.	3.	4.	5.	6.	1.	2.	3.	4.	5.	6.
30	97	90	79	66	55	50	22	43	61	75	84	87
32	98	91	80	68	57	53	22	42	60	73	82	85
34	98	91	81	71	60	56	22	42	59	71	80	83
36	98	92	82	72	62	59	21	40	57	70	78	81
38	98	92	83	73	65	62	20	39	56	68	76	79
40	98	92	84	75	67	64	20	38	54	66	74	77
42	98	93	85	77	69	67	19	37	53	64	72	74
44	98	93	86	78	72	69	19	36	51	62	69	72
46	98	93	87	80	74	72	18	36	49	60	67	69
48	98	94	88	81	76	74	18	33	46	58	65	67
50	99	95	89	83	78	77	17	32	45	56	62	64

USE OF THE SOLAR COMPASS.

182. To determine the latitude of a place. This requires to be determined frequently in the progress of the surveys—usually every clear day—since the latitude is required in running lines with the solar compass. The observations are best made at noon.

A short time before the anticipated occurrence of the sun noon, set up the instrument and level it. Set off on the declination circle the declination of the sun for that day and longitude, and correction for refraction. Set the hour circle at 12. Then turn the instrument until the solar image falls between the hour lines of the plate, and, by the tangent screw of the latitude circle, bring the image between the equatorial lines. As the time advances toward sun noon, the sun rises higher, and the image will fall below the equatorial lines. Follow the image with the instrument, constantly keeping it between the four lines. When the sun has reached its highest point, at sun noon, its further progress will make the image fall above the equatorial lines. Arrest the movement of the latitude circle at the point when the image ceases to move down, and begins to move up across the equatorial lines.

Read off on the vernier of the latitude circle the arc indicated. It must be the latitude of the place.

183. To determine the true time by the solar compass. It is necessary for the surveyor, in running lines, to use the time given by his watch or chronometer. In order that this may be relied upon, he must take frequent occasion to test and correct it.

In observing for latitude, as explained in the last article, the instant when the sun is at its greatest height is the instant of sun noon. Hence, the same observation will serve to determine the latitude and the time. Note the time by the watch or chronometer when the sun is at its meridian altitude. Correct this for the equation of time as given in the nautical almanac for that day. The result will be the reading of the watch or chronometer at true clock noon, and the difference between this and 12 will be the error of the watch or chronometer.

If the instrument is already set north and south, which may be done upon any meridian line, the hour can be determined at any hour as well as noon. Set the latitude and declination circles. Then turn the hour circle until the image of the sun falls truly between the lines. The hour read off on the vernier of the hour circle will be the sun time of the observation. Correct this for the equation of time, and compare with the time of the observation as given by the chronometer. The difference will be the error of the chronometer.

184. To run lines with the solar compass. The lines to be run in the government surveys are principally north and south lines or east and west lines. The instrument, however, can be used with equal facility for running any other lines.

North and south lines. Set the instrument over the starting-point and level it. Set off the latitude, declination, and hour upon their proper circles, and clamp them fast. Clamp the horizontal limb at zero, and bring the instrument approximately north and south by the magnetic needle. Lastly, by the tangent screw move the horizontal limb till the solar image falls precisely in its position between the lines. The instrument then points north and south, and may be used to locate points in the meridian.

East and west lines. Proceed as before, except that the horizontal limb must be clamped, not at zero, but at 90° . The line of sight then stands east and west when the solar apparatus stands north and south.

Oblique lines. To determine the bearing of any line with the meridian, set all the circles as before, and turn the telescope in the direction of the required line. Move the solar apparatus until the solar image falls in position. The angle now read on the horizontal limb will be the bearing required.

185. The magnetic needle in this instrument is of service to bring it into approximate position, and to run lines when the sun is obscured, or when less precision is required. The meander lines along the banks of rivers and lakes are usually thus run. The section lines, especially when no local attraction disturbs the action of the needle, may be run with the needle alone. But the township lines, and, more particularly, the standard parallels, are all to be run with the solar compass.

Before using the needle, the surveyor determines its variation. This is done by putting the instrument into the meridian by the sun, and then the reading of the needle will be its variation. The needle-box may then be turned by its tangent screw, until the needle reads zero; then the instrument is ready to be used as a vernier compass.

186. The telescope in the solar compass, as represented in the preceding plate, is placed at the circumference of the horizontal limb. The displacement of the line of sight from the centre of the instrument may be disregarded in long sights, or by a simple contrivance the target on the target-rod may be set as far out of line as the telescope is displaced.

The use of the telescope on the solar compass enables the surveyor to use the micrometer method of measuring distances, as described in article 43. In crossing rivers, swamps, lakes, and other impediments, this method becomes of the greatest service; its accuracy in skillful hands being almost, if not quite, equal to that of the best chain measurement.

CHAPTER XL

THE PLANE TABLE.

187. WHEN the object of the survey is to construct a map of the country surveyed, and not to obtain the dimensions of given lines and tracts of land, an instrument called a plane table has been used with great success. In the United States Coast Survey this instrument is relied on to furnish the details of topography, and under the skillful manipulation of its officers, the construction and use of the table have been carried to a high degree of perfection. We are chiefly indebted to the essay of Lieut. Harrison, published in the United States Coast Survey Report for 1865, for such details concerning its construction and use as we shall here give.

188. In the plate annexed, Fig. 97, which is copied by permission from the Coast Survey Report, will be seen the arrangement of a plane table as now employed. It consists of a well-seasoned board, about 24 by 30 inches, and three-fourths of an inch thick. To prevent warping, it is made in two layers, with the grain running in opposite directions.

It is supported on a tripod and tripod-head, and provided with leveling, clamp, and tangent screws, so that it may be levelled, clamped, and adjusted to any position.

The paper on which the map is to be drawn is spread upon the board, and held fast by spring clamps. When the sheet is larger than the board, it may be fastened in a roll under the edge of the board, and as fast as one part of the sheet is filled a new part may be unrolled, and the finished part rolled up. If a small sheet only is required, it may be fastened to the board by drawing-tacks.

189. The *alidade* used with the plane table is a brass ruler with bevelled edge, about 20 inches long, from which rises an upright standard bearing a telescope. Instead of the telescope, this ruler is sometimes provided with two upright sights at the ends. The telescope is the same in construction as that used in the transit, and may be provided with a micrometer to measure distances. An arc of a vertical circle attached to the axis of the telescope serves, when required, to measure the angles of elevation or depression of the line of sight. A circular spirit-level is inserted in the brass ruler, which serves to determine when the board is horizontal.

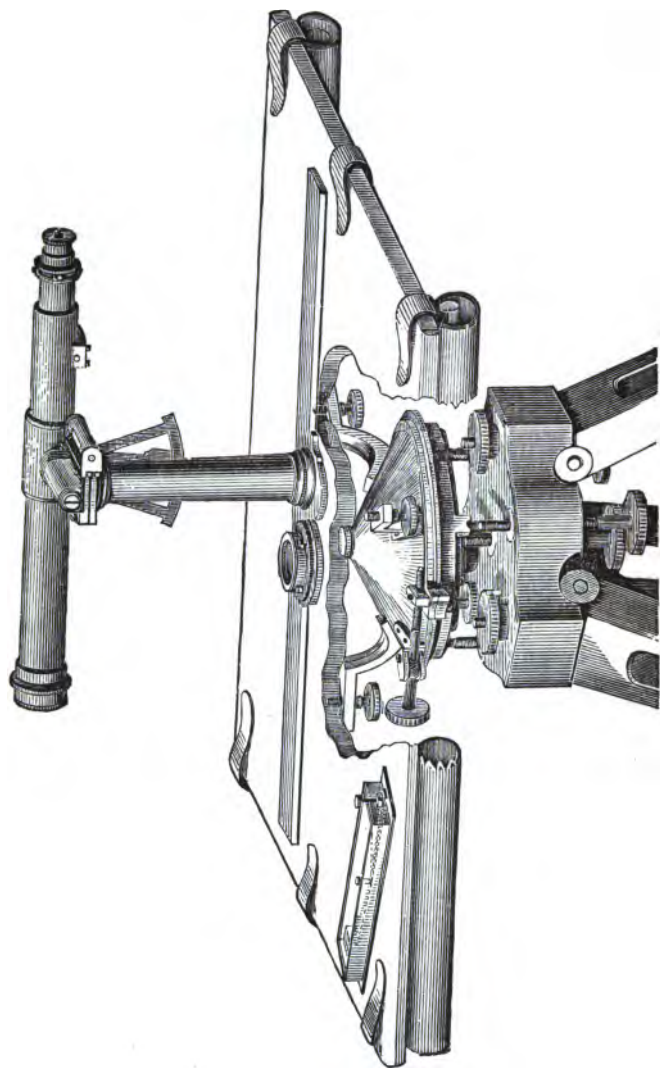


FIG. 97.

A box containing a magnetic needle also accompanies the board. By means of this the magnetic bearing of any line drawn upon the table can be ascertained, and the table itself can be adjusted to any position in the field.

In addition to the apparatus described, the surveyor must be provided with a pair of dividers and a scale, and with pencils and India-rubber. An oil-silk or India-rubber cover should also be at hand to protect the table and instruments from wet and dust.

190. The object of the field operations with the plane table is to construct a map or representation of points and lines upon the ground. It is really a drawing-board taken into the field, and as fast as the lines and points are determined they are plotted in their true location and dimensions upon the paper. The location of points in their true position with respect to one another is accomplished by four methods, which, in actual practice, are more or less used in conjunction. These have received the names of the method by **radiation**, the method by **progression**, the method by **prosection**, and the method by **resection**.

191. **By radiation.** Let it be required, for example, to make a map of the field ABCDE, Fig. 98. Set the plane table up at some point in the field from which all the corners of the field can be seen. Bring it to a level by means of the levelling screws, and clamp it fast in position. Point the alidade at each of the stations successively, and draw indefinite lines on the paper from p , the point of the paper directly over the point on the ground, toward each of the stations pA , pB , etc. Measure with the chain, or with the micrometer telescope, the distance from the station occupied by the plane table to each station, and lay off upon the drawer the lines drawn upon the paper, with a suitable scale, these measured distances, pa , pb , etc. Join the points a , b , c , etc., by straight lines, and the resulting figure $abcde$ will be a true representation of the field.

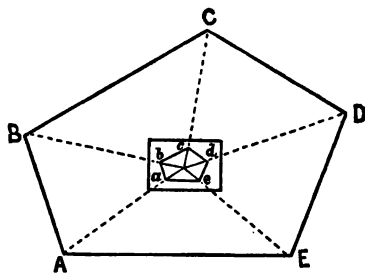


FIG. 98.

The position of any other objects, such as buildings, trees, bridges, etc., may be determined by the same method, by drawing with the alidade a line to each object to be located, and on this laying off to the same scale the measured distance.

When the micrometer telescope can be used for measuring the distances in this method, the operations are exceedingly rapid and easy. And when

When the micrometer telescope can be used for measuring the distances in this method, the operations are exceedingly rapid and easy. And when

extreme accuracy is not required, no way is more convenient for executing a sketch of a tract of ground.

192. By progression. In this method the plane table is carried forward and made to occupy successively the stations A, B, C, etc., Fig. 99. Set it up first at station A, and mark some point *a* on the paper to represent A.

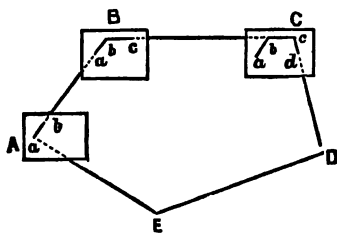


FIG. 99.

With the alidade draw through this point *a* an indefinite line directed toward the station B in the field. Measure with the chain or otherwise the distance AB, and lay off this distance *ab* with a suitable scale; *b* will be the location of the point B in the plot.

Remove the plane table to B. Lay on the alidade so as to coincide with *ba* on the paper, and turn the table to the right or left

until the alidade points back at the preceding station A. It is now in exactly the same position with respect to the points of the compass as when at A.

With the alidade draw a line through *b* directed toward C, and on this line lay off the measured distance BC to the same scale as the preceding line; *c* will then be the location of C in the plot. Proceeding thus, we can lay down every station of the field. When we have gone entirely round a field, the last line ought to close with the first station *a*.

193. The two preceding methods may often be used conjointly. Thus, in surveying and mapping a road and its adjacent objects, the line of the road itself may be traversed by the plane table, and each of its angles occupied as a station, as in the method of progression. At the same time, by the method of radiation, the location of adjacent objects, such as buildings, trees, etc., may be determined. The general features of the ground may further be sketched in by eye, and thus, while traversing the ground, a complete and faithful map of the line and all noteworthy adjacent objects may be executed.

194. By prosection. The principle depended on in this method for determining the location of points is the intersection of two lines drawn from two points toward a third. Let it be required to make a map of the stations A, B, C, etc., Fig. 190. Measure with great care a base-line MN, so situated that from its two extremities all the stations to be located can be seen. Set up the plane table at one of these stations, as M, and direct the alidade toward N, and draw an indefinite line. On this, set off to some convenient scale *mn*, representing MN.

With the table in the same position, direct the alidade toward each of the stations successively, and draw the indefinite lines mA , nB , etc. Remove the table to N , place the alidade so as to coincide with mn , and then turn the table until the alidade points back toward M . It is then situated in the field as at M . Through n draw lines with the alidade

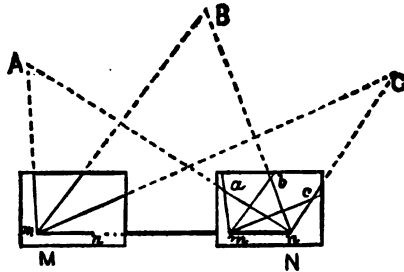


FIG. 100.

toward each station, nA , nB , etc. The points a , b , c , etc., where lines drawn toward corresponding stations from m and n intersect each other, will be the true location of the points A , B , C , etc., on the map; and their distances from each other, and from m and n , will be represented on the same scale as was used in constructing the base-line mn .

If we are surveying a definite tract of ground, as a field, one of the sides of the field might be occupied as a base-line when convenient. Or the base-line might be taken entirely outside of the space to be surveyed. The shores of a lake or harbor, or the features of some inaccessible island, may in this way be mapped without occupying any part of it.

195. By resection. In this method of using the plane table, one line of the field must be measured, as, for example, AB in Fig. 101. Lay off on the plane-table sheet a line ab , representing AB , to a suitable scale. Place the plane table at B , and with the alidade on the line ab direct the sights toward A . The line ab will then be parallel to AB . With the alidade directed toward C draw the indefinite line bC . Remove the table to C , and with the alidade on bC point back toward B . Turn the alidade so that, touching the point a , it will point toward A . It will intersect the line bC in some point c , which will be the true location of the point C . Through C draw a line toward D ; then remove the table to D , and proceed as at c .

While the leading points of the map are thus determined, the location of any other points on either side of the line may be determined by radi-

ation or by prosection. The minute details of the topography may also be sketched in by eye as the survey proceeds.

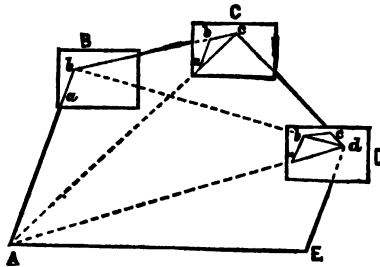


FIG. 101.

196. To orient the table. To place the table in such a position that the lines of the plot shall be parallel to the lines of the field, is called **orienting** the table. It signifies the adjustment of the table to the points of the compass, one of which is the orient or east.

In the preceding explanations we have shown how this could be done when the table is located at some point of a line on the ground, which has been drawn upon the table. We need only to lay the alidade on this line, and then turn the table until the telescope points at some other point in the same line.

But when the table is to be removed to some new point P, which is not connected with any other point in the field by a line already constructed, the method there used

will not apply. It can always be done approximately by using the compass-box which accompanies the plane table. At any preceding station where the table had been in position, lay the compass-box so that its straight edge shall coincide with any line on the table, and read the bearing of the needle. When the table has been removed to the new point P, lay the compass-box upon the same line and turn the table until the needle indicates the same bearing.

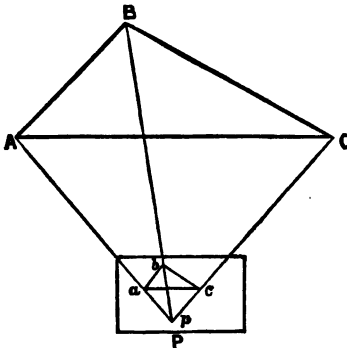


FIG. 102.

ing. The table must approximately be then in the same position as at the preceding station.

197. Three-point problem. Whenever we have three stations of the field already located on the plane-table sheet, we can always determine the location of any other point of the field from which these three points can be seen. This is called the three-point problem, and is of the most constant use in operations with the plane table. It includes, of course, the **orientation** of the table at the new station.

Let A, B, C, Fig. 103, be three stations whose positions on the sheet have already been settled at a , b , and c ; and let it be required to fix on the sheet the location of the station P, now occupied by the plane table. It is evident if the plane table could be **oriented**, the point would be determined by **resecting** from A through a , and from B through b , and from C through c . These lines ought all to intersect at one point, which would be p , the location P.

Having set the table over the station P, orient it approximately by the compass-box. Then with the alidade draw lines through a toward A, through b toward B, and through c toward C. If the orientation is not perfect, these lines will, by their intersections, form a small triangle, as shown in the figure, called the **triangle of error**.

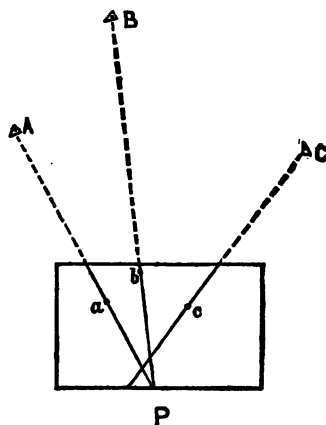


FIG. 103.

198. From this triangle of error and the three points a , b , and c , the true point can always be obtained by geometrical construction. Thus, let a , b , c , Fig. 104, be the locations of three points A, B, C. Describe a circle through a , b , and through the intersection of the resection lines drawn through these points. Describe another through a , c , and through the intersection of their resection lines, and a third through b , c , and the point bc . These circles will intersect each other at some point, which will be the location required.

This construction applies equally to the case where the station P lies within or without the triangle ABC. When the station is within the triangle, the point will be within the triangle of error, and when P is without ABC, will be found without the triangle of error.

The construction of the point, as above described, is attended with so many inconveniences in the field that it is rarely resorted to. The prac-

tised surveyor is able by trial to make the adjustment of the table with great rapidity, and with the additional convenience of avoiding the multiplication of lines upon the sheet.

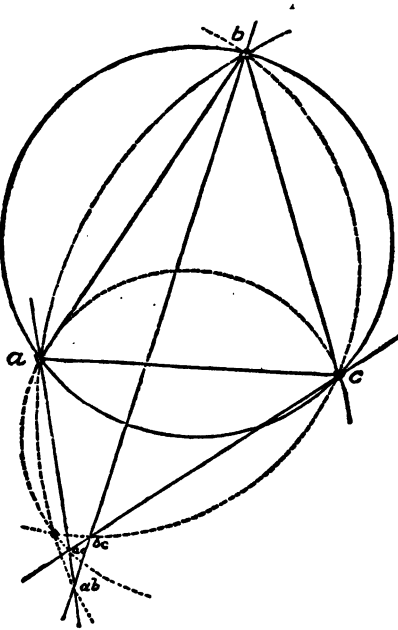


FIG. 104.

The method of determining the position of a point with reference to three points already located, is explained in Problem V., Chapter IV. When the angles from the required point to the known points have been measured, the location of the point may be obtained by the construction there illustrated. The usual plane-table method differs from that in not requiring the actual measurement of the angles.

CHAPTER XII.

LEVELLING.

199. THE process of **levelling** consists in finding the difference of level between two points, or in finding the heights above a fixed horizontal line of the successive points of a line on the surface of the earth. Two points are said to be in the same level when they are at the same distance from the centre of the earth. A level line or a level surface is one in which all points are equally distant from the centre of the earth. The surface of quiet water is a level surface. Any line or surface parallel to the surface of quiet water would be level.

The subject of levelling belongs to land surveying so far as it serves to determine the variations in the surface of the ground, and the consequent location of roads, drains, watercourses, channels for irrigation, etc. We propose, therefore, to discuss the subject only so far as it is connected with the surveying of lands, and not in reference to general engineering work.

200. According to our definition, a level line is a curved line having the centre of the earth for its centre. If a telescope be made level at the point A, Fig. 105, on the surface of the earth, its line of sight would be AT, which is a tangent line to the true level line at that point. The

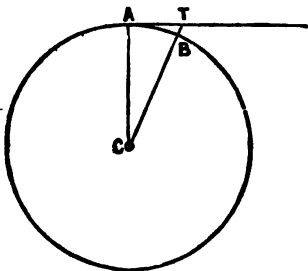


FIG. 105.

line AT is called the **apparent level** line, and AB the **true level**. At the distance AB they differ from each other by the distance BT.

From geometry we know

$$BT (2BC + BT) = AT^2.$$

Neglecting BT in comparison with 2BC, and representing the radius of the earth by R, the difference between true and apparent level BT by h , and the distance AB, nearly equal to AT, by d , we have

$$2R^2h = d^2$$

$$h = \frac{d^2}{2R}$$

For 1 mile, $h = 8$ inches, nearly.

For 2 miles, $h = 32$ inches, nearly.

For $\frac{1}{2}$ mile, $h = 2$ inches, nearly.

For $\frac{1}{4}$ mile, $h = \frac{1}{2}$ an inch, nearly.

In general, the value of h varies as the square of the distance. Hence, to find the value of h , multiply its value for one mile by the square of the number of miles.

201. The effect of the refraction produced on rays of light by the atmosphere is to decrease this difference between true and apparent level by a small amount. The annexed table gives the combined effect of the two corrections, at different distances.

CORRECTIONS OF APPARENT LEVEL, FOR THE EARTH'S CURVATURE AND FOR REFRACTION.

D = distance in feet ; h = correction in feet.

D.	h .	D.	h .	D.	h .	D.	h .
300	.002	1800	.066	3300	.223	4800	.472
400	.003	1900	.074	3400	.237	4900	.492
500	.005	2000	.082	3500	.251	5000	.512
600	.007	2100	.090	3600	.266	5100	.533
700	.010	2200	.099	3700	.281	5200	.554
800	.013	2300	.108	3800	.296	1m.	.571
900	.017	2400	.118	3900	.312	2m.	2.285
1000	.020	2500	.128	4000	.328	3m.	5.142
1100	.025	2600	.139	4100	.345	4m.	9.142
1200	.030	2700	.149	4200	.362	5m.	14.284
1300	.035	2800	.161	4300	.379	6m.	20.568
1400	.040	2900	.172	4400	.397	7m.	27.996
1500	.046	3000	.184	4500	.415	8m.	36.566
1600	.052	3100	.197	4600	.434	9m.	46.279
1700	.059	3200	.210	4700	.453	10m.	57.135

202. Plumb-line level. The direction of a plumb-line is vertical, and therefore at right angles to a level line. If, therefore, an instrument, having two arms AB and CD at right angles to each other, be so placed that the arm CD is vertical, the other arm AB will be horizontal. By suspending a plumb-line at C, we can determine when CD is vertical. Such an instrument might be employed for finding a level line.

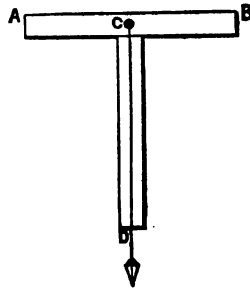


FIG. 106.

203. Water level. A tube bent upward at its two ends, and filled with a colored fluid, furnishes a simple expedient for obtaining level lines. The vertical parts of the tube must be of glass; the horizontal part may be of tin or lead, into which the glass ends may be cemented. The glass ends may be corked while the instrument is carried.

When the instrument is set up, and the ends uncorked, the fluid will at

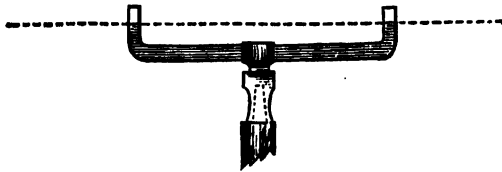


FIG. 107.

once come to the same level in the two ends. By sighting over the surfaces of the liquid in the two ends, a level line will be obtained.

204. Spirit level. A ground-glass tube slightly but uniformly bent, is filled with spirits, only leaving a small bubble of air, which will always occupy the highest point of the tube. This bent glass tube is then set in



FIG. 108.

a straight brass tube, cut away so as to show the convex side. This may be set in a block of wood having parallel sides, as in the mason's level, or

it may be attached to the horizontal plate of a compass or transit. Its use in this latter case has already been explained, in the chapter treating of angular instruments.

THE Y-LEVEL.

205. The instrument chiefly employed in surveying for determining the differences of level between points, is called the **Y-level**. It is shown in the accompanying plate. It consists of a telescope, somewhat longer than that used in the transit, mounted on two brass supports, from their shape called wyes (Y). The cross-bar to which the wyes are attached is at right angles to the axis or spindle on which it is turned in a horizontal plane. Directly beneath the telescope, and attached parallel to it, is the spirit level, by means of which the line of collimation of the telescope may be rendered horizontal. There is a scale placed on the brass tube of the spirit level, so marked as to show the position of the bubble.

The telescope is provided with a diaphragm at its focus, consisting of a horizontal and vertical thread. The instrument is supported on a tripod and tripod-head, and is levelled, clamped, and moved in the same way as the transit.

206. The adjustments of the level are three in number: first, to adjust the line of collimation; second, to render the spirit level parallel to the telescope; and, third, to render the axis perpendicular to the telescope and level.

To adjust the line of collimation. Set the tripod firmly, and loosen the collars holding the telescope in the wyes, so that it may be revolved freely. Bring the intersection of the threads upon some small, well-defined object. Then turn the telescope over in the wyes without disturbing the supports. If, on being revolved, the horizontal thread falls above or below the mark, the cross-threads are out of position in that direction. By the small screws provided for that purpose, bring the thread half-way back to the mark; and then, by the levelling-screws, bring it upon the mark. Repeat the same operation until the revolution of the telescope shows no disturbance of the pointing. The same process must be used to adjust the vertical thread.

207. To adjust the spirit level. The axis of the spirit level must be parallel to the line of collimation, so that it may serve as an indication of the telescope being level. Open the clips or collars holding the telescope. Bring the telescope over one pair of levelling-screws, and by means of them bring the bubble to the middle. Lift the telescope from the wyes, and replace it in a reversed position. If the level tube is paral-

lel to the telescope, the bubble will remain in the middle. If one end is nearer to the telescope than the other, the bubble will run to that end. Lower that end of the level until the bubble returns half-way back to the middle ; then, with the levelling-screws, bring the bubble to the middle. Repeat the operation until the reversal of the telescope in the wyes does not disturb the bubble.

The level tube may sometimes be out of adjustment by not being in the same vertical plane with the telescope, that is, the level may stand awry below the telescope. Roll the telescope a little in the wyes, so that the bubble tube is brought to one side of the cross-bar. If the bubble remains in the centre, the tube is not awry ; if the bubble runs to one end, that end projects too far, and must be brought back by the screws provided for the purpose.

208. To adjust the axis. The spindle of the instrument must be at right angles to the line of collimation of the telescope, so that when the telescope is levelled, and made to revolve on the spindle, it may describe a horizontal plane surface. The previous adjustments having been effected, bring the telescope directly over one pair of levelling-screws, and bring the bubble to the middle. Unclamp the axis and turn the telescope into a reverse position over the same pair of levelling-screws. If the bubble remains in the middle, the spindle is perpendicular. If it runs to one end, the wye supporting the telescope at that end is too high. By the nuts on the wyes, bring the bubble half-way back, and then, by the levelling-screws, bring it entirely to the middle. Repeat the operation until the reversal of the telescope on its spindle does not disturb the bubble. The adjustment is then complete.

209. Levelling-rod. In connection with the levelling instrument above described, a levelling-rod is required. It consists of a long rod of wood, Fig. 109, divided into feet, tenths, and hundredths. It is generally made in two parts, one of which slides down upon the other ; when drawn to its full length it is about 12 feet long. A target, or vane, is attached



FIG. 109.

to the rod, and slides up and down upon it. This target carries a small vernier by which the heights are read to thousandths of a foot.

The rod is to be held vertically by an assistant, over any station whose level is to be taken.

210. To find the difference of level between two stations. Let it be required to find the difference of level between two stations A and B, Fig. 110. Set up the levelling instrument at some point between the two

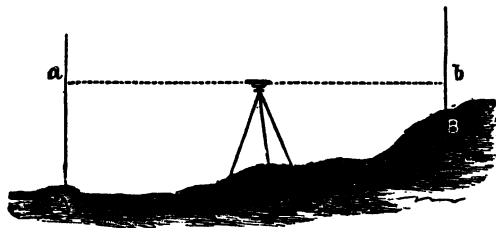


FIG. 110.

stations, and as nearly equally distant as can be estimated, and make it accurately level. Let an assistant hold a levelling-rod at A, and at a signal from his chief move the target up or down until the horizontal thread of the telescope exactly cuts it. The reading of the target will give the distance Aa , the distance of the line of sight of the telescope above A. The assistant now moves the rod to B, and the distance Bb is read. It is plain that

$$\text{The difference of level between A and B} = Aa - Bb.$$

It is not necessary in this operation that the level should be placed in a line between A and B. The plane of the instrument is the standard to which the height of both points is referred, and this may be placed anywhere within sight.

If, as was directed, the level is placed equally distant from the stations, the errors arising from the difference between true and apparent level exactly balance each other. For short distances the error may be disregarded. When long sights are used, the observations upon each station must be corrected by the table given in article 201.

211. When no station can be found for the level from which the two points can be seen, it is necessary to make use of intermediate stations. Thus, let it be required to find the difference of level between two distant points A and D, Fig. 111. Select some station from which A and also some intermediate station B, in the direction of D may be seen. The readings on A and B will give the difference of level between these two points. Move

the instrument to a new station from which B, and a new point C, further on, may be seen; the observations on these will give the difference be-

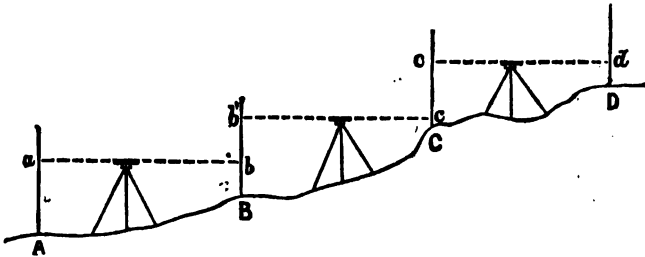


FIG. 111.

tween B and C, which being added to or subtracted from the difference between A and B, will give the difference between A and C. Proceeding thus, we finally obtain the difference between A and D.

212. The readings on stations looking backward toward the starting-point A, are called **back-sights**. Those taken on stations looking forward toward D, are called **fore-sights**. From an examination of the figure it will be plain that the difference of level between A and D will be found by taking the difference between the sum of the back-sights and the sum of the fore-sights. If the sum of the back-sights exceeds the sum of the fore-sights, the last point is higher than the first, and *vice versa*.

213. **To level for a section or profile of land.** When the relative elevations of a succession of points A, B, C, Fig. 112, etc., are required, it

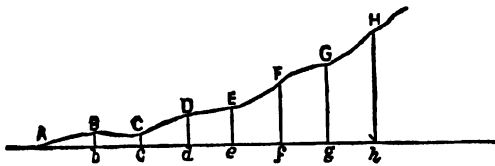


FIG. 112.

is customary to refer all the points to an imaginary horizontal line, which may be assumed to pass through the initial point, or, if more convenient, at any distance below it. This line is called the datum level.

When the surveyor is required to determine the rise and fall of a line running over the surface of the ground, such as a line of road or railroad,

he finds the height of a succession of points, generally at equal distances apart, above a horizontal line AX. Then, having drawn upon paper a horizontal line to represent AX, he lays off upon it the horizontal distances between the points to any suitable scale, and at these points erects perpendiculars representing the heights of the several points. A line joining the summits of these perpendiculars will show the changes in elevation of the ground along this line. Such a representation is called a **profile** or **section**.

Since the changes of elevation are usually much more minute than the horizontal distances, it is customary to use a more exaggerated scale for setting off the perpendicular heights than the horizontal distances.

214. In levelling over a line it is not necessary to change the position of the instrument for each new station. The same back-sight will serve to determine any number of stations on which fore-sights may be taken. The instrument should be changed when we are no longer able to see the rods on the forward stations, or when the error from the difference of true and apparent level becomes appreciable. Thus, in the figure, a back-sight

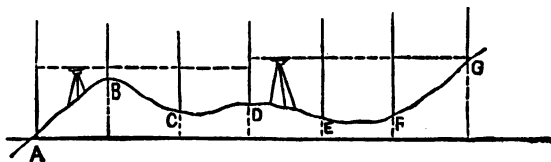


FIG. 113.

on A will determine the relative heights of B, C, and D, on which fore-sights can be taken from the same station. The instrument being moved, a back-sight must be taken on any station already determined, and then fore-sights on any number of succeeding stations.

215. Field-notes. The field-notes in levelling consist in a record of the back-sights and fore-sights, with such remarks as are necessary to explain them. When the difference between the first and last station alone is required, the difference between the footings of the back-sights and the fore-sights is all that is required. In levelling for section, the field-notes may be kept in the following form:

STA.	DIS.	B. S.	F. S.	CHANGE.	HEIGHT ABOVE DATUM.	REMARKS.
A	100	9.34			0.00	Datum level through A.
B	100		4.20	+ 5.14	5.14	
C	100		7.83	+ 1.51	1.51	
D	100	5.48	6.75	+ 2.59	2.59	
E	100		6.60	- 1.12	1.48	
F	100		6.35	- 0.87	1.72	
G	100		1.83	+ 3.65	6.24	

Column I. gives the stations occupied by the levelling-rod.

Column II. gives the distance from each station to the next.

Column III. gives the back-sights.

Column IV. gives the fore-sights.

Column V. gives the change of level between any station, and that on which the preceding back-sight was taken ; + signifies a rise, - a fall.

Column VI. gives the heights of the several points above the datum level. In this example, the datum level was assumed to pass through the first station A. Hence, its height is zero. The height of any station is got by adding with its proper sign the change of level to the height of that station on which the last back-sight was taken.

Column VII. contains any remarks or explanations.

216. In executing a series of levels, either for determining the difference in level between two points, or for delineating a section, the surveyor ought to make at various points permanent marks, called **bench-marks**, to which he can recur, when occasion requires. The starting-point, or some place near the starting-point, should be thus marked. At all points where, it may be anticipated, branch levels may diverge from, such marks should be left. They consist of some permanent object, such as a point of rock, a stone monument, a stone door-sill, a root of a tree, or other easily recognizable object. They must be marked so that they can easily and unmistakably be found, and described fully in the notes.

These bench-marks will serve, in case by any accident the points occupied in levelling have been lost, to recommence the line of levels. When the work has been left unfinished, a bench-mark will give a point from which to resume.

217. The occasions for determining the difference of level between two points occur frequently in the experience of a surveyor. To determine whether a spring of water will flow in a pipe from its source to any

given point, the surveyor must obtain by levelling the difference in height between the two points. To determine the head of water which can be got in any given case, requires the difference of level between two points. In draining a farm, the amount of descent from the ground to be drained to the proposed place of outflow must be ascertained. In the location of roads and railroads, in the improvement of watercourses, in the grading and sewerage of the streets of a city, this problem in levelling is of constant use.

Levelling for section is applicable in all branches of engineering work. The first step in preparing to build a road or railroad is to take a system of levels which will enable the surveyor to construct a section of the ground over which it is proposed to be run. For this purpose, he obtains the elevations of a series of points in this line above some assumed fixed level line. These, when plotted on paper, will show to the eye the changes of elevation along the line, and will show where in the construction the surface will require excavation, and where filling.

218. Topographical levelling. In order to ascertain and delineate the changes of level, and the comparative declivity of different parts of a piece of ground, we conceive the surface intersected by a series of horizontal planes, beginning with one passing through the lowest point, and each succeeding one being placed at a certain uniform distance above the preceding. The intersections of these planes with the ground-surface will give a series of irregular curve-lines, all the points in any one of which would be in the same level. These lines are termed **contour lines**, and when plotted on a horizontal plan of the ground, serve to show its topographical features. When the contour lines in such a plan approach near together, it shows the ground to be steeper than when they are more distant. The location of valleys, ridges, declivities, and plains is easily recognized from the distribution of the contour lines.

219. The chief object of topographical levelling is to locate these contour lines. When the ground is small in extent and easily seen from a small number of stations, the surveyor can proceed as follows:

Set up the level at some point commanding a view of as much space as possible, including the lowest point of the ground. Take a back-sight upon the lowest point. Then lower the target by whatever interval is to be left between the horizontal planes; for example, one foot. Let the rod be moved to higher ground until the target is again brought to the level of the telescope; this is one point in the one-foot contour line. With the target at the same point, move the rod until another point is found. Mark the points thus found; they will constitute the one-foot contour line. The target may now be lowered another equal interval, and the points of the second contour line be located.

When the surveyor can no longer follow the rod from his first position, he moves to a new position from which he can see the rod on some point already occupied, and as much additional space as possible. Placing the rod on the known point, the target is adjusted to the new position of the level, and then the work proceeds as before.

The location of these lines on a map may be effected by running them with a compass and chain.

220. The method employed when the ground is of considerable extent will be seen from the following example :

Let it be required to locate the contour lines on a plot of ground

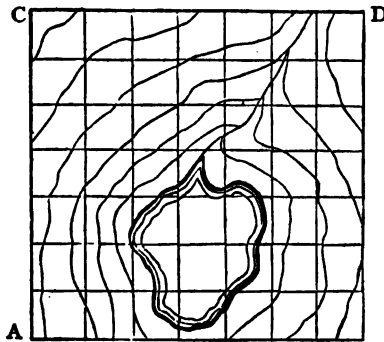


FIG. 114.

ABCD. With the transit or otherwise run two sets of parallel lines dividing the field into equal squares. At the intersections set pegs and number them. Then with the level determine the height of each peg above some standard ; for example, the surface of the pond.

Having drawn a plot of the ground, lay down upon it in light pencil-lines the squares as run in the field. From the field-notes we know the height of each point, and therefore we know how much the ground rises along each line from peg to peg. From this we can estimate where each of the contour lines cuts the parallel lines. Having thus found where the one-foot contour line cuts each of the lines in the field, draw an irregular curve-line through these points, and it will be the contour line required. In the same way find the points and draw each contour line. In general the estimate can be made with sufficient accuracy by eye.

221. Numerous applications of the principle of topographical levelling will be met with by the surveyor. In surveying a city for the purpose of laying down water-pipes, or for devising a system of sewerage,

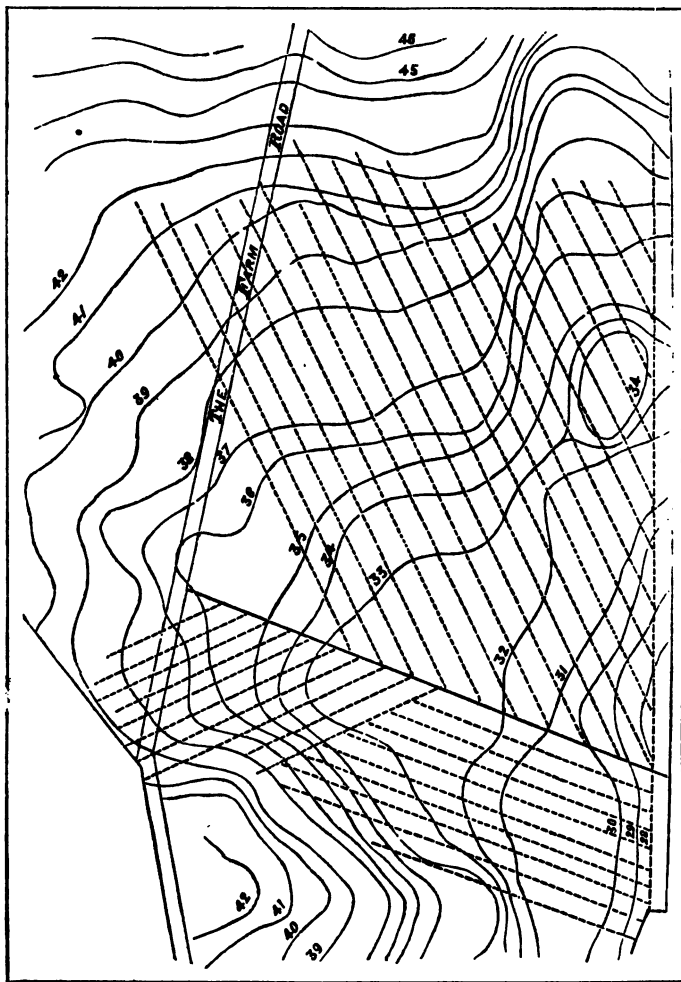


FIG. 115.—MAP OF NEW JERSEY AGRICULTURAL COLLEGE FARM, SHOWING THE CONTOUR LINES.



LEVELLING.

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the heights of all the street intersections above the lowest point, or above tidewater, are to be determined by the level. With these a system of contour lines, rising one above another at uniform intervals, can be laid down upon a city-map. Such a map will show at a glance the fall along any street, the proper outlet for the sewerage of any district, and the proper streets to be connected together in one district. ●

The same method, if applied to a farm, would show the relative heights of all parts, and the proper location and arrangement of a system of drains. To illustrate the use of such a survey we insert a map of a portion of the New Jersey Agricultural College Farm, at New Brunswick, Fig. 115. The contour lines are drawn at intervals of one foot. The heavy line shows an open ditch for receiving the drainage, and the broken lines the underdrains so far as executed. In laying down a system of drains it is necessary to make the lines of drainage cut the contour lines as nearly at right angles as possible, because this, being the line of quickest descent, will secure a most rapid flow of the water.

CHAPTER XIII.

UNDERGROUND SURVEYING.

222. FOR the purpose of delineating the excavations of mines, tunnels, etc., the operations of surveying must frequently be conducted underground. To meet the exigencies of these cases some modifications in the instruments and methods employed in surface surveying are rendered necessary. Ordinary flags or other signals cannot be seen; hence lanterns with various colored lights are substituted. The compasses, theodolites, and transits used are arranged, graduated, and mounted in a manner suited to the changed circumstances.

223. To measure the lines of an underground survey, a Gunter's chain, or the 100-foot engineer's chain, is used. The chain is not held horizontal, but the measurement is made along the slope of the gallery. At the same time the angle of inclination of each slope is measured by means of a semicircle, and recorded opposite to the measured distance. The horizontal distances may be deduced by multiplying the length of each line by the cosine of the inclination, and the rise or fall for each slope may be deduced by multiplying the length by the sine of the inclination. The following table shows how such distances and their reductions may be recorded. The positive sign signifies a rise, and the negative a fall.

COURSES.	DISTANCES.	INCLINATIONS.	HOR. DIS.	RISE OR FALL.
AB	56.4	- 16° 30'	54.1	- 18.2
BC	86.0	+ 20 15	80.7	+ 29.8
CD	21.8	0	21.8	0
CE	40.8	- 18 45	38.6	- 13.1

224. The terminations of the courses may be marked by iron pins driven into holes drilled for the purpose; or when these points are only temporary and are not required for future reference, they may be marked with chalk. The chainmen may also mark the ends of the successive chain-lengths with chalk. Lamps are employed as signals at the successive stations to guide the chainmen in running the lines, and also for signals in measuring the angles.

225. When the dimensions of the cross-section of the gallery are required, they are measured at right angles to the traverse line. A hand-

diagram of the cross-section is made in the note-book, and on it are written the measurements necessary to determine its area. The distance of each cross-section from the last preceding station is recorded in the notes and noted on the sketch.

Fig. 116 shows the sketch of a cross-section. The dimensions marked upon the diagram are sufficient to determine the area. It must also be marked in such a way as to indicate at what point of the traverse it was taken.



FIG. 116.

226. The instrument used for measuring angles in subterranean surveying is usually either the magnetic compass or the transit. When ferruginous substances are present the compass cannot be conveniently employed, on account of the deflection of the needle. Whenever it is employed, care must be used by taking back-sights as well as fore-sights, to detect the presence of local attraction, and to eliminate the error. The angular instrument is mounted on a low tripod or on a box; or by some surveyors is suspended by a hook from a wire running through the gallery, and fastened from station to station of the traverse. In the latter case the instrument must be mounted so as to hang horizontal, somewhat after the manner of a ship-compass.



FIG. 117.

When the transit or a compass provided with a telescope is used, some method of illuminating the cross-wires is required. Instrument-makers furnish for this purpose a small reflector and lamp, to be suspended so as to throw a faint light through the object-glass upon the cross-wires.

227. To run the traverse with the compass. Place the compass at the station where the survey is to be begun, for example, in Fig. 117, at A, the centre of the shaft running up to the surface. Point the compass toward a signal-lamp at B, held at the same height as the compass. Read the bearing of the line AB from the magnetic meridian. Measure also the distance AB, and its angle of inclination. When required, measure also the cross-section at suitable intervals.

Remove the compass to B, and measure the reverse bearing of BA. The agreement of this with the direct bearing will test the local attraction. Measure also the direct bearing of BC, and so proceed till all the

lines and bearings are measured. A record like the following may be made of the work:

LIN.	DIS.	INCL.	DIRECT BEARING.	REVERSE BEARING.	REDUCED BEARING.	REMARKS.
AB	56.4	- 16° 30'	S. 48° 30' E.	48° 38'	S. 48° 34' E.	68 section. 5 section.
BC	86.0	+ 20 15	S. 10 48 W.	10 40	S. 10 44 W.	
CD	21.8	0	S. 80 18 W.	80 24	S. 80 21 W.	
CE	40.8	-18 45	S. 42 11 E.	42 15	S. 42 13 E.	

228. To discover at which station local attraction disturbs the magnetic needle, the surveyor sets up the compass at some intermediate station, and points it along the same line. If it agrees in reading with that at either end, the reading at that end may be regarded as free from disturbance, and may be used to correct the other. Thus, suppose in the preceding example it is found that the reverse bearing at station B, instead of exactly agreeing or nearly agreeing with the direct, is 49° 45'. The difference, 1° 15', is too large to be treated as an error of measurement, and must be regarded as the amount by which the north end of the needle is attracted to the east. Hence, when the compass is pointed along BC, the reading will be affected 1° 15' by this divergence of the needle. Since the south end of the needle points too far to the west, the reading of BC will be S. 9° 33' W., too small by 1° 15', and the true reading will be obtained by adding the correction.

The following rule will show how to apply this correction to the next bearing:

When the reverse bearing exceeds the direct, **subtract** the difference from the next direct bearing, if both the courses have both the letters alike, or both different; or **add** the difference when one set of letters are alike, but the other different.

When the direct bearing exceeds the reverse, **add** the difference to the next direct bearing, when both the courses have both the letters alike, or both different; or, **subtract** the difference when one set of letters are alike, but the other different.

229. By measuring the internal angles instead of the bearings of the courses, the magnetic compass may be used for running the traverse of an underground survey, even when the local attraction prevents our obtaining from it the true bearings of the courses. Set up the compass at the second station B, in the preceding example. The included angle ABC, measured with the compass, will be precisely the same, whether the needle is disturbed by local attraction or not. Thus:

The bearing of BA, N. 48° 30' W.

The bearing of BC, S. 10° 48' W

By Art. 53, angle ABC = $180^\circ - (48^\circ 30' + 10^\circ 48') = 120^\circ 42'$.

The disturbed bearing of BA, N. $49^\circ 45'$ W.

The disturbed bearing of BC, S. $9^\circ 33'$ W.

By Art. 53, the angle ABC = $180^\circ - (49^\circ 45' + 9^\circ 33') = 120^\circ 42'$.

In the same way the compass may be set up at each of the intermediate stations, and the angles between the courses determined, the result in each case being independent of any local attraction.

The bearings of the courses, in this case, with the meridian, will not be known. If, however, the true bearing of any one of the courses could be determined by connecting with known lines on the surface, the bearings of the remainder can be deduced from the known angles which they make with each other.

230. To run the traverse with the transit. Place the transit over the second station, as B, Fig. 117. Place signal-lamps at the preceding and succeeding stations, A and C, taking care to locate them at the same height above the ground as the telescope of the transit. Measure the horizontal angle ABC, and also, by means of the vertical circle of the transit, read the slope of the lines. Remove the transit to the next station C, and measure the horizontal and slope angles in the same way.

A convenient arrangement is for the surveyor to be provided with three tripods of equal height. They are so constructed that when the transit is taken off, a lantern may be put on which will stand at the same height as the telescope of the transit. The three tripods are set up at three continuous stations. On the middle one the transit is set, and lanterns on the others. When the angles at this station are measured, the tripod from the first station is carried forward to an advance station, the lantern from the third is placed on the second, and the transit is placed on the third tripod.

The transit used in subterranean surveying is commonly graduated from 0° to 360° in the direction of the numbering on a watch-face.

231. The field-notes of the survey may be kept in the same way as described for transit surveying in Art. 148. Or they may be kept in a tabular form, as follows:

STA.	DIS.	SLOPE.	HOR. DIS.	RISE. FALL.	B. S.	F. S.	INT. ANG.
A							
B	208	+ $3^\circ 45'$	207.5	+ 13.6	278° 30'	160° 0'	241° 30'
C	168	- 2 30	167.8	- 7.3	25 38	136° 52	111 14
D	300	- 2 50	299.6	- 14.8	348 36	249 40	261 4
E	316	0	316.0	0	323 51	106 7	142 16
F	114	+ $4^\circ 15'$	113.7	+ 8.4			

Column I contains the stations in their order of succession. In this example we have supposed the galleries to succeed each other. But often galleries branch off from the same station in different directions. The notes must indicate this.

Column II contains the distances measured along the slopes of the galleries. They are placed so as to show the distance of the station opposite to which they stand from the preceding.

Column III contains the angles by which the lines from the preceding station slope to the station named opposite. The positive sign indicates a rise, the negative a fall.

Column IV contains the horizontal distances obtained by multiplying the distances in Column II. by the cosine of the slope.

Column V contains the rise or fall of each station from the preceding, obtained by multiplying the distances in Column II. by the sine of the slope.

Columns VI and VII give the readings of the transit when pointed respectively upon the preceding and succeeding stations. They are set opposite to the station at which the angles were measured.

Column VIII is obtained by subtracting the back-sights in Column VI. from the fore-sights in Column VII. Whenever, in turning the instrument from the back-sight to the fore-sight, we pass the 360° point, we must add 360 to the reading of the fore-sight before we subtract.

When the remainder is less than 180° it shows that the next course turns to the left; when it is greater than 180° it shows that the next course turns to the right.

232. To reduce the traverse. Let it be required in the last example to determine the horizontal distance and direction of the last station from the first. Let ABCDEF, Fig. 118, represent a horizontal plot of the preceding example: it is required to find the line AF and the angle BAF. By referring all the courses to the meridian of the first course, it is plain that Af, the latitude of the last station, is the sum of all the latitudes of the courses, and that Ff, the

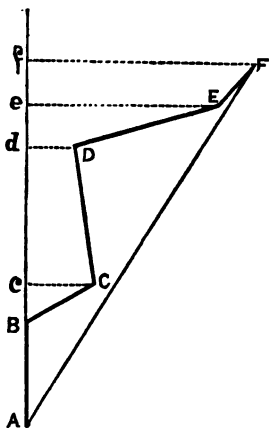


FIG. 118.

departure of the last course, is equal to the sum of the departures of all the courses. The length and bearing of AF can readily be obtained from the values of Af and Ff.

To obtain these, proceed as in transit surveying. The operation is summed up in the following table :

STA.	HOR. DIST.	HOR. ANG.	AZIMUTH WITH 1ST COURSE.	BEARINGS.	LATITUDE.		DEPARTURE.	
					N.	S.	E.	W.
A			0	N.	0			
B	207.5	241° 30'	241° 30'	N. 61° 30' E.	207.50			
C	167.8	111 14	172 44	N. 7 16 W.	80.07		147.46	
D	299.6	261 4	253 48	N. 73 48 E.	297.19			37.90
E	316.0	142 16	216 4	N. 36 4 E.	88.16		303.45	
F	113.7				91.91		66.64	
	1104.6				764.83	0	517.55	37.90
				$Af = 764.83$	$Ff = 479.65$			

The column of azimuths in this table is derived from the column of horizontal angles by the following rule :

Subtract 180° from the horizontal angle which any course makes with the preceding. The remainder may be + or -. Add the result, observing its proper sign, to the azimuth of the preceding course. If the sum is negative, add 360° to it ; if positive and greater than 360°, subtract 360°. The result will be the azimuth of the next course.

The bearings are marked from the first course as a meridian, and are obtained in a manner similar to that described in Art. 154.

The solution of the right-angled triangle AFf will give the unknown quantities AF and angle FAf .

To find the angle :

$$\tan FAf = \frac{R \times Ff}{Af} = 32^\circ 5' 35''$$

To find the distance :

$$AF = \frac{R \times Ff}{\sin A} = 1439.55$$

233. To connect the underground survey with lines on the surface.

In order to trace on the surface the extent and direction of the underground excavations, and also to ascertain the depth below the surface of any point, the surveyor must connect his traverse underground with lines on the surface. When the compass has been used in the survey, and we have obtained, free from local attraction, the magnetic bearing of the line

running from the bottom of the shaft, the extension of the survey to the surface may be effected thus :

Suspend a plumb-line from the surface down the shaft directly over station A. To keep it steady, it may be made to hang in a bucket of water. By means of this plumb-line, set the compass at the top of the shaft, directly over station A at the bottom. A temporary platform may be built over the shaft. Turn the compass until the bearing of the sights is the same as that of the first course of the underground survey. This line being run out would correspond with and be vertically over the first course. Precautions must be used to prevent the iron implements, etc., about the shaft, disturbing the needle.

When it is not convenient to set the compass directly over the shaft, the location of the first line may be got as follows :

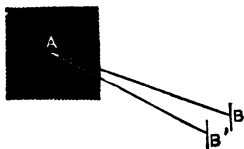


FIG. 119.

Set up the compass at any point B', Fig. 119, approximately in the required direction. Sight back on A, the suspended plumb-line; if the compass is truly in line, its bearing will correspond with the reverse bearing of the first course. By trial, move the compass until the bearing

corresponds ; it is then in the required line.

Having obtained the direction of the first line on the surface corresponding with the first line underground, the surveyor can repeat on the surface the entire traverse.

234. When the transit has been used for making the survey, the connection with the surface may be established by suspending two plumb-lines as shown in Fig. 120. Set up the transit at B, the second station, and point it back at A, the first station. From a rod reaching across the mouth of the shaft, suspend two plumb-lines CE and DF, as far apart as the width of the shaft will allow. Move the rod carrying the plumb-lines until they are both exactly in the line of sight of the telescope. The points CD will then be two points in the required surface-line. It is a serious objection to this method that the plumb-lines can only be placed at a small distance apart, and hence the surface-line cannot be regarded as fixed with great precision.

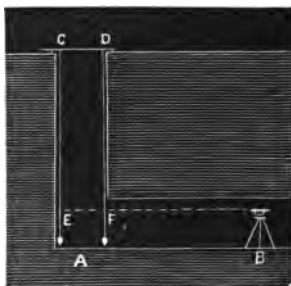


FIG. 120.

235. Fig. 121 shows a second method by which a transit may be employed to connect an underground with a surface survey. Set the transit on the edge of the shaft, in such a way as to be able to see diagonally across the shaft in the direction of the gallery. Fix a fine distinct mark at G in this line. Also fix a mark D upon the surface, in the same direction. From a straight-edge placed across the shaft from the instrument in the line CD, suspend a plumb-line EF. The line through the points F and G, in the underground survey, corresponds with CD on the surface. The direction which the line through FG makes with the first line of the survey can easily be determined.

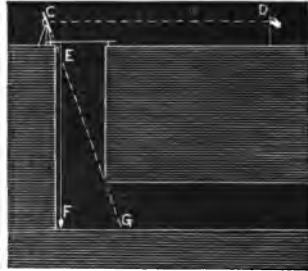


FIG. 121.

The transit may, in this operation, be placed at the bottom of the shaft G instead of the top. A diagonal eye-piece is then required to enable the surveyor to make observations nearly vertical up through the shaft.

236. Being able to locate on the surface the first line of the underground traverse, the surveyor can now locate points directly over each of the stations of the underground survey. In this operation it must be remembered that the horizontal distances between the stations are to be used.

Let it be required, in the preceding example, Fig. 118, to locate on the surface the last station F of the survey, and also to determine the depth below the surface. Having measured the horizontal distance of the first course, 207.5 at this point, measure a horizontal angle $241^{\circ} 30'$ from the preceding course, and measure the horizontal distance of the second course. Proceed thus to measure the horizontal angles and the horizontal distances of the traverse. Each of the stations on the surface will be directly over those of the underground survey.

Or, having determined the direction of the first course, and reduced the traverse as in Art. 232, measure the angle $BAF 32^{\circ} 5' 35''$, and also the horizontal distance AF 1439.5 feet. The point thus reached on the surface will be directly over F of the subterranean survey.

237. To determine the depth of the point F below the surface, find the difference of level between the mouth of the shaft and the point over F. This difference and the known depth of the shaft will give the difference of level between the bottom of the shaft and the required point. Finally, from the rise and fall in the gallery, we know the change of level

from the bottom of the shaft to the underground station, and this, applied to the previous difference, will give the depth required. Thus,

Required depth = depth of shaft + rise of surface (or — fall) — rise of gallery (or + fall).

238. The results of the survey may be represented on paper by making a **horizontal plan** of the traverse, and also laying down upon it the width of the gallery as determined by the measurement of the cross-section. This will show the horizontal extent and directions of the underground excavations. A **vertical section** is also constructed, running through the traverse line of the galleries, without taking into account its changes of horizontal direction. The vertical scale may be exaggerated over the horizontal, if that is necessary, to show the vertical changes. This section will show the bottom of the gallery with all changes of level, the height of the gallery, the depth of the shafts, and also the position of the surface directly over the gallery.

EXPLANATION OF TABLES.

TABLE I. USEFUL FORMULÆ.

239. THIS table gives, in a form convenient for reference, the principal formulæ of trigonometry and mensuration which are required in surveying. In the chapter on Trigonometry will be found the demonstration of those relating to triangles. Those relating to the areas and surfaces and contents of geometrical figures will be found demonstrated in the works on geometry. To these are added a few miscellaneous formulæ of frequent use.

TABLE II. USEFUL NUMBERS AND THEIR LOGARITHMS.

240. Numbers of frequent application are here given, together with their logarithms. They will save the computer some labor, and may be relied on as obtained from the most authentic sources.

TABLE III. LOGARITHMS OF NUMBERS.

241. This table gives the logarithms, carried to six decimal places, of all numbers consisting of four figures or less; and also furnishes the means for finding the logarithms of numbers of five or more figures. The characteristic, that is, the integral part of the logarithm, is not expressed in the table, but may always be found by the following rule:

The characteristic of the logarithm of a number greater than unity, is one less than the number of integral figures in the given number. Thus the characteristic of the logarithm of 365.25 is 2.

The characteristic of the logarithm of a decimal fraction is negative, and is equal to the number of places by which the first significant figure is removed from the place of units. Thus the characteristic of the logarithm of 0.0038 is $\bar{3}$. To distinguish the negative sign in this case where it only applies to the characteristic, and not to the decimal part of the logarithm, it is placed over the characteristic.

242. To find the logarithms of numbers consisting of three figures or less.

Find the number in the first column of the page headed N ; then the logarithm will be found opposite, in the second column headed O. To this must be prefixed the characteristic required by the rule.

Since the decimal part of the logarithm of a number of two figures, for example 36, is the same as that of the same number with a cipher attached, that is 360 ; when the number consists of less than three figures, we seek in the first column for these figures with a cipher or ciphers attached. Thus, to find the logarithm of 36, we seek opposite 360.

243. To find the logarithm of a number of four figures.

Seek the first three figures in the N column, and the fourth at the top of one of the columns. Opposite the three figures, and beneath the fourth, will be found the required logarithm.

244. To find the logarithm of a number of five or more figures.

Find the logarithm corresponding to the first four figures, as above. Take the number standing opposite in the difference column, and multiply this by the remaining figures of the number used as a decimal. The product will form a correction to be added to the logarithm of the four figures. Thus, to find the logarithm of 683237 :

Opposite 3862 we find	.586812
Correction = 112 × .37	41
Result with characteristic	5.586853

245. When the number is **wholly or partly decimal**, the decimal part of the logarithm is found as if the figures were all integral ; and then the proper characteristic is prefixed. Thus, the logarithm of 0.00486 is found opposite 486, and the characteristic $\bar{3}$ is prefixed. The logarithm for 96.83 is found opposite 9683, and the characteristic will be 1.

246. To find the number corresponding to any logarithm.

Seek in the table for the logarithm nearest less than the one given, and take out the three figures in the N column and the figure at the top. These are the first four figures of the required number. Subtract the tabular logarithm from the given logarithm, and divide the remainder with ciphers annexed by the number standing opposite in the difference column. The quotient will give figures to be annexed to the four already found. The division may be carried as far as the desired precision requires. The characteristic will indicate where the decimal point must be placed. Thus, let it be required to find the number corresponding to

Logarithm	3.486329
Nearest less logarithm486289
Remainder	40
40 ÷ 142	282
Number corresponding	3064.282

TABLE IV. LOGARITHMIC SINES, COSINES, TANGENTS, ETC.

247. This table gives the logarithms of the sines, cosines, tangents, and cotangents of all arcs from 0° to 90°, at intervals of one minute. They are computed for a radius of 10 000 000 000, whose logarithm is 10. When this table is used in formulæ involving the radius, the logarithmic radius 10 must be employed. The table is not carried beyond 90°, because the functions for arcs above 90° are the same as those of their supplements. Thus:

$$\begin{aligned} \text{Sin } 100^\circ &= \text{sin } 80^\circ \\ \text{Cos } 100 &= - \text{cos } 80 \\ \text{Tan } 100 &= - \text{tan } 80 \\ \text{Cot } 100 &= - \text{cot } 80 \end{aligned}$$

248. To find the logarithmic sine or tangent of any arc.

If the arc is less than 45°, find the number of degrees at the top of the page, and the minutes at the left hand. If the arc is more than 45°, find the degrees at the bottom of the page, and the minutes at the right hand. In the appropriate column opposite the given minutes will be found the required logarithmic sine or cosine.

When the arc contains seconds, take out the difference for 1'' opposite the logarithm for the given degrees and minutes. Multiply this by the given seconds, and the product forms a correction which must be added to the logarithm opposite the given degrees and minutes.

The cosines and cotangents are found from the tables in the same manner, except that the correction obtained for the seconds must be subtracted.

To find the sine of 25° 38' 16'':

$$\begin{array}{r} \text{Sin } 28^\circ 38' \dots\dots\dots 9.636097 \\ \text{Correction } 4.39 \times 16 \dots\dots \quad 70 \\ \hline \text{Sin } 28^\circ 38' 16'' \dots\dots\dots 9.636167 \end{array}$$

To find the cotangent of 119° 12' 22'':

$$\begin{array}{r} \text{Cot } 119^\circ 12' 22'' = - \text{cot } 60^\circ 47' 38'' \\ \text{Cot } 60^\circ 47' \dots\dots\dots 9.747616 \\ \text{Correction } 4.94 \times 38 \dots\dots \quad -188 \\ \hline \text{Cot } 60^\circ 47' 38'' \dots\dots\dots 9.747428 \end{array}$$

249. The secants and cosecants of arcs are not included in the table. They may be found from equations (2) and (4), Art. 13.

$$\text{Sec } a = \frac{R^2}{\cos a} ; \text{cosec } a = \frac{R^2}{\sin a}$$

Hence, $\log \sec a = 2 \log R - \log \cos a$
 and $\log \operatorname{cosec} a = 2 \log R - \log \sin a$.

Thus, to obtain the logarithmic secant of $36^\circ 16'$:

$2 \log R$	20.000000
$\operatorname{Cos} 36^\circ 16'$	9.906482
	10.093518
$\operatorname{Sec} 36^\circ 16'$	10.093518

250. To find the arc corresponding with any logarithmic sine and tangent.

Find in the proper column the logarithm next less than the one given; take the degrees and minutes corresponding; subtract the tabular logarithm from the one given; divide the remainder by the difference for 1" standing in the proper difference column; the quotient will be the seconds to be annexed to the degrees and minutes.

The method for finding the arc corresponding to a given cosine or cotangent is similar, except that we find the next greater logarithm instead of the next less.

Thus, to find the arc corresponding to a tangent 10.684639:

Given tangent	10.684639
Tabular tangent	10.684477 .. arc $78^\circ 19'$
	162
Remainder	162
Correction, $162 \div 10.61 =$	15"
Whole arc	$78^\circ 19' 15''$

To find the arc corresponding to a cosine 9.964418:

Given cosine	9.964418
Tabular cosine	9.964454 .. arc $22^\circ 52'$
Remainder	36
Correction, $36 \div .89 =$	40"
Whole arc	$22^\circ 52' 40''$

TABLE V. NATURAL SINES AND COSINES.

251. This table gives the sines and cosines of all arcs from 0° to 90° , at intervals of one minute to five decimal places. The radius for this table being unity, the values of all sines and cosines are less than one.

To find the sine and cosine for any arc:

If the arc is less than 45° , seek the degrees at the top, and the minutes at the left hand of the page. If the arc is more than 45° , seek the degrees at the bottom, and the minutes at the right hand of the page. The required sine or cosine corresponding to the given number of degrees

and minutes will be found in the column of the degrees and opposite the minutes.

When the given arc consists of degrees, minutes, and seconds, find the sine or cosine corresponding to the degrees and minutes. Subtract this sine or cosine from that belonging to the next greater arc. The difference is the change for 1 minute or 60 seconds. Multiply this difference by the given seconds, and divide by 60. The quotient is the correction which the sine or cosine requires for the additional seconds. In the case of the sine, add the correction; in the case of the cosine, subtract.

Thus, let it be required to find the sine of $28^{\circ} 14' 36''$:

$$\begin{aligned} \text{Sine } 28^{\circ} 14' &= .47306; \text{ diff.} = 26 \\ \text{Correction for } 36'' &= \frac{26 \times 36}{60} = \underline{16} \\ \text{Sine } 28^{\circ} 14' 36'' &= .47322 \end{aligned}$$

Let it be required to find the cosine of $61^{\circ} 29' 25''$:

$$\begin{aligned} \text{Cosine of } 61^{\circ} 29' &\dots\dots\dots .47741; \text{ diff.} = 26 \\ \text{Correction for } 25'' &= \frac{26 \times 25}{60} = \underline{-11} \\ \text{Cosine } 61^{\circ} 29' 25'' &\dots\dots\dots 47730 \end{aligned}$$

252. To find the arc corresponding to any given natural sine and cosine.

Find in the table the two consecutive sines or cosines between which the given one falls. The least of these, in the case of the sine, corresponds to the required degrees and minutes; in the case of cosines, the greater. Take the difference between the given sine and cosine, and that answering to the degrees and minutes. Take, also, the difference between the two consecutive sines and cosines. Multiply the first difference by 60, and divide the product by the second difference. The result will give the required seconds.

Thus, let it be required to find

$$\begin{aligned} \text{The arc whose sine is.} &\dots\dots\dots .60000 \\ \text{Nearest less tabular sine.} &\dots\dots\dots .59995 \dots 36^{\circ} 52' \\ \text{Difference.} &\dots\dots\dots 5 \\ \text{Difference between consec. sines.} &\dots\dots\dots 24 \\ \text{Required seconds} &= \frac{5 \times 60}{24} \dots\dots 12'' \\ \text{Required arc.} &\dots\dots\dots 36^{\circ} 52' 12'' \end{aligned}$$

Again, let it be required to find

The arc whose cosine is.....	.94660	
Nearest greater tabular cosine.....	.94665 ..	18° 48'
Difference.....	5	
Difference between successive cosines.....	9	
Required seconds = $\frac{5 \times 60}{9}$	33''	
Required arc.....	18° 48' 33''	

TABLE VI. NATURAL TANGENTS AND COTANGENTS.

253. This table gives the tangents and cotangents computed for a radius of unity for all arcs from 0° to 90°. The tangents vary between 0 and infinity, and the cotangents between infinity and radius. The variations of the tangents near the end of the quadrant and the cotangents near the beginning of the quadrant are so rapid that the table cannot be relied upon to give them for intermediate seconds when the arc lies within one or two degrees of those points. But angles of that character are very unusual in surveying, and when they are required, resort must be had to tables adapted for this purpose.

254. To find the tangent or cotangent corresponding to a given arc.

If the arc is less than 45°, seek the degrees at the top of the page, and the minutes at the left hand. If the arc is more than 45°, seek the degrees at the bottom of the page, and the minutes at the right hand. The required tangent or cotangent corresponding to a given number of degrees and minutes will be found in the column of the degrees and opposite the minutes.

When the given arc consists of degrees, minutes, and seconds, find in the table two consecutive tangents and cotangents between which the required one must fall. Take their difference, and multiply it by the given seconds, and divide by 60. The quotient will give a correction for the number opposite the degrees and minutes, to be added in case of the tangent, and subtracted in case of the cotangent.

Thus, let it be required to find the tangent of 48° 44' 23'' :

Tangent of 48° 44'.....	1.13961	diff. 67
Correction for 23'' = $\frac{67 \times 23}{60}$ =	26	
Tangent of 48° 44' 23''.....	1.13987	

Again, let it be required to find the cotangent of 74° 16' 45'' :

Cotangent of 74° 16'.....	0.28172	diff. 32
Correction for 45'' = $\frac{32 \times 45}{60}$ =	-24	
Cotangent of 74° 16' 45''.....	0.28148	

255. To find the arc corresponding to any given tangent or cotangent.

Find in the table the two consecutive tangents or cotangents between which the given one falls. The less of these in the case of the tangent corresponds to the required degrees and minutes; in the case of the cotangent, the greater. Take the difference between the given tangent or cotangent and that answering to the degrees and minutes. Take also the difference between the two consecutive tangents or cotangents. Multiply the first difference by 60, and divide by the second. The result will give the required **seconds**.

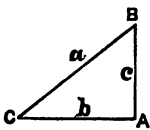
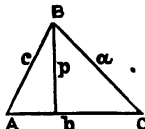
Thus, let it be required to find

The arc whose tangent is.....	3.48462
Nearest less tangent.....	3.48359..73° 59'
Difference.....	<u>103</u>
Difference between consecutive tangents...	382
Required seconds $\frac{103 \times 60}{382} =$	16''
Required arc.....	73° 59' 16''

Again, let it be required to find

The arc whose cotangent is.....	2.50000
Nearest greater cotangent.....	<u>2.50018..21° 48'</u>
Difference.....	18
Difference between consecutive cotangents..	211
Required seconds $\frac{18 \times 60}{211} =$	5''
Required arc.....	21° 48' 5''

TABLE I.
USEFUL FORMULÆ.
LINES AND ANGLES.

FIGURE.	GIVEN.	REQUIRED.	FORMULÆ.
<p>R. A. Triangle.</p> 	B, a.	C, b, c.	$C = 90^\circ - B$ $b = \frac{a \times \sin B}{R}$; $c = \frac{a \times \cos B}{R}$
	B, b.	C, a, c.	$C = 90^\circ - B$; $a = \frac{R \times b}{\sin B}$; $c = \frac{R \times b}{\tan B}$
	b, c.	B, C, a.	$\tan B = \frac{R \times b}{c}$; $\cot C = \frac{R \times b}{c}$ $a = \frac{R \times b}{\sin B}$ or $a = \sqrt{b^2 + c^2}$
	a, b.	B, C, c.	$\sin B = \frac{R \times b}{a}$; $\cos C = \frac{R \times b}{a}$ $c = \frac{a \times \sin C}{R}$ or $c = \sqrt{a^2 - b^2}$
<p>Triangle.</p> 	A, B, a.	C, b, c.	$C = 180^\circ - (A + B)$; $b = \frac{a \times \sin B}{\sin A}$; $c = \frac{a \times \sin C}{\sin A}$
	A, a, b.	B, C, c.	$\sin B = \frac{b \times \sin A}{a}$ $C = 180^\circ - (A + B)$; $c = \frac{a \times \sin C}{\sin A}$
	A, b, c.	B, C, a.	$\tan \frac{1}{2}(B - C) = \frac{b - c}{b + c} \tan \frac{1}{2}(B + C)$ $B = \frac{1}{2}(B + C) + \frac{1}{2}(B - C)$ $C = \frac{1}{2}(B + C) - \frac{1}{2}(B - C)$ $a = \frac{b \times \sin A}{\sin B}$
	a, b, c.	A, B, C.	$\sin \frac{1}{2}A = \sqrt{\frac{R^2(s - b)(s - c)}{b \times c}}$ $s = \frac{1}{2}(a + b + c)$

AREAS AND CONTENTS.

Triangle.	Area = $\frac{1}{2}$ base \times altitude Area = $\frac{1}{2} \frac{b \times c \times \sin A}{R}$
Parallelogram.	Area = $\sqrt{s(s-a)(s-b)(s-c)}$, $s = \frac{1}{2}(a+b+c)$ Area = base \times altitude Area = $\frac{\text{product of two sides} \times \text{sine of included angle}}{R}$
Trapezoid.	Area = $\frac{1}{2}$ sum of parallel sides \times altitude
Quadrilateral.	Area = $\frac{\text{product of diagonals} \times \text{sine of included angle}}{R}$
Circle.	Circumference = $2 \pi r$ Diameter = $\frac{\text{circumference}}{\pi}$ Length of an arc = $\frac{2 \pi r}{360} \times \text{No. of degrees}$ Area = $\pi \times r^2$ Area of sector = $\frac{\pi r^2}{360} \times \text{No. of degrees}$ Area of segment = sector - triangle Area of ring = $\pi (R^2 - r^2)$
Ellipse.	Area = πab , in which a and b are the semi-axes.
Parabola.	Area = $\frac{2}{3}$ base \times height = $\frac{2}{3} bh$
Right Prism.	Convex surface = perimeter \times altitude = $P \times h$ Contents = area of base \times altitude = $A \times h$
Regular Pyramid.	Convex surface = $\frac{1}{2}$ perimeter \times slant height = $\frac{1}{2} P \times l$ Contents = $\frac{1}{3}$ area of base \times altitude = $\frac{1}{3} A \times h$
Frustum of a Regular Pyramid.	Convex surface = $\frac{1}{2}$ sum of perimeters of bases \times slant height = $\frac{1}{2} (P + P') \times l$ Contents = $\frac{1}{3}$ alt. \times sum of upper base, lower base and a mean proportional = $\frac{1}{3} h (A + A' + \sqrt{A \cdot A'})$
Cylinder.	Convex surface = circumference \times alt. = $2 \pi r \times h$ Contents = area of base \times altitude = $\pi r^2 \times h$
Cone.	Convex surface = $\frac{1}{2}$ circumf. \times slant height = $\pi r l$ Contents = $\frac{1}{3}$ area of base \times altitude = $\frac{1}{3} \pi r^2 h$
Frustum of Cone.	Convex surface = $\frac{1}{2}$ sum of circumferences \times slant height = $\pi (r + r') \times l$ Contents = $\frac{1}{3}$ alt. \times (upper base + lower base + mean proportional) = $\frac{1}{3} h \pi (r^2 + r'^2 + rr')$
Sphere.	Surface = $\pi \times$ square of diameter = $4 \pi r^2$ Contents = $\frac{1}{6} \pi \times$ cube of diameter = $\frac{4}{3} \pi r^3$

MISCELLANEOUS FORMULÆ.

Falling Bodies.	$s = \text{space described} = \frac{1}{2} g t^2 = \frac{v^2}{2g}$
$g = 32.170 \text{ ft.}$	$v = \text{velocity acquired} = g t = \frac{2s}{t} = \sqrt{2gs}$
	$t = \text{time, in seconds} = \frac{v}{g} = \sqrt{\frac{2s}{g}}$
Discharge of Liquids.	Velocity per second = $\sqrt{2gh}$
	Quantity per second (theoretical) = $A\sqrt{2gh}$
	Quantity per second (actual) = $\frac{2}{3} A\sqrt{2gh}$
	$A = \text{area of orifice}$
	$h = \text{depth of orifice below the surface}$
Strength of Beams.	Supported at one end:
	$W = \frac{sbd^3}{2l}$ (rectangular)
	$W = \frac{\pi sr^3}{l}$ (cylindrical)
	Supported at both ends:
	$W = \frac{2sbd^3}{l}$ (rectangular)
	$W = \frac{4\pi sr^3}{l}$ (cylindrical).
	$s = \text{tensile strength for each unit of area}$
	$l = \text{length}; b = \text{breadth}; d = \text{depth}$
	$W = \text{breaking weight}$
Projectile.	Range = $\frac{v^2 \sin 2a}{g}$
	$v = \text{initial velocity}$
	$a = \text{angle of elevation}$

TABLE II.

USEFUL NUMBERS AND THEIR LOGARITHMS.

NAMES.	NUMBERS.	LOGS.
Ratio of circumference of circle to diameter, π	3.1415926	0.497150
Ratio of diameter of circle to circumference, $\frac{1}{\pi}$	0.318310	1.502850
Force of gravity in feet, at lat. 45°... g_{45}	32.17076	1.507461
Length of seconds' pendulum in inches, lat. 45°..... l_{45}	39.1156	1.592349
Equatorial radius of earth in miles.... a	3,962.80	3.598002
Polar radius of earth in miles..... b	3,949.55	3.596548
Radius of earth as a sphere, in miles....	3,958.	3.597476
Mean length of a degree on meridian, in miles.....	69.048	1.839151
Length of a degree of longitude, lat. 30°	59.944	1.777746
Length of a degree of longitude, lat. 35°	56.715	1.753698
Length of a degree of longitude, lat. 40°	53.053	1.724710
Length of a degree of longitude, lat. 45°	48.986	1.690072
Distance of the sun in miles.....	91,328,000	7.960604
Tropical year, in mean solar days.....	365.2422	2.562581
Statute mile, in feet.....	5,280	3.722634
Statute mile, in metres.....	1,609.41	3.206667
Gallon, in cubic inches.....	231	2.363612
French metre, in feet.....	3.280869	0.515989
French metre, in inches.....	39.370432	1.595170
French gramme, in grains.....	15.432349	1.188432
Velocity of light per second, in miles....	185,000	5.267172
Velocity per second of sound in air 32°, in feet.....	1,090	3.037427
Increase in velocity for each 1° rise in temperature.....	0.96	
One-horse power, or number of pounds raised 1 foot in 1 minute.....	33,000	4.518514
Mechanical equivalent of heat (foot-pound).....	772	2.887617

USEFUL NUMBERS—Continued.

SUBSTANCES.	SPECIFIC GRAVITY.	WT. PER CU- BIC FOOT.	TENSION PER SQ. INCH.
<i>Metals.</i>			
Brass.....	7.820	488.75	42,000
Copper, cast.....	8.788	549.25	24,250
Copper wire.....	8.880	555.00	61,200
Iron, mean American.....	7.690	590.62	31,829
Iron, best Swedish bar....	7.800	487.50	72,000
Platinum wire.....	20.337	1271.06	53,000
Steel, cast, mean.....	7.833	489.61	88,657
Tin, cast.....	7.390	461.82	5,000
Zinc.....	6.861	428.81	3,500
Wire ropes.....			37,000
<i>Timbers.</i>			
Ash.....	0.760	47.500	17,000
Cedar.....	0.561	35.060	11,400
Elm.....	0.540	33.750	13,400
Locust.....	0.950	59.375	20,500
Maple.....	0.637	39.503	10,500
Oak.....	0.687	42.937	11,500
Pine, red.....	0.660	41.250	10,000
Pine, American white.....	0.455	28.438	11,800
Spruce.....	0.500	31.250	10,298
Walnut.....	0.671	41.937	17,800
<i>Building Material.</i>			
Bluestone.....	2.688	168.00	
Brick, average.....	1.900	118.75	
Granite, Quincy.....	2.652	165.75	
Limestone.....	3.156	197.25	
Marble, Italian.....	2.708	169.25	
Sandstone.....	2.200	137.50	
Trap.....	2.720	170.00	

TABLE,

CONTAINING

THE LOGARITHMS OF NUMBERS

FROM 1 TO 10,000.

NUMBERS FROM 1 TO 100 AND THEIR LOGARITHMS,

WITH THEIR INDICES.

No.	Logarithm.	No.	Logarithm.	No.	Logarithm.	No.	Logarithm.	No.	Logarithm.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.906485
2	0.301030	22	1.343433	42	1.623240	62	1.792309	82	1.913814
3	0.477121	23	1.361798	43	1.633468	63	1.798341	83	1.921078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.928279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.935419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.942498
7	0.845008	27	1.431364	47	1.672098	67	1.826075	87	1.949519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.956483
9	0.954243	29	1.462396	49	1.690196	69	1.838849	89	1.963390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.970243
11	1.041393	31	1.491399	51	1.707570	71	1.851258	91	1.977041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.983783
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.990483
14	1.146193	34	1.531479	54	1.732394	74	1.869232	94	1.973129
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556303	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986779
18	1.255273	38	1.579784	58	1.763498	78	1.892095	98	1.991236
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

NOTE.—In the following part of the Table, the Indices are omitted, as they can be very easily supplied. Thus, the index of the logarithm of every integer number, consisting only of one number, is 0; of two figures, 1; of three figures, 2; of four figures, 3: being always a unit less than the number of figures contained in the integer number. The index to the logarithm of every number above 100, in the following part of the Table, is omitted; yet, in the operation, it must be prefixed, according to this remark: so that the logarithm of 600 is 2.77815, and that of 6000 is 3.77815, and so of the rest.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876	010300	010724	011147	011570	011993	012415	424
3	012837	013259	013680	014100	4521	4940	5360	5779	6197	6616	420
4	7033	7451	7868	8284	8700	9116	9532	9947	020361	020775	416
5	021189	021603	022016	022428	022841	023252	023664	024075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789	030195	030600	031004	031408	031812	032216	032619	033021	404
8	033424	033826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811	040207	040602	040998	397
110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
2	9218	9606	9993	050380	050766	051153	051538	051924	052309	052694	386
3	053078	053463	053846	4230	4613	4996	5378	5760	6142	6524	383
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	000320	379
5	060698	061075	061452	061829	062206	062582	062958	063333	063709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8156	8527	8898	9268	9638	070038	070407	070776	071145	071514	370
8	071882	072250	072617	072985	073352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426	360
1	082785	083144	083503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
4	093422	3772	4132	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681	100026	346
6	100371	100715	101059	101403	101747	102091	102434	102777	103119	342	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916	10253	338
9	110590	110926	111263	111599	111934	112270	112605	112940	113275	306	335
130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940	332
1	7271	7603	7934	8265	8595	8926	9256	9586	9915	102045	330
2	120574	120903	121231	121560	121888	122216	122544	122871	123198	325	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7195	7429	7753	8076	8399	8722	9045	9368	9690	130012	323
5	130334	130655	130977	131298	131619	131939	132260	132580	132900	321	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
8	9879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911	309
1	9219	9527	9835	150142	150449	150756	151063	151370	151676	151982	307
2	152288	152594	152900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9567	9868	10168	10469	10769	11068	301
5	161398	161667	161967	162266	162564	162863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170969	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	176091	176381	176670	176959	177348	177536	177825	178113	178401	178689	289
1	8077	9264	9552	9839	180126	180413	180699	180986	181272	181558	287
2	181844	182129	182415	182700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7803	8084	8366	8647	8928	9209	9490	9771	190051	281
5	190332	190612	190892	191171	191451	191730	192010	192289	192567	284	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755	200029	200303	200577	200850	201124	274
9	201397	201670	201943	202216	202488	2761	3033	3305	3577	3848	272

No.	0	1	2	3	4	5	6	7	8	9	Dist.
160	904180	904391	904603	904834	905004	905475	905746	906016	906280	906550	271
1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	200
2	9515	9783	10051	10319	10586	10853	11121	11389	11654	11921	207
3	132128	212454	2720	2826	2932	3318	3783	4049	4314	4579	230
4	4844	5109	5373	5636	5902	6166	6430	6694	6957	7221	254
5	7404	7747	8010	8273	8536	8799	9060	9323	9585	9846	262
6	99108	200570	200631	200692	201153	201414	201675	201936	202196	202456	261
7	3716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7687	8144	8400	8657	8913	9170	9426	9682	9938	230193	256
170	230449	230704	230960	231215	231470	231724	231979	232234	232488	232749	255
1	9896	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
3	8046	8297	8548	8799	9049	9299	9550	9800	940050	940300	250
4	940549	940799	941048	941297	941546	941795	942044	942293	9541	9790	249
5	20036	32983	3534	3782	4030	4277	4525	4772	5019	5266	248
6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8464	8709	8954	9198	9443	9687	9932	950176	245
8	250490	250664	250908	251151	251395	251638	251881	252125	252368	2610	244
9	9853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	952723	255514	255755	255996	256237	256477	256718	256958	257198	257439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	240
2	900071	900310	900548	900787	901025	901263	901501	901739	901976	902214	238
3	9451	9688	9925	3162	3399	3636	3873	4109	4346	4582	237
4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9980	970213	970446	970679	970912	971144	971377	971609	233
7	271842	272074	272306	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6492	6692	6891	7151	7360	7609	7838	8067	8296	8525	229
190	278754	278989	279221	279439	279667	279895	280123	280351	280578	280806	228
1	281033	281261	281488	281715	281942	282169	282396	282622	282849	283075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
5	900035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
6	3256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4486	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	300161	300378	300595	300813	218
200	301030	301947	301464	301681	301898	302114	302331	302547	302764	302980	217
1	3196	3412	3628	3844	4059	4275	4491	4708	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	310058	310269	310481	310693	310906	311118	311330	311542	212
5	311754	311966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3887	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	990146	290354	290562	290769	290977	291184	291391	291598	291805	292019	207
210	292219	292426	292633	292830	293046	293262	293478	293695	293911	294127	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8389	8583	8787	8991	9194	9398	9601	9805	330006	330211	203
4	330414	330617	330819	331023	331225	331427	331630	331832	2034	2236	202
5	9438	9640	9842	3044	3246	3447	3649	3850	4051	4253	202
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
7	6400	6600	6800	7000	7200	7400	7600	7800	8000	8200	200
8	8456	8656	8855	9054	9253	9451	9650	9849	340047	340246	199
9	340444	340643	340841	341039	341237	341435	341633	341830	9028	9225	198
No.	0	1	2	3	4	5	6	7	8	9	Dist.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
220	342423	342620	342817	343014	343212	343409	343606	343802	343999	344196	197
1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
3	8305	8500	8694	8889	9083	9278	9472	9666	9860	350054	194
4	350248	350442	350636	350829	351023	351216	351410	351603	351796	1989	193
5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
9	9835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
230	361728	361917	362105	362292	362482	362671	362859	363048	363236	363424	188
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
4	9216	9401	9587	9772	9958	370143	370328	370513	370698	370883	185
5	371068	371253	371437	371622	371806	1991	2175	2360	2544	2728	184
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
9	8398	8580	8761	8943	9124	9306	9487	9668	9849	380030	181
240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837	181
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
5	9166	9343	9520	9698	9875	390051	390228	390405	390582	390759	177
6	390935	391112	391288	391464	391641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	397940	398114	398287	398461	398634	398808	398981	399154	399328	399501	173
1	9674	9847	400020	400192	400365	400538	400711	400883	401056	401228	173
2	401401	401573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933	410102	410271	410440	410609	410777	410946	411114	411283	411451	169
8	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474	167
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956	420121	420286	420451	420616	420781	420945	421110	421275	421439	165
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752	9914	430075	430236	430398	430559	430720	430881	431042	431203	161
270	431364	431525	431685	431846	432007	432167	432328	432488	432649	432809	161
1	2960	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
2	4589	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
5	9333	9491	9648	9806	9964	440132	440279	440437	440594	440752	158
6	440909	441066	441224	441381	441538	1695	1852	2009	2166	2323	157
7	2489	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
9	5094	5260	5415	6071	6226	6382	6537	6692	6848	7003	155

No.	0	1	2	3	4	5	6	7	8	9	Diff.
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OF NUMBERS.

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1	8706	8961	9015	9170	9324	9478	9633	9787	9941	450005	154
2	450249	450403	450557	450711	450865	451018	451172	451326	451479	1633	154
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7892	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9302	9543	9694	9845	9995	460146	460296	460447	460597	460748	151
9	460808	461048	461198	461348	461499	1649	1799	1948	2098	2248	150
290	462308	462548	462697	462847	462997	463146	463296	463445	463594	463744	150
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6808	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
5	9822	9969	470116	470263	470410	470557	470704	470851	470998	471145	147
6	471292	471438	1585	1732	1878	2025	2171	2318	2464	2610	146
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	477121	477266	477411	477555	477700	477844	477989	478133	478278	478422	145
1	6366	6511	6655	6800	6943	7087	7230	7373	7516	7659	144
2	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958	490099	490239	490380	490520	490661	490801	490941	491081	491222	140
310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621	140
1	2700	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962	500099	500236	500374	500511	500648	500785	500922	137
7	501059	501196	501333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	505150	505286	505421	505557	505693	505828	505964	506099	506234	506370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874	510009	510143	510277	510411	134
4	510545	510679	510813	510947	511081	511215	1349	1482	1616	1750	134
5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	518514	518646	518777	518909	519040	519171	519303	519434	519566	519697	131
1	9838	9959	530090	530221	530353	530484	530615	530745	530876	531007	131
2	521138	521269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8789	128
8	8917	9045	9174	9302	9430	9559	9687	9815	9943	530072	128
9	530200	530323	530456	530584	530712	530840	530968	531096	531223	1351	128

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340	531479	531607	531734	531862	531990	532117	532245	532372	532500	532627	128
1	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
4	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
5	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
6	9076	9202	9327	9452	9578	9703	9829	9954	540079	540204	125
7	540329	540455	540580	540705	540830	540955	541080	541205	1330	1454	125
8	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	544068	544192	544316	544440	544564	544688	544812	544936	545060	545184	124
1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
4	9003	9126	9249	9371	9494	9616	9739	9861	9984	550106	123
5	550228	550351	550473	550595	550717	550840	550962	551084	551206	1328	122
6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
8	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
9	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	556303	556423	556544	556664	556785	556905	557026	557146	557267	557387	120
1	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
2	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
3	9907	560026	560146	560265	560385	560504	560624	560743	560863	560982	119
4	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
5	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568292	568319	568346	568374	568401	568428	568455	568482	568509	568536	117
1	9374	9491	9608	9725	9842	9959	570076	570193	570309	570426	117
2	570543	570660	570776	570893	571010	571126	1243	1359	1476	1592	117
3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
5	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
6	5128	5203	5278	5354	5430	5505	5580	5656	5731	5806	115
7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	579784	579898	580012	580126	580241	580355	580469	580583	580697	580811	114
1	580925	581039	1153	1267	1381	1495	1608	1722	1836	1950	114
2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
9	9950	590061	590173	590284	590396	590507	590619	590730	590842	590953	112
390	591065	591176	591287	591399	591510	591621	591732	591843	591955	592066	111
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
5	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
8	9883	9992	600101	600210	600319	600428	600537	600646	600755	600864	109
9	600973	601082	1191	1299	1408	1517	1625	1734	1843	1951	109
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Dis.
400	602000	602100	602277	602386	602404	602603	602711	602819	602928	603036	106
1	3144	3253	3361	3409	3577	3686	3794	3902	4010	4118	106
2	4236	4334	4442	4550	4658	4766	4874	4982	5089	5197	106
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	106
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
7	9594	9701	9808	9914	610021	610128	610234	610341	610447	610554	107
8	610660	610767	610873	610979	1066	1192	1298	1405	1511	1617	106
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	613784	612890	612906	613102	613207	613313	613419	613525	613630	613736	106
1	3642	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
5	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
6	9093	9198	9302	9406	9511	9615	9719	9824	9928	600032	104
7	600136	600240	600344	600448	600552	600656	600760	600864	600968	1073	104
8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	622249	622353	623456	623559	623663	623766	623869	623973	624076	624179	103
1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	103
5	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	103
6	9410	9512	9613	9715	9817	9919	630021	630123	630224	630326	103
7	630428	630530	630631	630733	630835	630936	1038	1139	1241	1342	103
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	633468	633569	633670	633771	633872	633973	634074	634175	634276	634376	101
1	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	101
2	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
3	6488	6588	6688	6788	6889	6989	7089	7189	7289	7390	100
4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
5	8489	8589	8689	8789	8889	8989	9088	9188	9287	9387	100
6	9486	9586	9686	9785	9885	9984	640084	640183	640283	640383	99
7	640481	640581	640680	640779	640879	640978	1077	1177	1276	1375	99
8	1474	1573	1673	1771	1871	1970	2069	2168	2267	2366	99
9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	643453	643551	643650	643749	643847	643946	644044	644143	644242	644340	98
1	4430	4537	4636	4734	4833	4931	5029	5127	5226	5324	98
2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
6	9335	9432	9530	9627	9724	9821	9919	650016	650113	650210	97
7	650308	650405	650502	650599	650696	650793	650890	9967	1084	1181	97
8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	653300	653405	653502	653598	653695	653791	653888	653984	654080	96
1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
2	5128	5225	5321	5427	5523	5619	5715	5810	5906	6002	96
3	6068	6164	6260	6356	6452	6547	6643	6739	6834	6930	96
4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
7	9916	600011	600106	600201	600296	600391	600486	600581	600676	600771	95
8	600865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95

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400	662758	662852	662947	663041	663135	663230	663324	663418	663512	663607	94
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
7	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	93
8	670246	670339	670431	670524	670617	670710	670802	670895	9988	1080	93
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
8	9428	9519	9610	9701	9791	9882	9973	680063	680154	680245	91
9	680336	680426	680517	680607	680698	680789	680879	0970	1060	1151	91
480	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055	90
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
9	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
9	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751	87
1	9838	9924	700011	700098	700184	700271	700358	700444	700531	700617	87
2	700704	700790	0877	0963	1050	1136	1222	1309	1395	1482	86
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	85
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	710033	85
3	710117	710202	710287	710371	710456	710540	710625	710710	710794	0879	85
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84

No.	0	1	2	3	4	5	6	7	8	9	Diff.
520	716003	716087	716170	716254	716337	716421	716504	716588	716671	716754	83
1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
3	8502	8585	8668	8751	8834	8917	9000	9083	9166	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
5	720159	720242	720325	720407	720490	720573	720655	720738	720821	0003	83
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013	82
1	5005	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9894	81
7	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
8	730782	0963	0944	1024	1105	1186	1266	1347	1428	1508	81
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732304	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
5	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
7	7997	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
9	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
2	9736	9814	9891	9968	750045	750123	750200	750277	750354	750431	77
3	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
5	9668	9743	9819	9894	9970	760045	760121	760197	760272	760347	75
6	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
580	763428	763503	763578	763653	763727	763802	763877	763952	764027	764101	75
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
5	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
8	9377	9451	9525	9599	9673	9746	9820	9894	9968	770042	74
9	770115	770189	770263	770336	770410	770484	770557	770631	770705	0778	74
590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514	74
1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	778151	778224	778296	778368	778441	778513	778585	778658	778730	778802	72
1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
2	9596	9669	9741	9813	9885	9957	780029	780101	780173	780245	72
3	780317	780389	780461	780533	780605	780677	0749	0821	0893	0965	72
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	785401	785472	785542	785615	785686	785757	785828	785899	785970	71
1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
6	9581	9651	9722	9792	9863	9933	790004	790074	790144	790215	70
7	790285	790356	790426	790496	790567	790637	0707	0778	0848	0918	70
8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	792392	792462	792532	792602	792672	792742	792812	792882	792952	793022	70
1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
3	4488	4558	4627	4697	4767	4836	4906	4976	5045	4115	70
4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	793341	793409	793478	793547	793616	793685	793754	793823	793892	793961	69
1	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	69
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	68
5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
9	5501	5569	5637	5705	5773	5841	5909	5976	6044	6112	68

OF NUMBERS.

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640	806180	806248	806316	806384	806451	806519	806587	806655	806723	806790	68
1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
2	7335	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
5	9560	9627	9694	9762	9829	9896	9964	810031	810098	810165	67
6	810233	810300	810367	810434	810501	810568	810636	0703	0770	0837	67
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514	67
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	65
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136	66
1	820201	820267	820333	820399	820464	820530	820595	0661	0727	0792	66
2	0858	0924	0990	1055	1120	1186	1251	1317	1382	1448	66
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
4	2198	2263	2329	2394	2460	2525	2590	2656	2721	2786	65
5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	826140	826206	826270	826334	826399	826464	826528	826593	826658	65
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
5	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
6	9947	830011	830075	830139	830204	830268	830332	830396	830460	830525	64
7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	832500	832573	832637	832700	832764	832828	832892	832956	833020	833083	64
1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	836849	836912	836975	837038	837101	837164	837227	837290	837352	837415	63
1	9478	9541	9604	9667	9729	9792	9855	9918	9981	840043	63
2	840196	840169	840232	840294	840357	840420	840482	840545	840608	0671	63
3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
700	845098	845160	845222	845284	845346	845408	845470	845532	845594	845656	62
1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
3	6055	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
8	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	61
9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809	61
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875	60
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	860038	860098	860158	860218	860278	60
5	860338	860398	860458	860518	860578	860638	860697	860757	860817	860877	60
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863858	59
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	869232	869290	869348	869408	869466	869525	869584	869642	869701	869760	59
1	9818	9877	9935	9994	870053	870111	870170	870228	870287	870345	59
2	870404	870462	870521	870579	0638	0696	0755	0813	0872	0930	58
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582	58
1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7890	58
5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
8	9679	9736	9794	9841	9898	9956	880013	880070	880127	880185	57
9	880242	880299	880356	880413	880471	880529	0355	0642	0699	0756	57
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700	880814	880871	880928	880985	881042	881099	881156	881213	881271	881328	57
1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
5	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	886491	886547	886604	886660	886716	886773	886829	886885	886942	886998	56
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
3	8179	8236	8292	8348	8404	8460	8515	8573	8629	8685	56
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974	800630	890086	890141	890197	890253	890309	890365	56
7	890421	890477	890533	0589	0645	0700	0756	0812	0868	0924	56
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892005	892150	892296	892362	892317	892373	892429	892484	892540	892595	56
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
4	9821	9875	9930	9985	900039	900094	900149	900203	900258	900312	55
5	900367	900422	900476	900531	0586	0640	0695	0749	0804	0859	55
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	903090	903144	903199	903253	903307	903361	903416	903470	903524	903578	54
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	910037	53
3	910091	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
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No.	0	1	2	3	4	5	6	7	8	9	Diff.
820	913814	913867	913920	913973	914026	914079	914132	914184	914237	914290	53
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
1	9601	9653	9706	9758	9810	9862	9914	9967	990019	990071	52
2	920123	920176	920228	920280	920332	920384	920436	920489	0541	0593	52
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
7	2725	2777	2829	2881	2933	2985	3037	3089	3141	3192	52
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	924331	924383	924434	924486	924538	924590	924641	924693	924744	52
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
1	9930	9981	930032	930083	930134	930185	930236	930287	930338	930389	51
2	930440	930491	0542	0592	0643	0694	0745	0796	0847	0898	51
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
5	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953	50
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
5	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	939519	939569	939619	939669	939719	939769	939819	939869	939919	939968	50
1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
No.	0	1	2	3	4	5	6	7	8	9	Diff.

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No.	0	1	2	3	4	5	6	7	8	9	Diff.
880	944483	944532	944581	944631	944680	944729	944779	944828	944877	944927	49
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
5	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
8	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949030	949439	949888	949536	949585	949634	949683	949731	949780	949829	49
1	9878	9926	9975	950024	950073	950121	950170	950219	950267	950316	49
2	950365	950414	950462	0511	0560	0608	0657	0706	0754	0803	49
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	954243	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
2	9995	969042	969090	969138	969185	969233	969281	969328	969376	969423	48
3	969471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	48
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
3	9882	9928	9975	970021	970068	970114	970161	970207	970254	970300	47
4	970347	970393	970440	0486	0533	0579	0626	0672	0719	0765	46
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46

No.	0	1	2	3	4	5	6	7	8	9	Diff.
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No.	0	1	2	3	4	5	6	7	8	9	Diff.
940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543	46
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
5	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
5	990003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	989271	989316	989362	989407	989452	989497	989543	989588	989633	989678	45
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175	45
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
7	9895	9939	9983	990028	990072	990117	990161	990206	990250	990294	44
8	990339	990383	990428	0472	0516	0561	0605	0650	0694	0738	44
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	44
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

TABLE IV.

LOGARITHMIC

SINES AND TANGENTS,

FOR EVERY

DEGREE AND MINUTE

OF

THE QUADRANT.

N. B. The minutes in the left-hand column of each page, increasing downwards, belong to the degrees at the top; and those increasing upwards, in the right-hand column, belong to the degrees below.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	0-00000		10-00000		0-00000		Infinite.	60
1	6-463796	501717	000000	00	6-463796	501717	13-536274	59
2	764756	293485	000000	00	764756	293483	235244	58
3	940647	908231	000000	00	940647	908231	059153	57
4	7-065786	161517	000000	00	7-065786	161517	19-934214	56
5	162996	131968	000000	00	162996	131969	837304	55
6	241877	111575	9-999999	01	241877	111578	758192	54
7	308324	96653	9-999999	01	308325	99653	601175	53
8	366816	83254	9-999999	01	366817	85254	633183	52
9	417968	70353	9-999999	01	417970	70263	582030	51
10	463725	68988	9-999999	01	463727	68989	536273	50
11	7-505118	62961	9-999999	01	7-505120	62961	19-494890	49
12	542906	57936	9-999997	01	542909	57933	457091	48
13	577668	53641	9-999997	01	577672	53642	422328	47
14	609653	49038	9-999996	01	609657	49039	390143	46
15	639616	46714	9-999996	01	639620	46715	360180	45
16	667845	43881	9-999995	01	667849	43882	332151	44
17	694173	41372	9-999995	01	694179	41373	305821	43
18	718997	39135	9-999994	01	719003	39136	280997	42
19	742477	37127	9-999993	01	742484	37128	257156	41
20	764754	35315	9-999993	01	764761	35136	235239	40
21	7-785943	33672	9-999992	01	7-785951	33673	19-214040	39
22	806146	32175	9-999991	01	806155	32176	163845	38
23	825451	30695	9-999990	01	825460	30696	174540	37
24	843934	29347	9-999989	02	843944	29349	150056	36
25	861662	28368	9-999988	02	861674	28369	135336	35
26	878695	27317	9-999988	02	878708	27318	121282	34
27	895065	26323	9-999987	02	895099	26325	104901	33
28	910879	25399	9-999986	02	910894	25401	089106	32
29	926119	24538	9-999985	02	926134	24540	073966	31
30	940842	23733	9-999983	02	940858	23735	059142	30
31	7-955082	22980	9-999982	02	7-955100	22981	19-044900	29
32	966870	22273	9-999981	02	966889	22275	031111	28
33	982233	21608	9-999980	02	982253	21610	017747	27
34	995196	20961	9-999979	02	995219	20963	004781	26
35	8-007787	20390	9-999977	02	8-007809	20392	11-992191	25
36	020021	19831	9-999976	02	020045	19833	979855	24
37	031919	19308	9-999975	02	031945	19305	969055	23
38	043501	18801	9-999973	02	043527	18803	956473	22
39	054781	18325	9-999972	02	054809	18327	945191	21
40	065776	17879	9-999971	02	065806	17874	934194	20
41	8-076500	17441	9-999969	02	8-076531	17444	11-923469	19
42	086965	17031	9-999968	02	086997	17034	913003	18
43	097183	16639	9-999966	02	097217	16642	902782	17
44	107167	16265	9-999964	03	107202	16268	892797	16
45	116926	15908	9-999963	03	116963	15910	883037	15
46	126471	15566	9-999961	03	126510	15568	873490	14
47	135810	15238	9-999959	03	135851	15241	864149	13
48	144953	14924	9-999958	03	144996	14927	855004	12
49	153907	14622	9-999956	03	153952	14627	846048	11
50	162661	14333	9-999954	03	162727	14336	837273	10
51	8-171280	14054	9-999952	03	8-171328	14057	11-828672	9
52	179713	13786	9-999950	03	179763	13790	820237	8
53	187965	13529	9-999948	03	188036	13532	811964	7
54	196102	13280	9-999946	03	196156	13284	803844	6
55	904070	13041	9-999944	03	904126	13044	795874	5
56	911895	12810	9-999942	04	911953	12814	788047	4
57	919581	12587	9-999940	04	919641	12590	780350	3
58	927134	12372	9-999938	04	927195	12376	772905	2
59	934557	12164	9-999936	04	934621	12168	765379	1
60	941855	11963	9-999934	04	941921	11967	758079	0

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	8-941855	11963	9-999934	04	8-941921	11967	11-758079	60
1	8-949033	11768	9-999932	04	8-949102	11773	758096	59
2	8-956004	11580	9-999929	04	8-956185	11584	743835	58
3	8-963042	11396	9-999927	04	8-963115	11409	736885	57
4	8-969881	11221	9-999925	04	8-969856	11225	730044	56
5	8-976614	11050	9-999923	04	8-976601	11054	723300	55
6	8-983243	10883	9-999920	04	8-983223	10887	716677	54
7	8-989773	10721	9-999918	04	8-989756	10726	710144	53
8	8-996207	10565	9-999915	04	8-996202	10570	703708	52
9	8-992546	10413	9-999913	04	8-992534	10418	697366	51
10	8-998794	10266	9-999910	04	8-998784	10270	691116	50
11	8-9314954	10122	9-999907	04	8-9315046	10126	11-684954	49
12	8-921027	9969	9-999905	04	8-921122	9967	678878	48
13	8-910716	9847	9-999902	04	8-91114	9851	672986	47
14	8-900224	9714	9-999899	05	8-903025	9719	666975	46
15	8-889753	9586	9-999897	05	8-88856	9590	661144	45
16	8-879204	9460	9-999894	05	8-874610	9465	655390	44
17	8-868681	9338	9-999891	05	8-865290	9343	649711	43
18	8-858183	9219	9-999888	05	8-855895	9224	644105	42
19	8-847715	9103	9-999885	05	8-846430	9108	638570	41
20	8-837277	8990	9-999882	05	8-836985	8995	633105	40
21	8-826871	8880	9-999879	05	8-827529	8885	11-627708	39
22	8-816490	8773	9-999876	05	8-817622	8777	622378	38
23	8-806132	8667	9-999873	05	8-808229	8672	617111	37
24	8-795796	8564	9-999870	05	8-796022	8570	611908	36
25	8-785481	8464	9-999867	05	8-785234	8470	606766	35
26	8-775187	8366	9-999864	05	8-775315	8371	601685	34
27	8-764914	8271	9-999861	05	8-765386	8276	596662	33
28	8-754661	8177	9-999858	05	8-755450	8182	591696	32
29	8-744426	8086	9-999855	05	8-745513	8091	586787	31
30	8-734207	7996	9-999851	06	8-735668	8002	581932	30
31	8-724004	7909	9-999848	06	8-725820	7914	11-577131	29
32	8-713816	7823	9-999844	06	8-716118	7830	572369	28
33	8-703642	7740	9-999841	06	8-706515	7745	567685	27
34	8-693481	7657	9-999838	06	8-697022	7663	563038	26
35	8-683334	7577	9-999834	06	8-687530	7583	558440	25
36	8-673201	7499	9-999831	06	8-678040	7505	553890	24
37	8-663081	7422	9-999827	06	8-668550	7428	549387	23
38	8-652974	7346	9-999823	06	8-659070	7352	544930	22
39	8-642881	7273	9-999820	06	8-649591	7279	540519	21
40	8-632801	7200	9-999816	06	8-640119	7206	536151	20
41	8-622734	7129	9-999812	06	8-630654	7135	11-531628	19
42	8-612681	7060	9-999809	06	8-621194	7066	527546	18
43	8-602641	6991	9-999805	06	8-611738	6996	523307	17
44	8-592614	6924	9-999801	06	8-592282	6921	519108	16
45	8-582601	6859	9-999797	07	8-582826	6865	514950	15
46	8-572601	6794	9-999793	07	8-573370	6801	510830	14
47	8-562614	6731	9-999790	07	8-563915	6736	506750	13
48	8-552641	6669	9-999786	07	8-554460	6676	502707	12
49	8-542681	6608	9-999782	07	8-545005	6615	498702	11
50	8-532734	6548	9-999778	07	8-535550	6555	494733	10
51	8-522801	6489	9-999774	07	8-526095	6496	11-490800	9
52	8-512881	6431	9-999770	07	8-516640	6439	486802	8
53	8-502974	6375	9-999765	07	8-507185	6382	482839	7
54	8-493081	6319	9-999761	07	8-497730	6326	478910	6
55	8-483201	6264	9-999757	07	8-488275	6279	475014	5
56	8-473334	6211	9-999753	07	8-478820	6218	471151	4
57	8-463481	6158	9-999748	07	8-469365	6165	467320	3
58	8-453641	6106	9-999744	07	8-459910	6113	463521	2
59	8-443814	6055	9-999740	07	8-450455	6062	459753	1
60	8-434001	6004	9-999735	07	8-441000	6012	456016	0
	Cosine	Sine		Tang.		Cotang.	M.	

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-542819	6004	9-999735	07	8-543084	6019	11-456916	60
1	546429	5955	999731	07	546691	5962	453309	59
2	549995	5906	999726	07	550268	5914	449732	58
3	553539	5858	999722	06	553817	5866	446183	57
4	557054	5811	999717	06	557336	5819	442664	56
5	560540	5765	999713	06	560898	5773	439179	55
6	563999	5719	999708	06	564491	5727	435709	54
7	567431	5674	999704	06	567797	5682	432273	53
8	570836	5630	999699	06	571137	5638	428863	52
9	574214	5587	999694	06	574580	5595	425480	51
10	577566	5544	999689	06	577877	5553	422123	50
11	9-580892	5500	9-999685	06	8-581308	5510	11-418792	49
12	584193	5460	999680	06	584514	5468	415436	48
13	587469	5419	999675	06	587795	5427	412105	47
14	590731	5379	999670	06	591051	5387	408849	46
15	593948	5339	999665	06	594293	5347	405717	45
16	597152	5300	999660	06	597492	5308	402566	44
17	600333	5261	999655	06	600677	5270	399393	43
18	603489	5223	999650	06	603839	5232	396161	42
19	606623	5186	999645	06	606978	5194	392922	41
20	609734	5149	999640	06	610094	5158	389690	40
21	9-612823	5112	9-999635	06	8-613189	5121	11-386811	39
22	615891	5076	999629	06	616262	5085	383736	38
23	618937	5041	999624	06	619313	5050	380687	37
24	621968	5006	999619	06	622343	5015	377657	36
25	624985	4972	999614	06	625352	4981	374648	35
26	627948	4938	999608	06	628340	4947	371660	34
27	630911	4904	999603	06	631308	4913	368699	33
28	633854	4871	999597	06	634256	4880	365744	32
29	636776	4839	999592	06	637184	4848	362816	31
30	639680	4806	999586	06	640093	4816	359907	30
31	9-642263	4775	9-999581	06	8-642662	4784	11-357018	29
32	645298	4743	999575	06	645653	4753	354147	28
33	648274	4712	999570	06	648704	4722	351306	27
34	651199	4682	999564	06	651537	4691	348463	26
35	653911	4652	999558	10	654352	4661	345648	25
36	656702	4622	999553	10	657149	4631	342851	24
37	659475	4592	999547	10	659928	4602	340079	23
38	662230	4563	999541	10	662689	4573	337311	22
39	664968	4535	999535	10	665433	4544	334567	21
40	667689	4506	999529	10	668160	4516	331840	20
41	9-670363	4479	9-999524	10	8-670670	4488	11-329130	19
42	673080	4451	999518	10	673363	4461	326437	18
43	675751	4424	999512	10	676239	4434	323761	17
44	678405	4397	999506	10	679090	4417	321100	16
45	681043	4370	999500	10	681544	4390	318456	15
46	683665	4344	999493	10	684172	4364	315826	14
47	686272	4318	999487	10	686784	4338	313216	13
48	688863	4292	999481	10	689381	4313	310619	12
49	691438	4267	999475	10	691963	4287	308037	11
50	693998	4242	999469	10	694529	4262	305471	10
51	9-696543	4217	9-999463	11	8-697081	4238	11-302919	9
52	699073	4192	999456	11	699617	4213	300383	8
53	701589	4168	999450	11	702139	4189	297861	7
54	704090	4144	999444	11	704646	4165	295354	6
55	706577	4121	999437	11	707140	4142	292860	5
56	709049	4097	999431	11	709618	4118	290389	4
57	711507	4074	999424	11	712083	4095	287917	3
58	713952	4051	999418	11	714534	4072	285465	2
59	716383	4029	999411	11	716972	4049	283028	1
60	718800	4006	999404	11	719396	4017	280604	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Size	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	8718900	4006	9999404	11	8719396	4017	11290604	60
1	721904	3984	999398	11	721806	3995	978194	59
2	723595	3969	999391	11	723494	3974	975796	58
3	725072	3941	999384	11	725096	3952	973419	57
4	726337	3919	999378	11	726950	3930	971041	56
5	730698	3898	999371	11	731317	3909	968683	55
6	733027	3877	999364	19	733663	3889	966337	54
7	735354	3857	999357	19	735906	3868	964004	53
8	737667	3836	999350	19	738317	3848	961683	52
9	739969	3816	999343	19	740626	3827	959374	51
10	742269	3796	999336	19	742923	3807	957078	50
11	8744536	3776	999329	19	8745207	3787	11222583	49
12	746802	3756	999322	19	747479	3768	954821	48
13	749053	3737	999315	19	749740	3749	952526	47
14	751297	3717	999308	19	751989	3729	949811	46
15	753528	3698	999301	19	754227	3710	947573	45
16	755747	3679	999294	19	756453	3692	945347	44
17	757955	3661	999286	19	758668	3673	943133	43
18	760151	3642	999279	19	760872	3655	940928	42
19	762337	3624	999272	19	763065	3636	938735	41
20	764511	3606	999265	19	765246	3618	936544	40
21	8766675	3588	999257	19	8767417	3600	11222583	39
22	766828	3570	999250	13	768958	3583	934322	38
23	770970	3553	999242	13	771727	3565	932173	37
24	773101	3535	999235	13	773906	3548	929614	36
25	775223	3518	999227	13	775995	3531	927040	35
26	777333	3501	999220	13	778114	3514	924466	34
27	779434	3484	999212	13	780222	3497	921878	33
28	781524	3467	999205	13	782320	3480	919278	32
29	783605	3451	999197	13	784408	3464	916662	31
30	785675	3431	999189	13	786486	3447	914034	30
31	8787736	3418	999181	13	8788554	3431	11211446	29
32	789767	3402	999174	13	790613	3414	911387	28
33	791828	3386	999166	13	792662	3399	908738	27
34	793859	3370	999158	13	794701	3383	906089	26
35	795981	3354	999150	13	796731	3368	903439	25
36	797994	3339	999142	13	798752	3352	900789	24
37	799997	3323	999134	13	800763	3337	898137	23
38	801992	3308	999126	13	802765	3322	895482	22
39	803978	3293	999118	13	804758	3307	892824	21
40	805952	3278	999110	13	806742	3292	890162	20
41	807919	3263	999102	13	808717	3278	11191283	19
42	809777	3249	999094	14	810683	3262	886317	18
43	811726	3234	999086	14	812641	3248	883459	17
44	813667	3219	999077	14	814589	3233	880591	16
45	815599	3205	999069	14	816529	3219	877714	15
46	817522	3191	999061	14	818461	3205	874829	14
47	819436	3177	999053	14	820384	3191	871936	13
48	821343	3163	999044	14	822298	3177	869037	12
49	823240	3149	999036	14	824205	3163	866132	11
50	825130	3135	999027	14	826103	3150	863221	10
51	827011	3122	999019	14	827992	3136	11179008	9
52	828884	3108	999010	14	829874	3123	859317	8
53	830749	3095	999002	14	831748	3110	856422	7
54	832607	3082	998993	14	833613	3096	853526	6
55	834456	3069	998984	14	835471	3083	850629	5
56	836297	3056	998976	14	837321	3070	847731	4
57	838130	3043	998967	15	839163	3057	844832	3
58	839956	3030	998958	15	840998	3045	841932	2
59	841774	3017	998950	15	842825	3032	839032	1
60	843585	3000	998941	15	844644	3019	836132	0

86 Degrees.



M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	8843585	3005	9989941	15	8844644	3019	11155356	60
1	845387	2999	9989932	15	846455	3007	153545	59
2	847183	2990	9989923	15	848900	2995	151740	58
3	848971	2967	9989914	15	850057	2982	149943	57
4	850751	2955	9989905	15	851846	2970	148154	56
5	852525	2943	9989896	15	853628	2958	146372	55
6	854291	2931	9989887	15	855403	2946	144597	54
7	856049	2919	9989878	15	857171	2935	142829	53
8	857801	2907	9989869	15	858932	2923	141068	52
9	859546	2896	9989860	15	860696	2911	139314	51
10	861283	2884	9989851	15	862433	2900	137567	50
11	8863014	9873	9989841	15	8864173	2888	11135927	49
12	864738	2861	9989832	15	865906	2877	134094	48
13	866455	2850	9989823	16	867632	2866	132268	47
14	868165	2839	9989813	16	869351	2854	130449	46
15	869868	2828	9989804	16	871064	2843	128636	45
16	871565	2817	9989795	16	872770	2832	127220	44
17	873255	2806	9989785	16	874469	2821	125531	43
18	874938	2795	9989776	16	876162	2811	123858	42
19	876615	2784	9989766	16	877849	2800	122151	41
20	878285	2773	9989757	16	879529	2789	120471	40
21	8879949	2763	9989747	16	8881202	2779	1118798	39
22	881607	2752	9989738	16	882809	2768	117131	38
23	883258	2742	9989728	16	884530	2758	115470	37
24	884903	2731	9989718	16	886185	2747	113815	36
25	886542	2721	9989708	16	887833	2737	112167	35
26	888174	2711	9989699	16	889476	2727	110524	34
27	889801	2700	9989689	16	891112	2717	108888	33
28	891421	2690	9989679	16	892742	2707	107258	32
29	893035	2680	9989669	17	894366	2697	105634	31
30	894643	2670	9989659	17	895984	2687	104016	30
31	896246	2660	9989649	17	897596	2677	1110240	29
32	897842	2651	9989639	17	899203	2667	100797	28
33	899432	2641	9989629	17	900803	2658	999197	27
34	901017	2631	9989619	17	902398	2648	997602	26
35	902596	2622	9989609	17	903987	2638	996013	25
36	904169	2613	9989599	17	905570	2629	994430	24
37	905736	2603	9989589	17	907147	2620	992853	23
38	907297	2593	9989578	17	908719	2611	991281	22
39	908853	2584	9989568	17	910285	2601	989715	21
40	910404	2575	9989558	17	911846	2592	988154	20
41	8911949	2566	9989548	17	8913401	2583	11086599	19
42	913488	2556	9989537	17	914951	2574	985049	18
43	915022	2547	9989527	17	916495	2565	983505	17
44	916550	2538	9989516	18	918034	2556	981966	16
45	918073	2529	9989506	18	919568	2547	980432	15
46	919591	2520	9989495	18	921096	2538	978904	14
47	921103	2512	9989483	18	922619	2530	977381	13
48	922610	2503	9989474	18	924136	2521	975864	12
49	924112	2494	9989464	18	925649	2512	974351	11
50	925600	2486	9989453	18	927156	2503	972844	10
51	8927100	2477	9989442	18	8928658	2495	11071342	9
52	928257	2469	9989431	18	930155	2486	969645	8
53	930068	2460	9989421	18	931647	2478	968353	7
54	931544	2452	9989410	18	933134	2470	966966	6
55	933015	2443	9989399	18	934616	2461	965594	5
56	934481	2435	9989388	18	936093	2453	963907	4
57	935942	2427	9989377	18	937565	2445	962435	3
58	937398	2419	9989366	18	939032	2437	960968	2
59	938850	2411	9989355	18	940494	2430	959506	1
60	940296	2403	9989344	18	941952	2421	958048	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	8-940896	2403	9-968344	19	8-941932	2421	11-058048	60
1	941738	2394	968333	19	943404	2413	058596	59
2	943174	2387	968322	19	944852	2405	055148	58
3	944606	2379	968311	19	946285	2397	053705	57
4	946034	2371	968300	19	947734	2390	062266	56
5	947456	2363	968289	19	949168	2382	050632	55
6	948874	2355	968277	19	950597	2374	049403	54
7	950287	2348	968266	19	952021	2366	047779	53
8	951696	2340	968255	19	953441	2358	046559	52
9	953100	2332	968243	19	954856	2351	045144	51
10	954499	2325	968232	19	956267	2344	043733	50
11	8-955894	2317	9-968220	19	8-957674	2337	11-042326	49
12	957294	2310	968209	19	959075	2329	040925	48
13	958670	2302	968197	19	960473	2323	039427	47
14	960052	2295	968186	19	961866	2314	038134	46
15	961429	2288	968174	19	963255	2307	036745	45
16	962801	2280	968163	19	964639	2300	035361	44
17	964170	2273	968151	19	966019	2293	033981	43
18	965534	2266	968139	20	967394	2286	032606	42
19	966893	2258	968128	20	968766	2279	031234	41
20	968249	2252	968116	20	970133	2271	029867	40
21	8-969600	2244	9-968104	20	8-971496	2265	11-028504	39
22	970947	2238	968092	20	972855	2257	027145	38
23	972289	2231	968080	20	974209	2251	025791	37
24	973628	2224	968068	20	975560	2244	024440	36
25	974969	2217	968056	20	976906	2237	023094	35
26	976293	2210	968044	20	978248	2230	021752	34
27	977619	2203	968032	20	979586	2223	020414	33
28	978941	2197	968020	20	980921	2217	019079	32
29	980259	2190	968008	20	982251	2210	017749	31
30	981573	2183	967996	20	983577	2204	016423	30
31	8-982883	2177	9-967984	20	8-984899	2197	11-015101	29
32	984189	2170	967972	20	986217	2191	013783	28
33	985491	2163	967959	20	987532	2184	012468	27
34	986789	2157	967947	20	988842	2178	011156	26
35	988083	2150	967935	21	990149	2171	009851	25
36	989374	2144	967922	21	991451	2165	008549	24
37	990660	2138	967910	21	992750	2158	007250	23
38	991943	2131	967897	21	994045	2152	005955	22
39	993222	2125	967885	21	995337	2146	004663	21
40	994497	2119	967872	21	996624	2140	003376	20
41	8-995768	2112	9-967860	21	8-997908	2134	11-002029	19
42	997036	2106	967847	21	999188	2127	000819	18
43	998299	2100	967835	21	1000465	2121	10-999535	17
44	999560	2094	967822	21	001738	2115	998269	16
45	9-000816	2087	9-967809	21	003007	2109	996993	15
46	002080	2082	967797	21	004272	2103	995728	14
47	003318	2076	967784	21	005534	2097	994466	13
48	004563	2070	967771	21	006792	2091	993206	12
49	005805	2064	967758	21	008047	2085	991953	11
50	007044	2058	967745	21	009298	2080	990709	10
51	8-008278	2052	8-967732	21	8-010546	2074	10-989454	9
52	009510	2046	967719	21	011790	2068	988210	8
53	010737	2040	967706	21	013031	2062	986969	7
54	011962	2034	967693	22	014268	2056	985732	6
55	013189	2029	967680	22	015502	2051	984498	5
56	014400	2023	967667	22	016732	2045	983268	4
57	015613	2017	967654	22	017959	2040	982041	3
58	016824	2012	967641	22	019183	2033	980817	2
59	018031	2006	967628	22	020403	2028	979597	1
60	019235	2000	967614	22	021620	2023	978380	0

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	0°19235	9000	9°97614	92	9°021690	9023	10°976390	60
1	090435	1905	997601	92	022634	9017	977166	59
2	081632	1909	997588	92	024044	9011	975956	58
3	022925	1914	997574	92	025251	9006	974749	57
4	024016	1918	997561	92	026455	9000	973545	56
5	025203	1923	997547	92	027655	1995	972345	55
6	026396	1927	997534	92	028852	1990	971148	54
7	027587	1931	997520	92	030046	1985	969954	53
8	028774	1937	997507	92	031237	1979	968763	52
9	029958	1941	997493	92	032425	1974	967575	51
10	031099	1947	997480	92	033609	1969	966391	50
11	9°032257	1941	9°997466	92	9°034791	1964	10°965209	49
12	033421	1936	997452	92	035969	1958	964031	48
13	034592	1930	997439	92	037144	1953	962856	47
14	035741	1925	997425	92	038316	1948	961684	46
15	036906	1920	997411	92	039485	1943	960515	45
16	038048	1915	997397	92	040651	1938	959349	44
17	039197	1910	997383	92	041813	1933	958187	43
18	040342	1905	997369	92	042973	1928	957027	42
19	041485	1899	997355	92	044130	1923	955870	41
20	042625	1894	997341	92	045284	1918	954716	40
21	9°043762	1889	9°997327	94	9°046434	1913	10°953566	39
22	044895	1884	997313	94	047582	1908	952418	38
23	046096	1879	997299	94	048727	1903	951273	37
24	047154	1875	997285	94	049869	1898	950131	36
25	048270	1870	997271	94	051008	1893	948992	35
26	049400	1865	997257	94	052144	1889	947856	34
27	050519	1860	997242	94	053277	1884	946723	33
28	051635	1855	997228	94	054407	1879	945593	32
29	052749	1850	997214	94	055535	1874	944465	31
30	053859	1845	997199	94	056659	1870	943341	30
31	054966	1841	9°997185	94	9°057781	1865	10°942219	29
32	056071	1836	997170	94	058900	1860	941100	28
33	057172	1831	997156	94	060016	1855	939984	27
34	058271	1827	997141	94	061130	1851	938870	26
35	059367	1822	997127	94	062240	1846	937760	25
36	060460	1817	997112	94	063348	1842	936652	24
37	061551	1813	997098	94	064453	1837	935547	23
38	062639	1808	997083	95	065556	1833	934444	22
39	063724	1804	997068	95	066655	1828	933345	21
40	064806	1799	997053	95	067752	1824	932248	20
41	9°065885	1794	9°997039	95	9°068846	1819	10°931154	19
42	066962	1790	997024	95	069938	1815	930062	18
43	068036	1786	997009	95	071027	1810	928973	17
44	069107	1781	996994	95	072113	1806	927887	16
45	070176	1777	996979	95	073197	1802	926803	15
46	071242	1773	996964	95	074278	1797	925722	14
47	072306	1768	996949	95	075356	1793	924644	13
48	073366	1763	996934	95	076432	1789	923568	12
49	074424	1759	996919	95	077505	1784	922495	11
50	075480	1755	996904	95	078578	1780	921424	10
51	9°076533	1750	9°996889	95	9°079644	1776	10°920356	9
52	077583	1746	996874	95	080710	1772	919290	8
53	078631	1742	996858	95	081773	1767	918227	7
54	079676	1738	996843	95	082833	1763	917167	6
55	080719	1733	996828	95	083891	1759	916109	5
56	081759	1729	996812	96	084947	1755	915053	4
57	082797	1725	996797	96	086000	1751	914000	3
58	083832	1721	996782	96	087050	1747	912950	2
59	084864	1717	996766	96	088098	1743	911902	1
60	085894	1713	996751	96	089144	1738	910856	0
	Cosine		Sine		Cotang.		Tang.	M.

M	Sine	D.	Coms	D.	Tang.	D.	Coang.	
0	0-085894	1713	0-996751	96	0-089144	1738	10-910856	60
1	0-086023	1700	0-996735	96	0-090187	1734	9-909813	59
2	0-087947	1704	0-996790	96	0-091228	1730	9-908772	58
3	0-089970	1700	0-996704	96	0-092266	1727	9-907734	57
4	0-090990	1696	0-996688	96	0-093302	1722	9-906698	56
5	0-091008	1692	0-996673	96	0-094336	1719	9-905664	55
6	0-092024	1688	0-996657	96	0-095367	1715	9-904633	54
7	0-093037	1684	0-996641	96	0-096395	1711	9-903605	53
8	0-094047	1680	0-996625	96	0-097422	1707	9-902578	52
9	0-095056	1676	0-996610	96	0-098446	1703	9-901554	51
10	0-096062	1673	0-996594	96	0-099468	1699	9-900532	50
11	0-097065	1668	0-996578	27	0-100487	1695	10-899513	49
12	0-098066	1665	0-996562	27	1-01504	1691	8-98496	48
13	0-099065	1661	0-996546	27	1-02519	1687	8-97481	47
14	1-000062	1657	0-996530	27	1-03532	1684	8-96468	46
15	1-001056	1653	0-996514	27	1-04542	1680	8-95458	45
16	1-002048	1649	0-996498	27	1-05550	1676	8-94450	44
17	1-003037	1645	0-996482	27	1-06556	1672	8-93444	43
18	1-004025	1641	0-996465	27	1-07559	1669	8-92441	42
19	1-005010	1638	0-996449	27	1-08560	1665	8-91440	41
20	1-005992	1634	0-996433	27	1-09559	1661	8-90441	40
21	0-106973	1630	0-996417	27	0-110556	1658	10-899444	39
22	0-107951	1627	0-996400	27	1-11551	1654	8-98440	38
23	0-108927	1623	0-996384	27	1-12543	1650	8-97457	37
24	0-109901	1619	0-996368	27	1-13533	1646	8-96467	36
25	1-110873	1616	0-996351	27	1-14521	1643	8-95479	35
26	1-111843	1612	0-996335	27	1-15507	1639	8-94493	34
27	1-112809	1608	0-996318	27	1-16491	1636	8-93509	33
28	1-113774	1605	0-996302	28	1-17472	1632	8-92528	32
29	1-114737	1601	0-996285	28	1-18452	1629	8-91548	31
30	1-115698	1597	0-996269	28	1-19429	1625	8-90571	30
31	0-116656	1594	0-996252	28	0-120404	1622	10-879596	29
32	1-117613	1590	0-996235	28	1-21377	1618	8-78923	28
33	1-118567	1587	0-996219	28	1-22348	1615	8-77952	27
34	1-119519	1583	0-996202	28	1-23317	1611	8-76983	26
35	1-20469	1580	0-996185	28	1-24284	1607	8-75716	25
36	1-21417	1576	0-996168	28	1-25249	1604	8-74751	24
37	1-22362	1573	0-996151	28	1-26211	1601	8-73789	23
38	1-23306	1569	0-996134	28	1-27172	1597	8-72828	22
39	1-24248	1566	0-996117	28	1-28130	1594	8-71870	21
40	1-25187	1562	0-996100	28	1-29087	1591	8-70913	20
41	0-126125	1559	0-996083	29	0-130041	1587	10-869059	19
42	1-27060	1556	0-996066	29	1-30994	1584	8-69006	18
43	1-27993	1552	0-996049	29	1-31944	1581	8-68058	17
44	1-28925	1549	0-996032	29	1-32893	1577	8-67107	16
45	1-29854	1545	0-996015	29	1-33839	1574	8-66161	15
46	1-30781	1542	0-995998	29	1-34784	1571	8-65216	14
47	1-31706	1539	0-995980	29	1-35726	1567	8-64274	13
48	1-32630	1535	0-995963	29	1-36667	1564	8-63333	12
49	1-33551	1532	0-995946	29	1-37605	1561	8-62395	11
50	1-34470	1529	0-995928	29	1-38542	1558	8-61458	10
51	0-135387	1525	0-995911	29	0-139476	1555	10-869024	9
52	1-36303	1522	0-995894	29	1-40409	1551	8-59591	8
53	1-37216	1519	0-995876	29	1-41340	1548	8-58660	7
54	1-38128	1516	0-995859	29	1-42269	1545	8-57731	6
55	1-39037	1512	0-995841	29	1-43196	1542	8-56804	5
56	1-39944	1509	0-995823	29	1-44121	1539	8-55879	4
57	1-40850	1506	0-995806	29	1-45044	1535	8-54956	3
58	1-41754	1503	0-995788	29	1-45966	1532	8-54034	2
59	1-42655	1500	0-995771	29	1-46885	1529	8-53115	1
60	1-43555	1496	0-995753	29	1-47803	1526	8-52197	0
	Coms	Sine	Coms	Tang.	Coms	Tang.	M.	

M.	Sine	D.	Cotang	D.	Tang.	D.	Cotang.	M.
0	9-143555	1486	9-985753	30	9-147803	1586	10-852197	00
1	144453	1483	985745	30	148718	1523	851282	50
2	145349	1480	985717	30	149632	1580	850368	58
3	146243	1477	985689	30	150544	1517	849456	57
4	147136	1484	985661	30	151454	1514	848546	56
5	148028	1481	985634	30	152363	1511	847637	55
6	148915	1478	985606	30	153269	1508	846731	54
7	149806	1475	985578	30	154174	1505	845826	53
8	150696	1472	985550	30	155077	1502	844922	52
9	151589	1469	985521	30	155978	1499	844022	51
10	152451	1466	985493	30	156877	1496	843123	50
11	9-153330	1463	9-985465	30	9-157775	1493	10-842225	49
12	154208	1460	985437	30	158671	1490	841329	48
13	155083	1457	985409	30	159565	1487	840435	47
14	155957	1454	985381	31	160457	1484	839543	46
15	156830	1451	985352	31	161347	1481	838653	45
16	157700	1448	985324	31	162236	1479	837764	44
17	158569	1445	985296	31	163123	1476	836877	43
18	159435	1442	985267	31	164008	1473	835992	42
19	160301	1439	985239	31	164892	1470	835108	41
20	161164	1436	985210	31	165774	1467	834226	40
21	9-162025	1433	9-985182	31	9-166654	1464	10-833346	39
22	162885	1430	985153	31	167532	1461	832468	38
23	163743	1427	985124	31	168409	1458	831591	37
24	164600	1424	985096	31	169284	1455	830716	36
25	165454	1422	985067	31	170157	1453	829843	35
26	166307	1419	985038	31	171029	1450	828971	34
27	167159	1416	985009	31	171899	1447	828101	33
28	168008	1413	984981	30	172767	1444	827233	32
29	168856	1410	984952	30	173634	1442	826366	31
30	169702	1407	984923	30	174499	1439	825501	30
31	9-170547	1405	9-984894	30	9-175362	1436	10-824638	29
32	171389	1402	984865	30	176224	1433	823776	28
33	172230	1399	984836	30	177084	1431	822916	27
34	173070	1396	984807	30	177942	1428	822058	26
35	173908	1394	984778	30	178799	1425	821201	25
36	174744	1391	984749	30	179655	1423	820345	24
37	175578	1388	984720	30	180508	1420	819492	23
38	176411	1386	984691	30	181360	1417	818640	22
39	177242	1383	984662	30	182211	1415	817789	21
40	178072	1380	984633	30	183059	1412	816941	20
41	9-178900	1377	9-984604	30	9-183907	1409	10-816093	19
42	179726	1374	984574	30	184752	1407	815248	18
43	180551	1372	984545	30	185597	1404	814403	17
44	181374	1369	984516	30	186439	1402	813559	16
45	182196	1366	984486	30	187280	1399	812717	15
46	183016	1364	984456	30	188120	1396	811876	14
47	183834	1361	984427	30	188958	1393	811034	13
48	184651	1359	984397	30	189794	1391	810192	12
49	185466	1356	984368	30	190629	1389	809351	11
50	186280	1353	984338	30	191462	1386	808510	10
51	9-187092	1351	9-984308	30	9-192294	1384	10-807706	9
52	187903	1348	984279	30	193124	1381	806876	8
53	188712	1346	984249	30	193953	1379	806047	7
54	189519	1343	984219	30	194780	1376	805220	6
55	190325	1341	984189	30	195606	1374	804394	5
56	191130	1338	984159	30	196430	1371	803570	4
57	191933	1336	984129	30	197253	1369	802747	3
58	192734	1333	984099	30	198074	1366	801926	2
59	193534	1330	984069	30	198894	1364	801106	1
60	194332	1328	984039	30	199713	1361	800287	0
	Cotang	Sine	Cotang		Tang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-194332	1324	9-994668	33	9-197713	1361	10-900287	90
1	195129	1326	994600	33	900589	1359	799471	59
2	195925	1323	994580	33	901345	1356	798655	58
3	196719	1321	994560	34	902159	1354	797841	57
4	197511	1318	994540	34	902971	1352	797029	56
5	198302	1316	994519	34	903788	1349	796218	55
6	199091	1313	994499	34	904592	1347	795406	54
7	199879	1311	994470	34	905400	1345	794600	53
8	200666	1308	994459	34	906207	1342	793793	52
9	201451	1306	994438	34	907013	1340	792987	51
10	202234	1304	994418	34	907817	1338	792183	50
11	9-203017	1301	9-994397	34	9-206619	1335	10-791381	49
12	203797	1299	994377	34	206480	1333	790580	48
13	204577	1296	994357	34	207290	1331	789780	47
14	205354	1294	994336	34	208101	1329	788982	46
15	206131	1292	994316	34	208915	1326	788185	45
16	206906	1289	994295	34	209731	1324	787389	44
17	207679	1287	994274	35	210545	1321	786595	43
18	208452	1285	994254	35	211369	1319	785801	42
19	209223	1282	994233	35	212192	1317	785011	41
20	209992	1280	994212	35	213015	1315	784220	40
21	9-210780	1278	9-994191	35	9-216568	1312	10-783432	39
22	211556	1275	994171	35	217386	1310	782644	38
23	212331	1273	994150	35	218202	1308	781858	37
24	213105	1271	994129	35	219026	1305	781074	36
25	213878	1268	994108	35	219847	1303	780290	35
26	214649	1266	994087	35	220669	1301	779508	34
27	215420	1264	994066	35	221492	1299	778728	33
28	216191	1261	994045	35	222315	1297	777948	32
29	216961	1259	994024	35	223138	1294	777170	31
30	217730	1257	994003	35	223960	1292	776394	30
31	9-218523	1255	9-993981	35	9-224782	1290	10-775618	29
32	219116	1253	993960	35	225605	1288	774844	28
33	219908	1250	993939	35	226428	1286	774071	27
34	220618	1248	993918	35	227250	1284	773300	26
35	221327	1246	993896	36	228071	1281	772529	25
36	222035	1244	993875	36	228892	1279	771761	24
37	222742	1242	993854	36	229713	1277	770993	23
38	223449	1239	993833	36	230534	1275	770227	22
39	224156	1237	993811	36	231355	1273	769461	21
40	224862	1235	993789	36	232176	1271	768696	20
41	9-225633	1233	9-993768	36	9-233005	1269	10-767935	19
42	226257	1231	993746	36	233826	1267	767174	18
43	226971	1228	993725	36	234646	1265	766414	17
44	227684	1226	993703	36	235465	1263	765655	16
45	228396	1224	993681	36	236284	1260	764897	15
46	229108	1222	993660	36	237103	1258	764141	14
47	229819	1220	993638	36	237921	1256	763386	13
48	230529	1218	993616	36	238739	1254	762633	12
49	231238	1216	993594	37	239557	1252	761880	11
50	231944	1214	993572	37	240373	1250	761128	10
51	9-233172	1212	9-993550	37	9-241202	1248	10-760378	9
52	233899	1209	993528	37	242021	1246	759629	8
53	234625	1207	993506	37	242840	1244	758882	7
54	235349	1205	993484	37	243658	1242	758135	6
55	236073	1203	993462	37	244475	1240	757390	5
56	236795	1201	993440	37	245292	1238	756646	4
57	237515	1199	993418	37	246107	1236	755903	3
58	238235	1197	993396	37	246922	1234	755161	2
59	238953	1195	993374	37	247737	1232	754421	1
60	239670	1193	993351	37	248551	1230	753681	0
	Cosine	Sine	Cotang.	Tang.				

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	929070	1183	9983351	37	9294319	1280	10753681	60
1	940888	1191	9983329	37	947057	1228	752943	59
2	941101	1189	9983307	37	947194	1226	752206	58
3	941814	1187	9983285	37	948530	1224	751470	57
4	942526	1185	9983262	37	949864	1222	750736	56
5	943237	1183	9983240	37	949998	1220	750002	55
6	943947	1181	9983217	38	950730	1218	749270	54
7	944656	1179	9983195	38	951461	1217	748539	53
8	945363	1177	9983172	38	952191	1215	747809	52
9	946069	1175	9983149	38	952920	1213	747080	51
10	946775	1173	9983127	38	953648	1211	746352	50
11	9247478	1171	9993104	38	9254374	1209	10745626	49
12	948181	1169	9983081	38	955100	1207	744900	48
13	948883	1167	9983059	38	955824	1205	744176	47
14	949583	1165	9983036	38	956547	1203	743453	46
15	950282	1163	9983013	38	957269	1201	742731	45
16	950980	1161	9982990	38	957990	1200	742010	44
17	951677	1159	9982967	38	958710	1198	741290	43
18	952373	1158	9982944	38	959429	1196	740571	42
19	953067	1156	9982921	38	960146	1194	739854	41
20	953761	1154	9982898	38	960863	1192	739137	40
21	9254453	1152	9992875	38	9261578	1190	10736429	39
22	955144	1150	9982852	38	961582	1189	737708	38
23	955834	1148	9982829	39	962305	1187	736986	37
24	956523	1146	9982806	39	963028	1185	736263	36
25	957211	1144	9982783	39	963750	1183	735542	35
26	957898	1142	9982759	39	964469	1181	734820	34
27	958583	1141	9982736	39	965188	1179	734103	33
28	959268	1139	9982713	39	965905	1178	733384	32
29	959951	1137	9982690	39	966621	1176	732673	31
30	960633	1135	9982666	39	967337	1174	731963	30
31	9261314	1133	9992643	39	9268671	1172	10731329	29
32	961319	1131	9982619	39	968057	1170	731252	28
33	962007	1130	9982596	39	970077	1169	729923	27
34	962695	1128	9982572	39	970779	1167	729221	26
35	963382	1126	9982549	39	971479	1165	728521	25
36	964067	1124	9982525	39	972178	1164	727822	24
37	964751	1122	9982501	39	972876	1162	727124	23
38	965435	1120	9982478	40	973573	1160	726427	22
39	966118	1119	9982454	40	974269	1158	725731	21
40	966801	1117	9982430	40	974964	1157	725036	20
41	9268065	1115	9992406	40	9275658	1155	10728342	19
42	967484	1113	9982382	40	976351	1153	724349	18
43	968166	1111	9982359	40	977043	1151	723657	17
44	970069	1110	9982335	40	977734	1150	722966	16
45	970735	1108	9982311	40	978424	1148	722276	15
46	971400	1106	9982287	40	979113	1147	721587	14
47	972064	1105	9982263	40	979801	1145	720899	13
48	972726	1103	9982239	40	980488	1143	720212	12
49	973388	1101	9982214	40	981174	1141	719526	11
50	974049	1199	9982190	40	981858	1140	718842	10
51	9274708	1098	9992166	40	9282542	1138	10717458	9
52	975307	1096	9982142	40	982545	1136	7181775	8
53	976024	1094	9982117	41	983230	1135	7174943	7
54	976738	1092	9982093	41	983918	1133	716812	6
55	977451	1091	9982069	41	984608	1131	716132	5
56	978161	1089	9982044	41	985297	1130	715453	4
57	978864	1087	9982020	41	985984	1128	714776	3
58	979567	1086	9981996	41	986671	1126	714100	2
59	980268	1084	9981971	41	987357	1125	713423	1
60	980969	1082	9981947	41	988042	1123	712748	0

SINES AND TANGENTS. (11 Degrees.)

19.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-280359	1062	9-991947	41	9-286632	1123	10-711348	60
1	281248	1061	991932	41	286336	1122	710674	59
2	281897	1070	991897	41	286990	1120	710001	58
3	282544	1077	991873	41	290671	1118	709329	57
4	283190	1076	991848	41	291342	1117	708658	56
5	283836	1074	991823	41	292013	1115	707987	55
6	284480	1072	991799	41	292689	1114	707318	54
7	285124	1071	991774	42	293350	1112	706650	53
8	285766	1069	991749	42	294017	1111	705983	52
9	286408	1067	991724	42	294684	1109	705316	51
10	287048	1066	991699	42	295349	1107	704651	50
11	9-287687	1064	9-991674	42	9-296013	1106	10-703987	49
12	288326	1063	991649	42	296677	1104	703322	48
13	288964	1061	991624	42	297339	1103	702661	47
14	289600	1059	991599	42	298001	1101	702000	46
15	290236	1058	991574	42	298662	1100	701338	45
16	290870	1056	991549	42	299322	1098	700678	44
17	291504	1054	991524	42	299980	1096	700020	43
18	292137	1053	991498	42	300638	1095	699362	42
19	292768	1051	991473	42	301295	1093	698705	41
20	293399	1050	991448	42	301951	1092	698049	40
21	9-294029	1048	9-991423	42	9-302607	1090	10-697393	39
22	294658	1046	991397	42	303261	1089	696739	38
23	295286	1045	991372	42	303914	1087	696086	37
24	295913	1043	991346	42	304567	1086	695433	36
25	296539	1042	991321	42	305218	1084	694782	35
26	297164	1040	991295	42	305869	1083	694131	34
27	297788	1039	991270	42	306519	1081	693481	33
28	298412	1037	991244	42	307168	1080	692832	32
29	299034	1036	991218	42	307815	1078	692185	31
30	299655	1034	991193	42	308463	1077	691537	30
31	9-300276	1032	9-991167	42	9-309109	1075	10-690691	29
32	300895	1031	991141	42	309754	1074	690046	28
33	301514	1029	991115	42	310398	1073	689402	27
34	302132	1028	991090	42	311042	1071	688758	26
35	302748	1026	991064	42	311685	1070	688115	25
36	303364	1025	991038	42	312327	1068	687473	24
37	303979	1023	991012	42	312967	1067	686833	23
38	304593	1022	990986	42	313608	1065	686192	22
39	305207	1020	990960	42	314247	1064	685553	21
40	305819	1019	990934	42	314885	1062	684915	20
41	9-306430	1017	9-990908	42	9-315522	1061	10-684477	19
42	307041	1016	990882	42	316159	1060	683841	18
43	307650	1014	990855	42	316795	1058	683205	17
44	308259	1013	990829	42	317430	1057	682570	16
45	308867	1011	990803	42	318064	1055	681936	15
46	309474	1010	990777	42	318697	1054	681303	14
47	310080	1008	990750	42	319329	1053	680671	13
48	310685	1007	990724	42	319961	1051	680039	12
49	311289	1005	990697	42	320592	1050	679408	11
50	311893	1004	990671	42	321222	1048	678778	10
51	9-312495	1003	9-990644	42	9-321851	1047	10-678149	9
52	313097	1001	990618	42	322479	1045	677521	8
53	313698	1000	990591	42	323106	1044	676894	7
54	314297	998	990565	42	323733	1043	676267	6
55	314897	997	990538	42	324358	1041	675642	5
56	315495	996	990511	42	324983	1040	675017	4
57	316092	994	990485	42	325607	1039	674393	3
58	316689	993	990458	42	326231	1037	673769	2
59	317284	991	990431	42	326853	1036	673147	1
60	317879	990	990404	42	327475	1035	672525	0
	Cosine		Sine		Cotang.		Tang.	M.

76 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-317879	990	9-990404	45	9-327474	1035	10-672586	60
1	318473	988	990378	45	328065	1033	671945	59
2	319066	987	990351	45	328715	1032	671285	58
3	319658	986	990324	45	329334	1030	670696	57
4	320249	984	990297	45	329953	1029	670047	56
5	320840	983	990270	45	330570	1028	669430	55
6	321430	982	990243	45	331187	1026	668813	54
7	322019	980	990215	45	331803	1025	668197	53
8	322607	979	990188	45	332418	1024	667582	52
9	323194	977	990161	45	333033	1023	666967	51
10	323780	976	990134	45	333646	1021	666354	50
11	9-324366	975	9-990107	46	9-334250	1020	10-665741	49
12	324950	973	990079	46	334871	1019	665129	48
13	325534	972	990052	46	335482	1017	664518	47
14	326117	970	990025	46	336093	1016	663907	46
15	326700	969	990007	46	336702	1015	663296	45
16	327281	968	989979	46	337311	1013	662689	44
17	327862	966	989952	46	337919	1012	662081	43
18	328442	965	989924	46	338527	1011	661473	42
19	329021	964	989897	46	339133	1010	660867	41
20	329609	963	989869	46	339739	1008	660261	40
21	9-330176	961	9-989833	46	9-340344	1007	10-659656	39
22	330753	960	989804	46	340948	1006	659052	38
23	331329	958	989777	46	341552	1004	658448	37
24	331903	957	989749	47	342155	1003	657845	36
25	332478	956	989721	47	342757	1002	657243	35
26	333051	954	989693	47	343358	1000	656642	34
27	333624	953	989665	47	343958	999	656042	33
28	334195	952	989637	47	344558	998	655442	32
29	334766	950	989609	47	345157	997	654843	31
30	335337	949	989582	47	345755	996	654245	30
31	9-335906	948	9-989553	47	9-346353	994	10-653647	29
32	336475	946	989525	47	346949	993	653051	28
33	337043	945	989497	47	347545	992	652455	27
34	337610	944	989469	47	348141	991	651859	26
35	338176	943	989441	47	348735	990	651265	25
36	338742	941	989413	47	349329	988	650671	24
37	339306	940	989384	47	349922	987	650078	23
38	339871	939	989356	47	350514	986	649486	22
39	340434	937	989328	47	351106	985	648894	21
40	340996	936	989300	47	351697	983	648303	20
41	9-341558	935	9-989271	47	9-352297	982	10-647713	19
42	342119	934	989243	47	352876	981	647124	18
43	342679	933	989215	47	353463	980	646535	17
44	343239	931	989188	47	354052	979	645947	16
45	343797	930	989157	47	354640	977	645360	15
46	344355	929	989129	48	355227	976	644773	14
47	344912	927	989100	48	355813	975	644187	13
48	345469	926	989071	48	356398	974	643602	12
49	346024	925	989042	48	356982	973	643018	11
50	346579	924	989014	48	357566	971	642434	10
51	9-347134	923	9-988985	48	9-358140	970	10-641851	9
52	347687	921	988956	48	358731	969	641269	8
53	348240	920	988927	48	359321	968	640687	7
54	348792	919	988898	48	359903	967	640107	6
55	349343	917	988869	48	360474	966	639526	5
56	349893	916	988840	48	361053	965	638947	4
57	350443	915	988811	49	361632	963	638368	3
58	350992	914	988782	49	362210	962	637790	2
59	351540	913	988753	49	362787	961	637213	1
60	352088	911	988724	49	363364	960	636636	0

Cosine

Sine

Cotang.

Tang.

M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-359066	911	9-988794	49	9-363364	960	10-636636	60
1	359035	910	988805	49	363840	959	636060	59
2	353181	909	988866	49	364515	958	635485	58
3	353736	908	988936	49	365090	957	634910	57
4	354271	907	989007	49	365664	955	634336	56
5	354815	905	989078	49	366237	954	633763	55
6	355358	904	989148	49	366810	953	633190	54
7	355901	903	989219	49	367383	952	632618	53
8	356443	902	989290	49	367953	951	632047	52
9	356984	901	989360	49	368524	950	631476	51
10	357524	899	989430	49	369094	949	630906	50
11	9-358084	898	9-989401	49	9-369663	948	10-630337	49
12	358003	897	989371	49	370232	946	629768	48
13	359141	896	989342	49	370799	945	629201	47
14	359678	895	989312	50	371367	944	628633	46
15	360215	893	989282	50	371933	943	628067	45
16	360752	892	989252	50	372499	942	627501	44
17	361287	891	989222	50	373064	941	626936	43
18	361822	890	989192	50	373629	940	626371	42
19	362355	889	989162	50	374193	939	625807	41
20	362889	888	989132	50	374756	938	625244	40
21	9-363422	887	9-989102	50	9-375319	937	10-624681	39
22	363454	885	989072	50	375881	935	624119	38
23	364485	884	989042	50	376443	934	623558	37
24	365016	883	989012	50	377003	933	622997	36
25	365546	882	988982	50	377563	932	622437	35
26	366075	881	988952	50	378122	931	621878	34
27	366604	880	988922	50	378681	930	621319	33
28	367132	879	988892	50	379239	929	620761	32
29	367659	877	988862	50	379797	928	620203	31
30	368185	876	988832	51	380354	927	619646	30
31	9-368711	875	9-988802	51	9-380910	926	10-619090	29
32	368226	874	988771	51	381466	925	618534	28
33	368761	873	988740	51	382020	924	617980	27
34	370285	872	988710	51	382575	923	617428	26
35	370808	871	988679	51	383129	922	616877	25
36	371330	870	988648	51	383683	921	616318	24
37	371852	869	988618	51	384234	920	615766	23
38	372373	867	988588	51	384786	919	615214	22
39	372894	866	988557	51	385337	918	614663	21
40	373414	865	988526	51	385888	917	614112	20
41	9-373933	864	9-988496	51	9-386439	915	10-613569	19
42	374452	863	988465	51	386997	914	613013	18
43	374970	862	988434	51	387556	913	612464	17
44	375487	861	988403	52	388104	912	611916	16
45	376003	860	988372	52	388651	911	611369	15
46	376519	859	988341	52	389198	910	610822	14
47	377035	858	988310	52	389734	909	610276	13
48	377549	857	988279	52	390270	908	609730	12
49	378063	856	988248	52	390815	907	609185	11
50	378577	854	988217	52	391360	906	608640	10
51	9-379089	853	9-988186	52	9-391903	905	10-608097	9
52	379601	852	988155	52	392447	904	607553	8
53	380113	851	988124	52	392990	903	607011	7
54	380624	850	988092	52	393531	902	606469	6
55	381134	849	988061	52	394073	901	605927	5
56	381643	848	988030	52	394614	900	605386	4
57	382152	847	988000	52	395154	899	604846	3
58	382661	846	987967	52	395694	898	604306	2
59	383168	845	987936	52	396233	897	603767	1
60	383675	844	987904	52	396771	896	603229	0

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-383675	844	9-988004	52	9-386771	806	10-603229	60
1	384182	843	988273	53	397309	806	602691	59
2	384687	842	988541	53	397846	805	602154	58
3	385192	841	988809	53	398383	804	601617	57
4	385697	840	989077	53	398919	803	601081	56
5	386201	839	989345	53	399455	802	600545	55
6	386704	838	989614	53	399990	801	600010	54
7	387207	837	989883	53	400524	800	599476	53
8	387709	836	990151	53	401058	800	598942	52
9	388210	835	990419	53	401591	800	598408	51
10	388711	834	990687	53	402124	807	597876	50
11	9-389211	833	9-990955	53	9-402656	806	10-597344	49
12	389711	832	991223	53	403187	805	596813	48
13	390210	831	991491	53	403718	804	596282	47
14	390708	830	991759	53	404249	803	595751	46
15	391206	828	992027	53	404778	802	595222	45
16	391703	827	992295	53	405308	801	594692	44
17	392199	826	992563	54	405836	800	594164	43
18	392695	825	992831	54	406364	879	593636	42
19	393191	824	993099	54	406892	878	593108	41
20	393685	823	993366	54	407419	877	592581	40
21	9-394179	822	9-993634	54	9-407945	876	10-592055	39
22	394673	821	993902	54	408471	875	591529	38
23	395166	820	994169	54	408997	874	591003	37
24	395658	819	994437	54	409521	874	590479	36
25	396150	818	994704	54	410045	873	589955	35
26	396641	817	994972	54	410569	872	589431	34
27	397132	817	995239	54	411092	871	588906	33
28	397621	816	995507	54	411615	870	588385	32
29	398111	815	995774	54	412137	869	587863	31
30	398600	814	996042	54	412658	868	587342	30
31	9-399088	813	9-996309	55	9-413179	867	10-586821	29
32	399575	812	996576	55	413699	866	586301	28
33	400062	811	996843	55	414219	865	585781	27
34	400549	810	997111	55	414738	864	585262	26
35	401035	809	997378	55	415257	864	584743	25
36	401520	808	997645	55	415775	863	584225	24
37	402005	807	997912	55	416293	862	583707	23
38	402489	806	998179	55	416810	861	583190	22
39	402972	805	998446	55	417328	860	582674	21
40	403455	804	998713	55	417845	859	582158	20
41	9-403938	803	9-998980	55	9-418358	858	10-581642	19
42	404420	802	999247	55	418873	857	581127	18
43	404901	801	999514	55	419387	856	580613	17
44	405382	800	999780	55	419901	855	580099	16
45	405862	799	999947	55	420415	855	579585	15
46	406341	798	999914	56	420927	854	579073	14
47	406820	797	999980	56	421440	853	578560	13
48	407299	796	999947	56	421952	852	578048	12
49	407777	795	999914	56	422463	851	577537	11
50	408254	794	999880	56	422974	850	577026	10
51	9-408731	794	9-999847	56	9-423484	849	10-576516	9
52	409207	793	999813	56	423993	848	576007	8
53	409682	792	999780	56	424503	848	575497	7
54	410157	791	999746	56	425011	847	574989	6
55	410632	790	999713	56	425519	846	574481	5
56	411106	789	999679	56	426027	845	573973	4
57	411579	788	999645	56	426534	844	573466	3
58	412052	787	999611	56	427041	843	572959	2
59	412524	786	999578	56	427547	843	572453	1
60	412996	785	999544	56	428052	842	571948	0

Cosine Sine Cotang Tang M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	0-412606	785	9-984944	57	9-429058	843	10-571948	00
1	413487	784	984910	57	429557	841	571443	59
2	413638	783	984876	57	429969	840	570938	58
3	414408	783	984842	57	429566	839	570434	57
4	414678	782	984808	57	430070	838	569930	56
5	415347	781	984774	57	430573	838	569427	55
6	415815	780	984740	57	431075	837	568925	54
7	416283	779	984706	57	431577	836	568423	53
8	416751	778	984672	57	432079	835	567921	52
9	417217	777	984637	57	432580	834	567420	51
10	417684	776	984603	57	433080	833	566920	50
11	9-418150	775	9-984569	57	9-433580	832	10-566420	49
12	418615	774	984535	57	434080	832	565920	48
13	419079	773	984500	57	434579	831	565421	47
14	419544	773	984466	57	435078	830	564923	46
15	420007	772	984432	58	435576	829	564424	45
16	420470	771	984397	58	436073	828	563927	44
17	420933	770	984363	58	436570	828	563430	43
18	421395	769	984328	58	437067	827	562933	42
19	421857	768	984294	58	437563	826	562437	41
20	422318	767	984259	58	438059	825	561941	40
21	9-422778	767	9-984224	58	9-438554	824	10-561446	39
22	423238	766	984190	58	439048	823	560952	38
23	423697	765	984155	58	439543	823	560457	37
24	424156	764	984120	58	440036	822	559964	36
25	424615	763	984085	58	440529	821	559471	35
26	425073	762	984050	58	441022	820	558978	34
27	425530	761	984015	58	441514	819	558486	33
28	425987	760	983981	58	442006	819	557994	32
29	426443	760	983946	58	442497	818	557503	31
30	426899	759	983911	58	442988	817	557012	30
31	9-427354	758	9-983875	58	9-443479	816	10-556521	29
32	427809	757	983840	59	443966	816	556029	28
33	428263	756	983805	59	444456	815	555542	27
34	428717	755	983770	59	444947	814	555053	26
35	429170	754	983735	59	445435	813	554565	25
36	429623	753	983700	59	445923	812	554077	24
37	430075	752	983664	59	446411	812	553589	23
38	430527	752	983629	59	446898	811	553102	22
39	430978	751	983594	59	447384	810	552616	21
40	431429	750	983558	59	447870	809	552130	20
41	9-431879	749	9-983523	59	9-448356	809	10-551644	19
42	432330	749	983487	59	448841	808	551159	18
43	432780	748	983452	59	449326	807	550674	17
44	433230	747	983416	59	449810	806	550190	16
45	433679	746	983381	59	450294	806	549706	15
46	434129	745	983345	59	450777	805	549223	14
47	434579	744	983309	59	451260	804	548740	13
48	435016	744	983273	60	451743	803	548257	12
49	435462	743	983238	60	452225	802	547775	11
50	435908	742	983202	60	452706	802	547294	10
51	9-436353	741	9-983166	60	9-453187	801	10-546811	9
52	436798	740	983130	60	453668	800	546328	8
53	437242	740	983094	60	454148	799	545845	7
54	437686	739	983058	60	454628	799	545372	6
55	438129	738	983022	60	455107	798	544893	5
56	438573	737	982986	60	455586	797	544414	4
57	439014	736	982950	60	456064	796	543936	3
58	439456	736	982914	60	456542	796	543458	2
59	439897	735	982878	60	457019	795	542981	1
60	440338	734	982842	60	457496	794	542504	0

Cosine Sine Cotang Tang M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-440338	734	9-989848	60	9-457486	794	10-549504	60
1	440778	733	982805	60	457973	793	549027	59
2	441218	732	982769	61	458449	793	541551	58
3	441658	731	982733	61	458925	792	541075	57
4	442098	731	982696	61	459400	791	540600	56
5	442535	730	982660	61	459875	790	540125	55
6	442973	729	982624	61	460349	790	539651	54
7	443410	728	982587	61	460823	789	539177	53
8	443847	727	982551	61	461297	788	538703	52
9	444284	727	982514	61	461770	788	538230	51
10	444720	726	982477	61	462243	787	537758	50
11	9-445155	725	9-982441	61	9-462714	786	10-537286	49
12	445590	724	982404	61	463186	785	536814	48
13	446025	723	982367	61	463658	785	536342	47
14	446459	723	982331	61	464130	784	535871	46
15	446893	722	982294	61	464599	783	535401	45
16	447326	721	982257	61	465069	783	534931	44
17	447759	720	982220	62	465539	782	534461	43
18	448191	720	982183	62	466009	781	533992	42
19	448623	719	982146	62	466476	781	533524	41
20	449054	718	982109	62	466945	780	533055	40
21	9-449485	717	9-982072	62	9-467413	779	10-532587	39
22	449915	716	982035	62	467880	778	532120	38
23	450345	716	981998	62	468347	778	531653	37
24	450775	715	981961	62	468814	777	531186	36
25	451204	714	981924	62	469280	776	530720	35
26	451632	713	981886	62	469746	775	530254	34
27	452060	713	981849	62	470211	775	529789	33
28	452488	712	981812	62	470676	774	529324	32
29	452915	711	981774	62	471141	773	528859	31
30	453342	710	981737	62	471605	773	528395	30
31	9-453768	710	9-981699	63	9-472068	772	10-527932	29
32	454194	709	981662	63	472532	771	527468	28
33	454619	708	981625	63	472995	771	527005	27
34	455044	707	981587	63	473457	770	526543	26
35	455468	707	981549	63	473919	769	526081	25
36	455893	706	981512	63	474381	769	525619	24
37	456316	705	981474	63	474842	768	525158	23
38	456739	704	981436	63	475303	767	524697	22
39	457162	704	981398	63	475763	767	524237	21
40	457584	703	981361	63	476223	766	523777	20
41	9-458006	702	9-981323	63	9-476683	765	10-523317	19
42	458427	701	981285	63	477142	765	522858	18
43	458848	701	981247	63	477601	764	522399	17
44	459268	700	981209	63	478059	763	521941	16
45	459688	699	981171	63	478517	763	521483	15
46	460108	698	981133	64	478975	762	521025	14
47	460527	698	981095	64	479432	761	520568	13
48	460946	697	981057	64	479889	761	520111	12
49	461364	696	981019	64	480345	760	519655	11
50	461782	695	980981	64	480801	759	519199	10
51	9-462199	695	9-980943	64	9-481257	759	10-518743	9
52	462616	694	980904	64	481712	758	518288	8
53	463032	693	980866	64	482167	757	517833	7
54	463448	693	980827	64	482621	757	517379	6
55	463864	692	980789	64	483075	756	516925	5
56	464279	691	980750	64	483529	755	516471	4
57	464694	690	980712	64	483982	755	516018	3
58	465108	690	980673	64	484435	754	515565	2
59	465522	689	980635	64	484887	753	515113	1
60	465935	688	980596	64	485339	753	514661	0
	Cosine		Sine		Cotang.		Tang.	M.

SINES AND TANGENTS. (17 Degrees.)

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	0-465035	686	0-980596	64	0-485339	755	10-514661	80
1	466348	686	980558	64	485791	752	514209	59
2	467671	687	980519	65	486242	751	513758	58
3	467173	686	980480	65	486693	751	513307	57
4	467585	685	980442	65	487143	750	512857	56
5	467996	685	980403	65	487593	749	512407	55
6	468407	684	980364	65	488043	749	511957	54
7	468817	683	980325	65	488492	748	511508	53
8	469227	683	980286	65	488941	747	511059	52
9	469637	682	980247	65	489390	747	510610	51
10	470046	681	980208	65	489839	746	510162	50
11	0-470455	680	0-980169	65	0-490288	746	10-500714	49
12	470963	680	980130	65	490733	745	509267	48
13	471471	679	980091	65	491180	744	508820	47
14	471979	678	980052	65	491627	744	508373	46
15	472486	678	980013	65	492073	743	507927	45
16	472992	677	979973	65	492519	743	507481	44
17	473498	676	979934	66	492965	742	507035	43
18	473904	676	979895	66	493410	741	506590	42
19	474310	675	979855	66	493854	740	506146	41
20	474715	674	979816	66	494299	740	505701	40
21	0-474519	674	0-979776	66	0-494743	740	10-505257	39
22	474923	673	979737	66	495186	739	504814	38
23	475327	673	979697	66	495630	738	504370	37
24	475730	673	979658	66	496073	737	503927	36
25	476133	671	979618	66	496515	737	503485	35
26	476536	670	979579	66	496957	736	503043	34
27	476938	669	979539	66	497399	736	502601	33
28	477340	669	979499	66	497841	735	502159	32
29	477741	668	979459	66	498282	734	501718	31
30	478142	667	979420	66	498722	734	501278	30
31	0-478542	667	0-979380	66	0-499163	733	10-500837	29
32	478942	666	979340	66	499603	733	500397	28
33	479342	665	979300	67	500042	732	499958	27
34	479741	665	979260	67	500481	731	499519	26
35	480140	664	979220	67	500920	731	499080	25
36	480539	663	979180	67	501359	730	498641	24
37	480937	663	979140	67	501797	730	498202	23
38	481334	662	979100	67	502235	729	497765	22
39	481731	661	979059	67	502672	728	497328	21
40	482128	661	979019	67	503109	728	496891	20
41	0-482525	660	0-978979	67	0-503546	727	10-496454	19
42	482921	659	978939	67	503982	727	496018	18
43	483316	659	978898	67	504418	726	495582	17
44	483712	658	978858	67	504854	725	495146	16
45	484107	657	978817	67	505289	725	494711	15
46	484501	657	978777	67	505724	724	494276	14
47	484895	656	978736	67	506159	724	493841	13
48	485289	655	978695	68	506593	723	493407	12
49	485682	655	978655	68	507027	722	492973	11
50	486075	654	978615	68	507460	722	492540	10
51	0-486467	653	0-978574	68	0-507893	721	10-492107	9
52	486860	653	978533	68	508326	721	491674	8
53	487251	652	978493	68	508759	720	491241	7
54	487643	651	978452	68	509191	719	490809	6
55	488034	651	978411	68	509622	719	490378	5
56	488424	650	978370	68	510054	718	489946	4
57	488814	650	978329	68	510485	718	489515	3
58	489204	649	978288	68	510916	717	489084	2
59	489593	648	978247	68	511346	716	488654	1
60	489982	648	978206	68	511776	716	488224	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-489082	648	9-978906	68	9-511776	716	10-489024	60
1	490371	648	978165	68	512006	716	487794	59
2	490759	647	978194	68	512635	715	487265	58
3	491147	646	978083	69	513064	714	486636	57
4	491535	646	978049	69	513493	714	486007	56
5	491922	645	978001	69	513921	713	485379	55
6	492308	644	977959	69	514349	713	484751	54
7	492695	644	977918	69	514777	712	484122	53
8	493081	643	977877	69	515204	712	483496	52
9	493468	642	977835	69	515631	711	482869	51
10	493851	642	977794	69	516057	710	482243	50
11	9-494236	641	9-977752	69	9-516484	710	10-482616	49
12	494621	641	977711	69	516910	709	482040	48
13	495005	640	977669	69	517335	709	481465	47
14	495388	639	977628	69	517761	708	480890	46
15	495772	639	977586	69	518185	708	481315	45
16	496154	638	977544	70	518610	707	481740	44
17	496537	637	977503	70	519034	706	481165	43
18	496919	637	977461	70	519458	706	480590	42
19	497301	636	977419	70	519882	705	480015	41
20	497682	636	977377	70	520305	705	479440	40
21	9-498064	635	9-977335	70	9-520728	704	10-479872	39
22	498444	634	977293	70	521151	703	479297	38
23	498825	634	977251	70	521573	703	478722	37
24	499204	633	977209	70	521995	703	478147	36
25	499584	632	977167	70	522417	702	477572	35
26	499963	632	977125	70	522838	702	477000	34
27	500342	631	977083	70	523259	701	476421	33
28	500721	631	977041	70	523680	701	475842	32
29	501099	630	976999	70	524100	700	475260	31
30	501476	629	976957	70	524520	699	474680	30
31	9-501854	629	9-976914	70	9-524939	699	10-475101	29
32	502231	628	976872	71	525359	698	474521	28
33	502607	628	976830	71	525778	698	473942	27
34	502984	627	976787	71	526197	697	473363	26
35	503360	626	976745	71	526615	697	472784	25
36	503735	626	976702	71	527033	696	472205	24
37	504110	625	976660	71	527451	696	471626	23
38	504485	625	976617	71	527868	695	471047	22
39	504860	624	976574	71	528285	695	470468	21
40	505234	623	976532	71	528702	694	469889	20
41	9-505608	623	9-976489	71	9-529119	693	10-470311	19
42	505981	622	976446	71	529535	693	470465	18
43	506354	622	976404	71	529950	693	470050	17
44	506727	621	976361	71	530366	692	469634	16
45	507099	620	976318	71	530781	691	469219	15
46	507471	620	976275	71	531196	691	468804	14
47	507843	619	976232	72	531611	690	468389	13
48	508214	619	976189	72	532025	690	467975	12
49	508585	618	976146	72	532439	689	467561	11
50	508956	618	976103	72	532853	689	467147	10
51	9-509326	617	9-976060	72	9-533266	688	10-466734	9
52	509696	616	976017	72	533679	688	466321	8
53	510065	616	975974	72	534092	687	465908	7
54	510434	615	975930	72	534504	687	465496	6
55	510803	615	975887	72	534916	686	465084	5
56	511172	614	975844	72	535328	686	464672	4
57	511540	613	975800	72	535739	685	464261	3
58	511907	613	975757	72	536150	685	463850	2
59	512275	612	975714	72	536561	684	463439	1
60	512642	612	975670	72	536972	684	463028	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-519642	612	9-975670	73	9-536972	684	10-463028	60
1	513009	611	975627	73	537382	683	462618	59
2	513741	611	975583	73	537798	683	462208	58
3	514472	610	975539	73	538202	682	461798	57
4	514107	609	975496	73	538611	682	461389	56
5	514472	609	975452	73	539020	681	460980	55
6	514837	608	975408	73	539429	681	460571	54
7	515202	608	975365	73	539837	680	460163	53
8	515566	607	975321	73	540245	680	459755	52
9	515930	607	975277	73	540653	679	459347	51
10	516294	606	975233	73	541061	679	458939	50
11	9-516657	605	9-975189	73	9-541468	678	10-458532	49
12	517020	605	975145	73	541875	678	458125	48
13	517382	604	975101	73	542281	677	457719	47
14	517745	604	975057	73	542688	677	457312	46
15	518107	603	975013	73	543094	676	456906	45
16	518468	603	974969	74	543499	676	456501	44
17	518829	602	974925	74	543905	675	456095	43
18	519190	601	974880	74	544310	675	455690	42
19	519551	601	974836	74	544715	674	455285	41
20	519911	600	974792	74	545119	674	454881	40
21	9-520271	600	9-974748	74	9-545524	673	10-454476	39
22	520631	599	974703	74	545928	673	454072	38
23	520990	599	974659	74	546331	672	453669	37
24	521349	598	974614	74	546735	672	453265	36
25	521707	598	974570	74	547138	671	452862	35
26	522066	597	974525	74	547540	671	452460	34
27	522424	596	974481	74	547943	670	452057	33
28	522781	596	974436	74	548345	670	451655	32
29	523138	595	974391	74	548747	669	451253	31
30	523495	595	974347	75	549149	669	450851	30
31	9-523852	594	9-974302	75	9-549550	668	10-450450	29
32	524208	594	974257	75	549951	668	450049	28
33	524564	593	974212	75	550352	667	449648	27
34	524920	593	974167	75	550752	667	449248	26
35	525275	592	974122	75	551152	666	448848	25
36	525630	591	974077	75	551552	666	448448	24
37	525984	591	974032	75	551952	665	448048	23
38	526339	590	973987	75	552351	665	447649	22
39	526693	590	973942	75	552750	665	447250	21
40	527046	589	973897	75	553149	664	446851	20
41	9-527400	589	9-973852	75	9-553548	664	10-446452	19
42	527753	588	973807	75	553946	663	446054	18
43	528105	588	973761	75	554344	663	445656	17
44	528458	587	973716	76	554741	662	445259	16
45	528810	587	973671	76	555139	662	444861	15
46	529161	586	973625	76	555536	661	444464	14
47	529513	586	973580	76	555933	661	444067	13
48	529864	585	973535	76	556329	660	443671	12
49	530215	585	973489	76	556725	660	443275	11
50	530565	584	973444	76	557121	659	442879	10
51	9-530915	584	9-973398	76	9-557517	659	10-442483	9
52	531265	583	973352	76	557913	659	442087	8
53	531614	582	973307	76	558308	658	441692	7
54	531963	582	973261	76	558702	658	441298	6
55	532312	581	973215	76	559097	657	440903	5
56	532661	581	973169	76	559491	657	440509	4
57	533009	580	973124	76	559885	656	440115	3
58	533357	580	973078	76	560279	656	439721	2
59	533704	579	973032	77	560673	655	439327	1
60	534052	578	972986	77	561066	655	438934	0

Cosine

Sine

Cosang

Tang.

M.

70 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-534053	578	9-973966	77	9-561066	655	10-438934	60
1	534260	577	973940	77	561459	654	438541	59
2	534475	577	973904	77	561851	654	438149	58
3	535092	577	973848	77	562244	653	437756	57
4	535438	576	973802	77	562636	653	437364	56
5	535783	576	973755	77	563028	653	436972	55
6	536129	575	973709	77	563419	652	436581	54
7	536474	574	973663	77	563811	652	436189	53
8	536818	574	973617	77	564202	651	435798	52
9	537163	573	973570	77	564592	651	435408	51
10	537507	573	973524	77	564983	650	435017	50
11	9-537851	572	9-972478	77	9-565373	650	10-434627	49
12	538194	572	972431	78	565763	649	434237	48
13	538538	571	972385	78	566153	649	433847	47
14	538880	571	972338	78	566542	649	433456	46
15	539223	570	972291	78	566932	648	433068	45
16	539565	570	972245	78	567320	648	432680	44
17	539907	569	972198	78	567709	647	432291	43
18	540249	569	972151	78	568098	647	431902	42
19	540590	568	972105	78	568486	646	431514	41
20	540931	568	972058	78	568873	646	431127	40
21	9-541272	567	9-971011	78	9-569261	645	10-430739	39
22	541613	567	971964	78	569648	645	430352	38
23	541953	566	971917	78	570035	645	429965	37
24	542293	566	971870	78	570422	644	429578	36
25	542632	565	971823	78	570809	644	429191	35
26	542971	565	971776	78	571195	643	428803	34
27	543310	564	971729	79	571581	643	428419	33
28	543649	564	971682	79	571967	642	428033	32
29	543987	563	971635	79	572352	642	427648	31
30	544325	563	971588	79	572738	642	427262	30
31	9-544663	562	9-971540	79	9-573123	641	10-426877	29
32	545000	562	971493	79	573507	641	426493	28
33	545338	561	971446	79	573892	640	426108	27
34	545674	561	971398	79	574276	640	425724	26
35	546011	560	971351	79	574660	639	425340	25
36	546347	560	971303	79	575044	639	424956	24
37	546683	559	971256	79	575427	639	424573	23
38	547019	559	971208	79	575810	638	424190	22
39	547354	558	971161	79	576193	638	423807	21
40	547689	558	971113	79	576576	637	423424	20
41	9-548024	557	9-971066	80	9-576958	637	10-423041	19
42	548359	557	971018	80	577341	636	422659	18
43	548693	556	970970	80	577723	636	422277	17
44	549027	556	970922	80	578104	636	421896	16
45	549360	555	970874	80	578486	635	421514	15
46	549693	555	970827	80	578867	635	421133	14
47	550026	554	970779	80	579248	634	420752	13
48	550359	554	970731	80	579629	634	420371	12
49	550692	553	970683	80	580009	634	419991	11
50	551024	553	970635	80	580389	633	419611	10
51	9-551356	552	9-970588	80	9-580769	633	10-419231	9
52	551687	552	970538	80	581149	632	418851	8
53	552018	552	970490	80	581528	632	418472	7
54	552349	551	970442	80	581907	632	418093	6
55	552680	551	970394	80	582286	631	417714	5
56	553010	550	970345	81	582665	631	417335	4
57	553341	550	970297	81	583043	630	416957	3
58	553670	549	970249	81	583422	630	416578	2
59	554000	549	970200	81	583800	629	416200	1
60	554329	548	970152	81	584177	629	415823	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-554390	548	9-970152	81	9-584177	620	10-415683	60
1	554658	548	970103	81	584555	620	415445	59
2	554987	547	969053	81	584932	620	415208	58
3	555315	547	970006	81	585300	620	414971	57
4	555643	546	969957	81	585668	627	414734	56
5	555971	546	969909	81	586036	627	413497	55
6	556299	545	969860	81	586403	627	413260	54
7	556628	545	969811	81	586771	626	413023	53
8	556953	544	969762	81	587139	626	412786	52
9	557280	544	969714	81	587506	625	412549	51
10	557606	543	969665	81	587874	625	412312	50
11	9-557932	543	9-969616	82	9-588236	625	10-411684	49
12	558258	543	969567	82	588601	624	411300	48
13	558583	542	969518	82	589066	624	410934	47
14	558909	542	969469	82	589440	623	410568	46
15	559234	541	969420	82	589814	623	410186	45
16	559558	541	969370	82	590188	623	409812	44
17	559883	540	969321	82	590562	622	409438	43
18	560207	540	969272	82	590935	622	409065	42
19	560531	539	969223	82	591308	622	408692	41
20	560855	539	969173	82	591681	621	408319	40
21	9-561178	538	9-969124	82	9-592054	621	10-407946	39
22	561501	538	969075	82	592426	620	407574	38
23	561824	537	969025	82	592798	620	407202	37
24	562146	537	968976	82	593170	619	406829	36
25	562468	536	968926	83	593542	619	406458	35
26	562790	536	968877	83	593914	618	406086	34
27	563112	536	968827	83	594285	618	405715	33
28	563433	535	968777	83	594656	618	405344	32
29	563755	535	968728	83	595027	617	404973	31
30	564075	534	968678	83	595398	617	404602	30
31	9-564396	534	9-968628	83	9-595768	617	10-404232	29
32	564716	533	968578	83	596138	616	403862	28
33	565036	533	968528	83	596508	616	403492	27
34	565356	532	968479	83	596878	616	403122	26
35	565676	532	968429	83	597247	615	402752	25
36	565995	531	968379	83	597616	615	402382	24
37	566314	531	968329	83	597985	615	402012	23
38	566632	531	968278	83	598354	614	401642	22
39	566951	530	968228	84	598723	614	401272	21
40	567269	530	968178	84	599091	613	400902	20
41	9-567587	529	9-968128	84	9-599459	613	10-400541	19
42	567904	529	968078	84	599827	613	400171	18
43	568222	528	968027	84	600194	612	399806	17
44	568539	528	967977	84	600562	612	399438	16
45	568856	528	967927	84	600929	611	399071	15
46	569172	527	967876	84	601296	611	398704	14
47	569488	527	967826	84	601662	611	398338	13
48	569804	526	967775	84	602029	610	397971	12
49	570120	526	967725	84	602395	610	397605	11
50	570435	525	967674	84	602761	610	397239	10
51	9-570751	525	9-967624	84	9-603127	609	10-396873	9
52	571066	524	967573	84	603493	609	396507	8
53	571380	524	967522	85	603858	609	396142	7
54	571695	523	967471	85	604223	608	395777	6
55	572009	523	967421	85	604588	608	395412	5
56	572323	523	967370	85	604953	607	395047	4
57	572636	522	967319	85	605317	607	394683	3
58	572950	522	967268	85	605682	607	394318	2
59	573263	521	967217	85	606046	606	393954	1
60	573575	521	967166	85	606410	606	393590	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9-573875	521	9-967168	85	9-608410	606	10-333590	60
1	573888	520	967115	85	608773	606	343327	59
2	574900	520	967064	85	607137	605	362263	58
3	574512	519	967013	85	607500	605	362500	57
4	574524	519	966961	85	607863	604	362137	56
5	575136	519	966910	85	608225	604	361775	55
6	575447	518	966859	85	608588	604	361412	54
7	575758	518	966808	85	608950	603	361050	53
8	576069	517	966756	86	609312	603	360688	52
9	576379	517	966705	86	609674	603	360326	51
10	576689	516	966653	86	610036	602	360064	50
11	9-576999	516	9-966602	86	9-610397	602	10-369603	49
12	577309	516	966550	86	610759	602	369241	48
13	577618	515	966499	86	611120	601	368880	47
14	577927	515	966447	86	611480	601	368520	46
15	578236	514	966395	86	611841	601	368159	45
16	578545	514	966344	86	612201	600	367799	44
17	578853	513	966292	86	612561	600	367439	43
18	579162	513	966240	86	612921	600	367079	42
19	579470	513	966188	86	613281	599	366719	41
20	579777	512	966136	86	613641	599	366359	40
21	9-580085	512	9-966085	87	9-614000	598	10-366000	39
22	580392	511	966033	87	614359	598	365641	38
23	580699	511	965981	87	614718	598	365282	37
24	581005	511	965929	87	615077	597	364923	36
25	581312	510	965876	87	615435	597	364563	35
26	581618	510	965824	87	615793	597	364203	34
27	581924	509	965772	87	616151	596	363843	33
28	582229	509	965720	87	616509	596	363481	32
29	582535	509	965668	87	616867	596	363123	31
30	582840	508	965615	87	617224	595	362776	30
31	9-583145	508	9-965563	87	9-617582	595	10-362418	29
32	583449	507	965511	87	617939	595	362061	28
33	583754	507	965458	87	618295	594	361705	27
34	584058	506	965406	87	618652	594	361348	26
35	584361	506	965353	88	619008	594	360992	25
36	584665	506	965301	88	619364	593	360636	24
37	584968	505	965248	88	619721	593	360279	23
38	585272	505	965195	88	620076	593	359924	22
39	585574	504	965143	88	620432	592	359568	21
40	585877	504	965090	88	620787	592	359213	20
41	9-586179	503	9-965037	88	9-621142	592	10-378858	19
42	586482	503	964984	88	621497	591	378503	18
43	586783	503	964931	88	621852	591	378148	17
44	587085	502	964879	88	622207	590	377793	16
45	587386	502	964826	88	622561	590	377439	15
46	587688	501	964773	88	622915	590	377085	14
47	587989	501	964719	88	623269	589	376731	13
48	588289	501	964666	89	623623	589	376377	12
49	588590	500	964613	89	623976	589	376024	11
50	588890	500	964560	89	624330	588	375670	10
51	9-589190	499	9-964507	89	9-624683	588	10-375317	9
52	589489	499	964454	89	625036	588	374964	8
53	589789	499	964400	89	625389	587	374612	7
54	590088	498	964347	89	625741	587	374259	6
55	590387	498	964294	89	626093	587	373907	5
56	590686	497	964240	89	626445	586	373555	4
57	590984	497	964187	89	626797	586	373203	3
58	591282	497	964133	89	627149	586	372851	2
59	591580	496	964080	89	627501	585	372499	1
60	591878	496	964026	89	627852	585	372148	0
	Cosine		Sine		Cotang.		Tang.	

M.	Sine	D.	Co-sine	D.	Tang.	D.	Cotang.	M.
0	0-501878	496	0-964096	80	0-607852	585	10-372148	60
1	502176	495	963973	80	622803	585	371797	59
2	502473	495	963910	80	628554	585	371446	58
3	502770	495	963865	80	633905	584	371095	57
4	503067	494	963811	80	639255	584	370745	56
5	503363	494	963757	80	644606	583	370394	55
6	503659	493	963704	80	649956	583	370044	54
7	503955	493	963650	80	655306	583	369694	53
8	504251	493	963596	80	660656	583	369344	52
9	504547	492	963542	80	666006	582	368995	51
10	504842	492	963488	80	671355	582	368645	50
11	0-505137	491	0-963434	90	0-631704	582	10-368296	49
12	505432	491	963379	90	637053	581	367947	48
13	505727	491	963325	90	642401	581	367597	47
14	506021	490	963271	90	647750	581	367248	46
15	506315	490	963217	90	653098	580	366902	45
16	506609	489	963163	90	658447	580	366553	44
17	506903	489	963108	91	663795	580	366205	43
18	507196	489	963054	91	669143	579	365857	42
19	507490	488	962999	91	674490	579	365510	41
20	507783	488	962945	91	679838	579	365162	40
21	0-508075	487	0-962890	91	0-635185	578	10-364815	39
22	508368	487	962836	91	685532	578	364468	38
23	508660	487	962781	91	690879	578	364121	37
24	508952	486	962727	91	696226	577	363774	36
25	509244	486	962672	91	701572	577	363428	35
26	509536	485	962617	91	706919	577	363081	34
27	509827	485	962562	91	712265	577	362735	33
28	600118	485	962508	91	717611	576	362389	32
29	600409	484	962453	91	722956	576	362044	31
30	600700	484	962398	92	728302	576	361698	30
31	0-600990	484	0-962343	92	0-639647	575	10-361353	29
32	601280	483	962288	92	733992	575	361008	28
33	601570	483	962233	92	739337	575	360663	27
34	601860	482	962178	92	744682	574	360318	26
35	602150	482	962123	92	750027	574	359973	25
36	602439	482	962067	92	755371	574	359629	24
37	602728	481	962012	92	760716	573	359284	23
38	603017	481	961957	92	766060	573	358940	22
39	603305	481	961902	92	771404	573	358596	21
40	603594	480	961846	92	776747	572	358253	20
41	0-603882	480	0-961791	92	0-642091	572	10-357909	19
42	604170	479	961735	92	782434	572	357566	18
43	604457	479	961680	92	787777	572	357223	17
44	604745	479	961624	93	793120	571	356880	16
45	605032	478	961569	93	798463	571	356537	15
46	605319	478	961513	93	803806	571	356194	14
47	605606	478	961458	93	809148	570	355852	13
48	605892	477	961402	93	814490	570	355510	12
49	606179	477	961347	93	819832	570	355168	11
50	606465	476	961290	93	825174	569	354826	10
51	0-606751	476	0-961235	93	0-645516	569	10-354484	9
52	607036	476	961179	93	830857	569	354143	8
53	607322	475	961123	93	836199	569	353801	7
54	607607	475	961067	93	841540	568	353460	6
55	607892	474	961011	93	846881	568	353119	5
56	608177	474	960955	93	852222	568	352778	4
57	608461	474	960899	93	857562	567	352438	3
58	608745	473	960843	94	862903	567	352097	2
59	609029	473	960786	94	868243	567	351757	1
60	609313	473	960730	94	873583	566	351417	0
	Co-sine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-00813	473	9-900730	94	9-648583	566	10-351417	60
1	604587	472	900674	94	648523	566	351077	59
2	604680	472	900618	94	648463	566	350737	58
3	610164	472	900561	94	648402	566	350396	57
4	610447	471	900505	94	648342	565	350055	56
5	610739	471	900448	94	648281	565	349719	55
6	611012	470	900392	94	650680	565	349380	54
7	611284	470	900335	94	650620	564	349041	53
8	611576	470	900279	94	651967	564	348703	52
9	611858	469	900222	94	651836	564	348364	51
10	612140	469	900165	94	651974	563	348026	50
11	9-612421	469	9-900109	95	9-652312	563	10-347688	49
12	612702	468	900052	95	652250	563	347350	48
13	612983	468	950995	95	652188	563	347012	47
14	613264	467	950938	95	653386	562	346674	46
15	613545	467	950882	95	653663	562	346337	45
16	613825	467	950825	95	654000	562	346000	44
17	614105	466	950768	95	654337	561	345663	43
18	614385	466	950711	95	654674	561	345326	42
19	614665	466	950654	95	655011	561	344989	41
20	614944	465	950596	95	655348	561	344652	40
21	9-615223	465	9-950539	95	9-655684	560	10-344316	39
22	615502	465	950482	95	656020	560	343978	38
23	615781	464	950425	95	656356	560	343644	37
24	616060	464	950368	95	656692	559	343306	36
25	616338	464	950310	96	657028	559	342972	35
26	616616	463	950253	96	657364	559	342636	34
27	616894	463	950195	96	657699	559	342301	33
28	617172	462	950138	96	658034	558	341966	32
29	617450	462	950081	96	658369	558	341631	31
30	617727	462	950023	96	658704	558	341296	30
31	9-618004	461	9-950965	96	9-659039	558	10-340961	29
32	618281	461	950908	96	659373	557	340627	28
33	618558	461	950850	96	659708	557	340292	27
34	618834	460	950792	96	660042	557	339958	26
35	619110	460	950734	96	660376	557	339624	25
36	619386	460	950677	96	660710	556	339290	24
37	619662	459	950619	96	661043	556	338957	23
38	619938	459	950561	96	661377	556	338623	22
39	620213	459	950503	97	661710	555	338290	21
40	620488	458	950445	97	662043	555	337957	20
41	9-620763	458	9-950387	97	9-662376	555	10-337624	19
42	621038	457	950329	97	662709	554	337291	18
43	621313	457	950271	97	663042	554	336958	17
44	621587	457	950213	97	663375	554	336625	16
45	621861	456	950154	97	663707	554	336293	15
46	622135	456	950096	97	664039	553	335961	14
47	622409	456	950038	97	664371	553	335629	13
48	622682	455	957979	97	664703	553	335297	12
49	622956	455	957921	97	665035	553	334965	11
50	623229	455	957863	97	665368	552	334634	10
51	9-623502	454	9-957804	97	9-665697	552	10-334303	9
52	623774	454	957746	98	666029	552	333971	8
53	624047	454	957687	98	666360	551	333640	7
54	624319	453	957628	98	666691	551	333309	6
55	624591	453	957570	98	667021	551	332979	5
56	624863	453	957511	98	667352	551	332648	4
57	625135	452	957452	98	667682	550	332318	3
58	625406	452	957393	98	668013	550	331987	2
59	625677	452	957335	98	668343	550	331657	1
60	625948	451	957276	98	668672	550	331328	0

SINES AND TANGENTS. (25 Degrees.)

M.	Sine	D.	Co-sine	D.	Tang.	D.	Co-tang.	M.
0	9625948	451	9557278	98	9625973	550	10331327	60
1	626219	451	957127	98	669002	549	330998	59
2	626490	451	957158	98	669332	549	330668	58
3	626760	450	957099	98	669661	549	330339	57
4	627030	450	957040	98	669991	548	330009	56
5	627300	450	956981	98	670320	548	329680	55
6	627570	449	956921	99	670649	548	329351	54
7	627840	449	956862	99	670977	548	329023	53
8	628109	449	956803	99	671306	547	328694	52
9	628378	448	956744	99	671634	547	328366	51
10	628647	448	956684	99	671963	547	328037	50
11	9628916	447	956625	99	962991	547	10327709	49
12	629185	447	956566	99	672319	546	327381	48
13	629453	447	956506	99	672647	546	327053	47
14	629721	446	956447	99	672974	546	326726	46
15	629989	446	956387	99	673302	546	326398	45
16	630257	446	956327	99	673629	545	326071	44
17	630524	446	956268	99	673957	545	325743	43
18	630792	445	956208	100	674284	545	325416	42
19	631059	445	956148	100	674610	544	325089	41
20	631326	445	956089	100	674937	544	324763	40
21	9631593	444	956029	100	9631593	544	10324436	39
22	631859	444	955969	100	675290	544	324110	38
23	632125	444	955909	100	675616	543	323784	37
24	632392	443	955849	100	675943	543	323457	36
25	632658	443	955789	100	676269	543	323131	35
26	632923	443	955729	100	676594	543	322806	34
27	633189	442	955669	100	676920	542	322480	33
28	633454	442	955609	100	677246	542	322154	32
29	633719	442	955548	100	677571	542	321829	31
30	633984	441	955488	100	677896	542	321504	30
31	9634949	441	955428	101	9634949	541	10321179	29
32	634514	440	955368	101	679146	541	320854	28
33	634778	440	955307	101	679471	541	320529	27
34	635042	440	955247	101	679795	541	320205	26
35	635306	439	955186	101	680120	540	319880	25
36	635570	439	955126	101	680444	540	319556	24
37	635834	439	955065	101	680768	540	319232	23
38	636097	438	955005	101	681092	540	318908	22
39	636360	438	954944	101	681416	539	318584	21
40	636623	438	954883	101	681740	539	318260	20
41	9636886	437	954823	101	9636886	539	10317937	19
42	637148	437	954762	101	682063	539	317613	18
43	637411	437	954701	101	682387	538	317290	17
44	637673	437	954640	101	682711	538	316967	16
45	637935	436	954579	101	683035	538	316644	15
46	638197	436	954518	102	683359	538	316321	14
47	638458	436	954457	102	683683	537	315999	13
48	638720	435	954396	102	684007	537	315676	12
49	638981	435	954335	102	684331	537	315354	11
50	639242	435	954274	102	684655	537	315032	10
51	9639503	434	954213	102	9639503	536	10314710	9
52	639764	434	954152	102	685012	536	314388	8
53	640024	434	954090	102	685334	536	314066	7
54	640284	433	954029	102	685657	536	313745	6
55	640544	433	953968	102	685979	535	313423	5
56	640804	433	953906	102	686302	535	313102	4
57	641064	432	953845	102	686624	535	312781	3
58	641324	432	953783	102	686946	534	312460	2
59	641584	432	953722	102	687269	534	312139	1
60	641842	431	953660	102	687591	534	311818	0

Co-sine | Sine | Co-tang. | Tang. | M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-641842	431	9-953660	103	9-668182	534	10-311818	60
1	642101	411	953590	103	668502	534	311498	59
2	642300	421	953537	103	668823	534	311177	58
3	642618	430	953475	103	669143	533	310857	57
4	642877	430	953413	103	669463	533	310537	56
5	643135	430	953352	103	669783	533	310217	55
6	643393	430	953290	103	690103	533	309897	54
7	643650	429	953228	103	690423	533	309577	53
8	643908	429	953166	103	690742	532	309258	52
9	644165	429	953104	103	691062	532	308938	51
10	644423	428	953042	103	691381	532	308619	50
11	9-644680	428	9-952980	104	9-691700	531	10-308300	49
12	644936	428	952918	104	692019	531	307981	48
13	645193	427	952855	104	692338	531	307662	47
14	645450	427	952793	104	692656	531	307344	46
15	645706	427	952731	104	692975	531	307025	45
16	645962	426	952669	104	693293	530	306707	44
17	646218	426	952606	104	693612	530	306388	43
18	646474	426	952544	104	693930	530	306070	42
19	646729	425	952481	104	694248	530	305752	41
20	646984	425	952419	104	694566	529	305434	40
21	9-647240	425	9-952256	104	9-694883	529	10-305117	39
22	647494	424	952204	104	695201	529	304799	38
23	647749	424	952141	104	695518	529	304482	37
24	648004	424	952078	105	695836	529	304164	36
25	648258	424	952016	105	696153	528	303847	35
26	648512	423	951953	105	696470	528	303530	34
27	648766	423	951890	105	696787	528	303213	33
28	649020	423	951827	105	697103	528	302897	32
29	649274	422	951764	105	697420	527	302580	31
30	649527	422	951701	105	697736	527	302264	30
31	9-649781	422	9-951728	105	9-698053	527	10-301947	29
32	650034	422	951665	105	698369	527	301631	28
33	650287	421	951602	105	698685	526	301315	27
34	650539	421	951539	105	699001	526	300999	26
35	650792	421	951476	105	699316	526	300684	25
36	651044	420	951412	105	699632	526	300368	24
37	651297	420	951349	106	699947	526	300053	23
38	651549	420	951286	106	700263	525	299737	22
39	651800	419	951222	106	700578	525	299422	21
40	652052	419	951159	106	700893	525	299107	20
41	9-652304	419	9-951096	106	9-701208	524	10-298792	19
42	652555	418	951032	106	701523	524	298477	18
43	652806	418	950968	106	701837	524	298163	17
44	653057	418	950905	106	702152	524	297848	16
45	653308	418	950841	106	702466	524	297534	15
46	653558	417	950778	106	702780	523	297220	14
47	653808	417	950714	106	703095	523	296905	13
48	654059	417	950650	106	703409	523	296591	12
49	654309	416	950586	106	703723	523	296277	11
50	654558	416	950522	107	704036	522	295964	10
51	654808	416	9-950458	107	9-704350	522	10-295650	9
52	655058	416	950394	107	704663	522	295337	8
53	655307	415	950330	107	704977	522	295023	7
54	655556	415	950266	107	705290	522	294710	6
55	655805	415	950202	107	705603	521	294397	5
56	656054	414	950138	107	705916	521	294084	4
57	656302	414	950074	107	706228	521	293772	3
58	656551	414	950010	107	706541	521	293459	2
59	656799	413	949945	107	706854	521	293146	1
60	657047	413	949881	107	707166	520	292834	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	0-657047	413	9-949681	107	9-707166	520	10-292834	60
1	657285	413	949616	107	707478	520	292522	59
2	657542	413	949552	107	707790	520	292210	58
3	657790	412	949488	108	708102	520	291898	57
4	658037	412	949423	108	708414	519	291586	56
5	658284	412	949358	108	708726	519	291274	55
6	658531	411	949294	108	709037	519	290963	54
7	658778	411	949229	108	709349	519	290651	53
8	659025	411	949164	108	709660	519	290340	52
9	659271	410	949100	108	709971	518	290029	51
10	659517	410	949035	108	710282	518	289718	50
11	9-659763	410	9-949170	108	9-710593	518	10-289407	49
12	660009	409	949105	108	710904	518	289096	48
13	660255	409	949040	108	711215	518	288785	47
14	660501	409	948975	108	711525	517	288475	46
15	660746	409	948910	108	711836	517	288164	45
16	660991	408	948845	108	712146	517	287854	44
17	661236	408	948780	109	712456	517	287544	43
18	661481	408	948715	109	712766	516	287234	42
19	661726	407	948650	109	713076	516	286924	41
20	661970	407	948584	109	713386	516	286614	40
21	9-662214	407	9-948519	109	9-713696	516	10-286304	39
22	662459	407	948454	109	714005	516	285995	38
23	662703	406	948388	109	714314	515	285686	37
24	662946	406	948323	109	714624	515	285376	36
25	663190	406	948257	109	714933	515	285067	35
26	663433	405	948192	109	715242	515	284758	34
27	663677	405	948126	109	715551	514	284449	33
28	663920	405	948060	109	715860	514	284140	32
29	664163	405	947995	110	716168	514	283832	31
30	664406	404	947929	110	716477	514	283523	30
31	9-664648	404	9-947863	110	9-716785	514	10-283213	29
32	664891	404	947797	110	717093	513	282907	28
33	665133	403	947731	110	717401	513	282599	27
34	665375	403	947665	110	717709	513	282291	26
35	665617	403	947600	110	718017	513	281983	25
36	665859	402	947533	110	718325	513	281675	24
37	666100	402	947467	110	718633	512	281367	23
38	666342	402	947401	110	718940	512	281060	22
39	666583	402	947335	110	719248	512	280752	21
40	666824	401	947269	110	719555	512	280445	20
41	9-667065	401	9-947203	110	9-719862	512	10-280138	19
42	667305	401	947136	111	720169	511	279831	18
43	667546	401	947070	111	720476	511	279524	17
44	667786	400	947004	111	720783	511	279217	16
45	668027	400	946937	111	721089	511	278911	15
46	668267	400	946871	111	721396	511	278604	14
47	668506	399	946804	111	721703	510	278298	13
48	668746	399	946738	111	722009	510	277991	12
49	668986	399	946671	111	722315	510	277685	11
50	669225	399	946604	111	722621	510	277379	10
51	9-669464	398	9-946538	111	9-722927	510	10-277073	9
52	669703	398	946471	111	723232	509	276768	8
53	669942	398	946404	111	723538	509	276462	7
54	670181	397	946337	111	723844	509	276156	6
55	670419	397	946270	112	724149	509	275851	5
56	670658	397	946203	112	724454	509	275546	4
57	670896	397	946136	112	724759	508	275241	3
58	671134	396	946069	112	725065	508	274935	2
59	671372	396	946002	112	725369	508	274631	1
60	671609	396	945935	112	725674	508	274326	0

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-671800	306	9-945035	119	9-725674	506	10-274326	60
1	671847	305	945068	119	725070	506	974021	59
2	671894	305	945100	119	726264	507	973716	58
3	671931	305	945133	119	726566	507	973412	57
4	671958	305	945166	119	726862	507	973108	56
5	671995	304	945198	119	727167	507	972803	55
6	672032	304	945231	119	727501	507	972499	54
7	672069	304	945264	119	727805	506	972195	53
8	672105	304	945296	119	728109	506	971891	52
9	672141	303	945328	119	728412	506	971588	51
10	672177	303	945361	119	728716	506	971284	50
11	9-6724213	303	9-9451193	113	9-729020	506	10-270680	49
12	672448	302	945152	113	729323	505	970677	48
13	672484	302	945088	113	729626	505	970374	47
14	6724919	302	944990	113	729929	505	970071	46
15	6725155	302	944922	113	730233	505	969767	45
16	6725390	301	944854	113	730535	505	969465	44
17	6725624	301	944786	113	730838	504	969162	43
18	6725859	301	944718	113	731141	504	968859	42
19	6726094	301	944650	113	731444	504	968556	41
20	6726328	300	944582	114	731746	504	968254	40
21	9-6726562	300	9-944514	114	9-732046	504	10-267952	39
22	6726796	300	944446	114	732351	503	967649	38
23	6727030	300	944377	114	732653	503	967347	37
24	6727264	300	944309	114	732955	503	967045	36
25	6727498	300	944241	114	733257	503	966743	35
26	6727731	300	944173	114	733558	503	966442	34
27	6727964	300	944104	114	733860	502	966140	33
28	6728197	300	944036	114	734162	502	965838	32
29	6728430	300	943967	114	734463	502	965537	31
30	6728663	300	943899	114	734764	502	965236	30
31	9-6728895	307	9-943830	114	9-735066	492	10-264034	29
32	6729128	307	943761	114	735367	502	964733	28
33	6729361	307	943693	115	735668	501	964432	27
34	6729594	307	943624	115	735969	501	964131	26
35	6729824	306	943555	115	736269	501	963831	25
36	6800556	306	943486	115	736570	501	963530	24
37	6800288	306	943417	115	736871	501	963230	23
38	6800519	305	943348	115	737171	500	962929	22
39	6800750	305	943279	115	737471	500	962629	21
40	6800982	305	943210	115	737771	500	962329	20
41	9-681213	305	9-943141	115	9-738071	500	10-261029	19
42	681443	304	943072	115	436371	500	961629	18
43	681674	304	943003	115	738671	499	961329	17
44	681905	304	942934	115	738971	499	961029	16
45	682135	304	942864	115	739271	499	960729	15
46	682365	303	942795	116	739570	499	960430	14
47	682595	303	942726	116	739870	499	960130	13
48	682825	303	942656	116	740169	499	959831	12
49	683055	303	942587	116	740468	498	959532	11
50	683284	302	942517	116	740767	498	959233	10
51	9-683514	302	9-942448	116	9-741066	498	10-258034	9
52	683743	302	942378	116	741365	498	958635	8
53	683972	302	942308	116	741664	498	958236	7
54	684201	301	942239	116	741962	497	957837	6
55	684430	301	942169	116	742261	497	957438	5
56	684658	301	942099	116	742559	497	957041	4
57	684887	300	942029	116	742858	497	956642	3
58	685115	300	941959	117	743156	497	956244	2
59	685343	300	941889	117	743454	497	955846	1
60	685571	300	941819	117	743752	496	955448	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-685571	380	9-941819	117	9-743752	496	10-256948	60
1	685799	379	941749	117	744050	496	255950	59
2	686027	379	941679	117	744348	496	255652	58
3	686254	379	941609	117	744645	496	255355	57
4	686482	379	941539	117	744943	496	255057	56
5	686709	378	941469	117	745240	496	254760	55
6	686936	378	941398	117	745538	495	254462	54
7	687163	378	941328	117	745835	495	254165	53
8	687389	378	941258	117	746132	495	253868	52
9	687616	377	941187	117	746429	495	253571	51
10	687843	377	941117	117	746726	495	253274	50
11	9-688069	377	9-941046	118	9-747023	494	10-252977	49
12	688295	377	940975	118	747319	494	252681	48
13	688521	376	940905	118	747616	494	252384	47
14	688747	376	940834	118	747913	494	252087	46
15	688972	376	940763	118	748209	494	251791	45
16	689198	376	940693	118	748505	493	251495	44
17	689423	375	940623	118	748801	493	251199	43
18	689648	375	940551	118	749097	493	250903	42
19	689873	375	940480	118	749393	493	250607	41
20	690098	375	940409	118	749689	493	250311	40
21	9-690323	374	9-940338	118	9-749985	493	10-250015	39
22	690548	374	940267	118	750281	492	249719	38
23	690772	374	940196	118	750576	492	249424	37
24	690996	374	940125	119	750872	492	249128	36
25	691220	373	940054	119	751167	492	248833	35
26	691444	373	939983	119	751462	492	248538	34
27	691668	373	939911	119	751757	492	248243	33
28	691892	373	939840	119	752052	491	247948	32
29	692115	372	939768	119	752347	491	247653	31
30	692339	372	939697	119	752642	491	247358	30
31	9-692562	372	9-939625	119	9-752937	491	10-247063	29
32	692785	371	939554	119	753231	491	246769	28
33	693008	371	939483	119	753526	491	246474	27
34	693231	371	939411	119	753820	490	246180	26
35	693453	371	939339	119	754115	490	245885	25
36	693676	370	939267	120	754409	490	245591	24
37	693898	370	939195	120	754703	490	245297	23
38	694120	370	939123	120	754997	490	245003	22
39	694342	370	939052	120	755291	490	244709	21
40	694564	369	938980	120	755585	489	244415	20
41	9-694786	369	9-938908	120	9-755878	489	10-244122	19
42	695007	369	938836	120	756172	489	243828	18
43	695229	369	938763	120	756465	489	243535	17
44	695450	368	938691	120	756759	489	243241	16
45	695671	368	938619	120	757052	489	242948	15
46	695892	368	938547	120	757345	488	242655	14
47	696113	368	938475	120	757638	488	242362	13
48	696334	367	938403	121	757931	488	242069	12
49	696554	367	938330	121	758224	488	241776	11
50	696775	367	938258	121	758517	488	241483	10
51	9-696995	367	9-938185	121	9-758810	488	10-241190	9
52	697215	366	938113	121	759102	487	240898	8
53	697435	366	938040	121	759395	487	240605	7
54	697654	366	937967	121	759687	487	240313	6
55	697874	366	937895	121	759979	487	240021	5
56	698094	365	937822	121	760272	487	239728	4
57	698313	365	937749	121	760564	487	239436	3
58	698532	365	937676	121	760856	486	239144	2
59	698751	365	937604	121	761148	486	238852	1
60	698970	364	937531	121	761439	486	238561	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Co-sine	D.	Tang.	D.	Cotang.	M.
0	9-606970	364	9-937531	191	9-761439	486	10-236561	00
1	609189	364	937458	192	761731	486	236269	59
2	609407	364	937385	193	762023	486	237977	58
3	609626	364	937312	194	762314	486	237686	57
4	609844	363	937239	195	762606	485	237394	56
5	700062	363	937165	196	762897	485	237103	55
6	700280	363	937092	197	763188	485	236811	54
7	700498	363	937019	198	763479	485	236521	53
8	700716	363	936946	199	763770	485	236230	52
9	700933	362	936872	199	764061	485	235939	51
10	701151	362	936799	199	764352	484	235648	50
11	9-701369	362	9-936725	199	9-764643	484	10-235357	49
12	701585	362	936652	199	764933	484	235067	48
13	701802	361	936578	199	765224	484	234776	47
14	702019	361	936505	199	765514	484	234486	46
15	702236	361	936431	199	765805	484	234195	45
16	702452	361	936357	199	766095	484	233905	44
17	702669	360	936284	199	766385	483	233615	43
18	702885	360	936210	199	766675	483	233325	42
19	703101	360	936136	199	766965	483	233035	41
20	703317	360	936062	199	767255	483	232745	40
21	9-703533	359	9-935988	199	9-767545	483	10-232455	39
22	703749	359	935914	199	767834	483	232166	38
23	703964	359	935840	199	768124	483	231876	37
24	704179	359	935766	194	768413	482	231587	36
25	704395	359	935692	194	768703	482	231297	35
26	704610	358	935618	194	768992	482	231008	34
27	704825	358	935543	194	769281	482	230719	33
28	705040	358	935469	194	769570	482	230430	32
29	705254	358	935395	194	769860	481	230140	31
30	705469	357	935320	194	770149	481	229852	30
31	9-705683	357	9-935246	194	9-770437	481	10-229563	29
32	705896	357	935171	194	770726	481	229274	28
33	706112	357	935097	194	771015	481	228985	27
34	706326	356	935022	194	771303	481	228697	26
35	706539	356	934948	194	771592	481	228408	25
36	706753	356	934873	194	771880	480	228120	24
37	706967	356	934798	195	772168	480	227832	23
38	707180	355	934723	195	772457	480	227543	22
39	707393	355	934649	195	772745	480	227255	21
40	707606	355	934574	195	773033	480	226967	20
41	9-707819	355	9-934499	195	9-773321	480	10-226679	19
42	708032	354	934424	195	773609	479	226392	18
43	708245	354	934349	195	773896	479	226104	17
44	708458	354	934274	195	774184	479	225816	16
45	708670	354	934199	195	774471	479	225529	15
46	708882	353	934123	195	774759	479	225241	14
47	709094	353	934048	195	775046	479	224954	13
48	709306	353	933973	195	775333	479	224667	12
49	709518	353	933898	196	775621	478	224379	11
50	709730	353	933822	196	775908	478	224092	10
51	9-709941	352	9-933747	196	9-776195	478	10-223805	9
52	710153	352	933671	196	776482	478	223518	8
53	710364	352	933596	196	776769	478	223231	7
54	710575	352	933520	196	777055	478	222945	6
55	710786	351	933445	196	777342	478	222658	5
56	710997	351	933369	196	777628	477	222373	4
57	711208	351	933293	196	777915	477	222085	3
58	711419	351	933217	196	778201	477	221799	2
59	711629	350	933141	196	778487	477	221513	1
60	711839	350	933066	196	778774	477	221226	0
	Co-sine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-711830	350	9-933066	136	9-778774	477	10-221226	60
1	713050	350	932860	137	779060	477	220940	59
2	715260	350	932614	137	779346	476	220654	58
3	717460	349	932368	137	779632	476	220368	57
4	719670	349	932122	137	779918	476	220082	56
5	721890	349	931875	137	780203	476	219797	55
6	723990	349	931629	137	780489	476	219511	54
7	726090	349	931383	137	780775	476	219225	53
8	728170	348	931137	137	781060	476	218940	52
9	730260	348	930890	137	781346	475	218654	51
10	732335	348	930644	137	781631	475	218369	50
11	9-714144	348	9-932398	137	9-781916	475	10-218084	49
12	714332	347	932151	137	782201	475	217799	48
13	714521	347	931905	136	782486	475	217514	47
14	714709	347	931658	136	782771	475	217229	46
15	714897	347	931411	136	783056	475	216944	45
16	715186	347	931164	136	783341	475	216659	44
17	715394	346	930917	136	783626	474	216374	43
18	715602	346	930670	136	783910	474	216089	42
19	715809	346	930423	136	784195	474	215805	41
20	716017	346	930176	136	784479	474	215521	40
21	9-716224	345	9-931460	136	9-784764	474	10-215236	39
22	716432	345	931213	136	785048	474	214952	38
23	716639	345	930966	136	785332	473	214668	37
24	716846	345	930719	136	785616	473	214384	36
25	717053	345	930472	136	785900	473	214100	35
26	717259	344	930225	136	786184	473	213816	34
27	717466	344	930008	136	786468	473	213532	33
28	717673	344	929791	136	786752	473	213248	32
29	717879	344	929574	136	787036	473	212964	31
30	718085	343	929357	136	787319	473	212681	30
31	9-718291	343	9-929068	136	9-787603	473	10-212397	29
32	718497	343	928811	136	787886	473	212114	28
33	718703	343	928554	136	788170	473	211830	27
34	718909	343	928297	136	788453	473	211547	26
35	719114	342	928040	136	788736	473	211264	25
36	719320	342	927783	136	789019	473	210981	24
37	719525	342	927526	136	789302	471	210698	23
38	719730	342	927269	136	789585	471	210415	22
39	719935	341	927012	136	789868	471	210132	21
40	720140	341	926755	136	790151	471	209849	20
41	9-720345	341	9-926498	136	9-790433	471	10-209567	19
42	720549	341	926241	136	790716	471	209284	18
43	720754	340	925984	136	790999	471	209001	17
44	720958	340	925727	136	791281	471	208719	16
45	721163	340	925470	136	791563	470	208437	15
46	721366	340	925213	136	791846	470	208154	14
47	721570	340	924956	136	792128	470	207873	13
48	721774	339	924699	136	792410	470	207590	12
49	721978	339	924442	131	792692	470	207308	11
50	722181	339	924185	131	792974	470	207026	10
51	9-722385	339	9-923928	131	9-793256	470	10-206744	9
52	722588	339	923671	131	793538	469	206462	8
53	722791	338	923414	131	793819	469	206181	7
54	722994	338	923157	131	794101	469	205899	6
55	723197	338	922900	131	794383	469	205617	5
56	723400	338	922643	131	794664	469	205336	4
57	723603	337	922386	131	794945	469	205055	3
58	723805	337	922129	131	795227	469	204773	2
59	724007	337	921872	131	795508	468	204492	1
60	724210	337	921615	131	795789	468	204211	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9734210	337	9998490	132	9798789	468	10304211	60
1	734412	337	9983423	132	7940070	468	3030300	59
2	734814	336	9982963	132	796351	468	3030449	58
3	734816	336	9981953	132	7966329	468	3030368	57
4	735017	336	9981104	132	796913	468	303067	56
5	735219	336	9980225	132	797194	468	3030906	55
6	735420	335	9979446	132	797475	468	303225	54
7	735622	335	9978667	132	797755	468	303245	53
8	735823	335	9977887	132	798036	467	301964	52
9	736024	335	9977108	132	798316	467	301684	51
10	736225	335	9976329	132	798596	467	301404	50
11	9736426	334	9997549	132	9798877	467	10301123	49
12	736826	334	997470	132	799157	467	300643	48
13	736827	334	997390	132	799437	467	300563	47
14	737027	334	997310	132	799717	467	300683	46
15	737228	334	997231	132	799997	466	300003	45
16	737429	333	997151	132	800277	466	199723	44
17	737629	333	997071	132	800557	466	199443	43
18	737830	333	996991	132	800836	466	199164	42
19	738030	333	996911	132	801116	466	198884	41
20	738231	333	996831	132	801396	466	198604	40
21	9738437	332	9996751	132	97991675	466	1030325	39
22	738626	332	996671	132	801655	466	198045	38
23	738625	332	996591	132	801934	465	197766	37
24	739024	332	996511	134	802213	465	197487	36
25	739223	331	996431	134	802492	465	197208	35
26	739423	331	996351	134	802771	465	196928	34
27	739621	331	996270	134	803051	465	196649	33
28	739820	331	996190	134	803330	465	196370	32
29	739918	330	996110	134	803609	465	196091	31
30	739216	330	996029	134	804187	465	195813	30
31	9739415	330	9995949	134	9799446	464	1030534	29
32	739613	330	995868	134	804745	464	195255	28
33	739811	330	995788	134	805023	464	194977	27
34	731009	329	995707	134	805302	464	194698	26
35	731206	329	995626	134	805580	464	194420	25
36	731404	329	995545	135	805859	464	194141	24
37	731602	329	995465	135	806137	464	193863	23
38	731799	329	995384	135	806415	463	193585	22
39	731996	328	995303	135	806693	463	193307	21
40	732193	328	995222	135	806971	463	193029	20
41	9739290	328	9995141	135	9799290	463	1030275	19
42	732587	328	995060	135	807527	463	192473	18
43	732784	328	994979	135	807805	463	192195	17
44	732980	327	994897	135	808083	463	191917	16
45	733177	327	994816	135	808361	463	191639	15
46	733373	327	994735	136	808639	462	191362	14
47	733569	327	994654	136	808916	462	191084	13
48	733765	327	994572	136	809193	462	190807	12
49	733961	326	994491	136	809471	462	190529	11
50	734157	326	994409	136	809748	462	190252	10
51	9739433	326	9994328	136	97991095	462	1030375	9
52	734549	326	994246	136	810302	462	189998	8
53	734744	325	994164	136	810580	462	189490	7
54	734939	325	994083	136	810857	462	189143	6
55	735135	325	994001	136	811134	461	188866	5
56	735330	325	993919	136	811410	461	188590	4
57	735525	325	993837	136	811687	461	188313	3
58	735719	324	993755	137	811964	461	188036	2
59	735914	324	993673	137	812241	461	187759	1
60	736109	324	993591	137	812517	461	187483	0

M.	Sine	D.	Co-sine	D.	Tang.	D.	Cotang.	M.
0	9-736109	394	9-923501	137	9-819517	461	10-187469	60
1	736303	394	923509	137	819794	461	187906	59
2	736496	394	923517	137	813070	461	186900	58
3	736692	393	923525	137	813347	460	186653	57
4	736886	393	923533	137	813623	460	186377	56
5	737080	393	923181	137	813899	460	186101	55
6	737274	393	923006	137	814175	460	185825	54
7	737467	393	923016	137	814458	460	185548	53
8	737661	392	923033	137	814736	460	185273	52
9	737855	392	923051	137	815004	460	184996	51
10	738048	392	923068	136	815279	460	184721	50
11	9-738941	392	9-923086	136	9-815555	459	10-184445	49
12	738434	392	923093	136	815831	459	184169	48
13	738627	391	923100	136	816107	459	183893	47
14	738820	391	923108	136	816383	459	183618	46
15	739013	391	923115	136	816658	459	183342	45
16	739206	391	923123	136	816933	459	183067	44
17	739398	391	923130	136	817209	459	182791	43
18	739590	390	923138	136	817484	459	182516	42
19	739783	390	923145	136	817759	459	182241	41
20	739975	390	923152	136	818035	458	181965	40
21	9-740167	390	9-923157	136	9-818310	458	10-181690	39
22	740359	390	923174	136	818585	458	181415	38
23	740550	319	923181	136	818860	458	181140	37
24	740742	319	923187	136	819135	458	180865	36
25	740934	319	923194	136	819410	458	180590	35
26	741125	319	923201	136	819684	458	180316	34
27	741316	319	923207	136	819959	458	180041	33
28	741508	318	923214	136	820234	458	179766	32
29	741699	318	923220	136	820508	457	179492	31
30	741889	318	923227	136	820783	457	179217	30
31	9-742080	318	9-923233	136	9-821057	457	10-178943	29
32	742271	318	923239	140	821332	457	178668	28
33	742462	317	923246	140	821606	457	178394	27
34	742652	317	923252	140	821880	457	178120	26
35	742842	317	923259	140	822154	457	177846	25
36	743033	317	923265	140	822429	457	177571	24
37	743223	317	923272	140	822703	457	177297	23
38	743413	316	923278	140	822977	456	177023	22
39	743603	316	923285	140	823252	456	176750	21
40	743793	316	923291	140	823526	456	176476	20
41	9-743984	316	9-923298	140	9-823798	456	10-176202	19
42	744174	316	923304	140	824073	456	175928	18
43	744364	315	923311	140	824348	456	175655	17
44	744554	315	919931	141	824619	456	175381	16
45	744743	315	919846	141	824893	456	175107	15
46	744932	315	919762	141	825166	456	174834	14
47	745121	315	919677	141	825439	455	174561	13
48	745309	314	919593	141	825713	455	174287	12
49	745494	314	919508	141	825986	455	174014	11
50	745683	314	919424	141	826259	455	173741	10
51	9-745871	314	9-919339	141	9-826533	455	10-173469	9
52	746059	314	919254	141	826805	455	173195	8
53	746248	313	919169	141	827078	455	172922	7
54	746436	313	919085	141	827351	455	172649	6
55	746624	313	919000	141	827624	455	172376	5
56	746812	313	918915	142	827897	454	172103	4
57	746999	313	918830	142	828170	454	171830	3
58	747187	312	918745	142	828443	454	171558	2
59	747374	312	918659	142	828715	454	171285	1
60	747562	312	918574	142	828987	454	171013	0
	Co-sine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-747592	312	9-918574	142	9-828987	454	10-171013	60
1	747749	312	918489	142	829290	454	170740	59
2	747936	312	918404	142	829532	454	170468	58
3	748123	311	918318	142	829805	454	170195	57
4	748310	311	918233	142	830077	454	169923	56
5	748497	311	918147	142	830349	453	169651	55
6	748683	311	918062	142	830621	453	169379	54
7	748870	311	917976	143	830893	453	169107	53
8	749056	310	917891	143	831165	453	168835	52
9	749243	310	917805	143	831437	453	168563	51
10	749429	310	917719	143	831709	453	168291	50
11	9-749615	310	9-917634	143	9-831981	453	10-168019	49
12	749801	310	917548	143	832253	453	167747	48
13	749987	309	917462	143	832525	453	167475	47
14	750173	309	917376	143	832796	453	167204	46
15	750358	309	917290	143	833068	452	166932	45
16	750543	309	917204	143	833339	452	166661	44
17	750729	309	917118	144	833611	452	166389	43
18	750914	309	917032	144	833882	452	166118	42
19	751100	308	916946	144	834154	452	165846	41
20	751284	308	916859	144	834425	452	165575	40
21	9-751469	308	9-916773	144	9-834696	452	10-165304	39
22	751654	308	916687	144	834967	452	165033	38
23	751839	308	916600	144	835238	452	164762	37
24	752023	307	916514	144	835509	452	164491	36
25	752208	307	916427	144	835780	451	164220	35
26	752392	307	916341	144	836051	451	163949	34
27	752576	307	916254	144	836322	451	163678	33
28	752760	307	916167	145	836593	451	163407	32
29	752944	306	916081	145	836864	451	163136	31
30	753128	306	915994	145	837134	451	162866	30
31	9-753312	306	9-915907	145	9-837405	451	10-162595	29
32	753495	306	915820	145	837675	451	162325	28
33	753679	306	915733	145	837946	451	162054	27
34	753862	305	915646	145	838216	451	161784	26
35	754046	305	915559	145	838487	450	161513	25
36	754229	305	915472	145	838757	450	161243	24
37	754412	305	915385	145	839027	450	160973	23
38	754595	305	915297	145	839297	450	160703	22
39	754778	304	915210	145	839568	450	160432	21
40	754960	304	915123	146	839838	450	160162	20
41	9-755143	304	9-915035	146	9-840109	450	10-159892	19
42	755326	304	914948	146	840378	450	159622	18
43	755508	304	914860	146	840647	450	159353	17
44	755690	304	914773	146	840917	449	159083	16
45	755872	303	914685	146	841187	449	158813	15
46	756054	303	914598	146	841457	449	158543	14
47	756236	303	914510	146	841726	449	158274	13
48	756418	303	914422	146	841996	449	158004	12
49	756600	303	914334	146	842266	449	157734	11
50	756782	302	914246	147	842535	449	157465	10
51	9-756963	302	9-914158	147	9-842805	449	10-157195	9
52	757144	302	914070	147	843074	449	156926	8
53	757326	302	913982	147	843343	449	156657	7
54	757507	302	913894	147	843612	449	156388	6
55	757688	301	913806	147	843882	448	156118	5
56	757869	301	913718	147	844151	448	155849	4
57	758050	301	913630	147	844420	448	155580	3
58	758230	301	913541	147	844689	448	155311	2
59	758411	301	913453	147	844958	448	155042	1
60	758591	301	913365	147	845227	448	154773	0
	Cosine		Sine		Cotang.		Tang.	M.

SINES AND TANGENTS. (36 Degrees.)

219

M.	Sine	D.	Co-sine	D.	Tang.	D.	Cotang.	M.
0	9-758501	301	9-913365	147	9-845237	448	10-154773	60
1	758772	300	913376	147	845406	448	154504	59
2	759058	300	913187	148	845764	448	154236	58
3	759132	300	913099	148	846033	448	153967	57
4	759212	300	913010	148	846302	448	153698	56
5	759492	300	912922	148	846570	447	153430	55
6	759672	300	912833	148	846839	447	153161	54
7	759853	300	912744	148	847107	447	152893	53
8	760031	300	912655	148	847376	447	152624	52
9	760211	300	912566	148	847644	447	152356	51
10	760390	300	912477	148	847913	447	152087	50
11	9-760569	300	9-912388	148	9-848181	447	10-151819	49
12	760748	300	912299	149	848449	447	151551	48
13	760927	300	912210	149	848717	447	151283	47
14	761106	300	912121	149	848986	447	151014	46
15	761285	300	912031	149	849254	447	150746	45
16	761464	300	911942	149	849522	447	150478	44
17	761642	300	911853	149	849790	446	150210	43
18	761821	300	911763	149	850058	446	149942	42
19	761999	300	911674	149	850325	446	149675	41
20	762177	300	911584	149	850593	446	149407	40
21	9-762356	300	9-911495	149	9-850861	446	10-149139	39
22	762534	300	911405	149	851129	446	148871	38
23	762712	300	911315	150	851396	446	148604	37
24	762890	300	911226	150	851664	446	148336	36
25	763067	300	911136	150	851931	446	148069	35
26	763245	300	911046	150	852199	446	147801	34
27	763422	300	910956	150	852466	446	147534	33
28	763600	300	910866	150	852733	445	147267	32
29	763777	300	910776	150	853001	445	146999	31
30	763954	300	910686	150	853268	445	146732	30
31	9-764131	300	9-910596	150	9-853535	445	10-146465	29
32	764308	300	910506	150	853802	445	146198	28
33	764485	300	910415	150	854069	445	145931	27
34	764662	300	910325	151	854336	445	145664	26
35	764838	300	910235	151	854603	445	145397	25
36	765015	300	910144	151	854870	445	145130	24
37	765191	300	910054	151	855137	445	144863	23
38	765367	300	909963	151	855404	445	144596	22
39	765544	300	909873	151	855671	444	144329	21
40	765720	300	909783	151	855938	444	144062	20
41	9-765896	300	9-909691	151	9-856204	444	10-143796	19
42	766073	300	909601	151	856471	444	143529	18
43	766247	300	909510	151	856737	444	143263	17
44	766423	300	909419	151	857004	444	142996	16
45	766598	300	909328	152	857270	444	142730	15
46	766774	300	909237	152	857537	444	142463	14
47	766949	300	909146	152	857803	444	142197	13
48	767124	300	909055	152	858069	444	141931	12
49	767300	300	908964	152	858336	444	141664	11
50	767475	301	908873	152	858602	443	141398	10
51	9-767649	301	9-908781	152	9-858868	443	10-141132	9
52	767824	301	908690	152	859134	443	140866	8
53	767999	301	908599	152	859400	443	140600	7
54	768173	301	908507	152	859666	443	140334	6
55	768348	300	908416	153	859932	443	140068	5
56	768522	300	908324	153	860198	443	139802	4
57	768697	300	908233	153	860464	443	139536	3
58	768871	300	908141	153	860730	443	139270	2
59	769045	300	908049	153	860995	443	139005	1
60	769219	300	907958	153	861261	443	138739	0
	Co-sine		Sine		Cotang.		Tang.	M.

64 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-769210		9-907958	153	9-861961	443	10-136739	60
1	769363	269	907866	153	861527	443	136473	59
2	769566	269	907774	153	861793	443	136906	58
3	769740	269	907682	153	862059	443	137949	57
4	769913	269	907590	153	862323	443	137677	56
5	770087	269	907498	153	862589	443	137411	55
6	770260	268	907406	153	862854	443	137146	54
7	770433	268	907314	154	863119	443	136881	53
8	770606	268	907222	154	863385	443	136615	52
9	770779	268	907130	154	863650	443	136350	51
10	770952	268	907037	154	863915	443	136085	50
11	9-771125	268	9-906945	154	9-864180	443	10-135890	49
12	771298	267	906852	154	864445	443	135555	48
13	771470	267	906760	154	864710	443	135220	47
14	771643	267	906667	154	864975	441	135025	46
15	771815	267	906575	154	865240	441	134770	45
16	771987	267	906482	154	865505	441	134465	44
17	772159	267	906390	155	865770	441	134220	43
18	772331	266	906296	155	866035	441	133965	42
19	772503	266	906204	155	866300	441	133700	41
20	772675	266	906111	155	866564	441	133436	40
21	9-772847	266	9-906018	155	9-866829	441	10-133171	39
22	773018	266	905925	155	867094	441	132906	38
23	773190	266	905832	155	867358	441	132642	37
24	773361	265	905739	155	867623	441	132377	36
25	773533	265	905645	155	867887	441	132113	35
26	773704	265	905552	155	868152	440	131848	34
27	773875	265	905459	155	868416	440	131584	33
28	774046	265	905366	156	868680	440	131320	32
29	774217	265	905272	156	868945	440	131055	31
30	774388	264	905179	156	869209	440	130791	30
31	9-774558	264	9-905085	156	9-869473	440	10-130527	29
32	774729	264	904992	156	869737	440	130263	28
33	774899	264	904898	156	870001	440	129999	27
34	775070	264	904804	156	870265	440	129735	26
35	775240	264	904711	156	870529	440	129471	25
36	775410	263	904617	156	870793	440	129207	24
37	775580	263	904523	156	871057	440	128943	23
38	775750	263	904429	157	871321	440	128679	22
39	775920	263	904335	157	871585	440	128415	21
40	776090	263	904241	157	871849	439	128151	20
41	9-776259	263	9-904147	157	9-872112	439	10-127888	19
42	776429	262	904053	157	872376	439	127624	18
43	776598	262	903959	157	872640	439	127360	17
44	776768	262	903864	157	872903	439	127097	16
45	776937	262	903770	157	873167	439	126833	15
46	777106	262	903676	157	873430	439	126570	14
47	777275	261	903581	157	873694	439	126306	13
48	777444	261	903487	157	873957	439	126043	12
49	777613	261	903392	158	874220	439	125780	11
50	777781	261	903298	158	874484	439	125516	10
51	9-777950	261	9-903203	158	9-874747	439	10-125253	9
52	778119	261	903108	158	875010	439	124990	8
53	778287	260	903014	158	875273	438	124727	7
54	778455	260	902919	158	875536	438	124464	6
55	778624	260	902824	158	875800	438	124200	5
56	778792	260	902729	158	876063	438	123937	4
57	778960	260	902634	158	876326	438	123674	3
58	779128	260	902539	159	876589	438	123411	2
59	779295	279	902444	159	876851	438	123149	1
60	779463	279	902349	159	877114	438	122886	0
	Cosine		Sine		Cotang.		Tang.	M.

SINES AND TANGENTS. (34 Degrees.)

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	0-779463	279	9-902349	159	9-877114	438	10-1292896	60
1	779631	279	9-925233	159	877377	438	1292823	59
2	779798	279	9-921158	159	877640	438	1292300	58
3	779966	279	9-920693	159	877903	438	1292077	57
4	780133	279	9-91807	159	878165	438	1215355	56
5	780300	278	9-918772	159	878428	438	1215772	55
6	780467	278	9-91776	159	878691	438	121309	54
7	780634	278	9-91681	159	878953	437	121047	53
8	780801	278	9-91585	159	879216	437	120784	52
9	780968	278	9-91490	159	879478	437	120522	51
10	781134	278	9-91394	160	879741	437	120259	50
11	9-781301	277	9-901298	160	9-880003	437	10-119907	49
12	781468	277	9-91302	160	880265	437	119735	48
13	781634	277	9-91106	160	880528	437	119472	47
14	781800	277	9-91010	160	880790	437	119210	46
15	781966	277	9-90914	160	881052	437	118948	45
16	782132	277	9-90818	160	881314	437	118686	44
17	782298	276	9-90722	160	881576	437	118424	43
18	782464	276	9-90626	160	881839	437	118161	42
19	782630	276	9-90529	160	882101	437	117899	41
20	782796	276	9-90433	161	882363	436	117637	40
21	9-782961	276	9-90037	161	9-882625	436	10-117375	39
22	783127	276	9-90240	161	882887	436	117113	38
23	783292	275	9-90144	161	883148	436	116852	37
24	783458	275	9-90047	161	883410	436	116590	36
25	783623	275	8-99951	161	883672	436	116328	35
26	783788	275	8-99854	161	883934	436	116066	34
27	783953	275	8-99757	161	884196	436	115804	33
28	784118	275	8-99660	161	884457	436	115543	32
29	784282	274	8-99564	161	884719	436	115281	31
30	784447	274	8-99467	162	884980	436	115020	30
31	9-784612	274	9-89970	162	9-885242	436	10-114758	29
32	784776	274	8-99873	162	885503	436	114497	28
33	784941	274	8-99776	162	885765	436	114235	27
34	785105	274	8-99678	162	886026	436	113974	26
35	785269	273	8-99581	162	886288	436	113712	25
36	785433	273	8-99484	162	886549	435	113451	24
37	785597	273	8-99387	162	886810	435	113190	23
38	785761	273	8-99289	162	887072	435	112928	22
39	785925	273	8-99192	162	887333	435	112667	21
40	786089	273	8-99094	163	887594	435	112406	20
41	9-786252	273	9-896397	163	9-887855	435	10-112145	19
42	786416	273	8-989290	163	888116	435	111884	18
43	786579	273	8-988202	163	888377	435	111623	17
44	786742	273	8-987104	163	888639	435	111361	16
45	786906	273	8-986006	163	888900	435	111100	15
46	787069	273	8-984908	163	889160	435	110840	14
47	787232	271	8-983810	163	889421	435	110579	13
48	787395	271	8-982712	163	889682	435	110318	12
49	787557	271	8-981614	163	889943	435	110057	11
50	787720	271	8-980516	163	890204	434	119796	10
51	9-787883	271	9-897418	164	9-890465	434	10-109535	9
52	788045	271	8-97330	164	890725	434	109275	8
53	788208	271	8-97222	164	890986	434	109014	7
54	788370	270	8-97113	164	891247	434	108753	6
55	788532	270	8-97005	164	891507	434	108493	5
56	788694	270	8-96896	164	891768	434	108232	4
57	788856	270	8-96788	164	892028	434	107972	3
58	789018	270	8-96679	164	892289	434	107711	2
59	789180	270	8-96571	164	892549	434	107451	1
60	789342	269	8-96462	164	892810	434	107190	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Co-sine	D.	Tang.	D.	Cotang.	M.
0	9789342	269	9896539	164	9892610	434	10107190	80
1	789504	269	806433	165	893070	434	106930	59
2	789865	269	806335	165	893331	434	106669	58
3	789927	269	806236	165	893591	434	106409	57
4	789989	269	806137	165	893851	434	106149	56
5	790149	269	806038	165	894111	434	105889	55
6	790310	268	805939	165	894371	434	105629	54
7	790471	268	805840	165	894632	433	105368	53
8	790632	268	805741	165	894892	433	105108	52
9	790793	268	805642	165	895152	433	104848	51
10	790954	268	805542	165	895412	433	104588	50
11	9791115	268	9895443	166	9895673	433	10104328	49
12	791275	267	805343	166	895632	433	104068	48
13	791436	267	805244	166	896192	433	103808	47
14	791596	267	805145	166	896452	433	103548	46
15	791757	267	805045	166	896712	433	103288	45
16	791917	267	804945	166	896971	433	103029	44
17	792077	267	804846	166	897231	433	102769	43
18	792237	266	804746	166	897491	433	102509	42
19	792397	266	804646	166	897751	433	102249	41
20	792557	266	804546	166	898010	433	101990	40
21	9792716	266	9894446	167	9896270	433	10101730	39
22	792576	266	804346	167	898530	433	101470	38
23	793035	266	804246	167	898789	433	101211	37
24	793195	265	804146	167	899049	432	100951	36
25	793354	265	804046	167	899308	432	100692	35
26	793514	265	803946	167	899568	432	100432	34
27	793673	265	803846	167	899827	432	100173	33
28	793832	265	803745	167	900086	432	999914	32
29	793991	265	803645	167	900346	432	999654	31
30	794150	264	803544	167	900605	432	999395	30
31	9794308	264	9893444	168	9900664	432	10090136	29
32	794467	264	803343	168	901124	432	998876	28
33	794626	264	803243	168	901383	432	998617	27
34	794784	264	803142	168	901642	432	998358	26
35	794942	264	803041	168	901901	432	998099	25
36	795101	264	802940	168	902160	432	997840	24
37	795259	264	802839	168	902419	432	997581	23
38	795417	263	802739	168	902679	432	997321	22
39	795575	263	802638	168	902938	432	997062	21
40	795733	263	802536	168	903197	431	996803	20
41	9795891	263	9892435	169	9903455	431	10096545	19
42	796049	263	802334	169	903714	431	996226	18
43	796206	263	802233	169	903973	431	996027	17
44	796364	262	802132	169	904232	431	995788	16
45	796521	262	802030	169	904491	431	995509	15
46	796679	262	801929	169	904750	431	995250	14
47	796836	262	801827	169	905008	431	994992	13
48	796993	262	801726	169	905267	431	994733	12
49	797150	261	801624	169	905526	431	994474	11
50	797307	261	801523	170	905784	431	994216	10
51	9797464	261	9891421	170	9906043	431	10093657	9
52	797621	261	801319	170	906302	431	993698	8
53	797777	261	801217	170	906560	431	993440	7
54	797934	261	801115	170	906819	431	993181	6
55	798091	261	801013	170	907077	431	992923	5
56	798247	261	800911	170	907336	431	992664	4
57	798403	260	800809	170	907594	431	992406	3
58	798560	260	800707	170	907852	431	992148	2
59	798716	260	800605	170	908111	430	991889	1
60	798873	260	800503	170	908369	430	991631	0
	Co-sine		Sine		Cotang.		Tang.	M.

SINES AND TANGENTS, (39 Degrees.)

223

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	0-798872	260	9-800503	170	0-908369	430	10-091631	60
1	799028	260	800400	171	908628	430	091372	59
2	799184	260	800298	171	908886	430	091114	58
3	799339	259	800195	171	909144	430	090856	57
4	799495	259	800093	171	909402	430	090598	56
5	799651	259	800090	171	909660	430	090340	55
6	799806	259	800088	171	909918	430	090082	54
7	799962	259	800085	171	910177	430	089823	53
8	800117	259	800082	171	910435	430	089565	52
9	800273	258	800079	171	910693	430	089307	51
10	800427	258	800077	171	910951	430	089049	50
11	9-800582	258	9-800374	172	9-911209	430	10-088791	49
12	800737	258	800271	172	911467	430	088533	48
13	800892	258	800168	172	911724	430	088275	47
14	801047	258	800064	172	911982	430	088018	46
15	801201	258	800061	172	912240	430	087760	45
16	801356	257	800058	172	912498	430	087502	44
17	801511	257	800055	172	912756	430	087244	43
18	801665	257	800051	172	913014	429	086986	42
19	801819	257	800048	172	913271	429	086729	41
20	801973	257	800044	172	913529	429	086471	40
21	9-802128	257	9-800341	173	9-913787	429	10-086213	39
22	802282	256	800237	173	914044	429	085956	38
23	802436	256	800134	173	914302	429	085698	37
24	802589	256	800030	173	914560	429	085440	36
25	802743	256	800026	173	914817	429	085183	35
26	802897	256	800022	173	915075	429	084925	34
27	803050	256	800018	173	915332	429	084668	33
28	803204	256	800014	173	915590	429	084410	32
29	803357	255	800010	173	915847	429	084153	31
30	803511	255	800006	174	916104	429	083896	30
31	9-803664	255	9-800302	174	9-916362	429	10-083638	29
32	803817	255	800198	174	916619	429	083381	28
33	803970	255	800093	174	916877	429	083123	27
34	804123	255	800089	174	917134	429	082866	26
35	804276	254	800085	174	917391	429	082609	25
36	804428	254	800080	174	917648	429	082352	24
37	804581	254	800076	174	917905	429	082095	23
38	804734	254	800071	174	918163	428	081837	22
39	804886	254	800066	174	918420	428	081580	21
40	805039	254	800062	175	918677	428	081323	20
41	9-805191	254	9-800257	175	9-918934	428	10-081066	19
42	805343	253	800152	175	919191	428	080809	18
43	805495	253	800047	175	919448	428	080552	17
44	805647	253	800042	175	919705	428	080295	16
45	805799	253	800037	175	919962	428	080038	15
46	805951	253	800032	175	920219	428	079781	14
47	806103	253	800027	175	920476	428	079524	13
48	806254	253	800022	175	920733	428	079267	12
49	806406	252	800016	175	920990	428	079010	11
50	806557	252	800011	176	921247	428	078753	10
51	9-806709	252	9-800205	176	9-921503	428	10-078497	9
52	806860	252	800100	176	921760	428	078240	8
53	807011	252	800094	176	922017	428	077983	7
54	807163	252	800089	176	922274	428	077726	6
55	807314	252	800083	176	922530	428	077470	5
56	807465	251	800077	176	922787	428	077213	4
57	807615	251	800072	176	923044	428	076956	3
58	807766	251	800066	176	923300	428	076700	2
59	807917	251	800060	176	923557	427	076443	1
60	808067	251	800054	177	923813	427	076187	0
	Cosine		Sine		Cotang.		Tang.	M.

50 Degree.

M.	Sine	D.	Coine	D.	Tang.	D.	Cotang.	M.
0	9-808067	251	9-884254	177	9-923813	427	10-076187	60
1	808218	251	884148	177	924070	427	075830	59
2	808368	251	884042	177	924327	427	075673	58
3	808519	250	883936	177	924583	427	075517	57
4	808669	250	883829	177	924840	427	075360	56
5	808819	250	883723	177	925096	427	075204	55
6	808969	250	883617	177	925352	427	075048	54
7	809119	250	883510	177	925609	427	074891	53
8	809269	250	883404	177	925865	427	074735	52
9	809419	249	883297	178	926122	427	074578	51
10	809569	249	883191	178	926378	427	074422	50
11	9-809718	249	9-883084	178	9-926634	427	10-073266	49
12	809868	249	882977	178	926890	427	073110	48
13	810017	249	882871	178	927147	427	072953	47
14	810167	249	882764	178	927403	427	072797	46
15	810316	248	882657	178	927659	427	072641	45
16	810465	248	882550	178	927915	427	072485	44
17	810614	248	882443	178	928171	427	072329	43
18	810763	248	882336	179	928427	427	072173	42
19	810912	248	882229	179	928683	427	072017	41
20	811061	248	882121	179	928940	427	071860	40
21	9-811210	248	9-882014	179	9-929196	427	10-070804	39
22	811358	247	881907	179	929452	427	070648	38
23	811507	247	881799	179	929708	427	070492	37
24	811655	247	881692	179	929964	426	070336	36
25	811804	247	881584	179	930220	426	069780	35
26	811852	247	881477	179	930475	426	069325	34
27	812000	247	881369	179	930731	426	068929	33
28	812148	247	881261	180	930987	426	068513	32
29	812296	246	881153	180	931243	426	068077	31
30	812444	246	881046	180	931499	426	067651	30
31	9-812592	246	9-880938	180	9-931755	426	10-066295	29
32	812740	246	880830	180	932010	426	067990	28
33	812888	246	880722	180	932266	426	067734	27
34	813135	246	880613	180	932522	426	067478	26
35	813283	246	880505	180	932778	426	067222	25
36	813430	245	880397	180	933033	426	066867	24
37	813578	245	880289	181	933289	426	066711	23
38	813725	245	880180	181	933545	426	066455	22
39	813872	245	880072	181	933800	426	066200	21
40	814019	245	879963	181	934056	426	065944	20
41	9-814166	245	9-879855	181	9-934311	426	10-065689	19
42	814313	245	879746	181	934567	426	065433	18
43	814460	244	879637	181	934823	426	065177	17
44	814607	244	879529	181	935078	426	064922	16
45	814753	244	879420	181	935333	426	064667	15
46	814900	244	879311	181	935589	426	064411	14
47	815046	244	879202	182	935844	426	064156	13
48	815193	244	879093	182	936100	426	063900	12
49	815339	244	878984	182	936355	426	063645	11
50	815485	243	878875	182	936610	426	063390	10
51	9-815631	243	9-878766	182	9-936866	425	10-063134	9
52	815778	243	878656	182	937121	425	062879	8
53	815924	243	878547	182	937376	425	062624	7
54	816069	243	878438	182	937632	425	062368	6
55	816215	243	878328	182	937887	425	062113	5
56	816361	243	878219	183	938142	425	061858	4
57	816507	242	878109	183	938398	425	061602	3
58	816652	242	877999	183	938653	425	061347	2
59	816798	242	877890	183	938908	425	061092	1
60	816943	242	877780	183	939163	425	060837	0

Coine

Sine

Cotang.

Tang.

M.

M.	Sine	D.	Secant	D.	Tang.	D.	Cotang.	M.
0	0-816943	242	0-877780	183	0-939163	425	10-060837	60
1	817088	242	877870	183	939418	425	060582	59
2	817233	242	877560	183	939673	425	060327	58
3	817379	242	877450	183	939928	425	060072	57
4	817524	241	877340	183	940183	425	059817	56
5	817668	241	877230	184	940438	425	059562	55
6	817813	241	877120	184	940694	425	059306	54
7	817958	241	877010	184	940949	425	059051	53
8	818103	241	876899	184	941204	425	058796	52
9	818247	241	876789	184	941458	425	058542	51
10	818392	241	876678	184	941714	425	058286	50
11	0-818536	240	0-876568	184	0-941968	425	10-058032	49
12	818681	240	876457	184	942223	425	057777	48
13	818825	240	876347	184	942478	425	057522	47
14	818969	240	876236	185	942733	425	057267	46
15	819113	240	876125	185	942988	425	057012	45
16	819257	240	876014	185	943243	425	056757	44
17	819401	240	875904	185	943498	425	056502	43
18	819545	239	875793	185	943752	425	056248	42
19	819689	239	875682	185	944007	425	055993	41
20	819832	239	875571	185	944262	425	055738	40
21	0-819976	239	0-875459	185	0-944517	425	10-055483	39
22	820120	239	875348	185	944771	424	055228	38
23	820263	239	875237	185	945026	424	054974	37
24	820406	239	875126	186	945281	424	054719	36
25	820550	238	875014	186	945535	424	054465	35
26	820693	238	874903	186	945790	424	054210	34
27	820836	238	874791	186	946045	424	053955	33
28	820979	238	874680	186	946300	424	053701	32
29	821122	238	874568	186	946554	424	053446	31
30	821265	238	874456	186	946808	424	053192	30
31	0-821407	238	0-874344	186	0-947063	424	10-052937	29
32	821550	238	874232	187	947318	424	052682	28
33	821693	237	874121	187	947572	424	052428	27
34	821835	237	874009	187	947826	424	052174	26
35	821977	237	873896	187	948081	424	051919	25
36	822120	237	873784	187	948336	424	051664	24
37	822262	237	873672	187	948590	424	051410	23
38	822404	237	873560	187	948844	424	051156	22
39	822546	237	873448	187	949099	424	050901	21
40	822688	236	873335	187	949353	424	050647	20
41	0-822830	236	0-873223	187	0-949607	424	10-050393	19
42	822972	236	873110	188	949862	424	050138	18
43	823114	236	872998	188	950116	424	049884	17
44	823255	236	872885	188	950370	424	049630	16
45	823397	236	872772	188	950625	424	049375	15
46	823539	236	872659	188	950879	424	049121	14
47	823680	235	872547	188	951133	424	048867	13
48	823821	235	872434	188	951388	424	048612	12
49	823963	235	872321	188	951642	424	048358	11
50	824104	235	872208	188	951896	424	048104	10
51	0-824245	235	0-872095	189	0-952150	424	10-047850	9
52	824386	235	871981	189	952405	424	047595	8
53	824527	235	871868	189	952659	424	047341	7
54	824668	234	871755	189	952913	424	047087	6
55	824808	234	871641	189	953167	423	046833	5
56	824949	234	871528	189	953421	423	046579	4
57	825090	234	871414	189	953675	423	046325	3
58	825230	234	871301	189	953929	423	046071	2
59	825371	234	871187	189	954183	423	045817	1
60	825511	234	871073	190	954437	423	045565	0

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-825511	234	9-871073	190	9-454437	423	10-045563	60
1	825651	233	870860	190	954601	423	045369	59
2	825791	233	870646	190	954845	423	045055	58
3	825931	233	870732	190	955000	423	044800	57
4	826071	233	870618	190	955454	423	044546	56
5	826211	233	870504	190	955707	423	044293	55
6	826351	233	870390	190	955961	423	044039	54
7	826491	233	870276	190	956215	423	043785	53
8	826631	233	870161	190	956469	423	043531	52
9	826770	232	870047	191	956723	423	043277	51
10	826910	232	869933	191	956977	423	043023	50
11	9-827049	228	9-869818	191	9-957231	423	10-042769	49
12	827189	229	869704	191	957485	423	042515	48
13	827328	229	869590	191	957739	423	042261	47
14	827467	229	869474	191	957993	423	042007	46
15	827606	229	869360	191	958246	423	041754	45
16	827745	229	869245	191	958500	423	041500	44
17	827884	231	869130	191	958754	423	041246	43
18	828023	231	869015	192	959008	423	040992	42
19	828162	231	868900	192	959262	423	040738	41
20	828301	231	868785	192	959516	423	040484	40
21	9-828439	231	9-868670	192	9-959769	423	10-040231	39
22	828578	231	868555	192	960023	423	039977	38
23	828716	231	868440	192	960277	423	039723	37
24	828855	230	868324	192	960531	423	039469	36
25	828993	230	868209	192	960784	423	039216	35
26	829131	230	868093	192	961038	423	038962	34
27	829269	230	867978	193	961291	423	038709	33
28	829407	230	867862	193	961545	423	038455	32
29	829545	230	867747	193	961799	423	038201	31
30	829683	230	867631	193	962052	423	037948	30
31	9-829821	229	9-867515	193	9-962306	423	10-037694	29
32	829959	229	867399	193	962560	423	037440	28
33	830097	229	867283	193	962813	423	037187	27
34	830234	229	867167	193	963067	423	036933	26
35	830372	229	867051	193	963320	423	036680	25
36	830509	229	866935	194	963574	423	036426	24
37	830646	229	866819	194	963827	423	036173	23
38	830784	229	866703	194	964081	423	035919	22
39	830921	228	866586	194	964335	423	035665	21
40	831058	228	866470	194	964588	423	035412	20
41	9-831195	228	9-866353	194	9-964842	423	10-035158	19
42	831332	228	866237	194	965095	423	034905	18
43	831469	228	866120	194	965349	423	034651	17
44	831606	228	866004	195	965602	423	034398	16
45	831742	228	865887	195	965855	423	034145	15
46	831879	228	865770	195	966109	423	033891	14
47	832015	227	865653	195	966362	423	033638	13
48	832152	227	865536	195	966616	423	033384	12
49	832288	227	865419	195	966869	423	033131	11
50	832425	227	865302	195	967123	423	032877	10
51	9-832561	227	9-865185	195	9-967376	423	10-032624	9
52	832697	227	865068	195	967630	423	032371	8
53	832833	227	864950	195	967883	423	032117	7
54	832969	226	864833	196	968136	423	031864	6
55	833105	226	864716	196	968389	423	031611	5
56	833241	226	864598	196	968643	423	031357	4
57	833377	226	864481	196	968896	423	031104	3
58	833512	226	864363	196	969149	423	030851	2
59	833648	226	864245	196	969403	423	030597	1
60	833783	226	864127	196	969656	423	030344	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M.
0	0-833783	226	0-864137	196	0-969656	422	10-030344	60
1	833919	225	864010	196	969909	422	030091	59
2	834054	225	863892	197	970162	422	029838	58
3	834189	225	863774	197	970416	422	029584	57
4	834325	225	863656	197	970669	422	029331	56
5	834460	225	863538	197	970922	422	029078	55
6	834595	225	863419	197	971175	422	028825	54
7	834730	225	863301	197	971429	422	028571	53
8	834865	225	863183	197	971682	422	028318	52
9	834999	224	863064	197	971935	422	028065	51
10	835134	224	862946	198	972188	422	027812	50
11	0-835269	224	0-862827	198	0-972441	422	10-027559	49
12	835403	224	862709	198	972694	422	027306	48
13	835538	224	862590	198	972948	422	027052	47
14	835672	224	862471	198	973201	422	026799	46
15	835807	224	862353	198	973454	422	026546	45
16	835941	224	862234	198	973707	422	026293	44
17	836075	223	862115	198	973960	422	026040	43
18	836209	223	861996	198	974213	422	025787	42
19	836343	223	861877	198	974466	422	025534	41
20	836477	223	861758	199	974719	422	025281	40
21	0-836611	223	0-861639	199	0-974972	422	10-025027	39
22	836745	223	861519	199	975226	422	024774	38
23	836878	223	861400	199	975479	422	024521	37
24	837012	222	861280	199	975732	422	024268	36
25	837146	222	861161	199	975985	422	024015	35
26	837279	222	861041	199	976238	422	023762	34
27	837412	222	860922	199	976491	422	023509	33
28	837546	222	860803	199	976744	422	023256	32
29	837679	222	860682	200	976997	422	023003	31
30	837812	222	860562	200	977250	422	022750	30
31	0-837945	222	0-860442	200	0-977503	422	10-022497	29
32	838078	221	860322	200	977756	422	022244	28
33	838211	221	860202	200	978009	422	021991	27
34	838344	221	860082	200	978262	422	021738	26
35	838477	221	859962	200	978515	422	021485	25
36	838610	221	859842	200	978768	422	021232	24
37	838742	221	859721	201	979021	422	020979	23
38	838875	221	859601	201	979274	422	020726	22
39	839007	221	859480	201	979527	422	020473	21
40	839140	220	859360	201	979780	422	020220	20
41	0-839272	220	0-859239	201	0-980033	422	10-019967	19
42	839404	220	859119	201	980286	422	019714	18
43	839536	220	858998	201	980538	422	019461	17
44	839668	220	858877	201	980791	421	019208	16
45	839800	220	858756	202	981044	421	018956	15
46	839932	220	858635	202	981297	421	018703	14
47	840064	219	858514	202	981550	421	018450	13
48	840196	219	858393	202	981803	421	018197	12
49	840328	219	858272	202	982056	421	017944	11
50	840459	219	858151	202	982309	421	017691	10
51	0-840591	219	0-858030	202	0-982562	421	10-017438	9
52	840723	219	857908	202	982814	421	017186	8
53	840854	219	857786	202	983067	421	016933	7
54	840985	219	857665	203	983320	421	016680	6
55	841116	218	857543	203	983573	421	016427	5
56	841247	218	857422	203	983826	421	016174	4
57	841378	218	857300	203	984079	421	015921	3
58	841509	218	857178	203	984331	421	015668	2
59	841640	218	857056	203	984584	421	015416	1
60	841771	218	856934	203	984837	421	015163	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D	Cosine	D.	Tang.	D.	Cotang.	M.
0	9-841771	218	9-855834	203	9-984837	421	10-015163	80
1	841902	218	855813	203	985090	421	014910	56
2	842033	218	855890	204	985343	421	014657	58
3	842163	217	855968	204	985596	421	014404	57
4	842294	217	856046	204	985848	421	014152	56
5	842424	217	856123	204	986101	421	013899	55
6	842555	217	856201	204	986354	421	013646	54
7	842685	217	856278	204	986607	421	013393	53
8	842815	217	856356	204	986860	421	013140	52
9	842946	217	856433	204	987112	421	012888	51
10	843076	217	856511	205	987365	421	012635	50
11	9-843206	216	9-855588	205	9-987618	421	10-012382	49
12	843336	216	855465	205	987871	421	012129	48
13	843466	216	855342	205	988123	421	011877	47
14	843595	216	855219	205	988376	421	011624	46
15	843725	216	855096	205	988629	421	011371	45
16	843855	216	854973	205	988882	421	011118	44
17	843984	216	854850	205	989134	421	010866	43
18	844114	215	854727	206	989387	421	010613	42
19	844243	215	854603	206	989640	421	010360	41
20	844372	215	854480	206	989893	421	010107	40
21	9-844502	215	9-854356	206	9-990145	421	10-009855	39
22	844631	215	854233	206	990398	421	009602	38
23	844760	215	854110	206	990651	421	009349	37
24	844899	215	853986	206	990903	421	009097	36
25	845018	215	853862	206	991156	421	008844	35
26	845147	215	853738	206	991409	421	008591	34
27	845276	214	853614	207	991662	421	008338	33
28	845405	214	853490	207	991914	421	008086	32
29	845533	214	853366	207	992167	421	007833	31
30	845662	214	853242	207	992420	421	007580	30
31	9-845790	214	9-853118	207	9-992672	421	10-007328	29
32	845919	214	852994	207	992925	421	007075	28
33	846047	214	852869	207	993178	421	006822	27
34	846175	214	852745	207	993430	421	006570	26
35	846304	214	852620	207	993683	421	006317	25
36	846432	213	852496	208	993936	421	006064	24
37	846560	213	852371	208	994189	421	005811	23
38	846688	213	852247	208	994441	421	005559	22
39	846816	213	852122	208	994694	421	005306	21
40	846944	213	851997	208	994947	421	005053	20
41	9-847071	213	9-851872	208	9-995199	421	10-004801	19
42	847199	213	851747	208	995452	421	004548	18
43	847327	213	851622	208	995705	421	004295	17
44	847454	212	851497	209	995957	421	004043	16
45	847582	212	851372	209	996210	421	003790	15
46	847709	212	851246	209	996463	421	003537	14
47	847836	212	851121	209	996715	421	003285	13
48	847964	212	850996	209	996968	421	003032	12
49	848091	212	850870	209	997221	421	002779	11
50	848218	212	850745	209	997473	421	002527	10
51	9-848345	212	9-850619	209	9-997726	421	10-002274	9
52	848472	211	850493	210	997979	421	002021	8
53	848599	211	850368	210	998231	421	001769	7
54	848726	211	850242	210	998484	421	001516	6
55	848852	211	850116	210	998737	421	001263	5
56	848979	211	849990	210	998990	421	001011	4
57	849106	211	849864	210	999242	421	000758	3
58	849232	211	849738	210	999495	421	000505	2
59	849359	211	849611	210	999748	421	000253	1
60	849485	211	849485	210	10-000000	421	000000	0

TABLE V.

NATURAL SINES AND COSINES.

TABLE XIV. NATURAL SINES AND COSINES.

M.	0°		1°		2°		3°		4°		M.
	Sine.	Co-in.	Sine	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99868	.06978	.99754	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99854	.07034	.99753	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99847	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99840	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99833	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99826	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99819	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03722	.99931	.05466	.99812	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99805	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99798	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99791	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99784	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99777	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99770	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99763	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99756	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99749	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99742	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99735	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99728	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05843	.99721	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05872	.99714	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04158	.99914	.05901	.99707	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04187	.99912	.05930	.99700	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04216	.99911	.05959	.99693	.07701	.99703	35
26	.00756	.99997	.02501	.99968	.04245	.99910	.05988	.99686	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04274	.99909	.06017	.99679	.07759	.99699	33
28	.00814	.99997	.02559	.99967	.04303	.99907	.06046	.99672	.07788	.99696	32
29	.00843	.99996	.02588	.99966	.04332	.99906	.06075	.99665	.07817	.99694	31
30	.00872	.99996	.02618	.99966	.04361	.99905	.06104	.99658	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04390	.99904	.06133	.99651	.07875	.99690	29
32	.00931	.99996	.02676	.99964	.04419	.99902	.06162	.99644	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04448	.99901	.06191	.99637	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04477	.99900	.06220	.99630	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04506	.99898	.06249	.99623	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04535	.99897	.06278	.99616	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04564	.99896	.06307	.99609	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04593	.99894	.06336	.99602	.08078	.99674	22
39	.01134	.99994	.02879	.99959	.04622	.99893	.06365	.99595	.08107	.99671	21
40	.01163	.99993	.02908	.99958	.04651	.99892	.06394	.99588	.08136	.99669	20
41	.01192	.99993	.02937	.99957	.04680	.99891	.06423	.99581	.08165	.99667	19
42	.01221	.99993	.02966	.99956	.04709	.99889	.06452	.99574	.08194	.99664	18
43	.01250	.99992	.02995	.99955	.04738	.99888	.06481	.99567	.08223	.99662	17
44	.01279	.99992	.03024	.99954	.04767	.99886	.06510	.99560	.08252	.99659	16
45	.01308	.99991	.03053	.99953	.04796	.99885	.06539	.99553	.08281	.99657	15
46	.01338	.99991	.03082	.99952	.04825	.99883	.06568	.99546	.08310	.99654	14
47	.01367	.99991	.03111	.99952	.04854	.99882	.06597	.99539	.08339	.99652	13
48	.01396	.99990	.03140	.99951	.04883	.99881	.06626	.99532	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04912	.99879	.06655	.99525	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04941	.99878	.06684	.99518	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04970	.99877	.06713	.99511	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05000	.99875	.06742	.99504	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05029	.99874	.06771	.99497	.08513	.99637	7
54	.01571	.99988	.03315	.99945	.05058	.99872	.06800	.99490	.08542	.99635	6
55	.01600	.99987	.03344	.99944	.05087	.99871	.06829	.99483	.08571	.99632	5
56	.01629	.99987	.03373	.99943	.05116	.99869	.06858	.99476	.08600	.99630	4
57	.01658	.99986	.03402	.99942	.05145	.99868	.06887	.99469	.08629	.99627	3
58	.01687	.99986	.03431	.99941	.05174	.99867	.06916	.99462	.08658	.99625	2
59	.01716	.99985	.03460	.99940	.05203	.99866	.06945	.99455	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05232	.99865	.06974	.99448	.08716	.99619	0

M. Sine. Co-in. Sine Cosin. Sine. Cosin. Sine. Cosin. Sine. Cosin. M.

84° 86° 87° 86° 85°

TABLE XIV. NATURAL SINES AND COSINES.

M.	5°		6°		7°		8°		9°		M.
	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Co-in.	
0	.08716	.99619	.10458	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10488	.99449	.12216	.99251	.13946	.99028	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14206	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15960	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98932	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14895	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14924	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14952	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14981	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15010	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15039	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15068	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16821	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99090	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	84°		83°		82°		81°		80°		

M.	10°		11°		12°		13°		14°		M.
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	.1736	.98451	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97080	90
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97073	89
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97066	88
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97059	87
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97052	86
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	85
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	84
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	83
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	82
9	.17622	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	81
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	80
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24502	.96952	79
12	.17708	.98421	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97736	.22863	.97351	.24559	.96937	47
14	.17765	.98410	.19481	.98084	.21189	.97729	.22892	.97344	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17881	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24701	.96902	42
19	.17910	.98384	.19623	.98056	.21332	.97699	.23033	.97311	.24729	.96894	41
20	.17939	.98378	.19652	.98050	.21360	.97693	.23062	.97304	.24757	.96887	40
21	.17968	.98373	.19680	.98044	.21388	.97687	.23090	.97298	.24785	.96880	39
22	.17997	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18026	.98363	.19737	.98033	.21445	.97674	.23146	.97284	.24841	.96866	37
24	.18055	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96859	36
25	.18084	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96852	35
26	.18113	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18142	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24953	.96837	33
28	.18171	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24981	.96830	32
29	.18200	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25009	.96822	31
30	.18229	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25037	.96815	30
31	.18258	.98320	.19965	.97987	.21673	.97623	.23373	.97230	.25065	.96807	29
32	.18287	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25093	.96800	28
33	.18316	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25121	.96792	27
34	.18345	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25149	.96785	26
35	.18374	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25177	.96778	25
36	.18403	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25205	.96771	24
37	.18432	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25233	.96764	23
38	.18461	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25261	.96756	22
39	.18490	.98277	.20193	.97940	.21899	.97573	.23600	.97176	.25289	.96749	21
40	.18519	.98272	.20222	.97934	.21928	.97566	.23628	.97169	.25317	.96742	20
41	.18548	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25345	.96734	19
42	.18577	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25373	.96727	18
43	.18606	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25401	.96719	17
44	.18635	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25429	.96712	16
45	.18664	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25457	.96705	15
46	.18693	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25485	.96697	14
47	.18722	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25513	.96690	13
48	.18751	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25541	.96682	12
49	.18780	.98223	.20478	.97881	.22183	.97508	.23881	.97106	.25569	.96675	11
50	.18809	.98218	.20507	.97875	.22212	.97502	.23909	.97100	.25597	.96667	10
51	.18838	.98213	.20535	.97869	.22240	.97496	.23938	.97093	.25625	.96660	9
52	.18867	.98207	.20564	.97863	.22268	.97489	.23966	.97086	.25653	.96653	8
53	.18896	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25681	.96645	7
54	.18925	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25709	.96638	6
55	.18954	.98191	.20649	.97845	.22353	.97470	.24051	.97065	.25737	.96630	5
56	.18983	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25765	.96623	4
57	.19012	.98179	.20706	.97833	.22410	.97457	.24107	.97051	.25793	.96615	3
58	.19041	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25821	.96608	2
59	.19070	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25849	.96600	1
60	.19099	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25877	.96593	0

79°

78°

77°

76°

75°

TABLE XIV. NATURAL SINES AND COSINES. 233

M.	15°		16°		17°		18°		19°		M.
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	25882	96599	27564	96126	29237	95680	30902	95106	32567	94652	60
1	25910	96585	27592	96118	29265	95692	30929	95097	32594	94642	59
2	25938	96578	27620	96110	29293	95681	30957	95088	32621	94633	58
3	25966	96570	27648	96102	29321	95665	30985	95079	32649	94623	57
4	25994	96562	27676	96094	29348	95656	31012	95070	32677	94614	56
5	26022	96555	27704	96086	29376	95648	31040	95061	32704	94604	55
6	26050	96547	27731	96078	29404	95639	31068	95052	32732	94595	54
7	26079	96540	27759	96070	29432	95631	31096	95043	32759	94585	53
8	26107	96532	27787	96062	29460	95622	31123	95034	32787	94576	52
9	26135	96524	27815	96054	29487	95614	31151	95024	32814	94566	51
10	26163	96517	27843	96046	29515	95605	31178	95015	32842	94557	50
11	26191	96509	27871	96037	29543	95596	31206	95006	32869	94547	49
12	26219	96502	27899	96029	29571	95588	31233	94997	32897	94538	48
13	26247	96494	27927	96021	29599	95579	31261	94988	32924	94528	47
14	26275	96486	27955	96013	29627	95571	31289	94979	32952	94518	46
15	26303	96479	27983	96005	29654	95562	31316	94970	32979	94509	45
16	26331	96471	28011	95997	29682	95554	31344	94961	33007	94500	44
17	26359	96463	28039	95989	29710	95545	31372	94952	33034	94490	43
18	26387	96456	28067	95981	29737	95537	31400	94943	33061	94480	42
19	26415	96448	28095	95972	29765	95528	31427	94934	33089	94470	41
20	26443	96440	28123	95964	29793	95519	31454	94924	33116	94460	40
21	26471	96433	28151	95956	29821	95510	31482	94915	33144	94451	39
22	26499	96425	28178	95948	29849	95501	31510	94906	33171	94442	38
23	26527	96417	28206	95940	29876	95493	31537	94897	33199	94432	37
24	26555	96410	28234	95931	29904	95484	31565	94888	33226	94422	36
25	26583	96402	28262	95923	29932	95475	31592	94879	33254	94413	35
26	26611	96394	28290	95915	29960	95467	31620	94870	33281	94403	34
27	26639	96386	28318	95907	29987	95458	31648	94861	33309	94393	33
28	26667	96379	28346	95898	30015	95449	31675	94851	33336	94384	32
29	26695	96371	28374	95890	30043	95440	31703	94842	33364	94374	31
30	26723	96363	28402	95882	30071	95432	31730	94832	33391	94364	30
31	26751	96355	28429	95874	30099	95423	31758	94823	33419	94354	29
32	26779	96347	28457	95865	30126	95414	31786	94814	33446	94345	28
33	26808	96340	28485	95857	30154	95405	31813	94805	33474	94335	27
34	26836	96332	28513	95849	30182	95397	31841	94796	33501	94325	26
35	26864	96324	28541	95841	30209	95388	31868	94787	33529	94315	25
36	26892	96316	28569	95832	30237	95379	31896	94777	33556	94305	24
37	26920	96308	28597	95824	30265	95370	31923	94768	33584	94295	23
38	26948	96301	28625	95816	30292	95361	31951	94759	33611	94285	22
39	26976	96293	28653	95807	30320	95352	31979	94749	33639	94275	21
40	27004	96285	28680	95799	30348	95343	32006	94740	33666	94265	20
41	27032	96277	28708	95791	30376	95334	32034	94730	33694	94255	19
42	27060	96269	28736	95782	30403	95325	32061	94721	33721	94245	18
43	27088	96261	28764	95774	30431	95316	32089	94711	33749	94235	17
44	27116	96253	28792	95766	30459	95307	32116	94702	33776	94225	16
45	27144	96245	28820	95757	30486	95298	32144	94693	33804	94215	15
46	27172	96238	28847	95749	30514	95289	32171	94684	33831	94205	14
47	27200	96230	28875	95740	30542	95280	32199	94674	33859	94195	13
48	27228	96222	28903	95732	30570	95271	32227	94665	33887	94185	12
49	27256	96214	28931	95724	30597	95262	32254	94656	33914	94175	11
50	27284	96206	28959	95715	30625	95253	32282	94646	33942	94165	10
51	27312	96198	28987	95707	30653	95244	32309	94637	33969	94155	9
52	27340	96190	29015	95698	30680	95235	32337	94627	33997	94145	8
53	27368	96182	29042	95690	30708	95226	32364	94618	34024	94135	7
54	27396	96174	29070	95681	30736	95217	32392	94608	34052	94125	6
55	27424	96166	29098	95672	30763	95208	32419	94599	34079	94115	5
56	27452	96158	29126	95664	30791	95199	32447	94590	34107	94105	4
57	27480	96150	29154	95655	30819	95190	32474	94580	34134	94095	3
58	27508	96142	29182	95647	30847	95181	32502	94571	34162	94085	2
59	27536	96134	29209	95639	30874	95172	32529	94561	34189	94075	1
60	27564	96126	29237	95630	30902	95163	32557	94552	34217	94065	0
M.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	M.

74°

73°

72°

71°

70°

TABLE XIV. NATURAL SINES AND COSINES.

M.	20°		21°		22°		23°		24°		M.
	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	
0	.34202	.93669	.35827	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93659	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93649	.35901	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93639	.35938	.93327	.37542	.92686	.39155	.92016	.40753	.91319	57
4	.34311	.93629	.35975	.93316	.37569	.92675	.39182	.92005	.40780	.91307	56
5	.34339	.93619	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93609	.36000	.93295	.37622	.92653	.39234	.91983	.40833	.91283	54
7	.34393	.93599	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93589	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93579	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93569	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93559	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93549	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93539	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93529	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93519	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93509	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93499	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93489	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93479	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93469	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93459	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93449	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93438	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93428	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93418	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93408	.36542	.93084	.38161	.92433	.39768	.91752	.41363	.91044	34
27	.34939	.93398	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93388	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93377	.36623	.93052	.38242	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93367	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93357	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93347	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93337	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93326	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93316	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93306	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93296	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93285	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93275	.36894	.92945	.38510	.92288	.40115	.91601	.41707	.90887	21
40	.35293	.93265	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93255	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93244	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93234	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93224	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93214	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93203	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93193	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93183	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93172	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93162	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93152	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93141	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93131	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93120	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93110	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93100	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93089	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93079	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93068	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93058	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0

M.	69°		68°		67°		66°		65°		M.
	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	
69	.35864	.93047	.37488	.92707	.39100	.92039	.40700	.91343	.42288	.90619	69
70	.35891	.93037	.37515	.92697	.39127	.92028	.40727	.91331	.42314	.90607	68
71	.35918	.93027	.37542	.92686	.39155	.92016	.40753	.91319	.42340	.90595	67
72	.35945	.93017	.37569	.92675	.39182	.92005	.40780	.91307	.42366	.90583	66
73	.35972	.93007	.37595	.92664	.39209	.91994	.40806	.91295	.42392	.90571	65
74	.36000	.92997	.37622	.92653	.39236	.91983	.40833	.91283	.42418	.90559	64
75	.36027	.92987	.37649	.92642	.39263	.91971	.40860	.91272	.42444	.90547	63
76	.36054	.92977	.37676	.92631	.39290	.91959	.40886	.91260	.42470	.90535	62
77	.36081	.92967	.37703	.92620	.39317	.91948	.40913	.91248	.42496	.90523	61
78	.36108	.92957	.37730	.92609	.39344	.91936	.40939	.91236	.42522	.90511	60
79	.36135	.92947	.37757	.92598	.39371	.91925	.40966	.91224	.42548	.90499	59
80	.36162	.92937	.37784	.92587	.39398	.91914	.40992	.91212	.42574	.90487	58
81	.36190	.92927	.37811	.92576	.39425	.91902	.41019	.91200	.42600	.90475	57
82	.36217	.92917	.37838	.92565	.39452	.91891	.41045	.91188	.42626	.90463	56
83	.36244	.92907	.37865	.92554	.39479	.91879	.41072	.91176	.42652	.90451	55
84	.36271	.92897	.37892	.92543	.39506	.91868	.41098	.91164	.42678	.90439	54
85	.36298	.92887	.37919	.92532	.39533	.91856	.41125	.91152	.42704	.90427	53
86	.36325	.92877	.37946	.92521	.39560	.91845	.41151	.91140	.42730	.90415	52
87	.36352	.92867	.37973	.92510	.39587	.91833	.41178	.91128	.42756	.90403	51
88	.36379	.92857	.37999	.92499	.39614	.91822	.41204	.91116	.42782	.90391	50
89	.36406	.92847	.38026	.92488	.39641	.91810	.41231	.91104	.42808	.90379	49
90	.36433	.92837	.38053	.92477	.39668	.91799	.41257	.91092	.42834	.90367	48
91	.36460	.92827	.38080	.92466	.39695	.91787	.41284	.91080	.42860	.90355	47
92	.36487	.92817	.38107	.92455	.39722	.91775	.41310	.91068	.42886	.90343	46
93	.36514	.92807	.38134	.92444	.39749	.91764	.41337	.91056	.42912	.90331	45
94	.36541	.92797	.38161	.92433	.39776	.91752	.41363	.91044	.42938	.90319	44
95	.36568	.92787	.38188	.92421	.39803	.91741	.41390	.91032	.42964	.90307	43
96	.36595	.92777	.38215	.92410	.39830	.91729	.41416	.91020	.42990	.90295	42
97	.36622	.92767	.38242	.92399	.39857	.91718	.41443	.91008	.43016	.90283	41
98	.36649	.92757	.38268	.92388	.39884	.91706	.41469	.90996	.43042	.90271	40
99	.36676	.92747	.38295	.92377	.39911	.91694	.41496	.90984	.43068	.90259	39
100	.36703	.92737	.38322	.92366	.39938	.91683	.41522	.90972	.43094	.90247	38
101	.36730	.92727	.38349	.92355	.39965	.91671	.41549	.90960	.43120	.90235	37
102	.36757	.92717	.38376	.92343	.39992	.91660	.41575	.90948	.43146	.90223	36
103	.36784	.92707	.38403	.92332	.40019	.91648	.41602	.90936	.43172	.90211	35
104	.36811	.92697	.38430	.92321	.40046	.91636	.41628	.90924	.43198	.90199	34
105	.36838	.92687	.38456	.92310	.40073	.91625	.41655	.90911	.43224	.90187	33
106	.36865	.92677	.38483	.92299	.40100	.91613	.41681	.90899	.43250	.90175	32
107	.36892	.92667	.38510	.92288	.40127	.91601	.41707	.90887	.43276	.90163	3

TABLE XIV. NATURAL SINES AND COSINES.

M.	25°		26°		27°		28°		29°		M.
	Sine.	Cos. In.	Sine.	Cos. In.	Sine.	Cos. In.	Sine.	Cos. In.	Sine.	Cos. In.	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88996	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88983	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88970	.47204	.88158	.48736	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88957	.47229	.88144	.48762	.87307	49
12	.42578	.90483	.44151	.89726	.45710	.88944	.47255	.88130	.48788	.87293	48
13	.42604	.90471	.44177	.89713	.45736	.88931	.47281	.88117	.48814	.87279	47
14	.42631	.90458	.44203	.89700	.45762	.88918	.47306	.88103	.48840	.87265	46
15	.42657	.90446	.44229	.89687	.45788	.88905	.47332	.88090	.48866	.87250	45
16	.42683	.90433	.44255	.89674	.45814	.88892	.47358	.88077	.48892	.87235	44
17	.42710	.90421	.44281	.89661	.45840	.88879	.47383	.88064	.48918	.87221	43
18	.42736	.90408	.44307	.89648	.45866	.88866	.47409	.88050	.48944	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88853	.47434	.88037	.48970	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88840	.47460	.88024	.48996	.87178	40
21	.42815	.90371	.44385	.89610	.45943	.88827	.47485	.88010	.49022	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88814	.47511	.88000	.49048	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88801	.47536	.87986	.49074	.87136	37
24	.42894	.90334	.44463	.89571	.46020	.88788	.47562	.87973	.49100	.87122	36
25	.42920	.90321	.44489	.89558	.46046	.88775	.47588	.87960	.49126	.87108	35
26	.42946	.90309	.44516	.89545	.46072	.88762	.47614	.87947	.49152	.87094	34
27	.42972	.90296	.44542	.89532	.46097	.88749	.47640	.87934	.49178	.87080	33
28	.42999	.90284	.44568	.89519	.46123	.88736	.47665	.87921	.49204	.87066	32
29	.43025	.90271	.44594	.89506	.46149	.88723	.47691	.87908	.49230	.87052	31
30	.43051	.90259	.44620	.89493	.46175	.88710	.47717	.87895	.49256	.87038	30
31	.43077	.90246	.44646	.89480	.46201	.88697	.47743	.87882	.49282	.87024	29
32	.43104	.90233	.44672	.89467	.46226	.88684	.47769	.87869	.49308	.87010	28
33	.43130	.90221	.44698	.89454	.46252	.88671	.47795	.87856	.49334	.86996	27
34	.43156	.90208	.44724	.89441	.46278	.88658	.47821	.87843	.49360	.86982	26
35	.43182	.90196	.44750	.89428	.46304	.88645	.47847	.87830	.49386	.86968	25
36	.43209	.90183	.44776	.89415	.46330	.88632	.47873	.87817	.49412	.86954	24
37	.43235	.90171	.44802	.89402	.46356	.88619	.47899	.87804	.49438	.86940	23
38	.43261	.90158	.44828	.89389	.46382	.88606	.47925	.87791	.49464	.86926	22
39	.43287	.90146	.44854	.89376	.46408	.88593	.47951	.87778	.49490	.86912	21
40	.43313	.90133	.44880	.89363	.46434	.88580	.47977	.87765	.49516	.86898	20
41	.43340	.90120	.44906	.89350	.46460	.88567	.48003	.87752	.49542	.86884	19
42	.43366	.90108	.44932	.89337	.46486	.88554	.48029	.87739	.49568	.86870	18
43	.43392	.90095	.44958	.89324	.46512	.88541	.48055	.87726	.49594	.86856	17
44	.43418	.90082	.44984	.89311	.46538	.88528	.48081	.87713	.49620	.86842	16
45	.43445	.90070	.45010	.89298	.46564	.88515	.48107	.87700	.49646	.86828	15
46	.43471	.90057	.45036	.89285	.46590	.88502	.48133	.87687	.49672	.86814	14
47	.43497	.90045	.45062	.89272	.46616	.88489	.48159	.87674	.49698	.86800	13
48	.43523	.90032	.45088	.89259	.46642	.88476	.48185	.87661	.49724	.86786	12
49	.43549	.90019	.45114	.89246	.46668	.88463	.48211	.87648	.49750	.86772	11
50	.43575	.90007	.45140	.89233	.46694	.88450	.48237	.87635	.49776	.86758	10
51	.43601	.89994	.45166	.89220	.46720	.88437	.48263	.87622	.49802	.86744	9
52	.43627	.89981	.45192	.89207	.46746	.88424	.48289	.87609	.49828	.86730	8
53	.43653	.89968	.45218	.89194	.46772	.88411	.48315	.87596	.49854	.86716	7
54	.43679	.89956	.45244	.89181	.46798	.88398	.48341	.87583	.49880	.86702	6
55	.43705	.89943	.45270	.89168	.46824	.88385	.48367	.87570	.49906	.86688	5
56	.43731	.89930	.45296	.89155	.46850	.88372	.48393	.87557	.49932	.86674	4
57	.43757	.89918	.45322	.89142	.46876	.88359	.48419	.87544	.49958	.86660	3
58	.43783	.89905	.45348	.89129	.46902	.88346	.48445	.87531	.49984	.86646	2
59	.43809	.89892	.45374	.89116	.46928	.88333	.48471	.87518	.50010	.86632	1
60	.43835	.89879	.45400	.89103	.46954	.88320	.48497	.87505	.50036	.86618	0
M.	Cos. In.		Cos. In.		Cos. In.		Cos. In.		Cos. In.		M.
	64°		63°		62°		61°		60°		

M.	30°		31°		32°		33°		34°		M.
	Sine.	Co. in.	Sine.	Co. in.	Sine.	Co. in.	Sine.	Co. in.	Sine.	Co. in.	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82823	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53385	.84557	.54853	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53410	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86311	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52027	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50577	.86266	.52076	.85370	.53558	.84448	.55023	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50653	.86222	.52151	.85325	.53631	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86149	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86134	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86044	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55629	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55653	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51353	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57239	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
M.	Co. in.	Sine.	Co. in.	Sine.	Co. in.	Sine.	Co. in.	Sine.	Co. in.	Sine.	M.
		55°		58°		57°		56°		55°	

TABLE XIV. NATURAL SINES AND COSINES.

M.	35°		36°		37°		38°		39°		M.
	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Co-in.	Sine.	Cosin.	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59575	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	54°		53°		52°		51°		50°		

M.	40°		41°		42°		43°		44°		M.
	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	
0	.64279	.76604	.65006	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65028	.75452	.66935	.74295	.68221	.73115	.69487	.71914	59
2	.64323	.76567	.65050	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65072	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65094	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65116	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65138	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65159	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65181	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65203	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65225	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65247	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76379	.65269	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65291	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65313	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65335	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65356	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65378	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.65400	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.65422	.75107	.67322	.73944	.68603	.72757	.69863	.71549	41
20	.64723	.76229	.65444	.75088	.67344	.73924	.68624	.72737	.69884	.71529	40
21	.64746	.76210	.65466	.75069	.67365	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76191	.65488	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.65510	.75030	.67408	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.65531	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.65553	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.65575	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.65597	.74953	.67495	.73787	.68772	.72597	.70029	.71387	33
28	.64900	.76078	.65618	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.65640	.74914	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.65662	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.65684	.74877	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.65706	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.65727	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.65749	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.65771	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.65793	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.65814	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.65836	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.65858	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.65880	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.65901	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.65923	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.65945	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.65966	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.65988	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66010	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66032	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66053	.74548	.67944	.73373	.69214	.72175	.70463	.70957	12
49	.65364	.75680	.66075	.74528	.67965	.73353	.69235	.72155	.70484	.70937	11
50	.65386	.75661	.66097	.74509	.67987	.73333	.69256	.72134	.70505	.70916	10
51	.65408	.75642	.66118	.74489	.68008	.73314	.69277	.72114	.70525	.70896	9
52	.65430	.75623	.66140	.74470	.68029	.73294	.69298	.72093	.70546	.70875	8
53	.65452	.75604	.66162	.74451	.68051	.73274	.69319	.72073	.70567	.70855	7
54	.65474	.75585	.66183	.74431	.68072	.73254	.69340	.72053	.70587	.70834	6
55	.65496	.75566	.66205	.74412	.68093	.73234	.69361	.72033	.70608	.70813	5
56	.65518	.75547	.66227	.74392	.68115	.73215	.69382	.72013	.70628	.70793	4
57	.65540	.75528	.66248	.74373	.68136	.73195	.69403	.71993	.70649	.70772	3
58	.65562	.75509	.66270	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66291	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66313	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.

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TABLE VI.

NATURAL TANGENTS AND COTANGENTS

240 TABLE XV. NATURAL TANGENTS AND COTANGENTS.

M.	0°		1°		2°		3°		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.00005	Infinite.	.01746	57.2900	.03492	29.6863	.05241	19.0611	60
1	.00029	8477.75	.01775	56.3506	.03512	29.3994	.05200	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03530	29.1664	.05159	18.8711	58
3	.00087	1145.42	.01833	54.5613	.03547	27.9372	.05118	18.7678	57
4	.00116	859.436	.01862	53.7086	.03563	27.7117	.05077	18.6656	56
5	.00145	687.549	.01891	52.8821	.03578	27.4899	.05036	18.5645	55
6	.00175	529.957	.01920	52.0807	.03592	27.2715	.04995	18.4645	54
7	.00204	401.106	.01949	51.3089	.03606	27.0566	.04954	18.3655	53
8	.00233	320.718	.01978	50.5485	.03620	26.8450	.04913	18.2677	52
9	.00262	281.971	.02007	49.8157	.03634	26.6367	.04872	18.1708	51
10	.00291	245.774	.02036	49.1089	.03648	26.4316	.04831	18.0750	50
11	.00320	212.521	.02066	48.4121	.03662	26.2296	.04790	17.9802	49
12	.00349	182.476	.02095	47.7395	.03676	26.0307	.04749	17.8865	48
13	.00378	154.441	.02124	47.0853	.03690	25.8348	.04708	17.7934	47
14	.00407	128.552	.02153	46.4489	.03704	25.6418	.04667	17.7015	46
15	.00436	104.192	.02182	45.8294	.03718	25.4517	.04626	17.6106	45
16	.00465	81.4658	.02211	45.2261	.03732	25.2644	.04585	17.5205	44
17	.00494	60.219	.02240	44.6386	.03746	25.0798	.04544	17.4314	43
18	.00523	40.934	.02269	44.0661	.03760	24.8978	.04503	17.3432	42
19	.00552	24.1932	.02298	43.5081	.03774	24.7185	.04462	17.2568	41
20	.00581	17.1885	.02327	42.9641	.03788	24.5418	.04421	17.1720	40
21	.00611	12.700	.02356	42.4335	.03802	24.3675	.04380	17.0887	39
22	.00640	9.259	.02385	41.9158	.03816	24.1957	.04339	17.0069	38
23	.00669	6.845	.02414	41.4106	.03830	24.0263	.04298	16.9265	37
24	.00698	4.457	.02443	40.9174	.03844	23.8593	.04257	16.8475	36
25	.00727	3.092	.02472	40.4358	.03858	23.6945	.04216	16.7698	35
26	.00756	2.259	.02501	39.9655	.03872	23.5318	.04175	16.6934	34
27	.00785	1.752	.02530	39.5059	.03886	23.3711	.04134	16.6182	33
28	.00814	1.274	.02559	39.0568	.03900	23.2123	.04093	16.5442	32
29	.00843	0.934	.02588	38.6177	.03914	23.0556	.04052	16.4712	31
30	.00872	0.639	.02617	38.1885	.03928	22.9008	.04011	16.3991	30
31	.00901	0.389	.02646	37.7686	.03942	22.7479	.03970	16.3279	29
32	.00930	0.249	.02675	37.3579	.03956	22.5960	.03929	16.2575	28
33	.00959	0.171	.02704	36.9560	.03970	22.4451	.03888	16.1879	27
34	.00988	0.107	.02733	36.5627	.03984	22.2951	.03847	16.1190	26
35	.01017	0.079	.02762	36.1776	.03998	22.1460	.03806	16.0508	25
36	.01046	0.059	.02791	35.8006	.04012	22.0000	.03765	15.9832	24
37	.01075	0.043	.02820	35.4313	.04026	21.8568	.03724	15.9162	23
38	.01104	0.032	.02849	35.0695	.04040	21.7164	.03683	15.8500	22
39	.01133	0.023	.02878	34.7151	.04054	21.5787	.03642	15.7845	21
40	.01162	0.016	.02907	34.3678	.04068	21.4437	.03601	15.7197	20
41	.01191	0.011	.02936	34.0273	.04082	21.3114	.03560	15.6556	19
42	.01220	0.007	.02965	33.6935	.04096	21.1817	.03519	15.5921	18
43	.01249	0.005	.02994	33.3662	.04110	21.0546	.03478	15.5292	17
44	.01278	0.003	.03023	33.0452	.04124	20.9300	.03437	15.4669	16
45	.01307	0.002	.03052	32.7303	.04138	20.8078	.03396	15.4052	15
46	.01336	0.001	.03081	32.4213	.04152	20.6880	.03355	15.3442	14
47	.01365	0.000	.03110	32.1181	.04166	20.5696	.03314	15.2838	13
48	.01394		.03139	31.8205	.04180	20.4526	.03273	15.2240	12
49	.01423		.03168	31.5284	.04194	20.3370	.03232	15.1648	11
50	.01452		.03197	31.2416	.04208	20.2228	.03191	15.1062	10
51	.01481		.03226	30.9599	.04222	20.1100	.03150	15.0482	9
52	.01510		.03255	30.6833	.04236	20.0000	.03109	14.9907	8
53	.01539		.03284	30.4116	.04250	19.8917	.03068	14.9337	7
54	.01568		.03313	30.1446	.04264	19.7850	.03027	14.8771	6
55	.01597		.03342	29.8822	.04278	19.6798	.02986	14.8210	5
56	.01626		.03371	29.6245	.04292	19.5761	.02945	14.7654	4
57	.01655		.03400	29.3711	.04306	19.4738	.02904	14.7102	3
58	.01684		.03429	29.1220	.04320	19.3729	.02863	14.6554	2
59	.01713		.03458	28.8771	.04334	19.2734	.02822	14.6010	1
60	.01742		.03487	28.6363	.04348	19.1753	.02781	14.5470	0

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TABLE XV. NATURAL TANGENTS AND COTANGENTS. 241

M.	4°		5°		6°		7°		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.06969	14.3007	.08749	11.4301	.10516	9.51436	.12979	8.14435	60
1	.07029	14.3111	.08778	11.3919	.10540	9.48771	.12908	8.13281	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12836	8.10886	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12767	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12701	8.06474	56
5	.07139	14.0179	.08895	11.2417	.10657	9.38307	.12636	8.04456	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12574	8.02548	54
7	.07197	13.8840	.08954	11.1681	.10716	9.33155	.12515	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30600	.12458	7.99058	52
9	.07256	13.7881	.09013	11.0954	.10775	9.28058	.12404	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12354	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12308	7.93438	49
12	.07344	13.6174	.09101	10.9883	.10863	9.20516	.12268	7.91582	48
13	.07373	13.5634	.09130	10.9532	.10893	9.18028	.12232	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12199	7.87895	46
15	.07431	13.4566	.09189	10.8839	.10952	9.13093	.12172	7.86064	45
16	.07461	13.4039	.09218	10.8493	.10981	9.10646	.12151	7.84244	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12131	7.82438	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12110	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12090	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11100	9.00983	.12070	7.77035	40
21	.07607	13.1461	.09364	10.6784	.11129	8.98598	.12050	7.75254	39
22	.07636	13.0953	.09394	10.6450	.11158	8.96227	.12030	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12010	7.71717	37
24	.07694	12.9968	.09453	10.5789	.11217	8.91520	.11990	7.69976	36
25	.07724	12.9489	.09482	10.5463	.11246	8.89185	.11970	7.68258	35
26	.07753	12.8981	.09511	10.5139	.11276	8.86862	.11950	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.11930	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.11910	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.11890	7.61297	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.11870	7.59675	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.11850	7.57872	29
32	.07929	12.6121	.09688	10.3224	.11453	8.73172	.11830	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11483	8.70931	.11810	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.11790	7.52806	26
35	.08017	12.4743	.09776	10.2294	.11541	8.66482	.11818	7.51132	25
36	.08046	12.4283	.09805	10.1988	.11570	8.64273	.11848	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.11877	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.11907	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.11937	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.11967	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.11997	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.12027	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.12057	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.12087	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.12109	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.12139	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.12169	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38635	.12199	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36585	.12229	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34546	.12259	7.26873	10
51	.08485	11.7853	.10246	9.76000	.12013	8.32446	.12289	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.12319	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.12349	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.12379	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24344	.12409	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.12439	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.12469	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.12499	7.14558	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.12529	7.13052	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.12559	7.11587	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.

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244 TABLE XV. NATURAL TANGENTS AND COTANGENTS.

M.	16°		17°		18°		19°		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.26975	3.48741	.30573	3.27085	.33493	3.07768	.34483	2.90421	60
1	.26706	3.48869	.30605	3.26745	.33524	3.07464	.34465	2.90147	59
2	.26738	3.47977	.30637	3.26406	.33556	3.07160	.34498	2.89873	58
3	.26769	3.47596	.30669	3.26067	.33588	3.06857	.34530	2.89600	57
4	.26800	3.47216	.30700	3.25729	.33621	3.06554	.34563	2.89327	56
5	.26832	3.46837	.30732	3.25392	.33653	3.06252	.34596	2.89055	55
6	.26864	3.46458	.30764	3.25055	.33685	3.05950	.34628	2.88783	54
7	.26896	3.46080	.30796	3.24719	.33717	3.05649	.34661	2.88511	53
8	.26927	3.45703	.30828	3.24383	.33749	3.05349	.34693	2.88239	52
9	.26958	3.45327	.30860	3.24049	.33782	3.05049	.34726	2.87970	51
10	.26990	3.44951	.30891	3.23714	.33814	3.04749	.34758	2.87700	50
11	.27021	3.44576	.30923	3.23381	.33846	3.04450	.34791	2.87430	49
12	.27053	3.44202	.30955	3.23048	.33878	3.04152	.34824	2.87161	48
13	.27084	3.43829	.30987	3.22715	.33911	3.03854	.34856	2.86892	47
14	.27116	3.43456	.31019	3.22384	.33943	3.03556	.34889	2.86624	46
15	.27147	3.43084	.31051	3.22053	.33975	3.03259	.34922	2.86356	45
16	.27179	3.42713	.31083	3.21722	.34007	3.02963	.34954	2.86089	44
17	.27210	3.42343	.31115	3.21392	.34040	3.02667	.34987	2.85822	43
18	.27242	3.41973	.31147	3.21063	.34073	3.02372	.35020	2.85555	42
19	.27274	3.41604	.31178	3.20734	.34104	3.02077	.35052	2.85289	41
20	.27305	3.41236	.31210	3.20406	.34136	3.01782	.35085	2.85023	40
21	.27337	3.40869	.31242	3.20079	.34169	3.01489	.35118	2.84756	39
22	.27368	3.40502	.31274	3.19752	.34201	3.01196	.35150	2.84489	38
23	.27400	3.40136	.31306	3.19426	.34233	3.00903	.35183	2.84223	37
24	.27432	3.39771	.31338	3.19100	.34266	3.00611	.35216	2.83956	36
25	.27463	3.39406	.31370	3.18775	.34298	3.00319	.35248	2.83690	35
26	.27495	3.39042	.31402	3.18451	.34330	3.00028	.35281	2.83424	34
27	.27526	3.38679	.31434	3.18127	.34363	2.99738	.35314	2.83158	33
28	.27558	3.38317	.31466	3.17804	.34395	2.99447	.35346	2.82894	32
29	.27590	3.37955	.31498	3.17481	.34427	2.99158	.35379	2.82628	31
30	.27621	3.37594	.31530	3.17159	.34460	2.98868	.35412	2.82361	30
31	.27653	3.37234	.31562	3.16838	.34492	2.98580	.35445	2.82100	29
32	.27685	3.36875	.31594	3.16517	.34524	2.98292	.35477	2.81840	28
33	.27716	3.36516	.31626	3.16197	.34557	2.98004	.35510	2.81581	27
34	.27748	3.36158	.31658	3.15877	.34589	2.97717	.35543	2.81322	26
35	.27780	3.35800	.31690	3.15558	.34621	2.97430	.35576	2.81063	25
36	.27811	3.35443	.31722	3.15240	.34654	2.97144	.35608	2.80803	24
37	.27843	3.35087	.31754	3.14923	.34686	2.96858	.35641	2.80544	23
38	.27875	3.34733	.31786	3.14605	.34718	2.96573	.35674	2.80285	22
39	.27906	3.34377	.31818	3.14288	.34751	2.96288	.35707	2.80026	21
40	.27938	3.34023	.31850	3.13973	.34783	2.96004	.35740	2.79768	20
41	.27970	3.33670	.31882	3.13658	.34816	2.95721	.35772	2.79510	19
42	.28001	3.33317	.31914	3.13344	.34848	2.95487	.35805	2.79251	18
43	.28033	3.32965	.31946	3.13031	.34881	2.95255	.35838	2.79003	17
44	.28065	3.32614	.31978	3.12718	.34913	2.95023	.35871	2.78764	16
45	.28097	3.32264	.32010	3.12400	.34945	2.94791	.35904	2.78525	15
46	.30128	3.31914	.32042	3.12087	.34978	2.94569	.35937	2.78286	14
47	.30160	3.31565	.32074	3.11775	.35010	2.94348	.35969	2.78048	13
48	.30192	3.31216	.32106	3.11464	.35043	2.94148	.36002	2.77811	12
49	.30224	3.30868	.32139	3.11153	.35075	2.93948	.36035	2.77575	11
50	.30255	3.30521	.32171	3.10842	.35108	2.93748	.36068	2.77340	10
51	.30287	3.30174	.32203	3.10532	.35140	2.93548	.36101	2.77104	9
52	.30319	3.29829	.32235	3.10223	.35173	2.93348	.36134	2.76869	8
53	.30351	3.29483	.32267	3.09914	.35205	2.93148	.36167	2.76633	7
54	.30383	3.29139	.32299	3.09606	.35238	2.92948	.36200	2.76397	6
55	.30414	3.28795	.32331	3.09298	.35270	2.92748	.36232	2.76161	5
56	.30446	3.28452	.32363	3.08991	.35303	2.92548	.36265	2.75924	4
57	.30478	3.28109	.32396	3.08685	.35335	2.92348	.36298	2.75688	3
58	.30509	3.27767	.32428	3.08379	.35368	2.92148	.36331	2.75451	2
59	.30541	3.27426	.32460	3.08073	.35400	2.91948	.36364	2.75214	1
60	.30573	3.27085	.32492	3.07768	.35433	2.91748	.36397	2.74978	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	73°		72°		71°		70°		

TABLE XV. NATURAL TANGENTS AND COTANGENTS. 245

30°		31°		32°		33°			
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.86397	2.74748	.86396	2.81500	.40408	2.47509	.49447	2.35886	60
1	.86490	2.74459	.86490	2.60289	.40436	2.47802	.49482	2.35899	59
2	.86483	2.74251	.86483	2.60357	.40470	2.47995	.49516	2.35905	58
3	.86496	2.74004	.86497	2.59881	.40504	2.48098	.49551	2.35915	57
4	.86529	2.73756	.86530	2.59606	.40538	2.48082	.49585	2.34925	56
5	.86562	2.73509	.86563	2.59499	.40572	2.46476	.49619	2.34936	55
6	.86595	2.73263	.86597	2.59156	.40606	2.46970	.49654	2.34447	54
7	.86628	2.73017	.86630	2.58963	.40640	2.46665	.49688	2.34958	53
8	.86661	2.72771	.86663	2.58708	.40674	2.46560	.49722	2.34069	52
9	.86694	2.72525	.86697	2.58484	.40707	2.46555	.49757	2.33681	51
10	.86727	2.72281	.86731	2.58261	.40741	2.45451	.49791	2.32693	50
11	.86760	2.72036	.86764	2.58038	.40775	2.45246	.49826	2.32505	49
12	.86793	2.71792	.86797	2.57815	.40809	2.45043	.49860	2.32317	48
13	.86826	2.71548	.86831	2.57592	.40843	2.44839	.49894	2.32130	47
14	.86859	2.71305	.86864	2.57371	.40877	2.44636	.49929	2.32943	46
15	.86892	2.71062	.86896	2.57150	.40911	2.44433	.49963	2.32756	45
16	.86925	2.70819	.86931	2.56928	.40945	2.44230	.49998	2.32570	44
17	.86958	2.70577	.86965	2.56707	.40979	2.44027	.50032	2.32383	43
18	.86991	2.70335	.86999	2.56487	.41013	2.43825	.50067	2.32197	42
19	.87024	2.70094	.87032	2.56266	.41047	2.43623	.50101	2.32011	41
20	.87057	2.69853	.87065	2.56046	.41081	2.43422	.50136	2.31826	40
21	.87090	2.69613	.87099	2.55827	.41115	2.43220	.50170	2.31641	39
22	.87123	2.69373	.87132	2.55608	.41149	2.43019	.50205	2.31456	38
23	.87157	2.69133	.87166	2.55389	.41183	2.42819	.50239	2.31271	37
24	.87190	2.68893	.87200	2.55170	.41217	2.42618	.50274	2.31086	36
25	.87223	2.68653	.87233	2.54952	.41251	2.42418	.50308	2.30902	35
26	.87256	2.68414	.87267	2.54734	.41285	2.42218	.50343	2.30718	34
27	.87289	2.68175	.87300	2.54516	.41319	2.42019	.50378	2.30534	33
28	.87322	2.67937	.87333	2.54299	.41353	2.41819	.50412	2.30351	32
29	.87355	2.67700	.87367	2.54082	.41387	2.41620	.50447	2.30167	31
30	.87388	2.67463	.87401	2.53865	.41421	2.41421	.50481	2.29984	30
31	.87421	2.67225	.87435	2.53648	.41455	2.41223	.50516	2.29801	29
32	.87455	2.66989	.87469	2.53432	.41490	2.41025	.50550	2.29619	28
33	.87488	2.66752	.87503	2.53217	.41524	2.40827	.50585	2.29437	27
34	.87521	2.66516	.87536	2.53001	.41558	2.40629	.50620	2.29254	26
35	.87554	2.66281	.87569	2.52786	.41592	2.40432	.50654	2.29073	25
36	.87588	2.66046	.87603	2.52571	.41626	2.40235	.50689	2.28891	24
37	.87621	2.65811	.87636	2.52356	.41660	2.40038	.50724	2.28710	23
38	.87654	2.65576	.87669	2.52142	.41694	2.39841	.50758	2.28528	22
39	.87687	2.65342	.87702	2.51929	.41728	2.39644	.50793	2.28346	21
40	.87720	2.65109	.87735	2.51715	.41763	2.39449	.50828	2.28167	20
41	.87754	2.64875	.87769	2.51502	.41797	2.39253	.50862	2.27987	19
42	.87787	2.64642	.87802	2.51289	.41831	2.39058	.50897	2.27809	18
43	.87820	2.64410	.87835	2.51076	.41865	2.38863	.50932	2.27626	17
44	.87853	2.64177	.87868	2.50864	.41899	2.38668	.50966	2.27447	16
45	.87887	2.63945	.87902	2.50652	.41933	2.38473	.44001	2.27267	15
46	.87920	2.63714	.87935	2.50440	.41968	2.38279	.44036	2.27088	14
47	.87953	2.63483	.87968	2.50229	.42002	2.38084	.44071	2.26909	13
48	.87986	2.63252	.87999	2.50018	.42036	2.37891	.44105	2.26730	12
49	.88020	2.63021	.40081	2.49807	.42070	2.37697	.44140	2.26552	11
50	.88053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.88086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.88120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.88153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.88186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.88220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.88253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.88286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.88320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.88353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.88386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.

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246 TABLE XV. NATURAL TANGENTS AND COTANGENTS.

M.	24°		25°		26°		27°		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.44528	2.24601	.46681	2.14451	.48773	2.05080	.50853	1.95381	60
1	.44558	2.24498	.46666	2.14298	.48809	2.04879	.50889	1.95180	59
2	.44588	2.24392	.46702	2.14125	.48845	2.04678	.50925	1.94979	58
3	.44617	2.24287	.46737	2.13963	.48881	2.04477	.50961	1.94778	57
4	.44647	2.23982	.46772	2.13801	.48917	2.04276	.50997	1.94577	56
5	.44677	2.2377	.46808	2.13639	.48953	2.04075	.51033	1.94376	55
6	.44707	2.23553	.46843	2.13477	.48989	2.03874	.51069	1.94175	54
7	.44737	2.23348	.46879	2.13315	.49025	2.03673	.51105	1.93974	53
8	.44767	2.23143	.46914	2.13154	.49061	2.03472	.51141	1.93773	52
9	.44797	2.22938	.46950	2.12992	.49097	2.03271	.51177	1.93572	51
10	.44827	2.22733	.46985	2.12831	.49133	2.03070	.51213	1.93371	50
11	.44857	2.22528	.47021	2.12671	.49169	2.02869	.51249	1.93170	49
12	.44887	2.22323	.47056	2.12511	.49205	2.02668	.51285	1.92969	48
13	.44917	2.22118	.47092	2.12350	.49241	2.02467	.51321	1.92768	47
14	.44947	2.21913	.47128	2.12190	.49277	2.02266	.51357	1.92567	46
15	.44977	2.21708	.47163	2.12030	.49313	2.02065	.51393	1.92366	45
16	.45007	2.21503	.47199	2.11871	.49349	2.01864	.51429	1.92165	44
17	.45037	2.21298	.47234	2.11711	.49385	2.01663	.51465	1.91964	43
18	.45067	2.21093	.47270	2.11552	.49421	2.01462	.51501	1.91763	42
19	.45097	2.20888	.47305	2.11392	.49457	2.01261	.51537	1.91562	41
20	.45127	2.20683	.47341	2.11233	.49493	2.01060	.51573	1.91361	40
21	.45157	2.20478	.47377	2.11075	.49529	2.00859	.51609	1.91160	39
22	.45187	2.20273	.47412	2.10916	.49565	2.00658	.51645	1.90959	38
23	.45217	2.20068	.47448	2.10758	.49601	2.00457	.51681	1.90758	37
24	.45247	2.19863	.47483	2.10600	.49637	2.00256	.51717	1.90557	36
25	.45277	2.19658	.47519	2.10442	.49673	2.00055	.51753	1.90356	35
26	.45307	2.19453	.47555	2.10284	.49709	1.99854	.51789	1.90155	34
27	.45337	2.19248	.47590	2.10126	.49745	1.99653	.51825	1.89954	33
28	.45367	2.19043	.47626	2.09968	.49781	1.99452	.51861	1.89753	32
29	.45397	2.18838	.47662	2.09811	.49817	1.99251	.51897	1.89552	31
30	.45427	2.18633	.47698	2.09654	.49853	1.99050	.51933	1.89351	30
31	.45457	2.18428	.47733	2.09498	.49889	1.98849	.51969	1.89150	29
32	.45487	2.18223	.47769	2.09341	.49925	1.98648	.52005	1.88949	28
33	.45517	2.18018	.47805	2.09184	.49961	1.98447	.52041	1.88748	27
34	.45547	2.17813	.47840	2.09028	.50004	1.98246	.52077	1.88547	26
35	.45577	2.17608	.47876	2.08872	.50040	1.98045	.52113	1.88346	25
36	.45607	2.17403	.47912	2.08716	.50076	1.97844	.52149	1.88145	24
37	.45637	2.17198	.47948	2.08560	.50112	1.97643	.52185	1.87944	23
38	.45667	2.16993	.47984	2.08405	.50148	1.97442	.52221	1.87743	22
39	.45697	2.16788	.48020	2.08250	.50184	1.97241	.52257	1.87542	21
40	.45727	2.16583	.48055	2.08094	.50220	1.97040	.52293	1.87341	20
41	.45757	2.16378	.48091	2.07939	.50256	1.96839	.52329	1.87140	19
42	.45787	2.16173	.48127	2.07783	.50292	1.96638	.52365	1.86939	18
43	.45817	2.15968	.48163	2.07628	.50328	1.96437	.52401	1.86738	17
44	.45847	2.15763	.48198	2.07472	.50364	1.96236	.52437	1.86537	16
45	.45877	2.15558	.48234	2.07317	.50400	1.96035	.52473	1.86336	15
46	.45907	2.15353	.48270	2.07161	.50436	1.95834	.52509	1.86135	14
47	.45937	2.15148	.48306	2.07004	.50472	1.95633	.52545	1.85934	13
48	.45967	2.14943	.48342	2.06848	.50508	1.95432	.52581	1.85733	12
49	.45997	2.14738	.48378	2.06692	.50544	1.95231	.52617	1.85532	11
50	.46027	2.14533	.48414	2.06537	.50580	1.95030	.52653	1.85331	10
51	.46057	2.14328	.48450	2.06381	.50616	1.94829	.52689	1.85130	9
52	.46087	2.14123	.48486	2.06226	.50652	1.94628	.52725	1.84929	8
53	.46117	2.13918	.48522	2.06070	.50688	1.94427	.52761	1.84728	7
54	.46147	2.13713	.48557	2.05915	.50724	1.94226	.52797	1.84527	6
55	.46177	2.13508	.48593	2.05759	.50760	1.94025	.52833	1.84326	5
56	.46207	2.13303	.48629	2.05604	.50796	1.93824	.52869	1.84125	4
57	.46237	2.13098	.48665	2.05448	.50832	1.93623	.52905	1.83924	3
58	.46267	2.12893	.48701	2.05293	.50868	1.93422	.52941	1.83723	2
59	.46297	2.12688	.48737	2.05137	.50904	1.93221	.52977	1.83522	1
60	.46327	2.12483	.48773	2.04982	.50940	1.93020	.53013	1.83321	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.

TABLE XV. NATURAL TANGENTS AND COTANGENTS. 247

M.	28°		29°		30°		31°		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.53171	1.89073	.55431	1.80405	.57735	1.73205	.60066	1.66428	60
1	.53208	1.87941	.55489	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55547	1.80158	.57813	1.72973	.60185	1.66209	58
3	.53283	1.87677	.55605	1.80034	.57851	1.72857	.60245	1.66100	57
4	.53320	1.87545	.55663	1.79911	.57890	1.72741	.60304	1.65990	56
5	.53358	1.87413	.55721	1.79787	.57929	1.72625	.60364	1.65881	55
6	.53395	1.87283	.55779	1.79665	.57968	1.72509	.60424	1.75773	54
7	.53432	1.87152	.55837	1.79543	.58007	1.72393	.60484	1.65663	53
8	.53470	1.87021	.55895	1.79421	.58046	1.72277	.60543	1.65554	52
9	.53507	1.86891	.55953	1.79299	.58085	1.72161	.60603	1.65445	51
10	.53545	1.86760	.56011	1.79177	.58124	1.72045	.60663	1.65337	50
11	.53582	1.86630	.56069	1.79055	.58162	1.71929	.60723	1.65228	49
12	.53620	1.86500	.56127	1.78933	.58201	1.71813	.60783	1.65120	48
13	.53657	1.86369	.56185	1.78811	.58240	1.71697	.60843	1.65011	47
14	.53695	1.86239	.56243	1.78689	.58279	1.71581	.60903	1.64903	46
15	.53732	1.86109	.56301	1.78567	.58318	1.71465	.60963	1.64795	45
16	.53770	1.85979	.56359	1.78445	.58357	1.71349	.61023	1.64687	44
17	.53807	1.85850	.56417	1.78323	.58396	1.71233	.61083	1.64579	43
18	.53844	1.85720	.56475	1.78201	.58435	1.71117	.61143	1.64471	42
19	.53882	1.85591	.56533	1.78079	.58474	1.71001	.61203	1.64363	41
20	.53920	1.85462	.56591	1.77957	.58513	1.70885	.61263	1.64255	40
21	.53957	1.85333	.56649	1.77835	.58552	1.70769	.61323	1.64147	39
22	.53995	1.85204	.56707	1.77713	.58591	1.70653	.61383	1.64040	38
23	.54032	1.85075	.56765	1.77591	.58630	1.70537	.61443	1.63932	37
24	.54070	1.84946	.56823	1.77469	.58669	1.70421	.61503	1.63824	36
25	.54107	1.84818	.56881	1.77347	.58708	1.70305	.61563	1.63716	35
26	.54145	1.84689	.56939	1.77225	.58747	1.70189	.61623	1.63608	34
27	.54183	1.84561	.56997	1.77103	.58786	1.70073	.61683	1.63500	33
28	.54220	1.84433	.57055	1.76981	.58825	1.69957	.61743	1.63392	32
29	.54258	1.84305	.57113	1.76859	.58864	1.69841	.61803	1.63284	31
30	.54296	1.84177	.57171	1.76737	.58903	1.69725	.61863	1.63176	30
31	.54333	1.84049	.57229	1.76615	.58942	1.69609	.61923	1.63068	29
32	.54371	1.83922	.57287	1.76493	.58981	1.69493	.61983	1.62960	28
33	.54409	1.83794	.57345	1.76371	.59020	1.69377	.62043	1.62852	27
34	.54446	1.83667	.57403	1.76249	.59059	1.69261	.62103	1.62744	26
35	.54484	1.83540	.57461	1.76127	.59098	1.69145	.62163	1.62636	25
36	.54522	1.83413	.57519	1.76005	.59137	1.69029	.62223	1.62528	24
37	.54560	1.83286	.57577	1.75883	.59176	1.68913	.62283	1.62420	23
38	.54597	1.83159	.57635	1.75761	.59215	1.68797	.62343	1.62312	22
39	.54635	1.83033	.57693	1.75639	.59254	1.68681	.62403	1.62204	21
40	.54673	1.82906	.57751	1.75517	.59293	1.68565	.62463	1.62096	20
41	.54711	1.82780	.57809	1.75395	.59332	1.68449	.62523	1.61988	19
42	.54748	1.82654	.57867	1.75273	.59371	1.68333	.62583	1.61880	18
43	.54786	1.82528	.57925	1.75151	.59410	1.68217	.62643	1.61772	17
44	.54824	1.82402	.57983	1.75029	.59449	1.68101	.62703	1.61664	16
45	.54862	1.82276	.58041	1.74907	.59488	1.67985	.62763	1.61556	15
46	.54900	1.82150	.58099	1.74785	.59527	1.67869	.62823	1.61448	14
47	.54938	1.82025	.58157	1.74663	.59566	1.67753	.62883	1.61340	13
48	.54975	1.81899	.58215	1.74541	.59605	1.67637	.62943	1.61232	12
49	.55013	1.81774	.58273	1.74419	.59644	1.67521	.63003	1.61124	11
50	.55051	1.81649	.58331	1.74297	.59683	1.67405	.63063	1.61016	10
51	.55089	1.81524	.58389	1.74175	.59722	1.67289	.63123	1.60908	9
52	.55127	1.81399	.58447	1.74053	.59761	1.67173	.63183	1.60800	8
53	.55165	1.81274	.58505	1.73931	.59800	1.67057	.63243	1.60692	7
54	.55203	1.81150	.58563	1.73809	.59839	1.66941	.63303	1.60584	6
55	.55241	1.81025	.58621	1.73687	.59878	1.66825	.63363	1.60476	5
56	.55279	1.80901	.58679	1.73565	.59917	1.66709	.63423	1.60368	4
57	.55317	1.80777	.58737	1.73443	.59956	1.66593	.63483	1.60260	3
58	.55355	1.80653	.58795	1.73321	.59995	1.66477	.63543	1.60152	2
59	.55393	1.80529	.58853	1.73199	.60034	1.66361	.63603	1.60044	1
60	.55431	1.80405	.58911	1.73077	.60073	1.66245	.63663	1.60000	0

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248 TABLE XV. NATURAL TANGENTS AND COTANGENTS.

32°		33°		34°		35°			
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.62487	1.60483	.64941	1.53985	.67451	1.48256	.70121	1.42815	60
1	.62527	1.59990	.64982	1.53986	.67492	1.48168	.70064	1.42726	59
2	.62568	1.59520	.65024	1.53987	.67533	1.48079	.70007	1.42638	58
3	.62609	1.59073	.65066	1.53988	.67574	1.47977	.70151	1.42550	57
4	.62649	1.58620	.65108	1.53989	.67615	1.47885	.70194	1.42462	56
5	.62689	1.59117	.65149	1.53990	.67656	1.47792	.70238	1.42374	55
6	.62730	1.59114	.65189	1.53991	.67697	1.47699	.70282	1.42286	54
7	.62770	1.59111	.65231	1.53992	.67738	1.47607	.70325	1.42198	53
8	.62811	1.59206	.65272	1.53993	.67779	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53994	.67820	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53995	.67861	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.53996	.67902	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.53997	.67943	1.47146	.70542	1.41759	48
13	.63014	1.58493	.65480	1.53998	.68004	1.47054	.70586	1.41672	47
14	.63055	1.58593	.65521	1.53999	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.53999	.68086	1.46870	.70673	1.41497	45
16	.63136	1.58998	.65604	1.53999	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58896	.65646	1.53999	.68172	1.46686	.70760	1.41322	43
18	.63217	1.58814	.65688	1.53999	.68215	1.46595	.70804	1.41235	42
19	.63258	1.59093	.65729	1.53999	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.53999	.68301	1.46411	.70892	1.41061	40
21	.63340	1.57879	.65813	1.53999	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.53999	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.53999	.68429	1.46137	.71022	1.40800	37
24	.63462	1.57575	.65938	1.53999	.68471	1.46046	.71066	1.40713	36
25	.63503	1.57474	.65980	1.53999	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.53999	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.53999	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.53999	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.53999	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56968	.66189	1.53999	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56968	.66230	1.53988	.68771	1.45410	.71372	1.40109	29
32	.63789	1.56767	.66272	1.53988	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.53988	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.53988	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.53988	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.53988	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56266	.66482	1.53988	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.53988	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.53988	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.53988	.69157	1.44598	.71769	1.39335	20
41	.64158	1.55866	.66650	1.53988	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.53988	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49649	.69286	1.44328	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49861	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49866	.69415	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69458	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49008	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72343	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72387	1.38144	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48443	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37723	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.

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TABLE XV. NATURAL TANGENTS AND COTANGENTS. 249

M.	36°		37°		38°		39°		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.73954	1.37638	.75855	1.32704	.78120	1.27994	.80978	1.23490	60
1	.73989	1.37554	.75401	1.32834	.78175	1.27917	.81037	1.23416	59
2	.74023	1.37470	.75447	1.32864	.78229	1.27841	.81097	1.23343	58
3	.74058	1.37386	.75493	1.32894	.78283	1.27764	.81158	1.23270	57
4	.74092	1.37302	.75538	1.32924	.78337	1.27688	.81217	1.23196	56
5	.74127	1.37218	.75584	1.32954	.78391	1.27611	.81278	1.23123	55
6	.74161	1.37134	.75629	1.32984	.78445	1.27535	.81338	1.23050	54
7	.74196	1.37050	.75675	1.33014	.78499	1.27458	.81399	1.22977	53
8	.74230	1.36966	.75721	1.33044	.78553	1.27382	.81459	1.22904	52
9	.74265	1.36882	.75767	1.33074	.78607	1.27305	.81519	1.22831	51
10	.74299	1.36798	.75813	1.33104	.78661	1.27229	.81579	1.22758	50
11	.74334	1.36714	.75858	1.33134	.78715	1.27152	.81639	1.22685	49
12	.74368	1.36630	.75904	1.33164	.78769	1.27077	.81699	1.22612	48
13	.74403	1.36546	.75950	1.33194	.78823	1.27000	.81759	1.22539	47
14	.74437	1.36462	.75996	1.33224	.78877	1.26924	.81819	1.22467	46
15	.74472	1.36378	.76042	1.33254	.78931	1.26848	.81879	1.22394	45
16	.74506	1.36294	.76088	1.33284	.78985	1.26771	.81939	1.22321	44
17	.74541	1.36210	.76134	1.33314	.79039	1.26695	.81999	1.22249	43
18	.74575	1.36126	.76180	1.33344	.79093	1.26619	.82059	1.22176	42
19	.74610	1.36042	.76226	1.33374	.79147	1.26543	.82119	1.22104	41
20	.74644	1.35958	.76272	1.33404	.79201	1.26467	.82179	1.22031	40
21	.74679	1.35874	.76318	1.33434	.79255	1.26391	.82239	1.21959	39
22	.74713	1.35790	.76364	1.33464	.79309	1.26315	.82299	1.21886	38
23	.74748	1.35706	.76410	1.33494	.79363	1.26239	.82359	1.21814	37
24	.74782	1.35622	.76456	1.33524	.79417	1.26163	.82419	1.21742	36
25	.74817	1.35538	.76502	1.33554	.79471	1.26087	.82479	1.21670	35
26	.74851	1.35454	.76548	1.33584	.79525	1.26011	.82539	1.21598	34
27	.74886	1.35370	.76594	1.33614	.79579	1.25935	.82599	1.21526	33
28	.74920	1.35286	.76640	1.33644	.79633	1.25859	.82659	1.21454	32
29	.74955	1.35202	.76686	1.33674	.79687	1.25783	.82719	1.21382	31
30	.74989	1.35118	.76732	1.33704	.79741	1.25707	.82779	1.21310	30
31	.75024	1.35034	.76778	1.33734	.79795	1.25631	.82839	1.21238	29
32	.75058	1.34950	.76824	1.33764	.79849	1.25555	.82899	1.21166	28
33	.75093	1.34866	.76870	1.33794	.79903	1.25479	.82959	1.21094	27
34	.75127	1.34782	.76916	1.33824	.79957	1.25403	.83019	1.21022	26
35	.75162	1.34698	.76962	1.33854	.80011	1.25327	.83079	1.20950	25
36	.75196	1.34614	.77008	1.33884	.80065	1.25251	.83139	1.20878	24
37	.75231	1.34530	.77054	1.33914	.80119	1.25175	.83199	1.20806	23
38	.75265	1.34446	.77100	1.33944	.80173	1.25100	.83259	1.20734	22
39	.75300	1.34362	.77146	1.33974	.80227	1.25024	.83319	1.20662	21
40	.75334	1.34278	.77192	1.34004	.80281	1.24948	.83379	1.20590	20
41	.75369	1.34194	.77238	1.34034	.80335	1.24872	.83439	1.20518	19
42	.75403	1.34110	.77284	1.34064	.80389	1.24797	.83499	1.20446	18
43	.75438	1.34026	.77330	1.34094	.80443	1.24721	.83559	1.20374	17
44	.75472	1.33942	.77376	1.34124	.80497	1.24645	.83619	1.20302	16
45	.75507	1.33858	.77422	1.34154	.80551	1.24570	.83679	1.20230	15
46	.75541	1.33774	.77468	1.34184	.80605	1.24494	.83739	1.20158	14
47	.75576	1.33690	.77514	1.34214	.80659	1.24418	.83799	1.20086	13
48	.75610	1.33606	.77560	1.34244	.80713	1.24343	.83859	1.20014	12
49	.75645	1.33522	.77606	1.34274	.80767	1.24267	.83919	1.19942	11
50	.75679	1.33438	.77652	1.34304	.80821	1.24191	.83979	1.19870	10
51	.75714	1.33354	.77698	1.34334	.80875	1.24115	.84039	1.19798	9
52	.75748	1.33270	.77744	1.34364	.80929	1.24040	.84099	1.19726	8
53	.75783	1.33186	.77790	1.34394	.80983	1.23964	.84159	1.19654	7
54	.75817	1.33102	.77836	1.34424	.81037	1.23888	.84219	1.19582	6
55	.75852	1.33018	.77882	1.34454	.81091	1.23813	.84279	1.19510	5
56	.75886	1.32934	.77928	1.34484	.81145	1.23737	.84339	1.19438	4
57	.75921	1.32850	.77974	1.34514	.81199	1.23661	.84399	1.19366	3
58	.75955	1.32766	.78020	1.34544	.81253	1.23585	.84459	1.19294	2
59	.75990	1.32682	.78066	1.34574	.81307	1.23510	.84519	1.19222	1
60	.76024	1.32598	.78112	1.34604	.81361	1.23434	.84579	1.19150	0
	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	58°		52°		51°		50°		

250 TABLE XV. NATURAL TANGENTS AND COTANGENTS.

M.	40°		41°		42°		43°		M.
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.83910	1.1915	.86929	1.15087	.90040	1.11061	.93252	1.07297	60
1	.83940	1.19105	.86980	1.14989	.90063	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14903	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14818	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14737	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14659	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14583	.90357	1.10672	.93578	1.06863	54
7	.84258	1.18684	.87287	1.14505	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14429	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14350	.90516	1.10478	.93743	1.06676	51
10	.84407	1.18474	.87441	1.14273	.90569	1.10414	.93797	1.06615	50
11	.84457	1.18404	.87493	1.14196	.90621	1.10349	.93852	1.06553	49
12	.84507	1.18334	.87545	1.14122	.90674	1.10285	.93906	1.06492	48
13	.84556	1.18264	.87595	1.14043	.90727	1.10221	.93961	1.06432	47
14	.84606	1.18194	.87646	1.14005	.90781	1.10156	.94016	1.06375	46
15	.84656	1.18125	.87698	1.14008	.90834	1.10091	.94071	1.06318	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06264	44
17	.84756	1.17986	.87801	1.13914	.90940	1.09963	.94180	1.06210	43
18	.84806	1.17916	.87853	1.13868	.90993	1.09899	.94235	1.06157	42
19	.84856	1.17846	.87904	1.13821	.91046	1.09834	.94290	1.06106	41
20	.84906	1.17777	.87955	1.13784	.91100	1.09770	.94345	1.06054	40
21	.84956	1.17708	.88007	1.13737	.91153	1.09706	.94400	1.06002	39
22	.85006	1.17638	.88059	1.13691	.91206	1.09642	.94455	1.05950	38
23	.85057	1.17569	.88111	1.13644	.91259	1.09578	.94510	1.05899	37
24	.85107	1.17500	.88163	1.13598	.91312	1.09514	.94565	1.05847	36
25	.85157	1.17430	.88214	1.13551	.91365	1.09450	.94620	1.05795	35
26	.85207	1.17361	.88265	1.13505	.91418	1.09386	.94675	1.05744	34
27	.85257	1.17292	.88317	1.13458	.91471	1.09322	.94731	1.05693	33
28	.85306	1.17223	.88369	1.13412	.91524	1.09258	.94786	1.05643	32
29	.85356	1.17154	.88421	1.13366	.91578	1.09195	.94841	1.05593	31
30	.85406	1.17085	.88473	1.13320	.91631	1.09131	.94896	1.05543	30
31	.85456	1.17016	.88524	1.13273	.91684	1.09067	.94952	1.05493	29
32	.85507	1.16947	.88576	1.13227	.91740	1.09003	.95007	1.05443	28
33	.85557	1.16878	.88628	1.13181	.91794	1.08940	.95063	1.05394	27
34	.85609	1.16809	.88680	1.13135	.91847	1.08876	.95118	1.05345	26
35	.85660	1.16741	.88732	1.13089	.91901	1.08813	.95173	1.05295	25
36	.85710	1.16672	.88784	1.13043	.91955	1.08749	.95229	1.05246	24
37	.85761	1.16603	.88836	1.12997	.92008	1.08686	.95284	1.05197	23
38	.85811	1.16535	.88888	1.12951	.92062	1.08622	.95340	1.05148	22
39	.85862	1.16466	.88940	1.12905	.92116	1.08559	.95395	1.05099	21
40	.85912	1.16398	.88992	1.12859	.92170	1.08496	.95451	1.05050	20
41	.85963	1.16330	.89045	1.12813	.92224	1.08432	.95506	1.04999	19
42	.86014	1.16261	.89097	1.12767	.92277	1.08369	.95562	1.04948	18
43	.86064	1.16192	.89149	1.12721	.92331	1.08306	.95618	1.04897	17
44	.86115	1.16124	.89201	1.12675	.92385	1.08243	.95673	1.04846	16
45	.86166	1.16056	.89253	1.12629	.92439	1.08179	.95729	1.04795	15
46	.86216	1.15987	.89306	1.12583	.92493	1.08116	.95785	1.04744	14
47	.86267	1.15919	.89358	1.12537	.92547	1.08053	.95841	1.04693	13
48	.86318	1.15851	.89411	1.12491	.92601	1.07990	.95897	1.04642	12
49	.86368	1.15783	.89463	1.12445	.92655	1.07927	.95953	1.04591	11
50	.86419	1.15715	.89515	1.12399	.92709	1.07864	.96008	1.04540	10
51	.86470	1.15647	.89567	1.12353	.92763	1.07801	.96064	1.04489	9
52	.86521	1.15579	.89620	1.12307	.92817	1.07738	.96120	1.04438	8
53	.86572	1.15511	.89672	1.12261	.92871	1.07675	.96176	1.04387	7
54	.86623	1.15443	.89725	1.12215	.92925	1.07612	.96232	1.04336	6
55	.86674	1.15375	.89777	1.12169	.92980	1.07549	.96288	1.04285	5
56	.86725	1.15308	.89830	1.12123	.93034	1.07487	.96344	1.04234	4
57	.86776	1.15240	.89883	1.12077	.93088	1.07425	.96400	1.04183	3
58	.86827	1.15172	.89936	1.12031	.93143	1.07362	.96457	1.04132	2
59	.86878	1.15104	.89988	1.11985	.93197	1.07299	.96513	1.04081	1
60	.86929	1.15037	.90041	1.11939	.93252	1.07237	.96569	1.04030	0

TABLE XV. NATURAL TANGENTS AND COTANGENTS. 251

44°				44°				44°			
M.	Tang.	Cotang.	M.	M.	Tang.	Cotang.	M.	M.	Tang.	Cotang.	M.
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01170	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03373	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02712	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02652	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02592	44	36	.98613	1.01406	24	56	.99768	1.00232	4
17	.97529	1.02532	43	37	.98671	1.01347	23	57	.99826	1.00173	3
18	.97586	1.02472	42	38	.98728	1.01288	22	58	.99884	1.00115	2
19	.97643	1.02412	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02352	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
M.	Cotang.	Tang.	M.	M.	Cotang.	Tang.	M.	M.	Cotang.	Tang.	M.
45°				45°				45°			

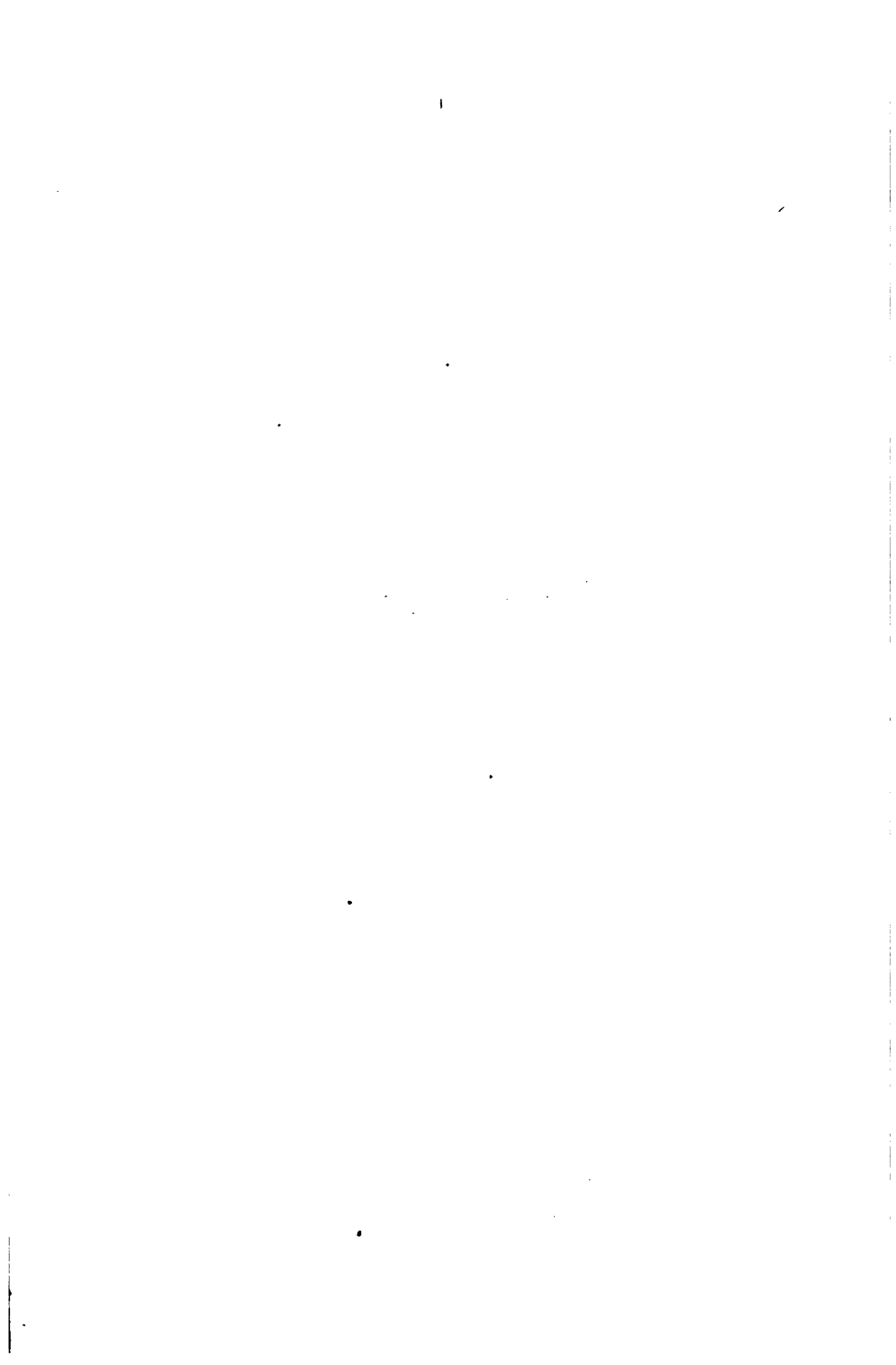


TABLE VII.

TRAVERSE TABLE.

TRAVERSE TABLE.

Course	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		Course		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.			
0	15	1.0000	0.0044	2.0000	0.0087	3.0000	0.0131	4.0000	0.0175	5.0000	0.0218	89	45
	30	0000	0087	1.9999	0175	2.9999	0262	3.9998	0349	4.9998	0436	80	30
	45	09999	0131	9998	0262	9997	0393	9997	0524	9996	0654	15	15
1	0	9998	0175	9997	0349	9995	0524	9994	0698	9992	0873	80	0
	15	9997	0218	9995	0436	9993	0654	9990	0873	9988	1091	45	45
	30	9997	0262	9993	0524	9990	0785	9986	1047	9983	1309	30	30
	45	9995	0315	9991	0611	9886	0916	9981	1222	9977	1527	15	15
2	0	9994	0349	9988	0698	9982	1047	9976	1396	9970	1745	80	0
	15	9992	0393	9985	0785	9977	1178	9969	1570	9961	1968	45	45
	30	9990	0436	9981	0872	9971	1309	9962	1745	9952	2181	30	30
	45	9988	0490	1.9977	1.0960	2.9965	0.1499	3.9951	0.1919	4.9942	0.2399	15	15
3	0	9986	0523	9973	1047	9959	1570	9945	2098	9931	2617	80	0
	15	9984	0567	9968	1194	9952	1701	9936	2298	9920	2895	45	45
	30	9981	0610	9963	1281	9944	1931	9925	2442	9907	3092	30	30
	45	9979	0654	9957	1308	9936	1962	9914	2616	9903	3270	15	15
4	0	9976	0698	9951	1395	9927	2093	9903	2790	9878	3488	80	0
	15	9973	0741	9945	1482	9918	2228	9888	2964	9863	3705	45	45
	30	9969	0785	9938	1569	9908	2354	9877	3188	9846	3923	30	30
	45	9966	0828	9931	1656	9897	2484	9863	3312	9828	4140	15	15
5	0	9962	0872	9924	1743	9886	2615	9848	3486	9810	4358	80	0
	15	9958	0915	1.9916	0.1890	2.9874	0.2745	3.9832	0.3660	4.9790	0.4575	45	45
	30	9954	0958	9908	1917	9862	2875	9816	3824	9770	4709	30	30
	45	9950	1002	9899	2004	9849	3006	9799	4006	9748	5098	15	15
6	0	9945	1045	9891	2091	9836	3186	9781	4181	9736	5296	80	0
	15	9941	1089	9881	2177	9822	3366	9762	4355	9703	5443	45	45
	30	9936	1132	9871	2264	9807	3526	9743	4528	9679	5660	30	30
	45	9931	1175	9861	2351	9792	3626	9723	4701	9653	5877	15	15
7	0	9925	1219	9851	2437	9776	3656	9702	4876	9627	6098	80	0
	15	9920	1262	9840	2524	9767	3786	9680	5048	9600	6310	45	45
	30	9914	1305	9829	2611	9743	3916	9658	5221	9572	6526	30	30
	45	9909	0.1349	1.9817	0.2897	2.9726	0.4046	3.9635	0.5394	4.9543	0.6743	15	15
8	0	9903	1392	9805	2783	9708	4175	9611	5567	9518	6959	80	0
	15	9897	1435	9793	2870	9690	4305	9586	5740	9483	7175	45	45
	30	9890	1478	9780	2956	9670	4434	9561	5912	9451	7390	30	30
	45	9884	1521	9767	3042	9651	4564	9534	6085	9418	7616	15	15
9	0	9877	1564	9754	3129	9631	4693	9518	6257	9384	7822	80	0
	15	9870	1607	9740	3215	9610	4822	9490	6490	9350	8037	45	45
	30	9863	1650	9726	3301	9589	4951	9451	6692	9314	8252	30	30
	45	9856	1693	9711	3387	9567	5080	9422	6774	9278	8467	15	15
10	0	9848	1736	9696	3473	9544	5209	9392	6946	9240	8682	80	0
	15	0.9840	0.1779	1.9681	0.3559	2.9521	0.5398	3.9362	0.7118	4.9202	0.8897	45	45
	30	9833	1822	9665	3645	9498	5407	9320	7299	9163	9112	30	30
	45	9825	1865	9649	3730	9474	5596	9298	7461	9123	9296	15	15
11	0	9816	1908	9633	3816	9449	5724	9265	7629	9061	9540	80	0
	15	9808	1951	9616	3902	9424	5858	9231	7804	9029	9755	45	45
	30	9799	1994	9598	3987	9398	5981	9197	7975	8996	9966	30	30
	45	9790	2036	9581	4073	9371	6109	9162	8146	8952	1.0182	15	15
12	0	9781	2079	9563	4158	9344	6237	9126	8316	8907	0.9967	80	0
	15	9772	2122	9545	4244	9317	6365	9089	8487	8862	0.9849	45	45
	30	9763	2164	9526	4329	9289	6493	9052	8658	8815	0.9720	30	30
	45	0.9753	0.2207	1.9507	0.4414	2.9360	0.6621	3.9014	0.8828	4.8767	1.1082	15	15
13	0	9744	2250	9485	4499	9231	6749	8975	8998	8719	1.1448	80	0
	15	9734	2292	9468	4584	9201	6876	8935	9168	8669	1460	45	45
	30	9724	2334	9447	4669	9171	7009	8895	9338	8618	1472	30	30
	45	9713	2377	9427	4754	9140	7151	8854	9507	8567	1684	15	15
14	0	9703	2419	9406	4838	9109	7298	8812	9677	8515	2096	80	0
	15	9692	2462	9385	4923	9077	7355	8769	9846	8462	2308	45	45
	30	9681	2504	9363	5008	9044	7511	8726	1.0015	8407	2519	30	30
	45	9670	2546	9341	5092	9011	7638	8682	0.1824	8352	2730	15	15
15	0	9659	2588	9319	5176	8978	7765	8637	0.8538	8296	2941	80	0
		Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.		
		Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.			

TRAVERSE TABLE.

Course	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		Course
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
0 15	5.9999	0.0262	6.9999	0.0305	7.9999	0.0349	8.9999	0.0393	9.9999	0.0436	89 45
30	9998	0524	9997	0611	9997	0698	9997	0785	9996	0873	30
45	9995	0785	9994	0916	9993	1047	9992	1178	9991	1309	15
1 0	9991	1047	9989	1222	9988	1396	9986	1571	9985	1745	89 0
15	9986	1309	9983	1527	9981	1745	9979	1963	9976	2181	45
30	9979	1571	9976	1832	9973	2064	9969	2356	9966	2618	30
45	9972	1832	9967	2138	9963	2443	9958	2748	9953	3054	15
2 0	9963	2094	9957	2443	9951	2792	9945	3141	9939	3490	89 0
15	9954	2356	9946	2748	9938	3141	9931	3533	9923	3926	45
30	9943	2617	9933	3053	9924	3490	9914	3926	9905	4362	30
45	5.9931	0.2379	6.9919	0.3358	7.9906	0.4388	8.9896	0.4913	9.9885	0.4778	15
3 0	9918	3140	9904	3664	9890	4187	9877	4710	9863	5234	87 0
15	9904	3402	9887	3963	9871	4535	9855	5102	9839	5669	45
30	9898	3663	9869	4272	9851	4884	9832	5494	9813	6105	30
45	9872	3924	9850	4578	9829	5232	9807	5896	9786	6540	15
4 0	9854	4185	9829	4883	9805	5581	9781	6278	9756	6976	86 0
15	9835	4447	9808	5188	9780	5929	9758	6670	9725	7411	45
30	9815	4708	9784	5492	9753	6277	9729	7061	9692	7846	30
45	9794	4968	9760	5797	9725	6625	9691	7453	9657	8281	15
5 0	9772	5229	9734	6101	9696	6972	9658	7844	9619	8710	85 0
15	5.9748	0.5490	6.9706	0.6405	7.9664	0.7320	8.9622	0.8235	9.9580	0.9150	45
30	9724	575	9678	6709	963	7668	9586	8626	9540	9638	30
45	9698	611	9648	7013	9597	8015	9547	9017	9497	1.0119	15
6 0	9671	6373	9617	7317	9562	8362	9507	9408	9452	1.0453	84 0
15	9643	6532	9584	7621	9525	8709	9465	9798	9406	1.0887	45
30	9614	6732	9550	7924	9486	9056	9421	1.0188	9857	1.1320	30
45	9584	7052	9515	8228	9445	9403	9376	0578	9807	1.1754	15
7 0	9553	7312	9478	8531	9404	9750	9329	0968	9255	2.1287	83 0
15	9520	7572	9440	8834	9360	1.0096	9280	1358	9200	2.0200	45
30	9487	7832	9401	9137	9316	1.0442	9220	1747	9144	3.0053	0
45	5.9452	0.8091	6.9361	0.9440	7.9269	1.0788	8.9178	1.2187	9.9087	1.3485	15
8 0	9416	8350	9319	9742	9221	1134	9124	2526	9027	3917	82 0
15	9379	8610	9276	1.0444	9172	1479	9069	2914	8965	4349	45
30	9341	8869	9231	1.0947	9121	1825	9011	3308	8902	4781	30
45	9302	9127	9185	1.0649	9069	2170	8953	3691	8839	5212	15
9 0	9261	9386	9138	1.0850	9015	2515	8892	4079	8769	5643	81 0
15	9220	9645	9090	1232	8960	2859	8830	4467	8700	6074	45
30	9177	9903	9040	1533	8913	3204	8766	4854	8629	6505	30
45	9133	1.0161	8989	1834	8844	3548	8700	5241	8566	6936	15
10 0	9088	0419	8937	2135	8785	3892	8633	5626	8481	7363	80 0
15	5.9042	1.0677	6.8883	1.2456	7.8723	1.4235	8.8564	1.6015	9.8404	1.7794	45
30	8995	1.934	8828	2756	8660	4579	8493	6401	8325	8224	30
45	8947	1191	8772	3057	8596	4922	8421	6787	8245	8652	15
11 0	8893	1449	8714	3357	8530	5265	8346	7173	8163	9081	79 0
15	8847	1705	8655	3656	8463	5607	8271	7558	8079	9509	45
30	8795	1962	8585	3954	8394	5949	8193	7943	7991	9937	30
45	8743	2219	8533	4255	8324	6291	8114	8328	7915	2.0364	15
12 0	8689	2475	8470	4554	8252	6633	8033	8712	7815	0791	73 0
15	8634	2731	8406	4852	8178	6974	795	9096	7723	1318	45
30	8578	2986	8341	5151	8104	7315	7867	9480	7630	1644	30
45	5.8521	1.242	6.8274	1.5149	7.8027	1.7857	8.7781	1.9803	9.7534	2.2070	15
13 0	8463	3497	8206	5747	7950	7996	7693	2.0246	7437	2495	77 0
15	8403	3752	8137	6044	7870	8336	7604	0628	7338	1.920	45
30	8342	4007	8066	6341	7790	8676	7418	1010	7237	3345	30
45	8281	4261	7994	6638	7707	9015	7251	1392	7134	3769	15
14 0	8218	4515	7921	6935	7624	9354	7327	1773	7080	4192	76 0
15	8154	4769	7846	7231	7539	9692	7231	2154	6923	4615	45
30	8089	5023	7770	7527	7452	2.0090	7193	2534	6815	5088	30
45	8023	5276	7693	7822	7364	0968	7034	2914	6705	5460	15
15 0	7956	5529	7615	8117	7274	0706	6933	3294	6593	5882	75 0
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Course
	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		

TRAVERSE TABLE.

Course	Dist. 1.			Dist. 2.			Dist. 3.			Dist. 4.			Dist. 5.		
	Lat.	Dep.		Lat.	Dep.		Lat.	Dep.		Lat.	Dep.		Lat.	Dep.	
15	0.9648	0.2630		1.9296	0.5261		2.8944	0.7891		3.8591	1.0521		4.8239	1.3152	74 45
30	9636	2672		9273	5345		8909	8017		8545	9690		8182	3392	30
45	9625	2714		9249	5429		8874	8143		8498	9858		8123	3572	15
16	0.9613	0.2759		1.9225	0.5513		2.8838	0.8269		3.8450	1.025		4.8063	1.2892	74 0
15	9600	2798		9201	5597		8801	8395		8402	1193		8002	3991	45
30	9588	2840		9176	5680		8765	8529		8353	1361		7941	4201	30
45	9576	2882		9151	5764		8727	8646		8303	1528		7879	4410	15
17	0.9563	0.2924		1.9126	0.5847		2.8689	0.8771		3.8252	1.0695		4.7815	1.5619	73 0
15	9550	2965		9100	5931		8651	8896		8201	1862		7751	4827	45
30	9537	3007		9074	6014		8612	9021		8149	2028		7686	5035	30
45	9524	3049		9048	6097		2.8572	0.9146		3.8096	1.2195		4.7620	1.5943	15
18	0.9511	0.3090		1.9021	0.6180		2.8532	0.9271		3.8042	1.2361		4.7553	1.6151	72 0
15	9497	3132		8994	6263		8491	9395		7988	2527		7485	5658	45
30	9483	3173		8966	6346		8450	9519		7933	2692		7416	5865	30
45	9469	3214		8939	6429		8408	9643		7877	2858		7347	6072	15
19	0.9455	0.3256		1.8910	0.6511		2.8366	0.9767		3.7821	1.3023		4.7276	1.6378	71 0
15	9441	3297		8882	6594		8323	9891		7764	3188		7204	6485	45
30	9426	3338		8853	6676		8279	1.0114		7706	3352		7132	6690	30
45	9412	3379		8824	6758		8235	0.138		7647	3517		7059	6896	15
20	0.9397	0.3420		1.8794	0.6840		2.8191	1.0261		3.7588	1.3845		4.6985	1.7101	70 0
15	9382	3461		1.8764	0.6922		2.8146	1.0384		3.7528	1.3845		4.6910	1.7306	45
30	9367	3502		8733	7004		8100	0.506		7467	4008		6834	7510	30
45	9351	3543		8703	7086		8054	069.9		7405	4172		6757	7715	15
21	0.9336	0.3584		1.8672	0.7167		2.8007	0.751		3.7443	1.4335		4.6819	1.7918	69 0
15	9320	3624		8640	7249		7960	087.2		7380	4498		6700	8122	45
30	9304	3665		8608	7330		7913	0965		7317	4660		6621	8325	30
45	9288	3706		8576	7411		7864	1117		7252	4822		6540	8528	15
22	0.9272	0.3746		1.8544	0.7492		2.7816	1.238		3.7087	1.4984		4.6539	1.8730	68 0
15	9255	3786		8511	7573		7766	1359		7022	5146		6477	8932	45
30	9239	3827		8478	7654		7716	1481		6955	5307		6414	9134	30
45	9222	3867		1.8444	0.7734		2.7666	1.1601		3.6888	1.5468		4.6110	1.9236	15
23	0.9205	0.3907		1.8410	0.7815		2.7615	1.1722		3.6820	1.5629		4.6025	1.9537	67 0
15	9188	3947		8376	7896		7564	1842		6752	5790		5940	9737	45
30	9171	3987		8341	7975		7512	1962		6682	5950		5853	9937	30
45	9153	4027		8306	8055		7459	2082		6612	6110		5766	1.0137	15
24	0.9135	0.4067		1.8271	0.8135		2.7406	1.2202		3.6542	1.6369		4.5677	2.0337	66 0
15	9118	4107		8235	8214		7353	2322		6470	6429		5688	1.0536	45
30	9100	4147		8199	8294		7299	2441		6398	6588		5598	1.0735	30
45	9081	4187		8163	8373		7244	2560		6327	6746		5507	1.0933	15
25	0.9063	0.4226		1.8126	0.8452		2.7189	1.2679		3.6252	1.6905		4.5215	1.31	65 0
15	9045	4266		1.8089	0.8531		2.7134	1.2797		3.6178	1.7063		4.5223	2.1328	45
30	9026	4305		8052	8610		7078	2915		6105	7220		5129	1.5226	30
45	9007	4344		8014	8689		7021	3033		6028	7378		5035	1.722	15
26	0.8988	0.4384		1.7976	0.8767		2.6964	1.3151		3.5952	1.7335		4.4940	1.919	64 0
15	8969	4423		7937	8846		6906	3269		5875	7692		4844	2.114	45
30	8949	4462		7899	8924		6848	3386		5797	7848		4747	2310	30
45	8930	4501		7860	9002		6789	3503		5719	8004		4649	2505	15
27	0.8910	0.4540		1.7820	0.9080		2.6700	1.3620		3.5640	1.8160		4.4520	2.00	63 0
15	8890	4579		7780	9157		6671	3736		5561	8315		4451	2594	45
30	8870	4617		7740	9235		6610	3852		5480	8470		4351	2687	30
45	8850	0.4656		1.7700	0.9312		2.6550	1.3968		3.5400	1.8625		4.4249	2.3281	15
28	0.8829	0.4695		1.7659	0.9389		2.6488	1.4084		3.5318	1.8779		4.4147	2.3474	62 0
15	8809	4733		7618	9466		6427	4200		5236	8933		4045	3666	45
30	8788	4772		7576	9543		6365	4315		5153	9086		3941	3858	30
45	8767	4810		7535	9620		6302	4430		5069	9240		3836	4049	15
29	0.8746	0.4848		1.7492	0.9696		2.6239	1.4544		3.4985	1.9392		4.3731	2.4240	61 0
15	8725	4886		7450	9772		6175	4659		4900	9515		3625	4431	45
30	8704	4924		7407	9848		6111	4773		4814	9697		3518	4621	30
45	8682	0.4962		7364	9924		6046	4886		4728	9849		3410	4811	15
30	8660	5000		7321	1.0000		5981	5000		4641	2.0000		3201	5000	60 0
	D. p.	Lat.		Dep.	Lat.		Dep.	Lat.		Dep.	Lat.		Dep.	Lat.	Course
	Dist. 1.			Dist. 2.			Dist. 3.			Dist. 4.			Dist. 5.		

TRAVERSE TABLE.

Course	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		Course
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
15 15	5.7887	1.5782	6.7535	1.8412	7.7183	2.1042	8.6831	2.3673	9.6479	2.6303	74 45
30	7818	6034	7454	8707	7090	1379	6727	4051	6363	6724	30
45	7747	6286	7372	9001	6996	1715	6621	4430	6246	7144	15
16 0	7676	6538	7288	9295	6901	2051	6514	4807	6126	7564	74 0
15	7603	6790	7203	95-8	6804	2386	6404	5185	6005	7983	45
30	7529	7041	7117	9881	6706	2721	6284	5561	5882	8402	30
45	7454	7292	7030	2.0174	6606	3056	6181	5938	5757	8820	15
17 0	7378	7542	6941	0466	6504	3390	6067	6313	5630	9237	73 0
15	7301	7792	6851	0758	6402	3723	5952	6689	5502	9654	45
30	7223	8042	6760	1049	6297	4056	5835	7064	5372	3.0071	30
45	5.7144	1.8292	6.6668	2.1341	7.6192	2.4389	8.5716	2.7438	9.5240	3.0486	15
18 0	7063	8541	6574	1631	6085	4721	5595	7812	5106	0902	72 0
15	6982	8790	6479	1921	5976	5052	5473	8185	4970	1316	45
30	6899	9038	6383	2211	5866	5384	5349	8557	4822	1730	30
45	6816	9286	6285	2501	5754	5715	5224	8930	4669	2144	15
19 0	6731	9534	6186	2790	5641	6045	5097	9301	4522	2537	71 0
15	6645	9781	6086	3078	5527	6375	4968	9672	4409	2969	45
30	6558	2.0028	5985	3366	5411	6705	4838	3.0043	4264	3381	30
45	6471	0275	5882	3654	5294	7033	4706	0413	4118	3792	15
20 0	6382	0521	5778	3941	5175	7362	4572	0782	3969	4202	70 0
15	5.6291	2.0767	6.5673	2.4228	7.5055	2.7689	8.4437	3.1151	9.3819	3.4612	45
30	6200	1012	5567	4515	4984	8017	4300	1519	3667	5021	30
45	6108	1257	5459	4800	4811	8343	4162	1836	3514	5429	15
21 0	6015	1502	5351	5086	4686	8669	4022	2253	3358	5837	60 0
15	5920	1746	5241	5371	4561	8995	3881	2619	3201	6244	45
30	5825	1990	5129	5655	4433	9320	3738	2985	3042	6650	30
45	5729	2233	5017	5939	4305	9645	3593	3350	2881	7056	15
22 0	5631	2476	4903	6222	4175	9969	3447	3715	2718	7461	68 0
15	5532	2719	4788	6505	4043	3.0292	3299	4078	2554	7865	45
30	5433	2961	4672	6788	3910	0615	3149	4442	2388	8268	30
45	5.5332	2.3203	6.4554	2.7070	7.3776	3.0937	8.2998	3.4804	9.2220	3.8671	15
23 0	5290	3444	4435	7351	3640	1258	2845	5166	2050	9073	67 0
15	5127	3685	4315	7632	3503	1580	2691	5527	1879	9474	45
30	5024	3925	4194	7912	3365	1900	2535	5857	1706	9875	30
45	4919	4165	4072	8192	3225	2220	2378	6247	1531	10275	15
24 0	4813	4404	3948	8472	3084	2539	2219	6606	1355	0674	66 0
15	4706	4643	3823	8750	2941	2858	2059	6965	1176	1072	45
30	4598	4882	3697	9029	2797	3175	1887	7322	0966	1469	30
45	4489	5120	3570	9306	2651	3493	1733	7679	0814	1866	15
25 0	4378	5357	3442	9583	2505	3809	1568	8036	0631	2262	65 0
15	5.4267	2.5594	6.3212	2.9800	7.2356	3.4125	8.1401	3.8891	9.0446	4.2657	45
30	4155	5831	3181	3.0136	2207	4441	1233	8746	0259	3051	30
45	4042	6067	3049	0411	2056	4756	1063	9100	0070	3445	15
26 0	3928	6302	2916	0686	1904	5070	0891	9453	8.9879	3837	64 0
15	3812	6537	2781	0960	1750	5383	0719	9806	9687	4229	45
30	3696	6772	2645	1234	1595	5696	0.44	4.0158	9493	4620	30
45	3579	7006	2509	1507	1438	6008	0368	0509	9298	5010	15
27 0	3460	7239	2370	1779	1281	6319	0191	0859	9101	5399	63 0
15	3341	7472	2231	2051	1121	6630	0012	1209	8902	5787	45
30	3221	7705	2091	2322	0961	6940	7.9831	1557	8701	6175	30
45	5.3009	2.7937	6.1949	3.2593	7.0799	3.7249	7.9649	4.1905	8.8499	4.6561	15
28 0	2977	8168	1806	2863	0636	7558	9465	2252	8295	6947	62 0
15	2853	8399	1662	3132	0471	7866	9280	2599	8189	7332	45
30	2729	8630	1517	3401	0305	8173	9094	2944	7882	7716	30
45	2604	8859	1371	3669	0138	8479	8005	3289	7673	8069	15
29 0	2477	9089	1223	3937	6.9970	8785	8716	3633	7462	8481	61 0
15	2350	9317	1075	4205	9800	9090	8525	3976	7250	8862	45
30	2221	9545	0925	4470	9628	9394	8352	4518	7036	9242	30
45	2092	9773	0774	4735	9456	9697	8138	4659	6820	9622	15
30 0	1962	3.0000	0622	5000	9282	4.0000	7942	5000	6003	5.0000	60 0
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Course
	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		

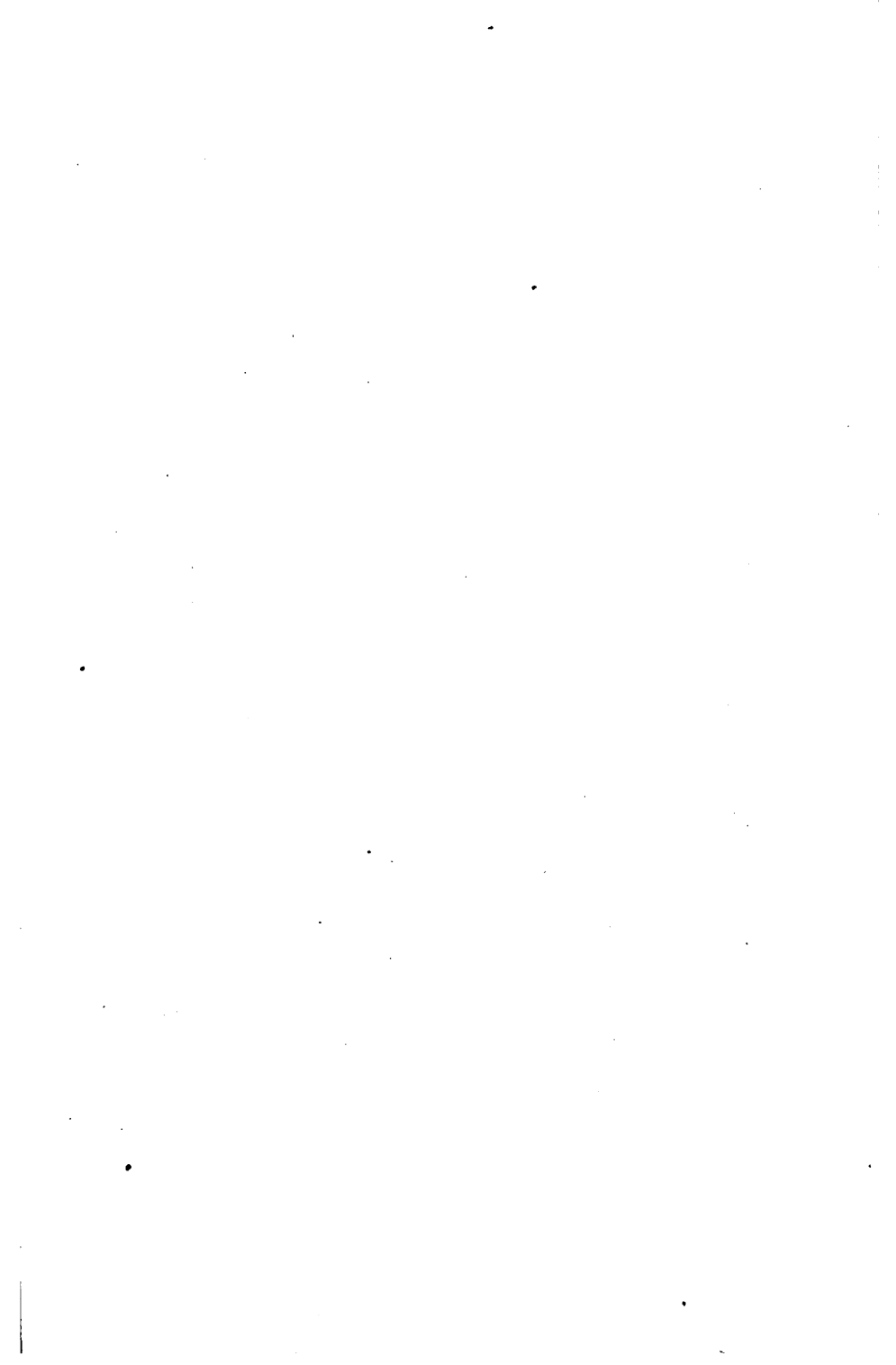


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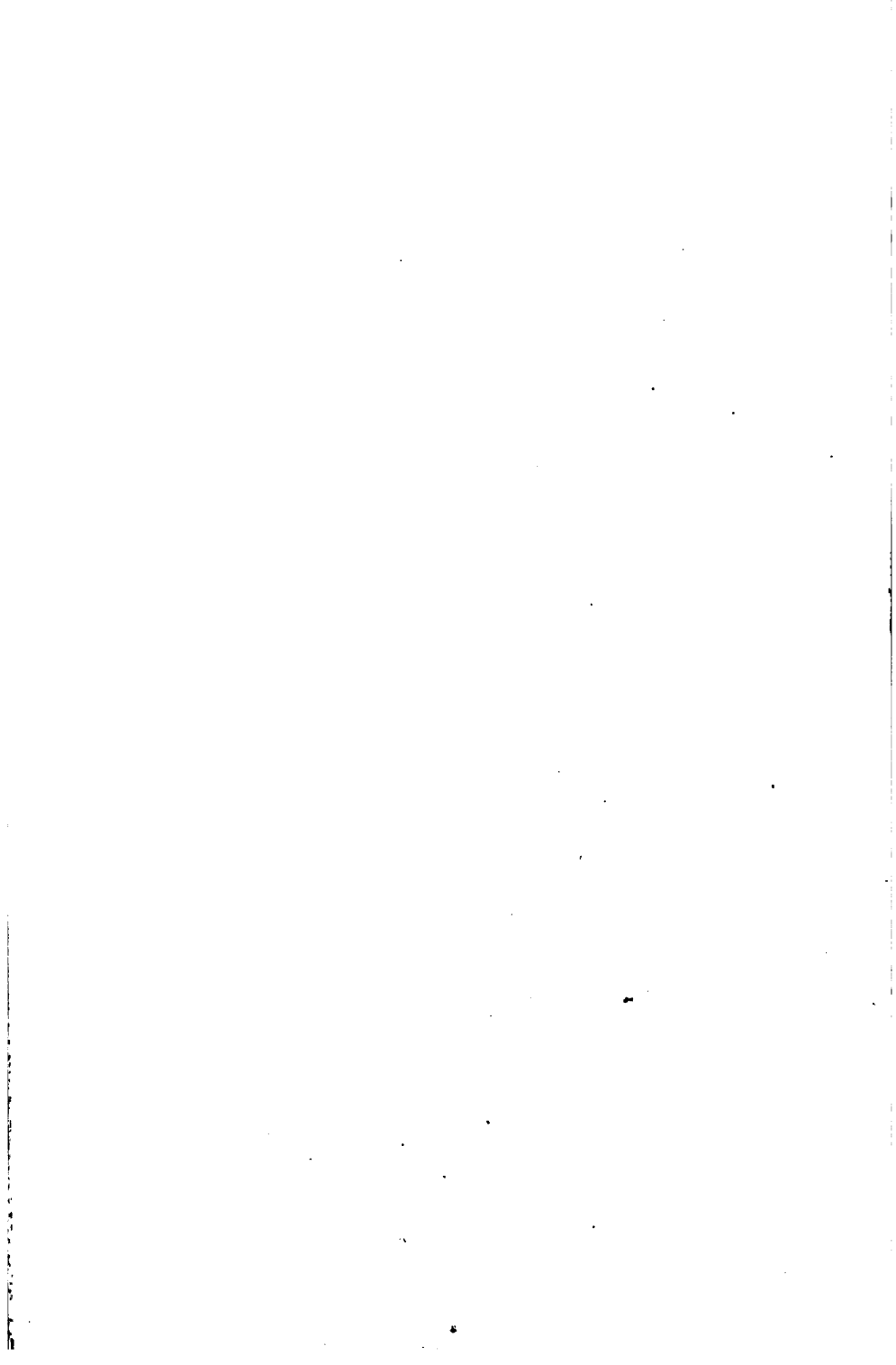
Course	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		Course
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
30 15	0.8638	0.5038	1.7277	1.0075	2.5915	1.5113	3.4553	2.0151	4.3192	2.5189	59 45
30	8616	5075	7293	0151	5849	5226	4405	0802	3081	5377	30
45	8594	5113	7188	0226	5782	5339	4376	0452	2970	5565	15
21 0	8572	5150	7142	0301	5715	5451	4287	0602	2858	5752	50 0
15	8549	5188	7098	0375	5647	5563	4196	0751	2746	5929	45
30	8526	5225	7053	0450	5579	5675	4106	0900	2632	6125	30
45	8504	5262	7007	0524	5511	5786	4014	1049	2518	6311	15
32 0	8480	5299	6961	0598	5441	5898	3922	1197	2402	6496	58 0
15	8457	5336	6915	0672	5372	6008	3829	1345	2286	6681	45
30	8434	5373	6868	0746	5302	6119	3736	1492	2170	6865	30
45	0.8410	0.5410	1.6821	1.0819	2.5231	1.4929	3.3642	2.1639	4.2052	2.7049	15
33 0	8387	5446	6773	0893	5160	6329	3547	1786	1934	7232	57 0
15	8363	5483	6726	0966	5089	6449	3451	1932	1814	7415	45
30	8339	5519	6678	1039	5017	6558	3355	2077	1694	7597	30
45	8315	5556	6629	1111	4944	6667	3259	2223	1573	7779	15
24 0	8290	5592	6581	1184	4871	6776	3162	2368	1452	7960	56 0
15	8266	5628	6532	1256	4798	6884	3064	2512	1329	8140	45
30	8241	5664	6483	1328	4724	6992	2965	2656	1206	8320	30
45	8216	5700	6433	1400	4649	7100	2866	2800	1082	8500	15
35 0	8192	5736	6385	1472	4575	7207	2766	2943	0958	8679	55 0
15	0.8166	0.5771	1.6333	1.1543	2.4409	1.7314	3.2666	2.3086	4.0882	2.8857	45
30	8141	5807	6282	1614	4423	7421	2665	3228	0706	9035	30
45	8116	5842	6231	1685	4347	7527	2463	3370	0579	9212	15
36 0	8090	5878	6180	1756	4271	7634	2261	3511	0451	9389	54 0
15	8064	5913	6129	1826	4193	7739	2058	3652	0322	9565	45
30	8039	5948	6077	1896	4116	7845	2154	3793	0193	9741	30
45	8013	5983	6025	1966	4038	7950	2050	3933	0063	9916	15
37 0	7986	6018	5973	2036	3959	8054	1945	4073	3.9932	3.0091	53 0
15	7960	6053	5920	2106	3880	8159	1840	4212	2800	0 75	45
30	7934	6088	5867	2175	3801	8263	1734	4350	9668	0428	30
45	0.7907	0.6122	1.5814	1.2244	2.3721	1.8967	3.1628	2.4489	3.9534	3.0611	15
38 0	7880	6157	5760	2213	3640	8470	1620	4626	9400	0782	52 0
15	7853	6191	5706	2282	3560	8573	1413	4764	9296	0955	45
30	7826	6225	5652	2450	3478	8675	1304	4901	9130	1126	30
45	7799	6259	5598	2518	3397	8778	1195	5037	8994	1296	15
39 0	7771	6293	5543	2586	3314	8880	1086	5173	8857	1466	51 0
15	7744	6327	5488	2654	3232	8981	0976	5308	8730	1635	45
30	7716	6361	5432	2722	3149	9082	0865	5443	8581	1804	30
45	7688	6394	5377	2789	3065	9183	0754	5578	8442	1972	15
40 0	7660	6428	5321	2856	2981	9284	0642	5719	8292	2139	50 0
15	0.7632	0.6461	1.5265	1.2922	2.2897	1.9384	3.0529	2.5845	3.8162	3.2306	45
30	7604	6494	5208	2989	2812	9483	0416	5978	8020	2472	30
45	7576	6528	5151	3055	2727	9585	0303	6110	7878	2658	15
41 0	7547	6561	5094	3121	2641	9682	0188	6242	7735	2803	49 0
15	7518	6593	5037	3187	2555	9780	0074	6374	7592	2967	45
30	7490	6626	4979	3252	2469	9879	2.9958	6505	7448	3131	30
45	7461	6659	4921	3318	2382	9976	9842	6635	7303	3294	15
42 0	7431	6691	4863	3383	2294	2.0074	9726	6765	7157	3457	48 0
15	7402	6724	4804	3447	2207	0171	9609	6885	7011	3618	45
30	7373	6756	4746	3512	2118	0268	9491	7024	6864	3780	30
45	0.7343	0.6788	1.4686	1.3576	2.2030	2.0364	2.9373	2.7152	3.6716	3.3940	15
43 0	7314	6820	4627	3640	1941	0460	9254	7280	6568	4100	47 0
15	7284	6852	4567	3704	1851	0557	9135	7407	6419	4259	45
30	7254	6884	4507	3767	1761	0651	9015	7534	6269	4418	30
45	7224	6915	4447	3830	1671	0745	8895	7661	6118	4576	15
44 0	7193	6947	4387	3893	1580	0840	8774	7786	5967	4733	46 0
15	7163	6978	4326	3956	1489	0934	8652	7912	5815	4890	45
30	7133	7009	4265	4018	1398	1027	8530	8036	5663	5045	30
45	7102	7040	4204	4080	1306	1120	8407	8161	5509	5201	15
45 0	7071	7071	4142	4142	1213	1213	8284	8284	5355	5355	45 0
	Dep.	Lat.	D-p.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Course
	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		

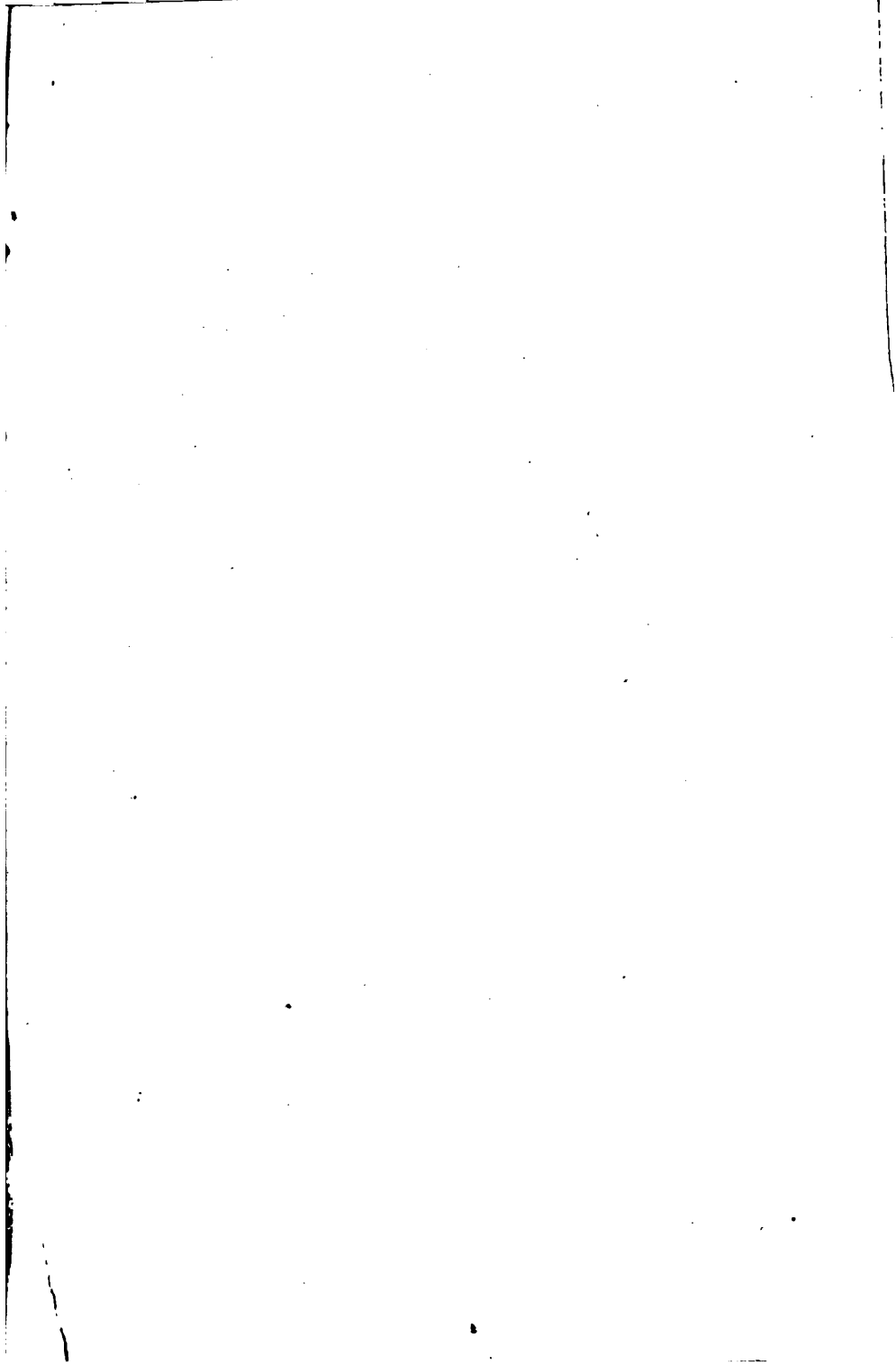
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Course	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
30 15	5.1890	3.0226	6.0468	3.5264	6.9107	4.0302	7.7745	4.5340	8.6384	5.0377	59 45
30	1698	0452	0314	5528	8930	0603	7547	5678	6163	0754	30
45	1564	0678	0158	5791	8753	0903	7347	6016	5941	1129	15
31 0	1430	0902	0002	6053	8573	1203	7145	6353	5717	1504	59 0
15	1295	1126	5.9844	6314	8393	1502	6942	6690	5491	1877	45
30	1158	1350	9685	6575	8211	1800	6738	7025	5264	2250	30
45	1021	1573	9525	6835	8028	2097	6532	7359	5035	2621	15
32 0	0883	1795	9363	7094	7844	2394	6324	7693	4805	2992	58 0
15	0744	2017	9201	7353	7658	2689	6116	8025	4573	3361	45
30	0603	2238	9037	7611	7471	2984	5905	8357	4339	3730	30
45	5.0462	3.2458	5.8873	3.7868	6.7283	4.3278	7.5694	4.8688	8.4104	5.4007	15
33 0	0320	2678	8707	8125	7094	3571	5480	9018	3867	4464	57 0
15	0177	2898	8540	8381	6903	3363	5267	9346	3629	4829	45
30	0033	3116	8372	8636	6711	4155	5050	9674	3389	5194	30
45	4.9888	3324	8203	8890	6518	4446	4832	5.0001	3147	5537	15
34 0	9742	3552	8033	9144	6323	4735	4613	0327	2904	5919	56 0
15	9595	3768	7861	9396	6127	5024	4393	0652	2659	6280	45
30	9448	3984	7689	9648	5930	5312	4171	0977	2413	6641	30
45	9299	4200	7515	9900	5732	5600	3948	1300	2165	7000	15
35 0	9149	4415	7341	4.0150	5532	5886	3724	1622	1915	7358	55 0
15	4.8998	3.4629	5.7165	4.0400	6.5331	4.6172	7.3498	5.1943	8.1664	5.7715	45
30	8847	4842	6988	0649	5129	6456	3270	2263	1412	8070	30
45	8694	5055	6810	0897	4926	6740	2042	2582	1157	8425	15
36 0	8541	5267	6631	1145	4721	7023	2812	2901	0902	8779	54 0
15	8387	5479	6451	1392	4516	7305	2580	3218	0644	9131	45
30	8231	5689	6270	1638	4309	7586	2347	3534	0386	9482	30
45	8075	5899	6088	1883	4100	7866	2113	3849	0125	9832	15
37 0	7918	6109	5904	2127	3891	8145	1877	4163	7.9864	6.0182	53 0
15	7760	6318	5720	2371	3680	8424	1640	4476	9600	0529	45
30	7601	6526	5535	2613	3468	8701	1402	4789	9335	0766	30
45	4.7441	3.6733	5.5348	4.2855	6.3255	4.8977	7.1162	5.5100	7.9069	6.1222	15
38 0	7281	6940	5161	3096	3041	9253	0921	5410	8801	1566	52 0
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30	6297	8165	4014	4525	1730	0886	9446	7247	7162	3608	30
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40 0	5963	8567	3623	4995	1284	1423	8944	7851	6604	4279	50 0
15	4.5794	3.3767	5.3426	4.5229	6.1059	5.1690	6.8691	5.8151	7.6323	6.4612	45
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41 0	5283	9364	2830	5924	0377	2485	7924	9045	5471	5606	49 0
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42 0	4589	4.0148	2020	6839	9452	3530	6883	6.0222	4314	6913	48 0
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30	4237	0535	1609	7291	8982	4047	6355	0803	3728	7559	30
45	4.059	4.0728	5.1403	4.7516	5.8746	5.4304	6.6089	6.1092	7.2432	6.7880	15
43 0	3881	0920	1195	7740	8508	4560	5822	1320	3135	8200	47 0
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